

COMP 1805 Discrete Structures I

Study Session Questions

December 27, 2025

Proofs

1. PROVE/DISPROVE: If x and y are rational numbers, then $x \cdot y$ is a rational number.

2. Prove or disprove the converse of (a).

3. For all integers $n > 1$, n can be written as a product of primes.

Section 1: Predicate Calculus

4. Given $\alpha = \text{True}$, $\beta = \text{False}$, and $\delta = \text{False}$, which statements are true? Select all that apply.

- $(\alpha \vee \beta) \wedge \neg\delta$
- $(\alpha \wedge \beta) \vee (\neg\delta \wedge \alpha)$
- $(\beta \Rightarrow \alpha) \wedge (\delta \vee \alpha)$
- $(\alpha \wedge \delta) \vee (\beta \wedge \neg\delta)$
- $(\alpha \vee \neg\beta) \wedge (\delta \vee \beta)$
- none of these

5. Consider the statement: *If it rains, then the ground is wet.* Which of the following conclusions are valid?

- If the ground is wet, it has rained.
- If the ground is not wet, then it has not rained.
- If it does not rain, the ground will never be wet.
- It is not possible that the ground is dry and it is raining.
- none of these

6. Match each logical expression to its type.

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| 1. $\neg((\beta \rightarrow \alpha)) \wedge (\beta \wedge \alpha)$ | A. Contingency |
| 2. $(\alpha \rightarrow \neg\beta) \vee \neg(\alpha \vee \beta)$ | B. Tautology |
| 3. $\neg((\neg\beta \wedge \neg\alpha) \wedge \neg(\alpha \rightarrow \beta))$ | C. Contradiction |

7. Which of the following are TRUE of a logically valid argument? Select all that apply.

- The conclusion must be true if the premises are true.
- It cannot have false premises and a true conclusion.
- A valid argument is also sound if all premises are true.
- It is always a tautology.
- none of these

8. Which of the following equivalences are true for $P \Leftrightarrow Q$? Select all that apply.

- $(P \wedge Q) \vee (\neg P \wedge Q)$
- $(P \vee \neg Q) \wedge (\neg P \vee Q)$
- $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
- $\neg(P \oplus Q)$
- False
- none of these

9. Let $P(x)$ be “ x has a parking pass” and $T(x)$ be “ x has an O-Train pass”. Match each statement to its meaning.

- 1.** $\forall x(P(x) \Rightarrow \neg T(x))$
- 2.** $\exists x(P(x) \Rightarrow \neg T(x))$
- 3.** $\forall x(T(x) \Rightarrow \neg P(x))$
- 4.** $\exists x(P(x) \vee T(x))$

- A.** Everybody who has an O-Train pass cannot have a parking pass.
- B.** Someone has an O-Train pass or a parking pass.
- C.** Someone has a parking pass but no O-Train pass.
- D.** Everybody who has a parking pass cannot have an O-Train pass.

10. Which of the following are logically equivalent to $\neg((P \Rightarrow Q) \wedge (\neg R \vee S))$? Select all that apply.

- $(P \wedge \neg Q) \vee (R \wedge \neg S)$
- $(P \wedge \neg Q) \wedge (R \wedge \neg S)$
- $(P \wedge \neg Q) \vee (\neg(\neg R \vee S))$
- $(\neg P \wedge R) \vee (P \wedge \neg Q \wedge \neg S)$
- $(P \Rightarrow Q) \wedge (\neg R \vee S)$
- none of these

11. Let $P(x)$ represent the statement “ x is a student at CU”. Let $Q(x)$ represent the statement “ x has a valid bus pass”. If $\forall x(P(x) \Rightarrow Q(x))$ is true, then which of the following must be true?

