Homework 5: Probability and Bayesian Network

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Throughout, all numbers are rounded to two digits.

1 Probability

You are watching your favorite contestant answer trivia questions on a TV game show. The questions come from three categories: Science, History, and Entertainment. The show is not perfectly balanced, so 35% of the questions are Science, 33% are History, and 32% are Entertainment. Over the course of this year, your contestant has answered 74 out of 93 Science questions correctly, 31 out of 88 History questions correctly, and 55 out of 84 Entertainment questions correctly. Answer the following questions using the statistics above.

A. Derive the conditional probabilities that the contestant answers the next question correctly, given that it is from a specific category. Do this for each of the three categories.

Q represent the question it can be T or F. S, H, E each represents Science, History, and Entertainment respectively. The categories are represented by C and it can be S, H, or E.

$$
\begin{aligned}
P(Q|S) &= \frac{74}{93} = 0.80 \\
P(Q|H) &= \frac{31}{88} = 0.35 \\
P(Q|E) &= \frac{55}{84} = 0.65
\end{aligned} \tag{1}
$$

B. Of course you as an observer have no idea what category the next question will be. What is the probability that the contestant will answer the next question right?

$$
\begin{aligned}
P(Q) = \sum_{C\in\{S,H,E\}} P(Q|C)P(C) &= P(Q|S)P(S) + P(Q|H)P(H) + P(Q|E)P(E) \\
&= 0.80 \times 0.35 + 0.35 \times 0.33 + 0.65 \times 0.32 = 0.60
\end{aligned} \tag{2}
$$

C. You get up to get more chips and dip and missed the last question. Your brainy friend who knows nothing about entertainment (and would thus have a 0% chance of getting an entertainment question right) knew the answer (another friend of yours, who is a reliable source, confirms this). What is the probability that the contestant also answered the question correctly?

According to the description, we know it was not an entertainment question. So it can be Science or History question.

D. You get up again to get your friends more chips and dip (you're such a good friend) and miss the next question too. From the shouts of dismay you hear from the other room, you figure out that the contestant answered incorrectly. What is the probability that the question was a history question?

$$P(H|\neg Q) = \frac{P(\neg Q|H)P(H)}{P(\neg Q|H)P(H) + P(\neg Q|S)P(S) + P(\neg Q|E)P(E)} \tag{3}$$

where,

$$\begin{aligned} P(\neg Q|S) &= 1 - 0.80 = 0.20 \\ P(\neg Q|H) &= 1 - 0.35 = 0.65 \\ P(\neg Q|E) &= 1 - 0.65 = 0.35 \end{aligned} \tag{4}$$

we get,

$$P(H|\neg Q) = \frac{0.65 \times 0.33}{0.65 \times 0.33 + 0.2 \times 0.35 + 0.35 \times 0.32} = 0.54 \tag{5}$$

E. You are deciding whether or not you should bet one of your friends \$100 that the contestant will answer the last question of the show correctly. That is, if the contestant answers correctly, your friend will pay you \$100; otherwise, you will have to pay your friend \$100. You know a show employee who can tell you for sure what the category of the last question will be. Now you are considering whether you should call her up and offer her a bribe to tell you. How much, if anything, should you be willing to pay for this information? (Assume your interest is in making money.)

In case of no action,

$$\langle U_1 \rangle = 100\$P(Q) - 100\$P(\neg Q) = 100\$ \times 0.6 - 100\$ \times 0.4 = 20\$ \tag{6}$$

In case of bribing the employee for amount of $X\$$ there is chance of 35% of the question is Science, 33% is History, and 32% is Entertainment. I would play if the question was Science or Entertainment but not if the question was from History. So the expected utility is

$$\langle U \rangle = P(S)(100\$P(Q|S) - 100\$P(\neg Q|S) + P(E)(100\$P(Q|E) - 100\$P(\neg Q|E) - X\$ \tag{7}$$

The above equation reads,

$$\langle U_2 \rangle = 0.35(100\$ \times 0.8 - 100\$ \times 0.2) + 0.32(100\$ \times 0.65 - 100\$ \times 0.35) - X\$ = 30.6\$ - X\$ \quad (8)$$

If the bribing fee is less than 10.6$ then it worth bribe the employee.

2 Building a Bayes Net

Jordy drives to work every weekday (Monday-Friday). You will build a Bayes net to describe Jordy's music choices in the car (whether, on any given day, it is pop music or not). 10% of the time, Jordy gets a coffee before work. Coffee makes Jordy feel slightly more hipster and thus with some probability more adventurous: in particular, Jordy feels adventurous 70% of the time after having coffee, in comparison with only 40% of the time after not having coffee.

Three (random) days a week, Jordy's roommate carpools to work with Jordy. Whenever Jordy's roommate rides in, she says she wants to listen to classical music 90% of the time. However, she likes the Friday morning program (where the station switches over to jazz) a lot less, so on a Friday she would only have a 30% chance of saying she wants to listen to that station.

