COMP 1805 Discrete Structures I

Study Session Questions

December 27, 2025

Proofs

1. PROVE/DISPROVE: If x and y are rational numbers, then $x \cdot y$ is a rational number.

2. Prove or disprove the converse of (a).

3. For all integers $n > 1$, n can be written as a product of primes.

Section 1: Predicate Calculus

4. Given $\alpha =$ True, $\beta =$ False, and $\delta =$ False, which statements are true? Select all that apply.
   - ◯ $(\alpha \vee \beta) \wedge \neg\delta$
   - ◯ $(\alpha \wedge \beta) \vee (\neg\delta \wedge \alpha)$
   - ◯ $(\beta \Rightarrow \alpha) \wedge (\delta \vee \alpha)$
   - ◯ $(\alpha \wedge \delta) \vee (\beta \wedge \neg\delta)$
   - ◯ $(\alpha \vee \neg\beta) \wedge (\delta \vee \beta)$
   - ◯ none of these

5. Consider the statement: *If it rains, then the ground is wet.* Which of the following conclusions are valid?
   - ◯ If the ground is wet, it has rained.
   - ◯ If the ground is not wet, then it has not rained.
   - ◯ If it does not rain, the ground will never be wet.
   - ◯ It is not possible that the ground is dry and it is raining.
   - ◯ none of these

6. Match each logical expression to its type.

1. $\neg((\beta \rightarrow \alpha)) \wedge (\beta \wedge \alpha)$	**A.** Contingency
2. $(\alpha \rightarrow \neg\beta) \vee \neg(\alpha \vee \beta)$	**B.** Tautology
3. $\neg((\neg\beta \wedge \neg\alpha) \wedge \neg(\alpha \rightarrow \beta))$	**C.** Contradiction

7. Which of the following are TRUE of a logically valid argument? Select all that apply.
   - ◯ The conclusion must be true if the premises are true.
   - ◯ It cannot have false premises and a true conclusion.
   - ◯ A valid argument is also sound if all premises are true.
   - ◯ It is always a tautology.
   - ◯ none of these

8. Which of the following equivalences are true for $P \Leftrightarrow Q$? Select all that apply.

   ◯ $(P \wedge Q) \vee (\neg P \wedge Q)$

   ◯ $(P \vee \neg Q) \wedge (\neg P \vee Q)$

   ◯ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

   ◯ $\neg(P \oplus Q)$

   ◯ False

   ◯ none of these

9. Let $P(x)$ be "x has a parking pass" and $T(x)$ be "x has an O-Train pass". Match each statement to its meaning.

1. $\forall x(P(x) \Rightarrow \neg T(x))$	**A.** Everybody who has an O-Train pass cannot have a parking pass.
2. $\exists x(P(x) \Rightarrow \neg T(x))$	**B.** Someone has an O-Train pass or a parking pass.
3. $\forall x(T(x) \Rightarrow \neg P(x))$	**C.** Someone has a parking pass but no O-Train pass.
4. $\exists x(P(x) \vee T(x))$	**D.** Everybody who has a parking pass cannot have an O-Train pass.

10. Which of the following are logically equivalent to $\neg((P \Rightarrow Q) \wedge (\neg R \vee S))$? Select all that apply.

   ◯ $(P \wedge \neg Q) \vee (R \wedge \neg S)$

   ◯ $(P \wedge \neg Q) \wedge (R \wedge \neg S)$

   ◯ $(P \wedge \neg Q) \vee (\neg(\neg R \vee S))$

   ◯ $(\neg P \wedge R) \vee (P \wedge \neg Q \wedge \neg S)$

   ◯ $(P \Rightarrow Q) \wedge (\neg R \vee S)$

   ◯ none of these

11. Let $P(x)$ represent the statement "x is a student at CU". Let $Q(x)$ represent the statement "x has a valid bus pass". If $\forall x(P(x) \Rightarrow Q(x))$ is true, then which of the following must be true?

