AXIS Progress Report 1*

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Nomenclature

V_C	Ionization chamber volume
ρ_P	Propellant density
M_P	Propellant molar mass
M_{RI}	Radioisotope (RI) molar mass
I	Ionization efficiency (ionizations/α)
M_0	Initial RI mass
Δm_{RI}	Change in RI mass needed for full chamber ionization
Δt_{ionize}	Change in time needed for full chamber ionization
m_{RI}	RI mass, time-dependent
m_P	Propellant mass, time-dependent
t	time (mission duration)
$t_{1/2}$	RI half-life
A	RI specific activity
E_α	RI α energy
$\overline{E_{I_{ab}}}$	Mean ionization energy from energy level a to b
E_{I_a}	Ionization energy at energy level a

*The Alpha-Xenon Ionization Simulation (AXIS) subproject started as an investigation into the optimal radioisotope geometry as the primary ionizer in the SEPTIR ion thruster for picosats on the FIRESTORM mission. As questions arose, the focus of the project shifted toward the a combination of the original objective within the context of the ionization efficiency fundamentally available from the propellant and radioisotope used, and the method used to investigate the ionization of xenon were similarly applied to other options for a gaseous propellant. While the inclusion of other propellants would make "Alpha-Propellant Ionization Simulation" a more suitable project title, "APIS" is not quite as euphonious as "AXIS", so the latter, despite being a misnomer, will be used to refer to the subproject throughout this and subsequent reports.

1 Introduction

This study briefly explored the metrics needed to compare different radioisotope-propellant combinations for the proposed SEPTIR design to be implemented into the FIRESTORM mission. Geant4 simulations were used to determine the ionization efficiency I and stopping ranges of the α particles emitted from the radioisotopes into the propellant gas. For each radioisotope-propellant combination, I was combined with the natural decay rate of the radioisotopes to inform the maximum propellant mass flow rate deliverable by the SEPTIR system, while the stopping ranges offer a preliminary insight into the optimal radioisotope-ionization chamber geometry, but more simulations and empirical work must be done to investigate the latter.

2 Methodology and Results

2.1 Ionization efficiency

The Geant4 simulation toolkit was used record the total number of ionization processes that occurred when α particles were emitted into a volume of gaseous propellant. For each ionization process, the energy lost by the α particle due to the specific interaction was divided by either the first ionization energy E_{I_1} or the mean ionization energy $\overline{E_{I_{ab}}}$ of the gas as taken from Wikipedia (1). The ionization efficiencies for each radioisotope are recorded in Table 1.

Table 1: First, Mean Ionization Efficiencies; Mean ionization energy level is denoted by (a,b)

	Cesium (132.9 au)	Bismuth (209.0 au)	Xenon (200.6 au)	Mercury (131.3 au)	Iodine (126.9 au)
E_{I_1}	3.894 eV	7.286 eV	10.44 eV	12.13 eV	10.45 eV
^{209}Po (4.9 MeV)	1.26×10^6	6.73×10^5	4.04×10^5	4.69×10^5	4.69×10^5
^{208}Po (5.1 MeV)	1.31×10^6	7.00×10^5	4.20×10^5	4.88×10^5	4.88×10^5
^{241}Am (5.5 MeV)	1.41×10^6	7.55×10^5	4.53×10^5	5.27×10^5	5.26×10^5
^{238}Pu (5.6 MeV)	1.44×10^6	7.69×10^5	4.62×10^5	5.36×10^5	5.36×10^5
^{244}Cm (5.8 MeV)	1.49×10^6	7.96×10^5	4.78×10^5	5.56×10^5	5.55×10^5
$\overline{E_{I_{ab}}}$	2.0765×10^{-5}	1.6512×10^{-5}	4.2156×10^{-5}	2.182×10^{-5}	2.0847×10^{-5}
(a,b)	(1,3)	(1,3)	(1,6)	(1,3)	(1,3)
^{209}Po (4.9 MeV)	2.36×10^5	2.97×10^5	2.25×10^5	1.16×10^5	2.35×10^5
^{208}Po (5.1 MeV)	2.46×10^5	3.09×10^5	2.34×10^5	1.21×10^5	2.45×10^5
^{241}Am (5.5 MeV)	2.65×10^5	3.33×10^5	2.52×10^5	1.30×10^5	2.64×10^5
^{238}Pu (5.6 MeV)	2.70×10^5	3.39×10^5	2.57×10^5	1.33×10^5	2.69×10^5
^{244}Cm (5.8 MeV)	2.79×10^5	3.51×10^5	2.66×10^5	1.38×10^5	2.78×10^5

