A MULTIPHASE APPROACH TO THE CONSTRUCTION OF POD-ROM FOR FLOWS INDUCED BY ROTATING SOLIDS

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Introduction

- **Objective**: Construct reduced order models (ROM) for the simulation of turbomachinery with *imposed rotation velocity* by proper orthogonal decomposition (POD).
- **Difficulty**: The POD yields a *spatial* basis from temporal correlations (here of the velocity).
- **Approach**:
  1. Extend the Navier-Stokes equations to the solid (rotor) domain by the multiphase approach. The body velocity is enforced via distributed Lagrange multipliers.
  2. Build a single POD basis for the multiphase velocity and project the governing equations.

References

[1] Glowinski, R., Pan, T. W., Hesla, T. I., & Joseph, D. D. (1999). A distributed Lagrange multiplier/fictitious domain method for particulate flows. International Journal of Multiphase Flow, 25(5), 755-794.

[2] Liberge, E., & Hamdouni, A. (2010). Reduced order modelling method via proper orthogonal decomposition (POD) for flow around an oscillating cylinder. Journal of fluids and structures, 26(2), 292-311.

[3] Mosquera, R. , Hamdouni, A., El Hamidi, A., Allery, C. (2018). POD Basis Interpolation via Inverse Distance Weighting on Grassmann Manifolds. Manuscript submitted to AIMS Journals.

1. Multiphase approach

Level-set Signed distance to the fluid/solid interface $\Gamma_{\mathrm{I}}(t)$:

$$\chi(\boldsymbol{x},t)=\begin{cases}+d\big(\boldsymbol{x},\Gamma_{\mathrm{I}}(t)\big) & \text{if} \quad \boldsymbol{x}\in\Omega_{\mathrm{S}}(t)\cup\Gamma_{\mathrm{I}}(t),\\ -d\big(\boldsymbol{x},\Gamma_{\mathrm{I}}(t)\big) & \text{if} \quad \boldsymbol{x}\in\Omega_{\mathrm{F}}(t)\end{cases}$$

Smoothed Heaviside (immersion depth ϵ)

$$h_\epsilon(\bullet)=\frac{1}{2}\left(1+\tanh\left(\frac{\pi\,\bullet}{\epsilon}\right)\right).$$

Membership function for $\Omega_{\mathrm{S}}(t)$

$$\mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})=h_\epsilon\big(\chi(\boldsymbol{x},t)\big).$$

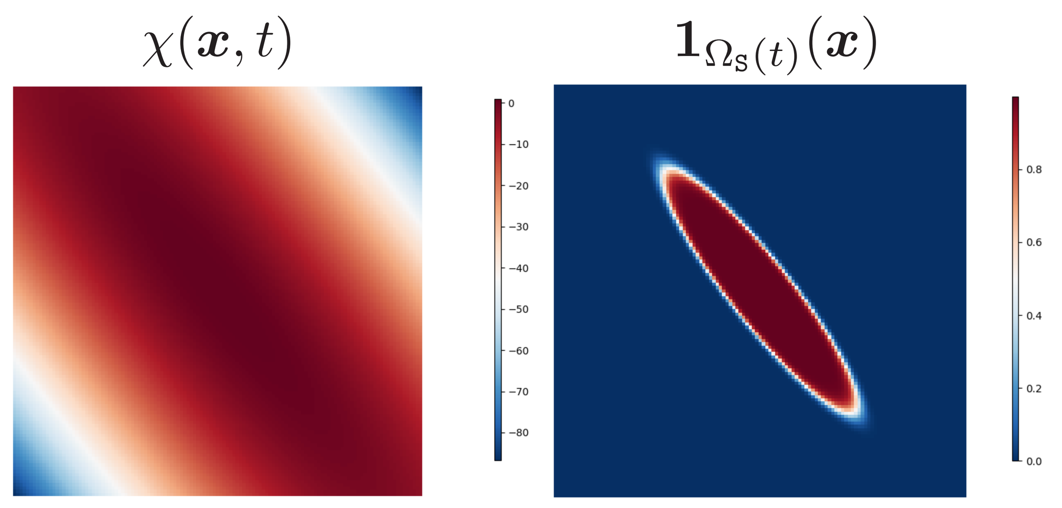
Multiphase quantities

Velocity field over $\Omega=\Omega_{\mathrm{S}}(t)\cup\Omega_{\mathrm{F}}(t)$:

