$\boxed{1.2}$

# $\boxed{\text{ZERO-PADDING}}$

$$I = \begin{array}{c|ccc} & -1 & 0 & \\ \hline -1 & a & b & c \\ 0 & d & e & g \\ 1 & h & m & n \end{array} \qquad f = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}$$

$I(-1,-1)$: $1^{st}$ element:

$$\begin{array}{ccc|c} 0\cdot p & 0\cdot q & 0\cdot r & \\ 0\cdot s & a\cdot t & b\cdot u & c \\ 0\cdot v & d\cdot w & e\cdot x & g \\ & & & \\ h & m & n & \end{array}$$

$I(-1,0)$: $2^{nd}$ element:

$$\begin{array}{c|ccc} 0 & 0\cdot p & 0\cdot q & 0\cdot r \\ 0 & a\cdot s & b\cdot t & c\cdot u \\ 0 & d\cdot v & e\cdot w & g\cdot x \\ & & & \\ & h & m & n \end{array}$$

$I(-1,1)$: $3^{rd}$ element:

$$\begin{array}{cc|ccc} 0 & 0 & 0\cdot p & 0\cdot q & 0\cdot r \\ 0 & a & b\cdot s & c\cdot t & 0\cdot u \\ 0 & d & e\cdot v & g\cdot w & 0\cdot x \\ & & & & \\ & h & m & n & \end{array} \quad \cdots \quad \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & a & b & c & 0 \\ 0 & d & e & g & 0 \\ 0 & h & m & n & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$\boxed{1.2}$

$$I = \begin{bmatrix} 0.5 & 2 & 1.5 \\ 0.5 & 1 & 0 \\ 2 & 0.5 & 1 \end{bmatrix} \quad f = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

WITH ZERO-PADDING: 
$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 2 & 1.5 & 0 \\ 0 & 0.5 & 1 & 0 & 0 \\ 0 & 2 & 0.5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I \odot f = \begin{bmatrix} ((0.5\cdot1)+(2\cdot0.5)+(1\cdot0.5)+(0\cdot0.5)) & ((2\cdot1)+(1.5\cdot0.5)+(0\cdot0.5)+(1\cdot0)+(0.5\cdot0.5)+(0\cdot0.5)) & ((1.5\cdot1)+(0\cdot0)+(1\cdot0.5)+(2\cdot0)) \\ ((0.5\cdot1)+(0.5\cdot1)+(2\cdot0)+(1\cdot0.5)+(0.5\cdot0.5)+(2\cdot0)) & ((1\cdot1)+(0.5\cdot0)+(0.5\cdot0.5)+(2\cdot1)+(1.5\cdot0)+(0\cdot0.5)+(1\cdot0.5)\ldots+(0.5\cdot0)+(2\cdot0.5)) & ((0\cdot1)+(1.5\cdot1)+(2\cdot0.5)+(1\cdot0)+(0.5\cdot0.5)+(1\cdot0)) \\ ((2\cdot1)+(0.5\cdot1)+(1\cdot0)+(0.5\cdot0.5)) & ((0.5\cdot1)+(1\cdot1)+(0\cdot0)+(1\cdot0.5)+(2\cdot0)+(0.5\cdot0.5)) & ((1\cdot1)+(0.5\cdot0)+(1\cdot0.5)+(0\cdot1)) \end{bmatrix}$$

$$= \begin{bmatrix} (0.5+1+0.5+0) & (2+0.75+0+0+0.25+0) & (1.5+0+0.5+0) \\ (0.5+0.5+0+0.5+0.25+0) & (1+0+0.25+2+0+0+0.5+0+1) & (0+1.5+1+0+0.25+0) \\ (2+0.5+0+0.25) & (0.5+1+0+0.5+0+0.25) & (1+0+0.5+0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 2 \\ 1.75 & 4.75 & 2.75 \\ 2.75 & 2.25 & 1.5 \end{bmatrix}$$

$$f \odot I = \begin{bmatrix} ((0.5\cdot1)+(0\cdot1)+(1\cdot1)+(0\cdot0.5)) & ((1\cdot1)+(0.5\cdot0.5)+(0\cdot2)+(1\cdot0.5)+(0.5\cdot1)+(0\cdot0)) & ((0\cdot1)+(1\cdot0.5)+(1\cdot2)+(0.5\cdot0.5)) \\ ((0\cdot1)+(0.5\cdot2)+(1\cdot1.5)+(1\cdot0)+(0\cdot1)+(0.5\cdot0.5)) & ((1\cdot1)+(0\cdot0.5)+(0.5\cdot0.5)+(1\cdot2)+(0\cdot1.5)+(0.5\cdot0)+(0.5\cdot1)\ldots+(0\cdot0.5)+(0.5\cdot2)+(0\cdot0.5)+(0.5+0.5)) & ((0.5\cdot1)+(0\cdot2)+(1\cdot0.5)+(1\cdot0.5)+(0\cdot2)+(0.5\cdot0.5)) \\ ((0.5+0+1.5+0)) & ((0\cdot1)+(0.5\cdot0.5)+(0\cdot0.5)+(1\cdot2)+(0.5\cdot1.5)+(0.5\cdot0)) & ((1.5\cdot1)+(0\cdot0.5)+(1\cdot0.5)+(0.5\cdot2)) \end{bmatrix}$$