If Jordy has a friend demanding to listen to another station and is feeling adventurous, Jordy will always be a good friend (and also explore new music) and listen to whatever that friend wants to listen to. If Jordy is not feeling adventurous but a friend demands to listen to another station, however, half the time Jordy will invoke driver's right to choose the music and listen to pop music anyway. And if no one else is in the car or expresses a preference, Jordy will always listen to pop music when not feeling adventurous. If Jordy happens to be feeling adventurous and no one else expresses a musical preference, Jordy just hits a random radio preset button and listens to whatever music that station is playing. There are five preset buttons, two of which are set to the two pop music stations in town.

Represent this information as a Bayesian network. Use node titles that represent the action (e.g. GetsCoffee for Jordy getting coffee) and include all CPTs.

Here are priors: P(Friday = T) = 1/5=20%, P(Carpool = T) = 3/5=60%, and P(GetCoffee = T) = 10%. Here are the conditional probability tables.

Table 1: P(Adventurous | GetCoffee)

GetCoffee	P(Adventurous | GetCoffee)
T	70%
F	40%

Table 2: P(DemandClassic | Carpool , Friday)

Carpool	Friday	P(DemandClassic | Carpool , Friday)
T	T	30%
T	F	90%
F	T	0%
F	F	0%

Table 3: P(Pop | Adventurous, DemandClassic)

DemandClassic	Adventurous	P(Pop | Adventurous, DemandClassic)
T	T	0%
T	F	50%
F	T	40%
F	F	100%

and the Bayesian Network looks like:

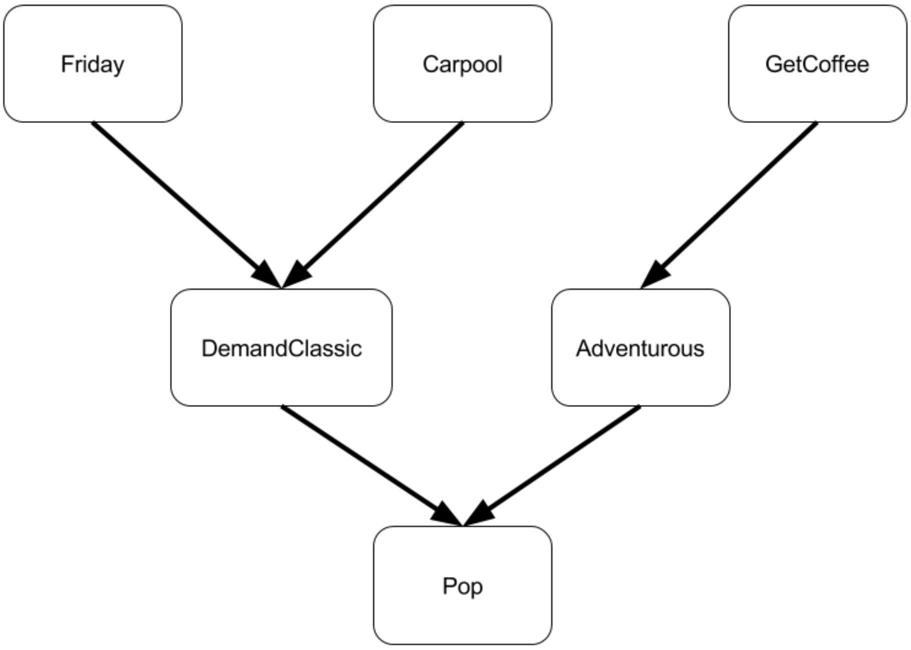


Figure 1: Bayesian Network

3 Using Bayesian Networks

Consider the following Bayesian network about the weather in a certain small town in Michigan. Answer the following questions by hand, and show all your work:

