A Port-Hamiltonian version of the Thiele-Small model of loudspeakers

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1 System dimensions

$\dim(\mathbf{x}) = n_{\mathbf{x}} = 3;$
$\dim(\mathbf{w}) = n_{\mathbf{w}} = 0;$
$\dim(\mathbf{y}) = n_{\mathbf{y}} = 1;$
$\dim(\mathbf{p}) = n_{\mathbf{p}} = 0;$

2 System variables

State variable $\mathbf{x} = \begin{pmatrix} x_{\mathrm{L}} \\ x_{\mathrm{K}} \\ x_{\mathrm{M}} \end{pmatrix};$
Input $\mathbf{u} = \begin{pmatrix} u_1 \end{pmatrix};$
Output $\mathbf{y} = \begin{pmatrix} y_1 \end{pmatrix};$

3 Constitutive relations

Hamiltonian $\mathtt{H}(\mathbf{x}) = \frac{K}{2} \cdot x_{\mathrm{K}}^2 + \frac{invL}{2} \cdot x_{\mathrm{L}}^2 + \frac{invM}{2} \cdot x_{\mathrm{M}}^2;$
Hamiltonian gradient $\nabla\mathtt{H}(\mathbf{x}) = \begin{pmatrix} invL \cdot x_{\mathrm{L}} \\ K \cdot x_{\mathrm{K}} \\ invM \cdot x_{\mathrm{M}} \end{pmatrix};$

4 System parameters

4.1 Constant

parameter	value (SI)
L :	0.011
R :	5.7
K :	40000000.0
M :	0.019
A :	0.406
invL :	90.90909090909092
invM :	52.631578947368425

5 System structure

$$\mathbf{M} = \begin{pmatrix} -R & 0 & -B \cdot e^{-x_{\mathrm{K}}^{2}} & -1 \\ 0 & 0 & 1 & 0 \\ B \cdot e^{-x_{\mathrm{K}}^{2}} & -1 & -A & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{M_{xx}} = \begin{pmatrix} -R & 0 & -B \cdot e^{-x_{\mathrm{K}}^{2}} \\ 0 & 0 & 1 \\ B \cdot e^{-x_{\mathrm{K}}^{2}} & -1 & -A \end{pmatrix};$$

$$\mathbf{M_{xy}} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix};$$

$$\mathbf{M_{yx}} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix};$$

$$\mathbf{M_{yy}} = \begin{pmatrix} 0 \end{pmatrix};$$

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & -1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^{2}} & -1.0 \\ 0 & 0 & 1.0 & 0 \\ 1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^{2}} & -1.0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{J_{xx}} = \begin{pmatrix} 0 & 0 & -1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^{2}} \\ 0 & 0 & 1.0 \\ 1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^{2}} & -1.0 & 0 \end{pmatrix};$$

$$\mathbf{J_{xy}} = \begin{pmatrix} -1.0 \\ 0 \\ 0 \end{pmatrix};$$

$$\mathbf{J_{yy}} = \begin{pmatrix} 0 \end{pmatrix};$$

$$\mathbf{R} = \begin{pmatrix} 1.0 \cdot R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 \cdot A & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{R_{xx}} = \begin{pmatrix} 1.0 \cdot R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.0 \cdot A \end{pmatrix};$$

$$\mathbf{R_{xy}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix};$$

$$\mathbf{R_{yy}} = \begin{pmatrix} 0 \end{pmatrix};$$