2.2 Translating Ionization Efficiency to Mission Lifetime

The thrust SEPTIR will deliver at a given moment is dependent on the mass flow rate out of the engine, and is therefore dependent on the density of propellant gas in the ionization chamber. Assuming one radioisotope atom produces one α particle per decay, (1) will express the changes in the radioisotope's mass as a function of the density of the propellant in the chamber; there is a direct linear correlation, as shown in Figure 1.

$$\Delta m_{RI}(t) = \frac{V_C M_{RI}}{I M_P} \rho_P(t) \tag{1}$$

To make the propellant density the only variable affecting thrust, the total ionization time should ideally be kept constant. Dividing the non-instantaneous Δm_{RI} by the non-instantaneous ionization time Δt_{ionize} gives the expression for the non-instantaneous, rate of change of the radioisotope's mass.

$$\frac{\Delta m_{RI}(t)}{\Delta t_{ionize}} = \frac{V_C M_{RI}}{I M_P \Delta t_{ionize}} \rho_P(t) \tag{2}$$

The decay rate of a mass of radioisotope is traditionally expressed as an exponential decay function that factors in the initial mass M_0 and the half-life $t_{1/2}$ of the radioisotope:

$$m_{RI}(t) = M_0 e^{-0.693t/t_{1/2}} \tag{3}$$

Differentiating (3) gives the instantaneous rate of change of the decaying radioisotope

$$|\frac{d}{dt} m_{RI}(t)| = \frac{0.693}{t_{1/2}} M_0 e^{-0.693t/t_{1/2}} \tag{4}$$

which we can equate to (2) and solve for the time-dependent density and mass of the propellant in the ionization chamber

$$\frac{V_C M_{RI}}{I M_P \Delta t_{ionize}} \rho_P(t) = \frac{0.693}{t_{1/2}} M_0 e^{-0.693t/t_{1/2}} \tag{5}$$

$$\rho_P(t) = \frac{0.693 I M_P \Delta t_{ionize}}{V_C M_{RI} t_{1/2}} M_0 e^{-0.693t/t_{1/2}} \tag{6}$$

$$m_P(t) = \frac{0.693 I M_P \Delta t_{ionize}}{M_{RI} t_{1/2}} M_0 e^{-0.693t/t_{1/2}} \tag{7}$$

The result is a time-dependent ionized propellant mass profile that mimics the decay profile of the radioisotope used to ionize the propellant.

2.3 Thruster Dynamics

As shown above, the only quantities that can be controlled to affect thrust via the injected propellant mass are the initial radioisotope mass M_0 and the ionization time Δt_{ionize}. The other values are intrinsic properties of the radioisotopes and their interactions with the gaseous propellants. More propellant can be ionized with a larger M_0 (there are more α particles emitted and their production rate (the decay rate of the radioisotope is larger), and with a longer Δt_{ionize} will give more time for alphas to be emitted and ionizations to occur. Fig. 2 depicts, for both the first- and mean-ionized cases, the amount of propellant ionized over the course of a 20 year mission, with $M_0 = 1$ g and $\Delta t_{ionize} = 1$ s. M_0 and Δt_{ionize} merely scale the m_P curves, so the trends observed in Fig. 2 would be consistent with different values.