$$= \begin{bmatrix} (0.5+0+1+0) & (1+0.25+0+0.5+0.5+0) & (0+0.5+2+0.25) \\ (0+1+1.5+0+0+0.25) & (1+0+0.25+2+0+0+0.5+0+1+0) & (0.5+0+0.5+0.5+0+0.25) \\ (0.5+0+1.5+0) & (0+0.25+0+2+0.75+0) & (0.5+0+0.5+1) \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 2.25 & 2.75 \\ 2.75 & 4.75 & 1.75 \\ 2 & 3 & 2 \end{bmatrix}$$

$$I \odot f = \begin{bmatrix} 2 & 3 & 2 \\ 1.75 & 4.75 & 2.75 \\ 2.75 & 2.25 & 1.5 \end{bmatrix} \neq \begin{bmatrix} 1.5 & 2.25 & 2.75 \\ 2.75 & 4.75 & 1.75 \\ 2 & 3 & 2 \end{bmatrix} = f \odot I$$

$\boxed{\text{NOT COMMUTATIVE}}$

2.1

# ZERO-PADDING & KERNEL FLIPPING FOR CONVOLUTION

$$I = \begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & a & b & c \\ 0 & d & e & g \\ 1 & h & m & n \end{array} \qquad f_x = \begin{bmatrix} x & y & z \end{bmatrix}$$

$I(-1,-1)$: 1st element:

$$\begin{matrix} \boxed{0 \cdot z \;\; a \cdot y \;\; b \cdot x} & c \\ d \;\; e \;\; g & \\ h \;\; m \;\; n & \end{matrix}$$

$I(-1,1)$:

... 3rd element:

$$\begin{matrix} 0 & a & \boxed{b \cdot z \;\; c \cdot y \;\; 0 \cdot x} \\ & d \;\; e \;\; g & \\ & h \;\; m \;\; n & \end{matrix} \quad \cdots \quad \begin{matrix} 0 & a & b & c & 0 \\ 0 & d & e & g & 0 \\ 0 & h & m & n & 0 \end{matrix}$$

---

$$f_y = \begin{bmatrix} q \\ r \\ s \end{bmatrix}$$

(for $I \otimes f_x \otimes f_y$)

$I(-1,-1)$:

1st element:

$$\begin{matrix} 0 & \boxed{\begin{matrix} 0 \cdot s \\ a \cdot r \\ d \cdot q \end{matrix}} & \begin{matrix} b & c \\ e & g \end{matrix} & 0 \\ 0 & h \;\; m & n & 0 \end{matrix} \quad \cdots \quad \begin{matrix} & 0 & 0 & 0 & \\ 0 & a & b & c & 0 \\ 0 & d & e & g & 0 \\ 0 & h & m & n & 0 \\ & 0 & 0 & 0 & \end{matrix}$$

$$
I = \begin{array}{c c}
 & \begin{array}{ccc} -1 & 0 & 1 \end{array} \\
\begin{array}{c} -1 \\ 0 \\ 1 \end{array} & \begin{bmatrix} a & b & c \\ d & e & g \\ h & m & n \end{bmatrix}
\end{array}
\qquad
f_{xy} = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}
$$

$I(-1,-1)$:

1st element:

$$
\begin{array}{cccc}
\left[\begin{array}{ccc} 0\cdot x & 0\cdot w & 0\cdot v \\ 0\cdot u & a\cdot t & b\cdot s \\ 0\cdot r & d\cdot q & e\cdot p \end{array}\right] & \begin{array}{c} \\ c \\ g \end{array} \\
\begin{array}{ccc} & h & m \end{array} & n
\end{array}
$$

$I(-1,1)$:

... 3RD element:

$$
\begin{array}{ccc}
\begin{array}{cc} 0 & 0 \\ 0 & a \\ 0 & d \end{array} & \left[\begin{array}{ccc} 0\cdot x & 0\cdot w & 0\cdot v \\ b\cdot u & c\cdot t & 0\cdot s \\ e\cdot r & g\cdot q & 0\cdot p \end{array}\right] \\
\begin{array}{cc} & h \end{array} & \begin{array}{ccc} m & n & \end{array}
\end{array}
$$

...