- A.** All students at CU have a valid bus pass.
- B.** Some students at CU have a valid bus pass.
- C.** All people have a valid bus pass.
- D.** Some people have a valid bus pass.
- E.** none of these

12. What is DNF of $\neg((\alpha \vee \beta) \wedge \neg \delta)$?

- A.** $\neg \alpha \wedge \neg \beta \wedge \delta$
- B.** $(\neg \alpha \wedge \neg \beta) \vee \delta$
- C.** $(\neg \alpha \wedge \delta) \vee (\neg \beta \wedge \delta)$
- D.** $(\neg \alpha \vee \neg \beta) \vee \delta$

13. Let $H(x, y)$ represent the statement “ x hates y ”. Match each statement with its meaning.

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| 1. $\exists x \forall y (H(x, y))$ | A. There is someone who hates everyone. |
| 2. $\forall x \exists y (H(x, y))$ | B. Everybody hates someone. |
| 3. $\forall y \exists x (H(x, y))$ | C. Everyone is hated by someone. |
| 4. $\exists y \forall x (H(x, y))$ | D. There is someone who is hated by everyone. |

Section 2: Set Theory & Graph Theory

14. Match each expression to its result. Let $A = \{31, 12, \{23, 29\}, \{12, 23\}\}$ and $B = \{31, \{12, 23\}, 10, 42, 58\}$.

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|-------------------------------------|--|
| 1. $A - B$ | A. $\{\{31\}, 12, \{23, 29\}\}$ |
| 2. $A \cap B$ | B. $\{10, 42, 58\}$ |
| 3. $B - A$ | C. $\{31, \{12, 23\}\}$ |
| 4. $(A \cup B) - (A \cap B)$ | D. $\{\{31\}, 12, \{23, 29\}, 31, \{12, 23\}\}$ |

15. Which of these are equivalent to $(A \cup C) - (B \cap C)$? Select all that apply.

- $((A - B) \cup (C - B)) \cap \overline{C}$
- $(A \cup C) \cap (\overline{B} \cup C)$
- $(A \cap \overline{B}) \cup C$
- $(A \cup C) \cap (\overline{B} \cap C)$
- $(A \cap C) \cup (B \cap C)$
- $(A \cap \overline{C}) \cup (\overline{B} \cap C)$

16. Match each set identity to its name.

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|--|-------------------------------|
| 1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | A. DeMorgan's Law |
| 2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ | B. Absorption Identity |
| 3. $\overline{(\overline{A})} = A$ | C. Distributive Law |
| 4. $A \cup (A \cap B) = A$ | D. Double Negation |

17. Write the graphs in order of chromatic number (starting with the largest): K_6 , W_{10} , C_5 , $K_{3,6}$.

18. A tree of n vertices has exactly $n - 1$ edges.

- True
- False

19. Perform a BFS on this graph, starting from 0. Indicate the order of vertices traversed (i.e. 0 1 2 3 4 5)

20. Perform a DFS on this graph, starting from 1. Indicate the order of vertices traversed (i.e. 1 2 3 4 5 6)

21. What's the adjacency list of the subgraph formed by the first 4 vertices found in a DFS, starting from (and including) 1?

- A. [[2], [5,6], [4,2], [7,2]]
- B. [[2], [1,5], [2,4], [5]]
- C. [[1], [1,2], [1,2,5], [1,2,5,4]]
- D. [[1], [1,2], [1,2,5,6], [1,2,5,6,4,7]]

22. What's the adj. matrix of the subgraph formed by the first 4 vertices found in a BFS, starting from (and including) 1?

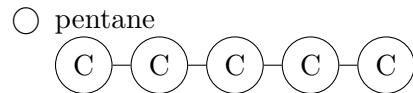
A. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

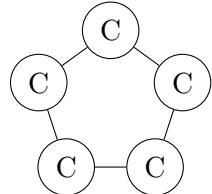
C. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

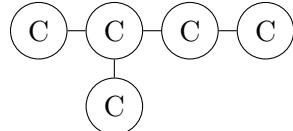
23. Which of these following chemical graphs are bipartite? Select all that apply.