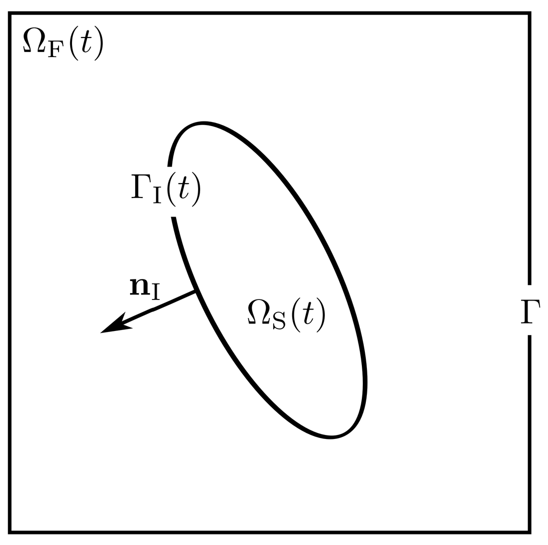
$$\begin{aligned}\boldsymbol{u}(\boldsymbol{x},t) \quad &= \quad \mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\,\boldsymbol{u}_{\mathrm{S}}(\boldsymbol{x},t)\\ &\quad +\big(\boldsymbol{I}-\mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\big)\,\boldsymbol{u}_{\mathrm{F}}(\boldsymbol{x},t).\end{aligned}$$

Material properties (density and viscosity):

$$\begin{aligned}\rho(\boldsymbol{x},t) \quad &= \quad \mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\,\rho_{\mathrm{S}}+\big(\boldsymbol{I}-\mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\big)\,\rho_{\mathrm{F}}\\ \nu(\boldsymbol{x},t) \quad &= \quad \mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\,\nu_{\mathrm{S}}+\big(\boldsymbol{I}-\mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\big)\,\nu_{\mathrm{F}}.\end{aligned}$$



2. Full order model [1]



Denoting $\boldsymbol{u}_{\boldsymbol{\omega}}$ the rotation velocity, $\boldsymbol{\lambda}$ the Lagrange multiplier and $\boldsymbol{\mu}$ the test function associated with the rotation constraint, the weak form of the coupled problem is

$$\begin{aligned}0 \quad = \quad & \int_\Omega \rho\left(\frac{\partial\boldsymbol{u}}{\partial t}+\nabla\boldsymbol{u}\cdot\boldsymbol{u}\right)\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x}-\int_\Omega \boldsymbol{f}\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x}\\ &+\int_\Omega 2\,\nu\,\operatorname{Tr}\left(\mathrm{D}\left(\boldsymbol{u}\right)\cdot\mathrm{D}\left(\boldsymbol{v}\right)\right)\,\mathrm{d}\boldsymbol{x}\\ &-\int_\Omega p\,\nabla\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x}+\int_{\Omega_{\mathrm{S}}(t)}\boldsymbol{\lambda}\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x}\\ &+\int_\Omega q\,\nabla\cdot\boldsymbol{u}\,\mathrm{d}\boldsymbol{x}+\int_{\Omega_{\mathrm{S}}(t)}\boldsymbol{\mu}\cdot(\boldsymbol{u}-\boldsymbol{u}_{\boldsymbol{\omega}})\,\mathrm{d}\boldsymbol{x},\end{aligned}$$

with an appropriate standard functional setting.