(for $I \otimes f_{xy}$)

$$
\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & a & b & c & 0 \\
0 & d & e & g & 0 \\
0 & h & m & n & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}
$$

$$I = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \\ 0 & 3 & -1 \end{bmatrix}$$

WITH ZERO-PADDING

$$\to I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \qquad f_y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

WHEN FLIPPED

$$f_x = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \qquad f_y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\boxed{2.1}$ $g_1 = I \otimes f_x \otimes f_y$ $\quad g_2 = I \otimes f_{xy}$ $\quad f_{xy} = f_x \otimes f_y$

$$I \otimes f_x = \begin{bmatrix} ((0\cdot0)+(1\cdot-1)+(1\cdot0)) & ((1\cdot0)+(-1\cdot-1)+(1\cdot0)) & ((-1\cdot0)+(-1\cdot0)+(1\cdot1)) \\ ((2\cdot0)+(1\cdot0)+(1\cdot-1)) & ((1\cdot0)+(2\cdot1)+(0\cdot-1)) & ((0\cdot0)+(1\cdot1)+(0\cdot-1)) \\ ((0\cdot0)+(1\cdot0)+(3\cdot-1)) & ((3\cdot0)+(0\cdot1)+(-1\cdot-1)) & ((-1\cdot0)+(3\cdot1)+(0\cdot-1)) \end{bmatrix}$$

$$= \begin{bmatrix} (0-1+0) & (0+1+0) & (0+0+1) \\ (0+0-1) & (0+2+0) & (0+1+0) \\ (0+0-3) & (0+0+1) & (0+3+0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$$(I \otimes f_x) \otimes f_y = \begin{bmatrix} ((-1\cdot1)+(1\cdot0)+(-1\cdot1)) & ((1\cdot1)+(1\cdot0)+(2\cdot1)) & ((1\cdot1)+(1\cdot1)+(1\cdot0)) \\ ((-1\cdot1)+(-1\cdot1)+(-3\cdot1)) & ((2\cdot1)+(1\cdot1)+(1\cdot1)) & ((1\cdot1)+(1\cdot1)+(3\cdot1)) \\ ((-3\cdot1)+(-1\cdot1)+(0\cdot1)) & ((1\cdot1)+(2\cdot1)+(0\cdot1)) & ((3\cdot1)+(1\cdot0)+(1\cdot1)) \end{bmatrix}$$

$$= \begin{bmatrix} (-1+0-1) & (1+0+2) & (1+1+0) \\ (-1-1-3) & (2+1+1) & (1+1+3) \\ (-3-1+0) & (1+0+2) & (3+0+1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

$$f_{xy} = f_x \otimes f_y$$

$$f_x \otimes f_y = \begin{bmatrix} (-1\cdot 1) & (0\cdot 1) & (1\cdot 1) \\ (-1\cdot 1) & (0\cdot 1) & (1\cdot 1) \\ (-1\cdot 1) & (0\cdot 1) & (1\cdot 1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$I \otimes f_{xy} = \begin{bmatrix} ((0\cdot 0)+(1\cdot -1)+(1\cdot -1)+(2\cdot 0)) & ((1\cdot 0)+(-1\cdot -1)+(0\cdot -1)+(1\cdot 0)+(2\cdot 1)+(0\cdot 1)) & ((-1\cdot 0)+(1\cdot 1)+(1\cdot 1)+(0\cdot 0)) \\ ((2\cdot 0)+(0\cdot 0)+(1\cdot -1)+(1\cdot -1)+(3\cdot -1)+(0\cdot 0)) & ((1\cdot 0)+(2\cdot 1)+(0\cdot 1)+(1\cdot 0)+(-1\cdot -1)+(0\cdot -1)+(-1\cdot -1)+(3\cdot 0)+(0\cdot 1)) & ((0\cdot 0)+(-1\cdot 0)+(1\cdot 1)+(1\cdot 1)+(3\cdot 1)+(-1\cdot 0)) \\ ((0\cdot 0)+(2\cdot 0)+(1\cdot -1)+(3\cdot -1)) & ((3\cdot 0)+(0\cdot 1)+(2\cdot 1)+(1\cdot 0)+(0\cdot -1)+(-1\cdot -1)) & ((-1\cdot 0)+(0\cdot 0)+(1\cdot 1)+(3\cdot 1)) \end{bmatrix}$$

$$= \begin{bmatrix} (0-1-1+0) & (0+1+0+0+2+0) & (0+1+1+0) \\ (0+0-1-1-3+0) & (0+2+0+0+1+0+1+0+0) & (0+0+1+1+3+0) \\ (0+0-1-3) & (0+0+2+0+0+1) & (0+0+1+3) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

$$g_1 = I \otimes f_x \otimes f_y = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix} = I \otimes f_{xy} = g_2$$

$\boxed{\text{YES, ASSOCIATIVE}}$

$\boxed{2.2}$

$$I \otimes f_x : \begin{bmatrix} \text{3 mul. 2 add.} & \text{3 mul. 2 add.} & \text{3 mul. 2 add.} \\ \text{3 mul. 2 add.} & \text{3 mul. 2 add.} & \text{3 mul. 2 add.} \\ \text{3 mul. 2 add.} & \text{3 mul. 2 add.} & \text{3 mul. 2 add.} \end{bmatrix} \begin{matrix} = \text{9 mul. 6 add.} \\ = \text{9 mul. 6 add.} \\ = \text{9 mul. 6 add.} \end{matrix} = \text{27 mul. 18 add.}$$

$$(I \otimes f_x) \otimes f_y : \begin{bmatrix} \text{"} \end{bmatrix} = \text{"} = \text{27 mul. 18 add.}$$

$$\boxed{\begin{matrix} g_1 : \text{54 mul.} \\ \text{36 add.} \\ \text{190 total operations} \end{matrix}}$$

