COMP 1805 Discrete Structures I

Study Session Questions

December 27, 2025

Proofs

1. PROVE/DISPROVE: If x and y are rational numbers, then $x \cdot y$ is a rational number.

2. Prove or disprove the converse of (a).

3. For all integers $n > 1$, n can be written as a product of primes.

Section 1: Predicate Calculus

4. Given $\alpha = $ True, $\beta = $ False, and $\delta = $ False, which statements are true? Select all that apply.

    ○ $(\alpha \lor \beta) \land \neg \delta$

    ○ $(\alpha \land \beta) \lor (\neg \delta \land \alpha)$

    ○ $(\beta \Rightarrow \alpha) \land (\delta \lor \alpha)$

    ○ $(\alpha \land \delta) \lor (\beta \land \neg \delta)$

    ○ $(\alpha \lor \neg \beta) \land (\delta \lor \beta)$

    ○ none of these

5. Consider the statement: *If it rains, then the ground is wet.* Which of the following conclusions are valid?

    ○ If the ground is wet, it has rained.

    ○ If the ground is not wet, then it has not rained.

    ○ If it does not rain, the ground will never be wet.

    ○ It is not possible that the ground is dry and it is raining.

    ○ none of these

6. Match each logical expression to its type.

    1. $\neg((\beta \rightarrow \alpha)) \land (\beta \land \alpha)$ **A.** Contingency

    2. $(\alpha \rightarrow \neg \beta) \lor \neg(\alpha \lor \beta)$ **B.** Tautology

    3. $\neg((\neg \beta \land \neg \alpha) \land \neg(\alpha \rightarrow \beta))$ **C.** Contradiction

7. Which of the following are TRUE of a logically valid argument? Select all that apply.

    ○ The conclusion must be true if the premises are true.

    ○ It cannot have false premises and a true conclusion.

    ○ A valid argument is also sound if all premises are true.

    ○ It is always a tautology.

    ○ none of these

8. Which of the following equivalences are true for $P \Leftrightarrow Q$? Select all that apply.

   ○ $(P \wedge Q) \vee (\neg P \wedge Q)$

   ○ $(P \vee \neg Q) \wedge (\neg P \vee Q)$

   ○ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

   ○ $\neg(P \oplus Q)$

   ○ False

   ○ none of these

9. Let $P(x)$ be "x has a parking pass" and $T(x)$ be "x has an O-Train pass". Match each statement to its meaning.

   1. $\forall x(P(x) \Rightarrow \neg T(x))$ **A.** Everybody who has an O-Train pass cannot have a parking pass.

   2. $\exists x(P(x) \Rightarrow \neg T(x))$ **B.** Someone has an O-Train pass or a parking pass.

   3. $\forall x(T(x) \Rightarrow \neg P(x))$ **C.** Someone has a parking pass but no O-Train pass.

   4. $\exists x(P(x) \vee T(x))$ **D.** Everybody who has a parking pass cannot have an O-Train pass.

10. Which of the following are logically equivalent to $\neg((P \Rightarrow Q) \wedge (\neg R \vee S))$? Select all that apply.

    ○ $(P \wedge \neg Q) \vee (R \wedge \neg S)$

    ○ $(P \wedge \neg Q) \wedge (R \wedge \neg S)$

    ○ $(P \wedge \neg Q) \vee (\neg(\neg R \vee S))$

    ○ $(\neg P \wedge R) \vee (P \wedge \neg Q \wedge \neg S)$

    ○ $(P \Rightarrow Q) \wedge (\neg R \vee S)$

    ○ none of these

11. Let $P(x)$ represent the statement "x is a student at CU". Let $Q(x)$ represent the statement "x has a valid bus pass". If $\forall x(P(x) \Rightarrow Q(x))$ is true, then which of the following must be true?