3. Standard POD-ROM [2]

Mean field $\overline{\boldsymbol{u}}(\boldsymbol{x})=\dfrac{1}{N_T}\displaystyle\sum_{n=1}^{N_T}\boldsymbol{u}(\boldsymbol{x},t_n)$

Fluctuating filed $\widetilde{\boldsymbol{u}}(\boldsymbol{x},t)=\boldsymbol{u}(\boldsymbol{x},t)-\overline{\boldsymbol{u}}(\boldsymbol{x})$

Data matrix $U_{mn}\equiv\widetilde{\boldsymbol{u}}(\boldsymbol{x}_m,t_n)$

POD basis: left singular vectors of $\mathbf{U}$

$$\Phi=\Big(\boldsymbol{\phi}_i(\boldsymbol{x})\Big)_{1\leq i\leq N_\Phi}, \qquad N_\Phi << N_T << N_X$$

Decomposition

$$\boldsymbol{u}(\boldsymbol{x},t)\simeq\overline{\boldsymbol{u}}(\boldsymbol{x})+\sum_{i=1}^{N_\Phi}\boldsymbol{\phi}_i(\boldsymbol{x})\,\alpha_i(t)$$

Galerkin projection over the POD basis

$$\mathbf{A}(t)\cdot\frac{\mathrm{d}\boldsymbol{\alpha}(t)}{\mathrm{d}t}=\mathbf{B}(t)\cdot\boldsymbol{\alpha}(t)+\mathbf{C}(t):\boldsymbol{\alpha}(t)\otimes\boldsymbol{\alpha}(t)+\mathbf{F}(t),$$

⇒ Full projection at each timestep (cost $\sim N_X$).

$$\begin{aligned}A_{ij}(t) \quad &= \quad \int_\Omega \rho(\boldsymbol{x},t)\,\boldsymbol{I}\,\mathrm{d}\boldsymbol{x},\\ B_{ij}(t) \quad &= \quad \int_\Omega \rho(\boldsymbol{x},t)\,b^\rho_{i,j}(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}+\int_\Omega \nu(\boldsymbol{x},t)\,b^\nu_{i,j}(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x},\\ C_{ijk}(t) \quad &= \quad \int_\Omega \rho(\boldsymbol{x},t)\,c_{i,j,k}(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x},\\ F_i(t) \quad &= \quad \int_\Omega \rho(\boldsymbol{x},t)\,f^\rho_i(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}+\int_\Omega \nu(\boldsymbol{x},t)\,f^\nu_i(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}\\ &\quad +\int_{\Omega_{\mathrm{S}}(t)} f^\lambda_i(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}.\end{aligned}$$

4. Proposed POD-ROM

⇒ POD of the membership function

POD basis $\boldsymbol{\Lambda}=\Big(\Lambda_i(\boldsymbol{x})\Big)_{1\leq i\leq N_\Lambda}$, $N_\Lambda << N_X$.

Decomposition

$$\mathbf{1}_{\Omega_{\mathrm{S}}(t)}(\boldsymbol{x})\simeq\textstyle\sum_{i=1}^{N_\Lambda}\Lambda_i(\boldsymbol{x})\,\gamma_i(t)$$

Periodicity

Coefficients $\gamma_i(t)\to\widehat{\gamma}_i(\theta)$ determined *a priori*.

Insertion in the standard POD-ROM

$$\widehat{\mathbf{A}}(t)\cdot\frac{\mathrm{d}\boldsymbol{\alpha}(t)}{\mathrm{d}t}=\widehat{\mathbf{B}}(t)\cdot\boldsymbol{\alpha}(t)+\widehat{\mathbf{C}}(t):\boldsymbol{\alpha}(t)\otimes\boldsymbol{\alpha}(t)+\widehat{\mathbf{F}}(t),$$

⇒ Matrices evaluation at each timestep (cost $\sim N_\Lambda$).

$$\begin{aligned}\widehat{A}_{ij}(\theta) \quad &= \quad \overline{a}_{i,j}+\textstyle\sum_{k=1}^{N_\Lambda}\widetilde{a}_{i,j,k}\,\gamma_k(\theta),\\ \widehat{B}_{ij}(\theta) \quad &= \quad \overline{b}_{i,j}+\textstyle\sum_{k=1}^{N_\Lambda}\widetilde{b}_{i,j,k}\,\gamma_k(\theta),\\ \widehat{C}_{ijk}(\theta) \quad &= \quad \overline{c}_{i,j,k}+\textstyle\sum_{k=1}^{N_\Lambda}\widetilde{c}_{i,j,k,l}\,\gamma_k(\theta),\\ \widehat{F}_i(\theta) \quad &= \quad \overline{f}_i+\textstyle\sum_{j=1}^{N_\Lambda}\widetilde{f}_{i,j}\,\gamma_k(\theta)+\lambda_i.\end{aligned}$$