---

$$f_x \otimes f_y : \begin{bmatrix} \text{1 mul.} & \text{1 mul.} & \text{1 mul.} \\ \text{1 mul.} & \text{1 mul.} & \text{1 mul.} \\ \text{1 mul.} & \text{1 mul.} & \text{1 mul.} \end{bmatrix} \begin{matrix} = \text{3 mul.} \\ = \text{3 mul.} \\ = \text{3 mul.} \end{matrix} = \text{9 mul.}$$

$$I \otimes f_{xy} : \begin{bmatrix} \text{9 mul. 8 add.} & \text{9 mul. 8 add.} & \text{9 mul. 8 add.} \\ \text{9 mul. 8 add.} & \text{9 mul. 8 add.} & \text{9 mul. 8 add.} \\ \text{9 mul. 8 add.} & \text{9 mul. 8 add.} & \text{9 mul. 8 add.} \end{bmatrix} \begin{matrix} = \text{27 mul. 24 add.} \\ = \text{27 mul. 24 add.} \\ = \text{27 mul. 24 add.} \end{matrix} = \text{81 mul. 72 add.}$$

$$\boxed{\begin{matrix} g_2 : \text{90 mul.} \\ \text{72 add.} \\ \text{162 total operations} \end{matrix}}$$

$\boxed{2.3}$ HIGHLIGHTS VERTICAL EDGES

$\boxed{3}$

$$I = \begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix} \qquad g = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix}$$

WITH ZERO-PADDING: $I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 5 & 2 & 0 \\ 0 & 7 & 8 & 6 & 0 \\ 0 & 3 & 9 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$f_1 = 5 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I \otimes f_1 = \begin{bmatrix} 5 & 25 & 10 \\ 35 & 40 & 30 \\ 15 & 45 & 20 \end{bmatrix}$$

$$g - (I \otimes f_1) = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix} - \begin{bmatrix} 5 & 25 & 10 \\ 35 & 40 & 30 \\ 15 & 45 & 20 \end{bmatrix} = \begin{bmatrix} 24 & 18 & 0 \\ 27 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_2 = x \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I \otimes f_2 = \begin{bmatrix} 8x & 6x & 0 \\ 9x & 4x & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 24 & 18 & 0 \\ 27 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad x = 3$$

$$f = f_1 + f_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$I_Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad I_T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \quad I_M = \begin{bmatrix} 0.25 & 0.5 & 0.5 & 0.25 \\ 0.5 & 1 & 1 & 0.5 \\ 0.5 & 1 & 1 & 0.5 \\ 0.25 & 0.5 & 0.5 & 0.25 \end{bmatrix}$$

WHEN ZERO-PADDED:

$$I_Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{4.1}$

$$f_1 = \begin{bmatrix} a & b & c \\ d & e & r \\ g & p & q \end{bmatrix}$$

WHEN FLIPPED: $f_1 = \begin{bmatrix} q & p & g \\ r & e & d \\ c & b & a \end{bmatrix}$

row col.

$@I_Q(4,4)$ $(e\cdot 0)+(r\cdot 0)+(q\cdot 1)+(p\cdot 0)=0.25$

$\boxed{q = 0.25}$

$@I_Q(4,3)$ $(e\cdot 0)+(r\cdot 0)+(q\cdot 1)+(p\cdot 1)+(g\cdot 0)+(d\cdot 0)=0.5$

$0.25+p=0.5$

$\boxed{p=0.25}$

$@I_Q(4,2)$ $(e\cdot 0)+(r\cdot 0)+(q\cdot 1)+(p\cdot 1)+(g\cdot 1)=0.5$

$0.25+0.25+g=0.5$

$\boxed{g=0}$

$@I_Q(3,4)$ $(e\cdot 0)+(b\cdot 0)+(c\cdot 0)+(r\cdot 1)+(q\cdot 1)+(p\cdot 0)=0.5$

$r+0.25=0.5$

$\boxed{r=0.25}$

$@I_Q(3,3)$ $(e\cdot 1)+(r\cdot 1)+(q\cdot 1)+(p\cdot 1)+(g\cdot 0)+(d\cdot 0)+(a\cdot 0)+(b\cdot 0)+(c\cdot 0)=1$

$e+.25+.25+.25=1$

$\boxed{e=0.25}$

$@I_Q(3,1)$ $(e\cdot 1)+(p\cdot 1)+(g\cdot 1)+(d\cdot 1)+(a\cdot 0)+(b\cdot 0)=0.5$