    A. All students at CU have a valid bus pass.

    B. Some students at CU have a valid bus pass.

    C. All people have a valid bus pass.

    D. Some people have a valid bus pass.

    E. none of these

12. What is DNF of $\neg((\alpha \vee \beta) \wedge \neg \delta)$?

    A. $\neg \alpha \wedge \neg \beta \wedge \delta$

    B. $(\neg \alpha \wedge \neg \beta) \vee \delta$

    C. $(\neg \alpha \wedge \delta) \vee (\neg \beta \wedge \delta)$

    D. $(\neg \alpha \vee \neg \beta) \vee \delta$

13. Let $H(x, y)$ represent the statement "x hates y". Match each statement with its meaning.
    1. $\exists x \forall y (H(x, y))$ **A.** There is someone who hates everyone.
    2. $\forall x \exists y (H(x, y))$ **B.** Everybody hates someone.
    3. $\forall y \exists x (H(x, y))$ **C.** Everyone is hated by someone.
    4. $\exists y \forall x (H(x, y))$ **D.** There is someone who is hated by everyone.

Section 2: Set Theory & Graph Theory

14. Match each expression to its result. Let $A = \{31, 12, \{23, 29\}, \{12, 23\}\}$ and $B = \{31, \{12, 23\}, 10, 42, 58\}$.
    1. $A - B$ **A.** $\{\{31\}, 12, \{23, 29\}\}$
    2. $A \cap B$ **B.** $\{10, 42, 58\}$
    3. $B - A$ **C.** $\{31, \{12, 23\}\}$
    4. $(A \cup B) - (A \cap B)$ **D.** $\{\{31\}, 12, \{23, 29\}, 31, \{12, 23\}\}$

15. Which of these are equivalent to $(A \cup C) - (B \cap C)$? Select all that apply.
    ○ $((A - B) \cup (C - B)) \cap \overline{C}$
    ○ $(A \cup C) \cap (\overline{B} \cup C)$
    ○ $(A \cap \overline{B}) \cup C$
    ○ $(A \cup C) \cap (\overline{B} \cap C)$
    ○ $(A \cap C) \cup (B \cap C)$
    ○ $(A \cap \overline{C}) \cup (\overline{B} \cap C)$

16. Match each set identity to its name.
    1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **A.** DeMorgan's Law
    2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ **B.** Absorption Identity
    3. $\overline{(\overline{A})} = A$ **C.** Distributive Law
    4. $A \cup (A \cap B) = A$ **D.** Double Negation

17. Write the graphs in order of chromatic number (starting with the largest): K_6, W_{10}, C_5, $K_{3,6}$.

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18. A tree of n vertices has exactly $n - 1$ edges.
    ○ True
    ○ False

19. Perform a BFS on this graph, starting from 0. Indicate the order of vertices traversed (i.e. 0 1 2 3 4 5)

20. Perform a DFS on this graph, starting from 1. Indicate the order of vertices traversed (i.e. 1 2 3 4 5 6)

21. What's the adjacency list of the subgraph formed by the first 4 vertices found in a DFS, starting from (and including) 1?

       A. [[2], [5,6], [4,2], [7,2]]

       B. [[2], [1,5], [2,4], [5]]

       C. [[1], [1,2], [1,2,5], [1,2,5,4]]

       D. [[1], [1,2], [1,2,5,6], [1,2,5,6,4,7]]

22. What's the adj. matrix of the subgraph formed by the first 4 vertices found in a BFS, starting from (and including) 1?
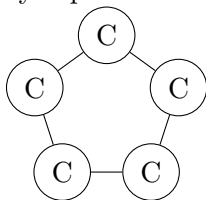
       A. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

       B. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

       C. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

       D. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

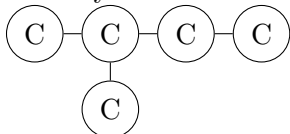23. Which of these following chemical graphs are bipartite? Select all that apply.
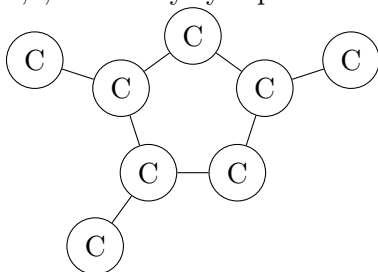
       ○ pentane
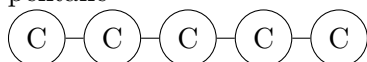
○ cyclopentane