^{208}Po ionizes more propellant than the other radioisotopes in the first 9 years, but m_P decreases rapidly due to its high specific activity (A $= 21.8 \times 10^{12}$ Bcq/g) and short half-life ($t_{1/2} = 2.9$ years). ^{244}Cm is can produce a much more gradually-declining propellant mass flow over 20 years, at the cost of being significantly less powerful than ^{208}Po in the first 9 years, but more so later in the mission. The trends previously shown between propellants display in Fig. 2 as well, with cesium being slightly more effective than bismuth in the first-ionized case, but only half as effective as bismuth in the mean-ionized case.

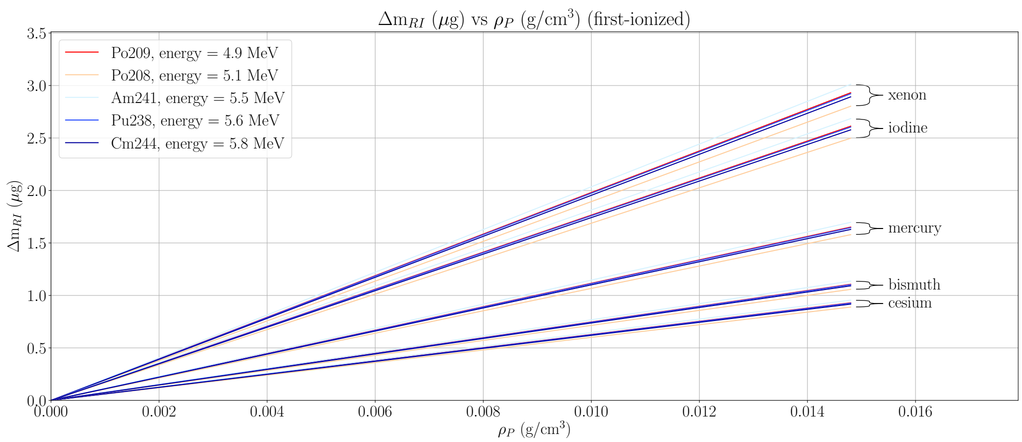
2.4 Radioisotope Geometry

Ideally, the radioisotope in the ionization chamber should be able to ionize gases in all parts of the chamber, meaning the emitted alphas do not stop before reaching the wall of the chamber. Fig. 3 in depicts the stopping ranges of α particles with decay energies in the different gaseous propellants at different densities. As expected, higher energy alphas penetrate further into the gas. These stopping ranges can inform the ideal radioisotope-chamber geometry. As demonstrated above, the density of the gas in the chamber decreases exponentially to ensure that (1) all of the propellant in the chamber is ionized and no excess propellant is injected, and (2) the full ionization time is constant. More work must be done to determine actual total number of ionizations that will occur at lower propellant densities in a fixed chamber volume and how the amount of gas injected into the chamber must change to accommodate the decrease in α particles. Currently, the maximum ionized m_P possible (constrained by the radioisotope's decay) are shown in Fig. 2, so the results of Fig. 3 will inform how the radioisotope should be shaped and placed into the chamber to maximize the propellant's exposure to the emitted α particles. Subsequent work may involve ionization simulations in a fixed volume and, later, empirical testing.

2.5 Figures

Figure 1: Assuming that a single α is produced per decay, then there is a linear relationship between the amount of RI mass used to fully ionize the gas in the ionization chamber and the density of the gas in the chamber (there is more propellant mass to be ionized). For all propellants, the resulting order of optimal RI is the same, being a combination of the ionization efficiencies of the RI and the ratio of the molar mass the radioisotope to the molar mass of the propellant. The optimal RI-propellant combination would require the smallest change in RI mass to ionize a full chamber of propellant at a certain density. So, ^{208}Po is the optimal radioisotope with any propellant. Cesium has a narrow lead on bismuth when only factoring in their first ionization energies, but bismuth displays an extensive lead when the first few ionization energies are averaged.