   A. All students at CU have a valid bus pass.

   B. Some students at CU have a valid bus pass.

   C. All people have a valid bus pass.

   D. Some people have a valid bus pass.

   E. none of these

12. What is DNF of $\neg((\alpha \vee \beta) \wedge \neg\delta)$?

   A. $\neg\alpha \wedge \neg\beta \wedge \delta$

   B. $(\neg\alpha \wedge \neg\beta) \vee \delta$

   C. $(\neg\alpha \wedge \delta) \vee (\neg\beta \wedge \delta)$

   D. $(\neg\alpha \vee \neg\beta) \vee \delta$

13. Let $H(x, y)$ represent the statement "x hates y". Match each statement with its meaning.

1. $\exists x \forall y (H(x, y))$
2. $\forall x \exists y (H(x, y))$
3. $\forall y \exists x (H(x, y))$
4. $\exists y \forall x (H(x, y))$

A. There is someone who hates everyone.
B. Everybody hates someone.
C. Everyone is hated by someone.
D. There is someone who is hated by everyone.

Section 2: Set Theory & Graph Theory

14. Match each expression to its result. Let $A = \{31, 12, \{23, 29\}, \{12, 23\}\}$ and $B = \{31, \{12, 23\}, 10, 42, 58\}$.

1. $A - B$
2. $A \cap B$
3. $B - A$
4. $(A \cup B) - (A \cap B)$

A. $\{\{31\}, 12, \{23, 29\}\}$
B. $\{10, 42, 58\}$
C. $\{31, \{12, 23\}\}$
D. $\{\{31\}, 12, \{23, 29\}, 31, \{12, 23\}\}$

15. Which of these are equivalent to $(A \cup C) - (B \cap C)$? Select all that apply.

- ◯ $((A - B) \cup (C - B)) \cap \overline{C}$
- ◯ $(A \cup C) \cap (\overline{B} \cup C)$
- ◯ $(A \cap \overline{B}) \cup C$
- ◯ $(A \cup C) \cap (\overline{B} \cap C)$
- ◯ $(A \cap C) \cup (B \cap C)$
- ◯ $(A \cap \overline{C}) \cup (\overline{B} \cap C)$

16. Match each set identity to its name.

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
3. $\overline{(\overline{A})} = A$
4. $A \cup (A \cap B) = A$

A. DeMorgan's Law
B. Absorption Identity
C. Distributive Law
D. Double Negation

17. Write the graphs in order of chromatic number (starting with the largest): K_6, W_{10}, C_5, $K_{3,6}$.

18. A tree of n vertices has exactly $n - 1$ edges.

- ◯ True
- ◯ False

19. Perform a BFS on this graph, starting from 0. Indicate the order of vertices traversed (i.e. 0 1 2 3 4 5)

20. Perform a DFS on this graph, starting from 1. Indicate the order of vertices traversed (i.e. 1 2 3 4 5 6)

21. What's the adjacency list of the subgraph formed by the first 4 vertices found in a DFS, starting from (and including) 1?

    A. [[2], [5,6], [4,2], [7,2]]
    B. [[2], [1,5], [2,4], [5]]
    C. [[1], [1,2], [1,2,5], [1,2,5,4]]
    D. [[1], [1,2], [1,2,5,6], [1,2,5,6,4,7]]

22. What's the adj. matrix of the subgraph formed by the first 4 vertices found in a BFS, starting from (and including) 1?

    A. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

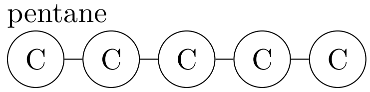
    B. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

    C. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

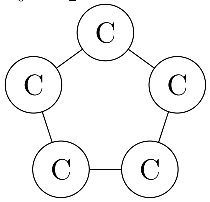
    D. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

23. Which of these following chemical graphs are bipartite? Select all that apply.

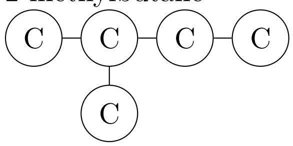
    ○ pentane



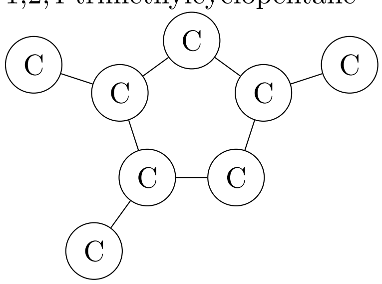
- ◯ cyclopentane



- ◯ 2-methylbutane



- ◯ 1,2,4-trimethylcyclopentane



24. If a graph contains an odd-length cycle, then it is not bipartite.
    - ◯ True
    - ◯ False
25. Every connected graph has a degree of at least 2.
    - ◯ True
    - ◯ False
26. In a simple graph with 6 vertices, there exists either a connected triangle or an unconnected set of 3 vertices.

