

COMP 1805 Discrete Structures I

Study Session Questions

December 27, 2025

Confidence Check

1. I am confident in my ability to understand propositional logic expressions and argue with quantified predicates.
2. I am confident in my ability to use set identity rules and apply concepts of graph theory.
3. I am confident in my ability to understand Big O notation, combinatorics, functions, relations, and summations.
4. I am confident in my ability to write direct proofs and proofs by *induction*, *contradiction*, and *counterexample*.

Proofs

5. PROVE/DISPROVE: If x and y are rational numbers, then $x \cdot y$ is a rational number.

6. Prove or disprove the converse of (a).

7. For all integers $n > 1$, n can be written as a product of primes.

Section 1: Predicate Calculus

8. Given $\alpha = \text{True}$, $\beta = \text{False}$, and $\delta = \text{False}$, which statements are true? Select all that apply.

- ☐ $(\alpha \vee \beta) \wedge \neg \delta$
☐ $(\alpha \wedge \beta) \vee (\neg \delta \wedge \alpha)$
☐ $(\beta \Rightarrow \alpha) \wedge (\delta \vee \alpha)$
☐ $(\alpha \wedge \delta) \vee (\beta \wedge \neg \delta)$
☐ $(\alpha \vee \neg \beta) \wedge (\delta \vee \beta)$
☐ none of these

9. Consider the statement: *If it rains, then the ground is wet.* Which of the following conclusions are valid?

- ☐ If the ground is wet, it has rained.
☐ If the ground is not wet, then it has not rained.
☐ If it does not rain, the ground will never be wet.
☐ It is not possible that the ground is dry and it is raining.
☐ none of these

10. Match each logical expression to its type.
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|--|--|
| <p>1. $\neg((\beta \rightarrow \alpha)) \wedge (\beta \wedge \alpha)$</p> <p>2. $(\alpha \rightarrow \neg\beta) \vee \neg(\alpha \vee \beta)$</p> <p>3. $\neg((\neg\beta \wedge \neg\alpha) \wedge \neg(\alpha \rightarrow \beta))$</p> | <p>A. Contingency</p> <p>B. Tautology</p> <p>C. Contradiction</p> |
|--|--|
11. Which of the following are TRUE of a logically valid argument? Select all that apply.
- ☐ The conclusion must be true if the premises are true.
 - ☐ It cannot have false premises and a true conclusion.
 - ☐ A valid argument is also sound if all premises are true.
 - ☐ It is always a tautology.
 - ☐ none of these
12. Which of the following equivalences are true for $P \Leftrightarrow Q$? Select all that apply.
- ☐ $(P \wedge Q) \vee (\neg P \wedge Q)$
 - ☐ $(P \vee \neg Q) \wedge (\neg P \vee Q)$
 - ☐ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
 - ☐ $\neg(P \oplus Q)$
 - ☐ False
 - ☐ none of these
13. Let $P(x)$ be “ x has a parking pass” and $T(x)$ be “ x has an O-Train pass”. Match each statement to its meaning.
- | | |
|---|--|
| <p>1. $\forall x(P(x) \Rightarrow \neg T(x))$</p> <p>2. $\exists x(P(x) \Rightarrow \neg T(x))$</p> <p>3. $\forall x(T(x) \Rightarrow \neg P(x))$</p> <p>4. $\exists x(P(x) \vee T(x))$</p> | <p>A. Everybody who has an O-Train pass cannot have a parking pass.</p> <p>B. Someone has an O-Train pass or a parking pass.</p> <p>C. Someone has a parking pass but no O-Train pass.</p> <p>D. Everybody who has a parking pass cannot have an O-Train pass.</p> |
|---|--|
14. Which of the following are logically equivalent to $\neg((P \Rightarrow Q) \wedge (\neg R \vee S))$? Select all that apply.
- ☐ $(P \wedge \neg Q) \vee (R \wedge \neg S)$
 - ☐ $(P \wedge \neg Q) \wedge (R \wedge \neg S)$
 - ☐ $(P \wedge \neg Q) \vee (\neg(\neg R \vee S))$
 - ☐ $(\neg P \wedge R) \vee (P \wedge \neg Q \wedge \neg S)$
 - ☐ $(P \Rightarrow Q) \wedge (\neg R \vee S)$
 - ☐ none of these
15. Let $P(x)$ represent the statement “ x is a student at CU”. Let $Q(x)$ represent the statement “ x has a valid bus pass”. If $\forall x(P(x) \Rightarrow Q(x))$ is true, then which of the following must be true?
- A. All students at CU have a valid bus pass.

- B. Some students at CU have a valid bus pass.
 C. All people have a valid bus pass.
 D. Some people have a valid bus pass.
 E. none of these
16. What is DNF of $\neg((\alpha \vee \beta) \wedge \neg\delta)$?
 A. $\neg\alpha \wedge \neg\beta \wedge \delta$
 B. $(\neg\alpha \wedge \neg\beta) \vee \delta$
 C. $(\neg\alpha \wedge \delta) \vee (\neg\beta \wedge \delta)$
 D. $(\neg\alpha \vee \neg\beta) \vee \delta$
17. Let $H(x, y)$ represent the statement “ x hates y ”. Match each statement with its meaning.
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|------------------------------------|---|
| 1. $\exists x \forall y (H(x, y))$ | A. There is someone who hates everyone. |
| 2. $\forall x \exists y (H(x, y))$ | B. Everybody hates someone. |
| 3. $\forall y \exists x (H(x, y))$ | C. Everyone is hated by someone. |
| 4. $\exists y \forall x (H(x, y))$ | D. There is someone who is hated by everyone. |

