MATH 1104 Linear Algebra

Study Session Questions

December 28, 2025

Learning Objectives

- You should be confident solving linear systems using row reduction and the matrix equation $Ax = b$, and you should know what linear dependence and span are.
- You should be confident applying matrix operations, including inverses, and you should know how to calculate determinants and use Cramer's rule.
- You should be confident analyzing vector spaces by identifying subspaces, computing dimension/rank, and finding bases for a space.
- You should be confident working with complex numbers and computing eigenvalues and eigenvectors to study diagonalizability.
- You should be confident calculating inner products, applying projections, and implementing the Gram-Schmidt process for orthonormal bases.

1. Find the general solution to the corresponding homogenous system of linear equations as defined below.

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 - 4x_3 &= 4 \\ x_1 + 2x_2 - 3x_3 &= 2 \end{aligned}$$

   A. no solutions
   B. $(x_1, x_2, x_3) = t(1, 0, 1) + s(0, 1, -2)$
   C. $(x_1, x_2, x_3) = t(-1, 2, 1)$
   D. $(x_1, x_2, x_3) = (1, 1, 0)$

2. How can we know the size of the solution set of a linear system? Select all true statements.

   ◯ An inconsistent system (i.e $[\,0\ 0\ 0 \mid 3\,]$) has no solution
   ◯ If the system is square and $\det(A) = 0$, the solution is unique
   ◯ If all columns in the coefficient matrix are pivots, the solution is unique
   ◯ If there are any free variables, the system has infinite solutions
   ◯ If the rank is less than the number of variables, there are no solutions
   ◯ A consistent system with fewer equations than variables has infinite sols.

3. Suppose $b = x_1 a_1 + x_2 a_2$ as defined below. What is the sum $x_1 + x_2$?

$$a_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 2 \\ 4 \end{bmatrix}$$

A. 3
B. 6
C. 9
D. 12

4. Given the vectors defined below, which of the following statements are true? Select all that apply.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

- ◯ $(3, -1, 3) \in \text{Span}\{v_1, v_2\}$
- ◯ The zero vector is in $\text{Span}\{v_1, v_3\}$
- ◯ $(0, 1, 2) \in \text{Span}\{v_1, v_3\}$
- ◯ $(7, -12, 4) \in \text{Span}\{v_1, v_2, v_3\}$
- ◯ All three vectors are linearly DEPENDENT.
- ◯ All three vectors span $\mathbb{R}^3$.

5. For what values (k, h) does the system defined below have a unique solution?

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 4 \\ 2 & 1 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} h \\ 5 \\ 7 \end{bmatrix}$$

A. $k = 2, \; h = -3$
B. $k \neq -2$, for all $h \in \mathbb{R}$
C. $k \neq -3$, for all $h \in \mathbb{R}$
D. $k \neq 3, \; k \neq 2$
E. $h = 4$, for all $k \in \mathbb{R}$
F. $k \neq 4$, for all $h \in \mathbb{R}$

6. For what values of m is the set $\{u, v\}$ linearly dependent? Type them in numerical order, comma and space separated.

$$u = \begin{bmatrix} m \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ m+1 \end{bmatrix}$$

7. Consider the following augmented matrix of a system of linear equations below. Select all statements that are TRUE.

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & a^2-4a & a-4 \end{array}\right]$$

- ◯ When $a = 0$, there are infinitely many solutions
- ◯ When $a \neq 0$, the system is consistent
- ◯ When $a = 2$, the system has no solutions
- ◯ When $a = 2$, the columns of the coefficient matrix are linearly independent
- ◯ When $a = 4$, there are infinitely many solutions
- ◯ When $a \neq 4$, the columns of the coefficient matrix are linearly independent

8. Compute the original matrix A given its inverse, A^{-1}, defined below.

$$A^{-1} = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

A. $\begin{bmatrix} -1 & 2 & -5 \\ 0 & -1 & 4 \\ 0 & 0 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$

E. the original matrix is the identity matrix

F. the original matrix does not exist

9. Which of the following matrices are invertible? Select all that apply.

- ◯ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- ◯ $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 0 & -1 & -2 \end{bmatrix}$
- ◯ $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- ◯ $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

10. Let A, B, and C each be 3×3 matrices. If $\det(A) = 2$, $\det(B) = 4$, and $\det(C) = 8$, select all responses that must be true.