(a)



(b)

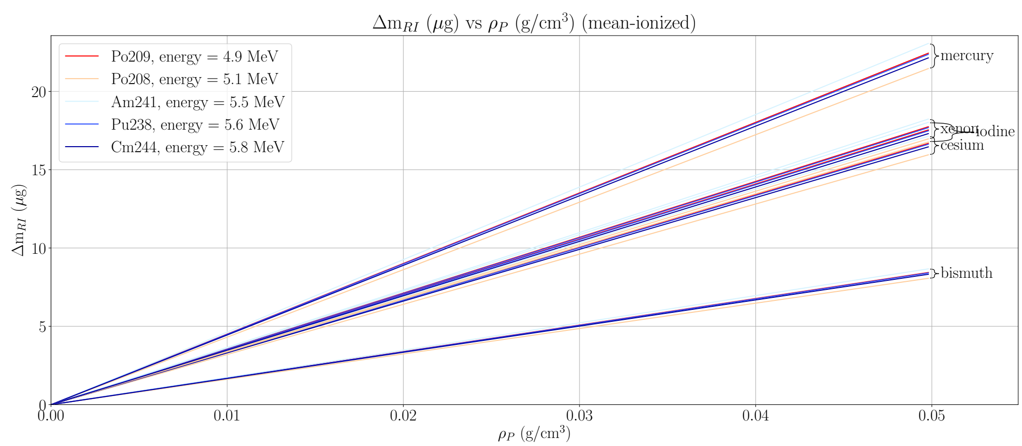
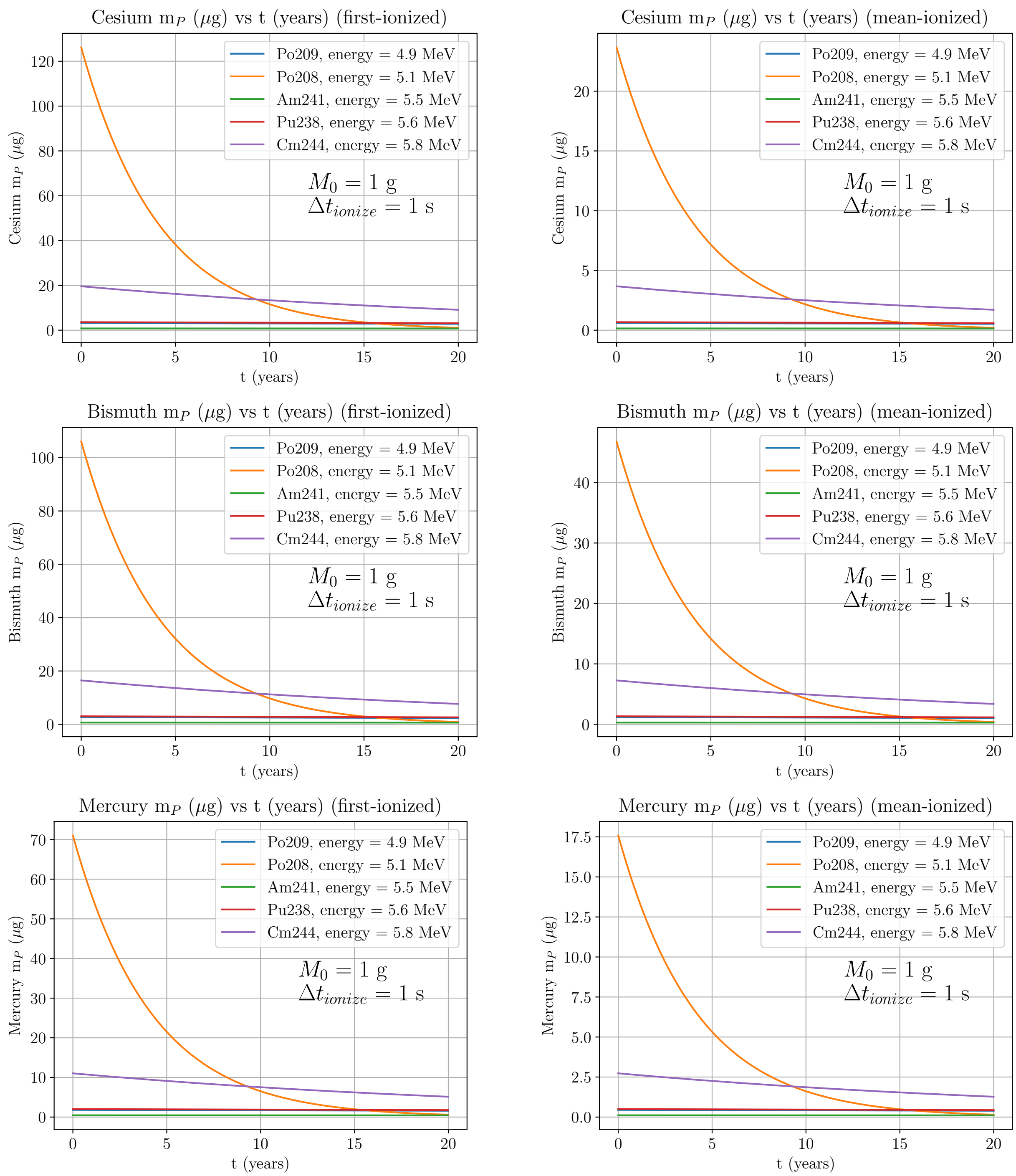