Section 2: Set Theory & Graph Theory

18. Match each expression to its result. Let $A = \{31, 12, \{23, 29\}, \{12, 23\}\}$ and $B = \{31, \{12, 23\}, 10, 42, 58\}$.
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|------------------------------|---|
| 1. $A - B$ | A. $\{\{31\}, 12, \{23, 29\}\}$ |
| 2. $A \cap B$ | B. $\{10, 42, 58\}$ |
| 3. $B - A$ | C. $\{31, \{12, 23\}\}$ |
| 4. $(A \cup B) - (A \cap B)$ | D. $\{\{31\}, 12, \{23, 29\}, 31, \{12, 23\}\}$ |
19. Which of these are equivalent to $(A \cup C) - (B \cap C)$? Select all that apply.
- ☐ $((A - B) \cup (C - B)) \cap \overline{C}$
 - ☐ $(A \cup C) \cap (\overline{B} \cup \overline{C})$
 - ☐ $(A \cap \overline{B}) \cup C$
 - ☐ $(A \cup C) \cap (\overline{B} \cap \overline{C})$
 - ☐ $(A \cap C) \cup (B \cap C)$
 - ☐ $(A \cap \overline{C}) \cup (\overline{B} \cap C)$
20. Match each set identity to its name.
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|---|------------------------|
| 1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | A. DeMorgan's Law |
| 2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ | B. Absorption Identity |
| 3. $\overline{(\overline{A})} = A$ | C. Distributive Law |
| 4. $A \cup (A \cap B) = A$ | D. Double Negation |

21.

22. A tree of n vertices has exactly $n - 1$ edges.☐ True☐ False

23.

24.

25. What's the adjacency list of the subgraph formed by the first 4 vertices found in a DFS, starting from (and including) 1?

A. $[[2], [5,6], [4,2], [7,2]]$ B. $[[2], [1,5], [2,4], [5]]$ C. $[[1], [1,2], [1,2,5], [1,2,5,4]]$ D. $[[1], [1,2], [1,2,5,6], [1,2,5,6,4,7]]$

26. What's the adj. matrix of the subgraph formed by the first 4 vertices found in a BFS, starting from (and including) 1?

A. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D.
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

27. Which of these following chemical graphs are bipartite? Select all that apply.

- ☐ pentane
- ☐ cyclopentane
- ☐ 2-methylbutane
- ☐ 1,2,4-trimethylpentane

28. If a graph contains an odd-length cycle, then it is not bipartite.

- ☐ True
- ☐ False

29. Every connected graph has a degree of at least 2.

- ☐ True
- ☐ False

30. In a simple graph with 6 vertices, there exists either a connected triangle or an unconnected set of 3 vertices.

Section 3: Everything Else

31. Determine the maximum number of distinct primes that can divide 10 consecutive numbers.

32. Match each expression to its series.
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|--|--|
| 1. $\frac{n(n+1)}{4}$ | A. $\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \cdots + \frac{n}{4}$ |
| 2. $\frac{1 - (\frac{1}{3})^n}{2}$ | B. $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots + \frac{1}{2^n}$ |
| 3. $\frac{n(n+1)}{5}$ | C. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \cdots + \frac{n}{5}$ |
| 4. $2 \left(1 - \left(\frac{1}{2}\right)^2\right)$ | D. $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}}$ |

33. PROVE or DISPROVE: For all integers a and b , if $a \mid b$ then $a \leq b$.

34. What can we say about $f(n) = 5n \log(n) + 7n + 18$? Select all that apply.

- ☐ $f(n) \in O(n \log(n))$
☐ $f(n) \in \Theta(n \log(n))$
☐ $f(n) \in \Omega(n \log(n))$
☐ $f(n) \in \Theta(n)$
☐ $f(n) \in \Omega(n)$

35. If $f(n) \in O(g(n))$ and $g(n) \in \Omega(h(n))$, which of the following must be true? Select all that apply.

- ☐ $f(n) \in O(h(n))$
☐ $f(n) \in \Omega(h(n))$
☐ $f(n) \in \Omega(g(n))$
☐ $g(n) \in \Omega(f(n))$
☐ $h(n) \in O(g(n))$
☐ none of the above

36. Which of the following pairs (c, k) can justify $2n^2 + 8n + 20 \in O(n^2)$? Select all that apply.

- ☐ $c = 2, k = 2^{18}$
☐ $c = 3, k = 10$
☐ $c = 4, k = 5$
☐ $c = 5, k = 5$
☐ $c = 2^{18}, k = 0$
☐ none of the above

37. Which of the following expressions are true? Select all that apply.

- ☐ $(10n - 5)^2 \in \Theta(n^2)$
- ☐ $\sqrt{n} \log(n) \in O(\log(n))$
- ☐ $4 \log(5n^4 + 43n^3) \in O(\log(n))$
- ☐ $4n^2 - n^{3/2} \in \Omega(n^3)$
- ☐ $30n^2 - 2n \in \Omega(1)$
- ☐ $36 + \frac{1}{n^2+n} \in \Theta(1)$

38. Determine the closed form of the given summation: $\sum_{i=1}^n \sum_{k=0}^{i-1} 2^k$

- A. $n(2^n - 1)$
- B. $2^{n+1} - n - 2$
- C. $\frac{2^{n+1}-2}{1-\frac{1}{2}}$
- D. $2^{n-1} + n$

39. Indicate whether each function is injective, surjective, both, or neither.

- 1. $f(x) = e^x$
- 2. $f(x) = x^3 - 3x$
- 3. $f(x) = 2x + 1$
- 4. $f(x) = \sin(x)$

40. Assign each relation based on its properties. Let $A = \{1, 2, 3\}$.

- 1. reflexive
- 2. anti-symmetric
- 3. transitive
- 4. symmetric