 ◯ $\det(3A) = 18$
 ◯ $\det(ABC) = 64$
 ◯ $\det\left(B^{-1}C^T\right) = 2$
 ◯ $\det\left((2B)^{-1}\right) = \frac{1}{8}$
 ◯ $\det\left[A^{-1}(B^T)^2\right] = 8$
 ◯ $\det(AI) = 2$

11. Let A and B be 3×3 matrices as defined below. If $\det(A) = 4$, determine $\det(B)$.

$$A = \begin{bmatrix} a & b & c \\ u & v & w \\ x & y & z \end{bmatrix}, \quad B = \begin{bmatrix} a & 3b+5a & c \\ u & 3v+5u & w \\ x & 3y+5x & z \end{bmatrix}$$

A. $\det(B) = -4$
B. $\det(B) = 60$
C. $\det(B) = 4$
D. $\det(B) = 8$
E. $\det(B) = 12$
F. $\det(B) = 20$

12. Let A and B be matrices as defined below. Find the matrix X such that $3X - B = AX + 2I$.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \quad 3X + B = AX + 2I$$

A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$

13. Find all values of k for which the matrix A defined below is NOT invertible.

$$A = \begin{bmatrix} 2 & 1 & k \\ 4 & k & 9 \\ k & 0 & 0 \end{bmatrix}$$

A. $k = -3$
B. $k = 3$

C. $k = -1$

D. $k = 1$

E. $k = 0$

F. such a k does not exist

14. How can Cramer's Rule be used to solve linear systems? Briefly explain in your own words.

15. What is the coordinate vector of $x = (5, 0, 4)$ relative to basis B as defined below?

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

A. $[x]_B = (5, 3, -3)$

B. $[x]_B = (3, -2, 2)$

C. $[x]_B = (1, -2, -4)$

D. $[x]_B = (-5, 0, -4)$

16. Which of the following statements about properties of transformations are TRUE? Select all that apply.

- ◯ A linear map is invertible only if it is both one-to-one and onto
- ◯ The kernel is the set of all vectors that map to the zero vector
- ◯ The range of a transformation is the span of the columns of its matrix
- ◯ A transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is always invertible if $n > m$
- ◯ Any set of independent vectors from the domain can form the standard matrix
- ◯ A transformation is one-to-one if its kernel contains only the zero vector

17. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation as defined below. Select the most true response.

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 3w \\ 2x - y + z \\ 4z - 5w \end{bmatrix}$$

A. T is one-to-one (injective).

B. T is onto (surjective).

C. T is neither injective nor surjective.

D. T is bijective, and thus, also invertible.

18. Let A be a 6×9 matrix such that row echelon form has 5 pivot positions. Which of these statements are TRUE?

    ○ $\dim(\mathrm{Nul}\, A) = 4$

    ○ $\mathrm{Rank}\, A = 5$

    ○ $\mathrm{Nul}\, A = \mathbb{R}^4$

    ○ $\dim(\mathrm{Col}\, A) = 5$

19. Let matrix A be as defined below. Which of the following sets is a basis for the null space of A?

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A. $\left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 1 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

20. Select all of the sets below that are vector spaces.

    ○ $\left\{ \begin{bmatrix} p \\ q \\ r \end{bmatrix} : 3p - 4q = r,\ 2q = p + 3q \right\}$

    ○ $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x - 2y + z = 5 \right\}$

    ○ $\left\{ \begin{bmatrix} 3 \\ c \\ d \end{bmatrix} : c, d \text{ real} \right\}$

    ○ $\left\{ \begin{bmatrix} -a + 2b \\ a - 2b \\ 3a - 6b \end{bmatrix} : a, b \text{ real} \right\}$

21. Let A be defined below and $w = (0, 0, 4, -2)$. Select the most correct response.

$$A = \begin{bmatrix} 3 & 0 & 4 & 8 \\ 1 & 0 & 3 & 6 \\ 4 & -2 & 2 & 4 \\ 4 & 1 & 0 & 0 \end{bmatrix}$$