$.25+.25+d=0.5$

$\boxed{d=0}$

$@I_Q(2,4)$ $(e\cdot 0)+(p\cdot 0)+(q\cdot 1)+(r\cdot 1)+(c\cdot 1)+(b\cdot 0)=0.5$

$.25+.25+c=0.5$

$\boxed{c=0}$

$@I_Q(2,3)$ $(e\cdot 1)+(r\cdot 1)+(q\cdot 1)+(p\cdot 1)+(g\cdot 0)+(d\cdot 0)+(a\cdot 0)+(b\cdot 1)+(c\cdot 1)=1$

$.25+.25+.25+.25+b=1$

$\boxed{b=0}$

$@I_Q(2,2)$ $(e\cdot 1)+(r\cdot 1)+(q\cdot 1)+(p\cdot 1)+(g\cdot 1)+(d\cdot 1)+(a\cdot 1)+(b\cdot 1)+(c\cdot 1)=1$

$.25+.25+.25+.25+a=1$

$\boxed{a=0}$

$$\boxed{f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .25 & .25 \\ 0 & .25 & .25 \end{bmatrix}}$$

$I_T$ is flipped version of $I_Q$

and $I_Q \otimes f_1 = I_T \otimes f_2 = I_M$

so $f_2$ is flipped $f_1$

$$\boxed{f_2 = \begin{bmatrix} .25 & .25 & 0 \\ .25 & .25 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$\boxed{4.2}$$

$$f = \frac{(f_1 + f_2)}{2} = \frac{\begin{bmatrix} 0 & 0 & 0 \\ 0 & .25 & .25 \\ 0 & .25 & .25 \end{bmatrix} + \begin{bmatrix} .25 & .25 & 0 \\ .25 & .25 & 0 \\ 0 & 0 & 0 \end{bmatrix}}{2} = \frac{\begin{bmatrix} .25 & .25 & 0 \\ .25 & .5 & .25 \\ 0 & .25 & .25 \end{bmatrix}}{2} = \begin{bmatrix} .125 & .125 & 0 \\ .125 & .25 & .125 \\ 0 & .125 & .125 \end{bmatrix}$$

$$I_Q' = I_Q \otimes f = \begin{bmatrix} ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(0\cdot.125)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot0)+(1\cdot.125)) \\ ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)+(0\cdot.125)+(1\cdot.125)+(1\cdot0)) & ((0\cdot.25)+(0\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(0\cdot.125)) \\ ((1\cdot.25)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(0\cdot.125)+(0\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)+(0\cdot.125)+(0\cdot.125)+(0\cdot0)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)+(0\cdot.125)+(0\cdot.125)+(0\cdot0)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)) \\ ((0\cdot.25)+(1\cdot.125)+(1\cdot0)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(0\cdot.125)) \end{bmatrix}$$

$$= \begin{bmatrix} (.25+.125+.125+.125) & (.25+.125+0+.125+.125+.125) & (.25+.125+0+.125+0+0) & (0+0+0+.125) \\ (.25+.125+.125+.125+0+.125) & (.25+.125+.125+.125+0+.125+.125+.125+0) & (.25+.125+.125+.125+0+0+0+.125+0) & (0+0+.125+.125+0+0) \\ (.25+.125+0+.125+0+0) & (.25+.125+.125+.125+0+.125+0+0+0) & (.25+.125+.125+.125+0+0+0+0+0) & (0+0+.125+.125+0+0) \\ (0+.125+0+0) & (0+0+.125+.125+0+0) & (0+0+.125+.125+0+0) & (0+0+.125+0) \end{bmatrix}$$

$$= \begin{bmatrix} .625 & .75 & .5 & .125 \\ .75 & 1 & .75 & .25 \\ .5 & .75 & .625 & .25 \\ .125 & .25 & .25 & .125 \end{bmatrix}$$

$$I'_T = I_T \otimes f$$

$$= \begin{bmatrix} ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)+(0\cdot.125)) & ((0\cdot.25)+(0\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(0\cdot.125)) & ((0\cdot.25)+(1\cdot.125)+(1\cdot0)+(0\cdot.125)) \\ ((0\cdot.25)+(0\cdot.125)+(0\cdot0)+(1\cdot.125)+(1\cdot.125)+(0\cdot.125)) & ((1\cdot.25)+(0\cdot.125)+(0\cdot.125)+(0\cdot.125)+(0\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)) & ((1\cdot.25)+(1\cdot.125)+(0\cdot.125)+(0\cdot.125)+(0\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)) & ((1\cdot.25)+(0\cdot.125)+(0\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)) \\ ((0\cdot.25)+(0\cdot.125)+(1\cdot0)+(1\cdot.125)+(1\cdot.125)+(0\cdot.125)) & ((1\cdot.25)+(0\cdot.125)+(0\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(0\cdot0)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)) \\ ((0\cdot.25)+(0\cdot.125)+(1\cdot0)+(1\cdot.125)) & ((1\cdot.25)+(0\cdot.125)+(0\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)+(1\cdot0)+(1\cdot.125)) & ((1\cdot.25)+(1\cdot.125)+(1\cdot.125)+(1\cdot.125)) \end{bmatrix}$$