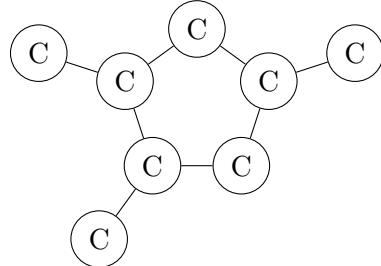
cyclopentane



2-methylbutane



1,2,4-trimethylcyclopentane



24. If a graph contains an odd-length cycle, then it is not bipartite.

True

False

25. Every connected graph has a degree of at least 2.

True

False

26. In a simple graph with 6 vertices, there exists either a connected triangle or an unconnected set of 3 vertices.

Section 3: Everything Else

27. Determine the maximum number of distinct primes that can divide 10 consecutive numbers.

28. Match each expression to its series.

1. $\frac{n(n+1)}{4}$

A. $\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \cdots + \frac{n}{4}$

2. $\frac{1-(\frac{1}{3})^n}{2}$

B. $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots + \frac{1}{2 \cdot 3^{n-1}}$

3. $\frac{n(n+1)}{5}$

C. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \cdots + \frac{n}{5}$

4. $2 \left(1 - \left(\frac{1}{2}\right)^2\right)$

D. $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}}$

29. PROVE or DISPROVE: For all integers a and b , if $a \mid b$ then $a \leq b$.

30. What can we say about $f(n) = 5n \log(n) + 7n + 18$? Select all that apply.

- $f(n) \in O(n \log(n))$
- $f(n) \in \Theta(n \log(n))$
- $f(n) \in \Omega(n \log(n))$
- $f(n) \in \Theta(n)$
- $f(n) \in \Omega(n)$

31. If $f(n) \in O(g(n))$ and $g(n) \in \Omega(h(n))$, which of the following must be true? Select all that apply.

- $f(n) \in O(h(n))$
- $f(n) \in \Omega(h(n))$

- $f(n) \in \Omega(g(n))$
- $g(n) \in \Omega(f(n))$
- $h(n) \in O(g(n))$
- none of the above
32. Which of the following pairs (c, k) can justify $2n^2 + 8n + 20 \in O(n^2)$? Select all that apply.
- $c = 2, k = 2^{18}$
- $c = 3, k = 10$
- $c = 4, k = 5$
- $c = 5, k = 5$
- $c = 2^{18}, k = 0$
- none of the above
33. Which of the following expressions are true? Select all that apply.
- $(10n - 5)^2 \in \Theta(n^2)$
- $\sqrt{n} \log(n) \in O(\log(n))$
- $4 \log(5n^4 + 43n^3) \in O(\log(n))$
- $4n^2 - n^{3/2} \in \Omega(n^3)$
- $30n^2 - 2n \in \Omega(1)$
- $36 + \frac{1}{n^2+n} \in \Theta(1)$
34. Determine the closed form of the given summation: $\sum_{i=1}^n \sum_{k=0}^{i-1} 2^k$
- A. $n(2^n - 1)$
- B. $2^{n+1} - n - 2$
- C. $\frac{2^{n+1}-2}{1-\frac{1}{2}}$
- D. $2^{n-1} + n$
35. Indicate whether each function is injective, surjective, both, or neither.
1. $f(x) = e^x$ A. injective
2. $f(x) = x^3 - 3x$ B. surjective
3. $f(x) = 2x + 1$ C. bijective
4. $f(x) = \sin(x)$ D. neither injective nor surjective
36. Assign each relation based on its properties. Let $A = \{1, 2, 3\}$.
1. reflexive A. $\{(1, 1), (2, 2), (3, 3), (1, 2)\}$
2. anti-symmetric B. $\{(1, 2), (2, 3)\}$
3. transitive C. $\{(1, 2), (2, 3), (1, 3)\}$
4. symmetric D. $\{(1, 2), (2, 1)\}$