○ 2-methylbutane

○ 1,2,4-trimethylcyclopentane

24. If a graph contains an odd-length cycle, then it is not bipartite.

○ True

○ False

25. Every connected graph has a degree of at least 2.

○ True

○ False

26. In a simple graph with 6 vertices, there exists either a connected triangle or an unconnected set of 3 vertices.

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Section 3: Everything Else

27. Determine the maximum number of distinct primes that can divide 10 consecutive numbers.

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28. Match each expression to its series.

   1. $\frac{n(n+1)}{4}$ **A.** $\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \cdots + \frac{n}{4}$

   2. $\frac{1-(\frac{1}{3})^n}{2}$ **B.** $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots + \frac{1}{2 \cdot 3^{n-1}}$

   3. $\frac{n(n+1)}{5}$ **C.** $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \cdots + \frac{n}{5}$

   4. $2\left(1 - (\frac{1}{2})^2\right)$ **D.** $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}}$

29. PROVE or DISPROVE: For all integers a and b, if $a \mid b$ then $a \leq b$.

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30. What can we say about $f(n) = 5n\log(n) + 7n + 18$? Select all that apply.

       ○ $f(n) \in O(n\log(n))$

       ○ $f(n) \in \Theta(n\log(n))$

       ○ $f(n) \in \Omega(n\log(n))$

       ○ $f(n) \in \Theta(n)$

       ○ $f(n) \in \Omega(n)$

31. If $f(n) \in O(g(n))$ and $g(n) \in \Omega(h(n))$, which of the following must be true? Select all that apply.

       ○ $f(n) \in O(h(n))$

       ○ $f(n) \in \Omega(h(n))$

○ $f(n) \in \Omega(g(n))$

○ $g(n) \in \Omega(f(n))$

○ $h(n) \in O(g(n))$

○ none of the above

32. Which of the following pairs (c, k) can justify $2n^2 + 8n + 20 \in O(n^2)$? Select all that apply.

○ $c = 2, k = 2^{18}$

○ $c = 3, k = 10$

○ $c = 4, k = 5$

○ $c = 5, k = 5$

○ $c = 2^{18}, k = 0$

○ none of the above

33. Which of the following expressions are true? Select all that apply.

○ $(10n - 5)^2 \in \Theta(n^2)$

○ $\sqrt{n} \log(n) \in O(\log(n))$

○ $4\log(5n^4 + 43n^3) \in O(\log(n))$

○ $4n^2 - n^{3/2} \in \Omega(n^3)$

○ $30n^2 - 2n \in \Omega(1)$

○ $36 + \frac{1}{n^2+n} \in \Theta(1)$

34. Determine the closed form of the given summation: $\sum_{i=1}^{n} \sum_{k=0}^{i-1} 2^k$

     A. $n(2^n - 1)$

     B. $2^{n+1} - n - 2$

     C. $\frac{2^{n+1}-2}{1-\frac{1}{2}}$

     D. $2^{n-1} + n$

35. Indicate whether each function is injective, surjective, both, or neither.

     1. $f(x) = e^x$ **A.** injective

     2. $f(x) = x^3 - 3x$ **B.** surjective

     3. $f(x) = 2x + 1$ **C.** bijective

     4. $f(x) = \sin(x)$ **D.** neither injective nor surjective

36. Assign each relation based on its properties. Let $A = \{1, 2, 3\}$.

     1. reflexive   **A.** $\{(1,1), (2,2), (3,3), (1,2)\}$

     2. anti-symmetric   **B.** $\{(1,2), (2,3)\}$

     3. transitive   **C.** $\{(1,2), (2,3), (1,3)\}$

     4. symmetric   **D.** $\{(1,2), (2,1)\}$