A. w is in $\operatorname{Col} A$.

B. w is in $\operatorname{Nul} A$.

C. w is in both $\operatorname{Col} A$ and $\operatorname{Nul} A$.

D. w is in neither $\operatorname{Col} A$ nor $\operatorname{Nul} A$.

22. Let $v = (0, 1, 1)$, an eigenvector of A. What is the corresponding eigenvalue?

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

A. -4

B. 4

C. -3

D. 3

E. -2

F. 2

23. What is the polar form of the complex number $z = 3 - 3i$?

A. $6(\cos(-\pi/4) - i\sin(-\pi/4))$

B. $3\sqrt{2}(\cos(-\pi/4) + i\sin(-\pi/4))$

C. $3\sqrt{2}(\cos(7\pi/4) - i\sin(7\pi/4))$

D. $3\sqrt{2}(\sin(\pi/4) + i\cos(\pi/4))$

24. Which of the following equations involving complex numbers are true? Select all that apply.

○ The modulus of $3 + 4i$ is 5.

○ $(2i)^2 = 4i$

○ $(2 + 3i)(2 - 3i) = 13$

○ $i^6 = i$

○ $\dfrac{3 + 4i}{1 + 2i} = i - 1$

○ $\dfrac{4 + 3i}{2 - i} = 5$

25. Suppose A is a 4×4 NON-invertible matrix. Which of the following cannot be the characteristic polynomial of A?

A. $\lambda^4(\lambda - 2)$

B. $-(\lambda + 3)^2$

C. $\lambda^2 + 4\lambda$

D. $-\lambda(\lambda^3 - 5)$

26. $\lambda = 4$ is an eigenvalue for the matrix B as defined below. Which one of the following sets is a basis for E_4?

$$B = \begin{bmatrix} 6 & -2 & 4 \\ 2 & 2 & 4 \\ 2 & -2 & 8 \end{bmatrix}$$

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

27. Exactly one of the following statements involving diagonalizability is TRUE. Which one?

A. Every invertible matrix is diagonalizable.

B. Every diagonalizable matrix is invertible.

C. All $n \times n$ diagonalizable matrices have n distinct eigenvalues.

D. All $n \times n$ diagonalizable matrices have n linearly independent eigenvectors.

28. Select the statement that is most true involving the diagonalizability of matrices A and B.

$$A = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$$

A. A is diagonalizable.

B. B is diagonalizable.

C. Both A and B are diagonalizable.

D. Neither A nor B are diagonalizable.

29. Let $u = (1, 2, 3)$ and $v = (3, 2, 1)$. Select all statements that are TRUE.

- ◯ $u \cdot v = 10$
- ◯ $\|u\| = \|v\|$
- ◯ The vectors u and v are orthogonal
- ◯ The sum $u + v$ is orthogonal to the difference $u - v$

30. Determine the value k for which $v = (k, 5, 7)$ lies in $W^{\perp}$ with W as defined below.

$$W = \text{Span}\left\{ \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 2 \end{bmatrix} \right\}$$

A. $k = -1$

B. $k = 1$

C. $k = -2$

D. $k = 2$

E. any k

F. no such k exists

31. Let $u = (2, 1)$ and $v = (1, 3)$. What is the angle between u and v?

A. $\pi/2$

B. $\pi/3$

C. $\pi/4$

D. $\pi/6$

32. Let $W = \text{Span}\{(1, 2, 2)\}$ and $x = (2, 3, 5)$. What is the distance from x to W?

A. $3\sqrt{3}$

B. $-\frac{7}{2}$

C. $2\sqrt{5}$

D. 2

33. Briefly explain how to use the Gram-Schmidt process to turn linearly-independent vectors into an orthonormal basis.

34. Suppose the columns of B form an orthogonal basis for $\mathbb{R}^4$. If $x = c_1b_1 + c_2b_2 + c_3b_3 + c_4b_4$, what is the value of c_2?

$$B = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & -1 & 8 & 2 \\ 3 & 0 & -4 & 9 \\ 4 & 0 & -2 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} 7 \\ 5 \\ -1 \\ 4 \end{bmatrix}$$

A. $\frac{5}{4}$

B. $-\frac{4}{25}$

C. $\frac{9}{5}$

D. 1