Figure 2: Below are plots of the maximum possible m_P (100 % ionized gas) as expressed by Eq. 7 with M_0 = 1 g and Δt_{ionize} = 1 s. ^{208}Po generates the most mass flow (and therefore thrust) in the first 9 years of constant propellant exposure and RI decay. ^{244}Cm, while ionizing significantly less propellant, is more consistent over the hypothetical 20 year mission lifetime. Additionally, the inter-propellant trends in Fig. 1 are evident here, where cesium is slightly more efficient than bismuth when considering only their first ionization energies, but bismuth becomes twice as efficient when factoring in their mean ionization energies



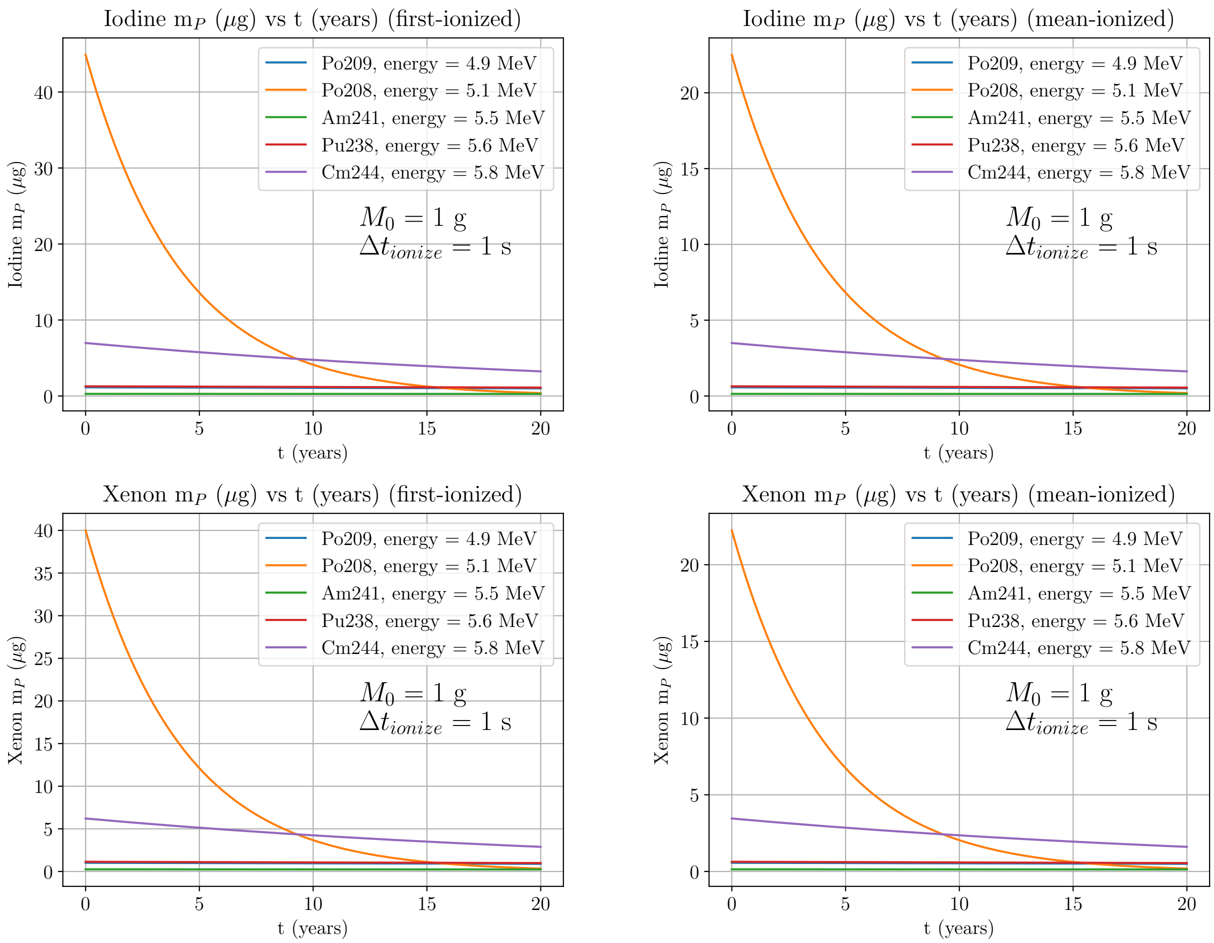
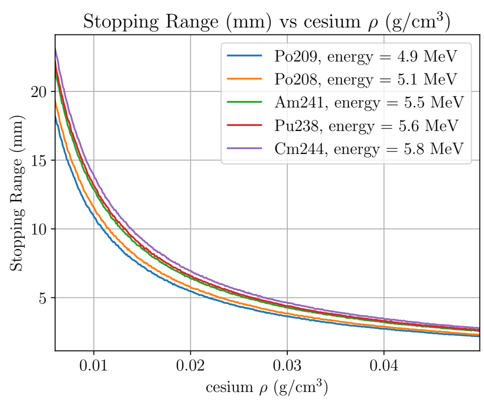
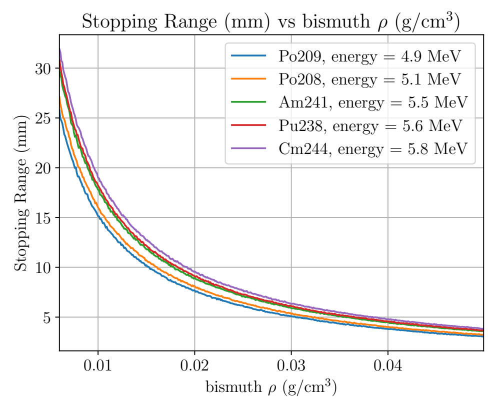

Iodine m$_P$ (μg) vs t (years) (first-ionized)
Iodine m$_P$ (μg) vs t (years) (mean-ionized)
Xenon m$_P$ (μg) vs t (years) (first-ionized)
Xenon m$_P$ (μg) vs t (years) (mean-ionized)
Iodine m$_P$ (μg)
Xenon m$_P$ (μg)
t (years)
Po209, energy = 4.9 MeV
Po208, energy = 5.1 MeV
Am241, energy = 5.5 MeV
Pu238, energy = 5.6 MeV
Cm244, energy = 5.8 MeV
$M_0 = 1$ g
$\Delta t_{ionize} = 1$ s