$$= \begin{bmatrix} (0+0+.125+0) & (0+0+.125+.125+0+0) & (0+0+.125+.125+0+0) & (0+.125+0+0) \\ (0+0+0+.125+.125+0) & (.25+0+0+0+0+.125+.125+.125+0) & (.25+.125+0+0+0+.125+.125+.125+0) & (.25+0+0+.125+0+.125) \\ (0+0+0+.125+.125+0) & (.25+0+0+.125+0+.125+.125+.125+0) & (.25+.125+.125+.125+0+.125+.125+.125+0) & (.25+.125+.125+.125+0+.125) \\ (0+0+0+.125) & (.25+0+0+.125+0+.125) & (.25+.125+.125+.125+0+.125) & (.25+.125+.125+.125) \end{bmatrix}$$

$$= \begin{bmatrix} .125 & .25 & .25 & .125 \\ .25 & .625 & .75 & .5 \\ .25 & .75 & 1 & .75 \\ .125 & .5 & .75 & .625 \end{bmatrix}$$

$$D(I'_Q, I'_T) = (.625-.125)^2 + (.75-.25)^2 + (.5-.25)^2 + (.125-.125)^2 + (.75-.75)^2 + (1-.625)^2 + (.75-.75)^2 + (.25-.5)^2 + (.5-.25)^2 + (.75-.75)^2 + (.625-1)^2 + (.25-.75)^2 + (.125-.125)^2 + (.25-.5)^2 + (.25-.75)^2 + (.125-.625)^2$$

$$= .25 + .25 + .0625 + 0 + .25 + .140625 + 0 + .0625 + .0625 + 0 + .140625 + .25 + 0 + .0625 + .25 + .25$$

$$\approx 2.0312$$

$$D(I_Q, I_T) = 10$$

$$D(I'_Q, I'_T) < D(I_Q, I_T)$$

AFTER CONVOLVING THE QUERY IMAGE AND TEMPLATE WITH A BLUR KERNEL, THE PIXELWISE DIFFERENCE WAS LOWER, INDICATING HIGHER CONFIDENCE IN THE PRESENCE OF A BARREL IN THE QUERY IMAGE, DUE TO THE SHIFT (MEAN-CENTERING) OF THE QUERY IMAGE AND TEMPLATE.

**5.1**

iPhone 7 plus

Web source: https://apple.stackexchange.com/questions/271301/do-the-iphone-7-and-7-plus-cameras-use-the-same-sensor

& for "front-facing": https://www.dpreview.com/news/3533076696/teardown-reveals-sony-image-sensors-in-iphone-7

Primary camera ("back"):

Width = 4.8 mm

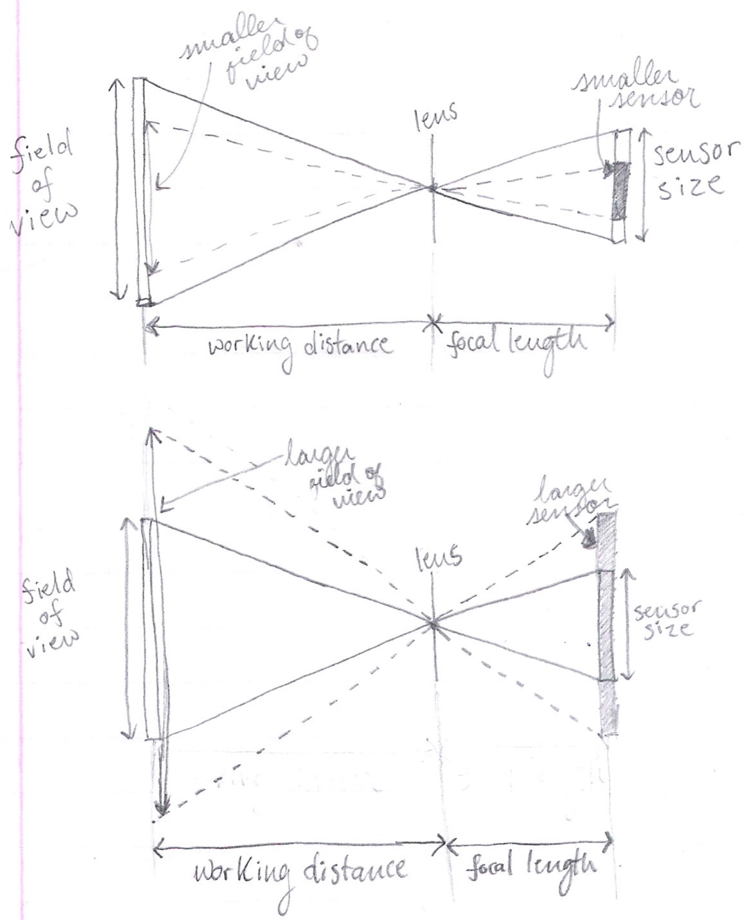
Height = 3.6 mm

Secondary camera ("front-facing"):

Width = 5.05 mm

Height = 3.72 mm

Assuming the same focal length and depth of field (working distance), the smaller the sensor, the smaller the field of view, and vice versa (the larger the sensor, the larger the field of view), e.g. :




This is because the size of the sensor determines the spatial extent of the image (i.e. incoming light) that can be captured.