Section 3: Everything Else

27. Determine the maximum number of distinct primes that can divide 10 consecutive numbers.

28. Match each expression to its series.

1. $\frac{n(n+1)}{4}$

2. $\frac{1-(\frac{1}{3})^n}{2}$

3. $\frac{n(n+1)}{5}$

4. $2\left(1-(\frac{1}{2})^2\right)$

A. $\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\cdots+\frac{n}{4}$

B. $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\cdots+\frac{1}{2\cdot 3^{n-1}}$

C. $\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\cdots+\frac{n}{5}$

D. $1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{n-1}}$

29. PROVE or DISPROVE: For all integers a and b, if $a \mid b$ then $a \leq b$.

30. What can we say about $f(n) = 5n\log(n) + 7n + 18$? Select all that apply.

- ◯ $f(n) \in O(n\log(n))$
- ◯ $f(n) \in \Theta(n\log(n))$
- ◯ $f(n) \in \Omega(n\log(n))$
- ◯ $f(n) \in \Theta(n)$
- ◯ $f(n) \in \Omega(n)$

31. If $f(n) \in O(g(n))$ and $g(n) \in \Omega(h(n))$, which of the following must be true? Select all that apply.

- ◯ $f(n) \in O(h(n))$

- ◯ $f(n) \in \Omega(h(n))$
- ◯ $f(n) \in \Omega(g(n))$
- ◯ $g(n) \in \Omega(f(n))$
- ◯ $h(n) \in O(g(n))$
- ◯ none of the above

32. Which of the following pairs (c, k) can justify $2n^2 + 8n + 20 \in O(n^2)$? Select all that apply.
    - ◯ $c = 2, k = 2^{18}$
    - ◯ $c = 3, k = 10$
    - ◯ $c = 4, k = 5$
    - ◯ $c = 5, k = 5$
    - ◯ $c = 2^{18}, k = 0$
    - ◯ none of the above

33. Which of the following expressions are true? Select all that apply.
    - ◯ $(10n - 5)^2 \in \Theta(n^2)$
    - ◯ $\sqrt{n}\log(n) \in O(\log(n))$
    - ◯ $4\log(5n^4 + 43n^3) \in O(\log(n))$
    - ◯ $4n^2 - n^{3/2} \in \Omega(n^3)$
    - ◯ $30n^2 - 2n \in \Omega(1)$
    - ◯ $36 + \frac{1}{n^2+n} \in \Theta(1)$

34. Determine the closed form of the given summation: $\sum_{i=1}^{n} \sum_{k=0}^{i-1} 2^k$

    A. $n(2^n - 1)$

    B. $2^{n+1} - n - 2$

    C. $\frac{2^{n+1}-2}{1-\frac{1}{2}}$

    D. $2^{n-1} + n$

35. Indicate whether each function is injective, surjective, both, or neither.

1. $f(x) = e^x$	**A.** injective
2. $f(x) = x^3 - 3x$	**B.** surjective
3. $f(x) = 2x + 1$	**C.** bijective
4. $f(x) = \sin(x)$	**D.** neither injective nor surjective

36. Assign each relation based on its properties. Let $A = \{1, 2, 3\}$.

1. reflexive	**A.** $\{(1,1),(2,2),(3,3),(1,2)\}$
2. anti-symmetric	**B.** $\{(1,2),(2,3)\}$
3. transitive	**C.** $\{(1,2),(2,3),(1,3)\}$
4. symmetric	**D.** $\{(1,2),(2,1)\}$