Figure 3: To inform the geometry of the isotope in the chamber, Geant4 was used to determine how far the emitted alphas would penetrate into the propellant gas. For example, assuming that the radioisotope is radiating alphas from center of a cylindrical volume, it would not make sense to increase the propellant density to such a point that the alphas cannot reach the extremities of the gaseous volume. By plotting the average stopping range of the alphas as a function of the propellant density, we can know the maximum propellant density that will still allow the full chamber of gas to be ionized. Note that alphas with higher energies penetrate further at given densities, as expected. As a sanity check on these results, the 2.4 mm stopping range of α in xenon at $\rho_P = 5 \times 10^{-2}$ was consistent with the empirical tests of the NEXT Collaboration's 2013 study (2)

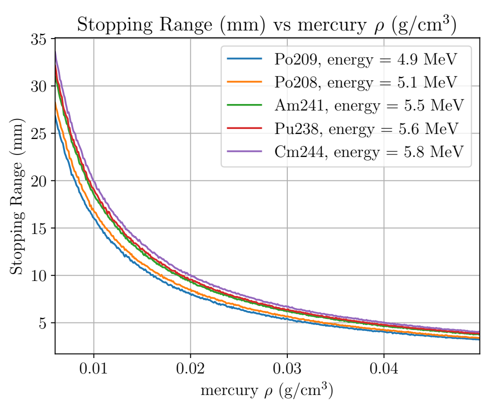
(a) Cesium



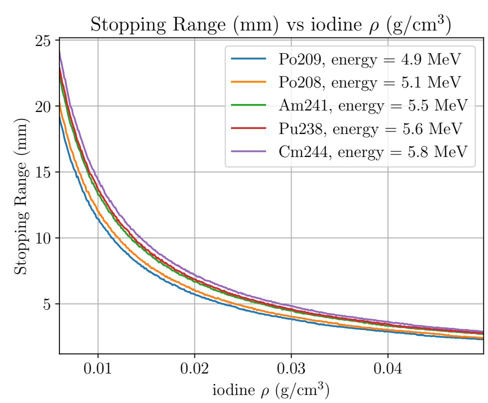
(b) Bismuth



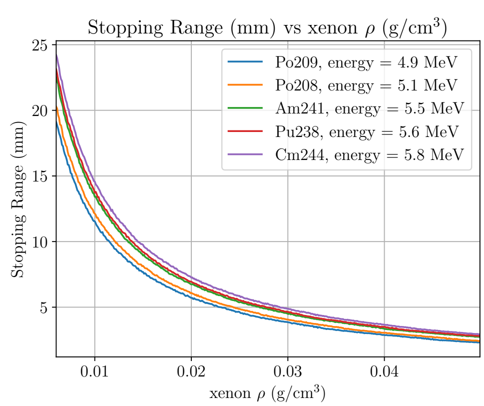
(c) Mercury



(d) Iodine



(e) Xenon



3 Conclusions

Bismuth appears to be an ideal propellant for SEPTIR, based on its high M_P and relatively low $\overline{E_{I_{ab}}}$ (and therefore high I, followed by cesium. ^{208}Po is an optimal radioisotope, ionizing significantly more propellant than the other radioisotopes within the first decade of a mission, but ionizes much less propellant afterward. ^{244}Cm is an alternative, ionizing less propellant overall but decreasing in efficiency much more gradually than ^{208}Po. A bismuth-^{208}Po may be an ideal radioisotope-propellant combination for a high-thrust alternative to cathode-based electric propulsion system for small spacecraft.

References

[1] Molar ionization energies of the elements. `https://en.wikipedia.org/wiki/Molar_ionization_energies_of_the_elements`.

[2] V Álvarez, F I G Borges, S Cárcel, S Cebrián, A Cervera, C A N Conde, T Dafni, J Díaz, M Egorov, R Esteve, and et al. Ionization and scintillation response of high-pressure xenon gas to alpha particles. *Journal of Instrumentation*, 8(05):P05025–P05025, May 2013.