**5.2**

Resolution = 12 megapixels
Pixel size = sensor area / total number of pixels
= (4.8mm x 3.6mm) / 12 MP
= 1.44 $mm^2$ / MP
= 0.0012 mm

**5.3**

Web source: [http://www.anandtech.com/show/10685/the-iphone-7-and-iphone-7-plus-review/6](http://www.anandtech.com/show/10685/the-iphone-7-and-iphone-7-plus-review/6)

Primary camera ("back"):
Focal length: 4 mm
Secondary camera ("front-facing"):
Focal length: 2.87 mm

I used a picture of an object of known width ($w$) at a known working distance ($y$) to measure the field of view ($FOV$):

$w = 0.4064\ m$
$y$ = 0.4828 m
$w_x$ = 2726 pixels
$w_{img}$ = 3024 pixels

$$FOV = \frac{w}{y} * \frac{w_{img}}{w_x} = \frac{.4064}{.4828} * \frac{3024}{2726} = 53.5 \text{ degrees}$$

Computed focal length:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right) = \frac{FOV}{2}$$

$$\frac{53.5}{2} = \tan^{-1}\left(\frac{4.8}{2f}\right)$$

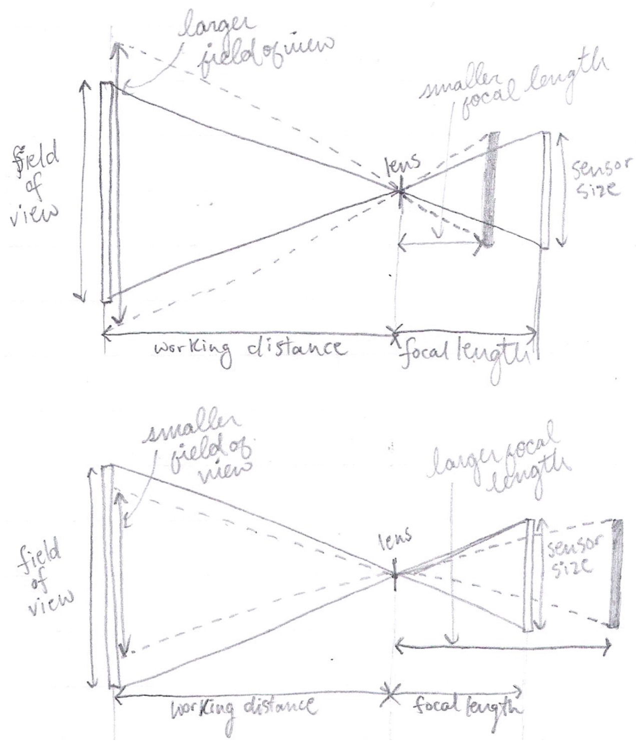
$$\tan\left(\frac{53.5}{2}\right) = \frac{4.8}{2f}$$

$$0.504 = \frac{4.8}{2f}$$

$$\boldsymbol{f = 4.76\ mm}$$

This computed focal length (4.8 mm) is very similar to the reported focal length (4 mm), with a difference of 0.8 mm.

The focal length has an inverse relationship with the field of view, in that the larger the focal length corresponds to a smaller field of view, and vice versa (the smaller the focal length, the larger the field of view), e.g. :




This is because the smaller the focal length, the wider the angular field of view.

**5.4**

$$f_x = f_y = \frac{4mm}{0.0012mm/pixel} = 3333.3\bar{3}\ pixels$$

$$s = 0$$

$$x_0 = \frac{4.8mm}{2} \Big/ 0.0012mm/pixel = 2000\ pixels$$

$$y_0 = \frac{3.6mm}{2} \Big/ 0.0012mm/pixel = 1500\ pixels$$

$$K = \begin{pmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3333.33 & 0 & 2000 \\ 0 & 3333.33 & 1500 \\ 0 & 0 & 1 \end{pmatrix}$$