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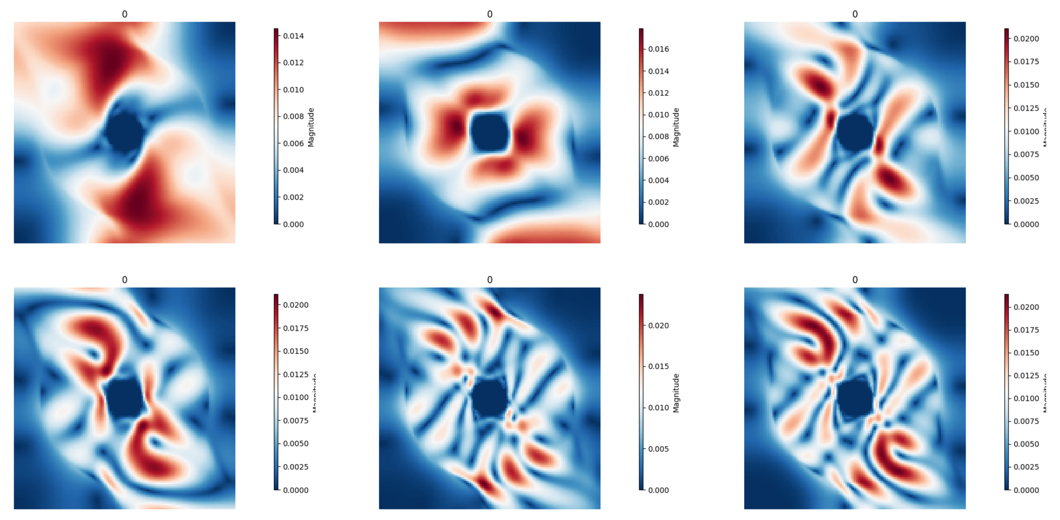
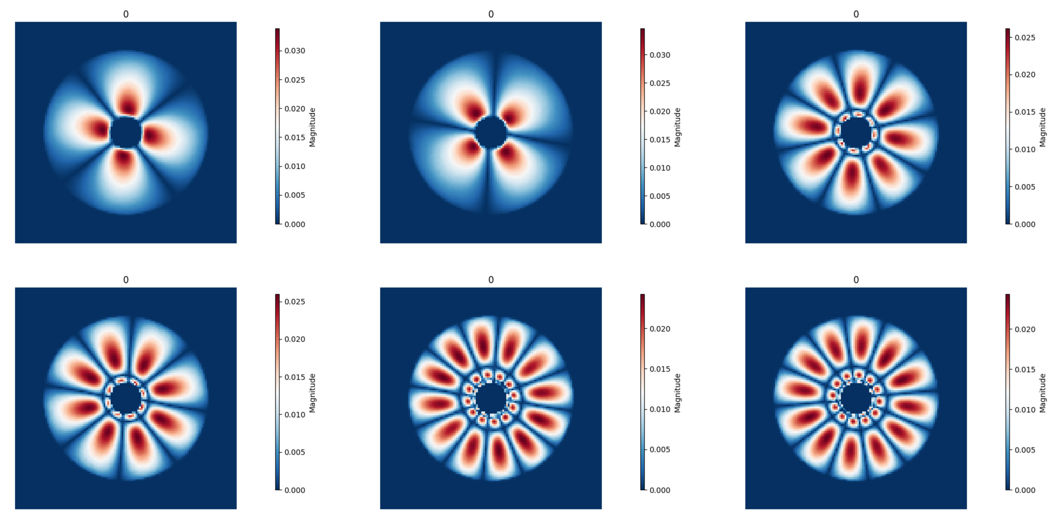
5. Results

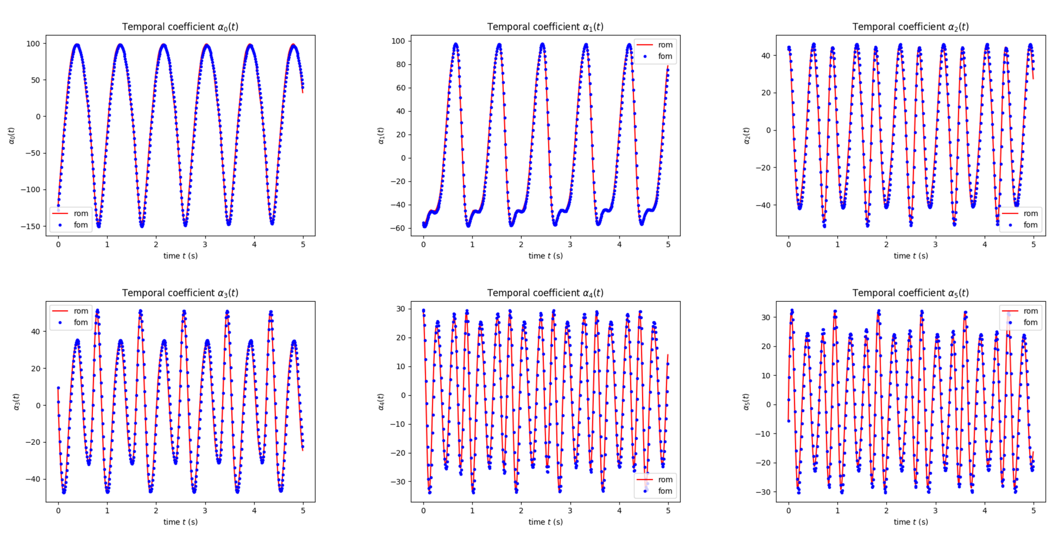
Φ: POD modes for the velocity



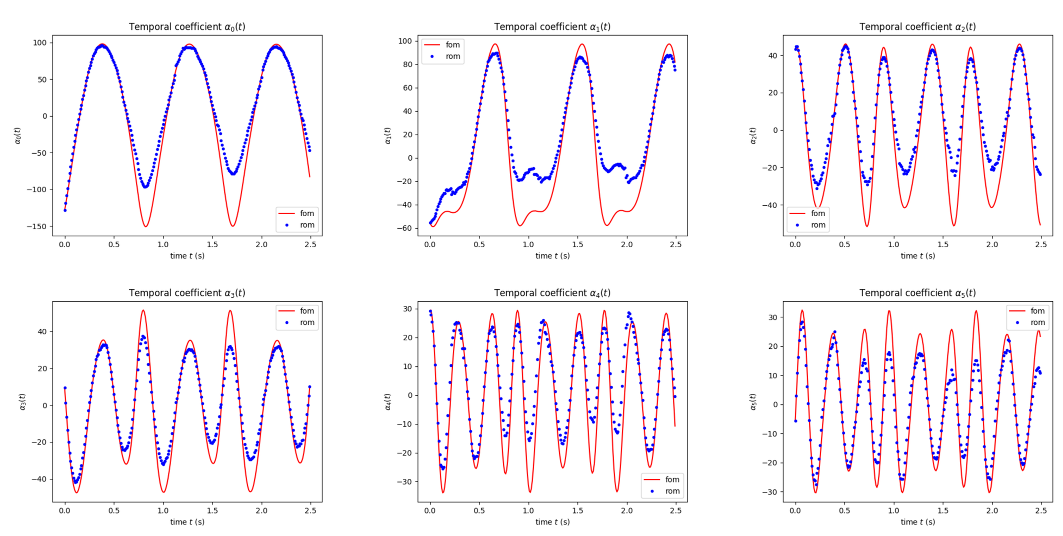
Λ: POD modes for the membership function



Coeff $\alpha_i(t)$: full order Vs. standard POD-ROM.



Coeff $\alpha_i(t)$: full order Vs. proposed POD-ROM.



Conclusion

Contributions

- Efficient reconstruction of the velocity in both the fluid and solid domains, while substantially reducing the computational cost.
- Very general: Any simulation code for the incompressible Navier-Stokes eq. can be used to generate the data $\big(\boldsymbol{u}(\boldsymbol{x},t_n)\big)_{1\leq n\leq N_T}$.

Perspectives

- Cope with the reconstruction of the velocity in the solid domain at each iteration by rewriting the governing equations for a rotating subdomain.
- Interpolate between the POD-ROMs over the Grassmann manifold (see *e.g.* [3]).