**5.5.1**

$H_F = 1.55\ m$
$h_F = 755\ pixels$
$h_{img} = 4032\ pixels$
$h_{ccd} = 0.0036\ m$
$f = 0.004\ m$

$$h_F = f\,\frac{h_{img}}{h_{ccd}}\frac{H_F}{Z}$$

$$Z = H_F\,\frac{f\frac{h_{img}}{h_{ccd}}}{h_F}$$

$$= 1.55\left(\frac{.004\left(\frac{4032}{.0036}\right)}{755}\right)$$

$$\mathbf{= 9.198\ m}$$

**5.5.2**

$H_B = 5.4102\ m$
$h_B = 1515\ pixels$
$h_{img} = 4032\ pixels$
$h_{ccd} = 0.0036\ m$
$f = 0.004\ m$

$$h_B = f\,\frac{h_{img}}{h_{ccd}}\frac{H_B}{Z}$$

$$Z = H_B\,\frac{f\frac{h_{img}}{h_{ccd}}}{h_B}$$

$$= 5.4102\left(\frac{.004\left(\frac{4032}{.0036}\right)}{1515}\right)$$

$$\mathbf{= 15.999\ m}$$

**5.6.1**

$h'_F$ = 509 pixels
$h_{img} = 4032\ pixels$
$h_{ccd} = 0.0036\ m$
$H_F = 1.55\ m$
$H_B = 5.4102\ m$
$d'_F = 9.198\ m$
$d'_B = 15.999\ m$
$f_1 = 0.004\ m$
$d''_F = d'_F + \Delta d$
$d''_B = d'_B + \Delta d$

$$d''_F = H_F\left(\frac{f_1 \frac{h_{img}}{h_{ccd}}}{h'_F}\right) = 1.55\left(\frac{.004\left(\frac{4032}{.0036}\right)}{509}\right) = 13.642$$

$$\Delta d = d''_F - d'_F = 13.642 - 9.198 = 4.444$$

$$h'_F = f_1 \frac{H_F}{d'_F} = f_2 \frac{H_F}{d''_F} = h''_F$$

$$f_1 \frac{H_F}{d'_F} = f_2 \frac{H_F}{d''_F}$$

$$\frac{f_1}{d'_F} = \frac{f_2}{d''_F}$$

$$f2 = d''_F\ \frac{f_1}{d'_F}$$
$$= (13.642)\ \frac{.004}{9.198}$$

$$= 0.0059$$

$$d''_B = d'_B + \Delta d = 15.999 + 4.444 = 20.443$$

$$\boldsymbol{h''_B} = f_2 \frac{h_{img}}{h_{ccd}} \frac{H_B}{d''_B} = (.0059) \frac{4032}{.0036} \frac{5.4102}{20.443} = \mathbf{1748.79}$$

**5.6.2**

$h''_B = 1631\ pixels$

The measured pixel height of the statue (1631 pixels) is reasonably close to the computed pixel height of the statue (1749 pixels).

**5.6.3**

$$d'_B = 15.999\ m$$
$$d'_F = 9.198\ m$$
$$f_1 = 0.004\ m$$

$$h'_F = f_1 \frac{H_F}{d'_F} = f_2 \frac{H_F}{d''_F} = h''_F$$

$$h'_B = f_1 \frac{H_B}{d'_B}$$

$$h''_B = f_2 \frac{H_B}{3(d'_B)}$$

$$d''_F = d'_F + \Delta d$$
$$d''_B = d'_B + \Delta d$$

$$f_1 \frac{H_F}{d'_F} = f_2 \frac{H_F}{d''_F}$$

$$\frac{f_1}{d'_F} = \frac{f_2}{d''_F}$$

$$f_2 = f_1 \frac{d''_F}{d'_F}$$

$$= f_1 \frac{d'_F + \Delta d}{d'_F}$$

$$\frac{f_1}{d'_B} = \frac{f_2}{3d'_B}$$

$$\frac{f_1}{d'_B} = \frac{f_1 \frac{d''_F}{d'_F}}{3d''_B}$$

$$= \frac{f_1 \frac{d'_F + \Delta d}{d'_F}}{3(d'_B + \Delta d)}$$

$$3\left(\frac{d'_B + \Delta d}{d'_B}\right) = \frac{d'_F + \Delta d}{d'_F}$$

$$3d'_B + 3\Delta d = \frac{d'_B d'_F + d'_B \Delta d}{d'_F}$$

$$3d'_B + 3\Delta d = d'_B + \frac{d'_B}{d'_F}\Delta d$$

$$\left(3 - \frac{d'_B}{d'_F}\right)\Delta d = d'_B - 3d'_B$$

$$\left(\frac{d'_B}{d'_F} - 3\right)\Delta d = 2d'_B$$

$$\Delta \boldsymbol{d} = \frac{2d'_B}{\left(\frac{d'_B}{d'_F} - 3\right)}$$

$$= \frac{2*15.999}{\left(\frac{15.999}{9.198} - 3\right)}$$

$$= \mathbf{-25.383}$$

$$\boldsymbol{f_2} = f_1 \frac{d'_F + \Delta d}{d'_F}$$

$$= .004\left(\frac{9.198 + (-25.383)}{9.198}\right)$$

$$= \mathbf{-0.007}$$

In order to increase the pixel height of the statue to three times the original pixel height while keeping the pixel height of the person the same in both images, the new camera position should be 25.383 meters in front of the original camera position, and the new focal length should be -0.007 meters from the original focal length. This would put the camera 9.384 meters past the statue location and the camera facing the opposite direction of its original position with a focal length of 0.003 meters.