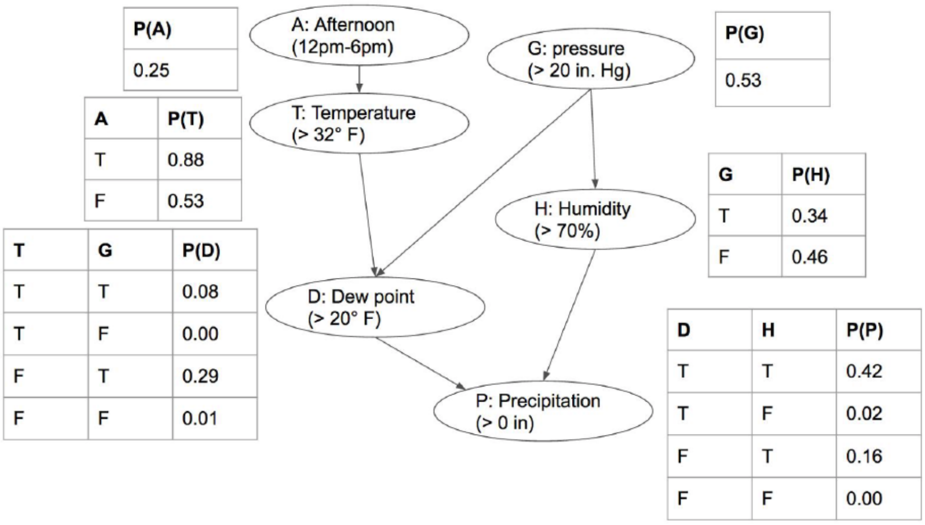


Figure 2: Bayesian Network

A. Only knowing that it is currently 3pm, what is the probability that it's raining – that is, that there are more than 0 inches of precipitation?

We want to calculate P(P=True|A=True) or simply P(P|A).

We may write it as following,

$$\sum_{d,h,g,t} P(P,d,h,g,t|A) = \sum_{d,h,g,t} P(P|h,d)\ P(h|g)\ P(d|t,g)\ P(t|A)\ P(g) =$$

We first marginalize over t to get the following table,

Table 4: P(D|A,G)

A	G	P(D|A,G)
T	T	0.1052
T	F	0.0012
F	T	0.1787
F	F	0.0047

it leads to the following equation

$$\sum_{d,h,g} P(P,d,h,g,t|A) = \sum_{d,h,g} P(P|h,d)\ P(h|g)\ P(d|A,g)\ P(g) = P(P|H,D)\ P(H|G)\ P(D|A,G)\ P(G) + P(P|H,D)\ P(H|\neg G)\ P(D|A,\neg G)\ P(\neg G) + P(P|H,\neg D)\ P(H|G)\ P(\neg D|A,G)\ P(G) + P(P|\neg H,D)\ P(\neg H|G)\ P(D|A,G)\ P(G) + P(P|H,\neg D)\ P(H|\neg G)\ P(\neg D|A,\neg G)\ P(\neg G) + P(P|\neg H,D)\ P(\neg H|\neg G)\ P(D|A,\neg G)\ P(\neg G) + P(P|\neg H,\neg D)\ P(\neg H|G)\ P(\neg D|A,G)\ P(G) + P(P|\neg H,\neg D)\ P(\neg H|\neg G)\ P(\neg D|A,\neg G)\ P(\neg G) = 0.42 * 0.34 * 0.1052 * 0.53 + 0.42 * 0.46 * 0.0012 * 0.47 + 0.16 * 0.34 * 0.8948 * 0.53 + 0.02 * 0.66 * 0.1052 * 0.53 + 0.16 * 0.46 * 0.9988 * 0.47 + 0.02 * 0.54 * 0.0012 * 0.47 + 0 * 0.66 * 0.8948 * 0.53 + 0 * 0.54 * 0.9988 * 0.47 = 0.0692$$

B. Suppose you are only interested in using the time of day and the atmospheric pressure to predict precipitation. Use variable elimination and node clustering to reduce the Bayes net to one with just the A, G, and P nodes.

We first marginalize over T to get the following table,

Table 5: P(D|A,G)

A	G	P(D|A,G)
T	T	0.1052
T	F	0.0012
F	T	0.1787
F	F	0.0047

Now we cluster D-H together to derive P(D-H|A,G),

Table 6: P(D-H|A,G)

A	G	D-H=TT	D-H=TF	D-H=FT	D-H=FF
T	T	0.0358	0.0694	0.3042	0.5907
T	F	0.0005	0.0006	0.4595	0.5394
F	T	0.0608	0.1179	0.2792	0.5421
F	F	0.0022	0.0025	0.4578	0.5375

Now we may marginalize over node P(D-H|A,G) to get

Table 7: P(P|A,G)

A	G	P(P|A,G)
T	T	0.0651
T	F	0.0737
F	T	0.0726
F	F	0.0742

C. Using the result from part b, if we know that it is currently 3pm and that it is snowing (ie. there is precipitation), what is the probability that the pressure is >20 inHg?

$$\mathrm{P(G|A,P)} = \frac{\mathrm{P(P|A,G)P(G)}}{\mathrm{P(P|A,G)P(G)+P(P|A,\neg G)P(\neg G)}} = \frac{0.0651\times 0.53}{0.0651\times 0.53+0.0737\times 0.47} = 0.499$$

4 JavaBayes

You should use the JavaBayes software package for this task. It can be found at http://www.cs.cmu.edu/ jav abayes/Home/ and can be run as an applet from the link http://www.cs.cmu.edu/ javabayes/Home/applet.ht (no software to install, but note that modern security protocols in your browser may not allow the Java applet to run). Documentation can be found on the first webpage. There are a few things to keep in mind:

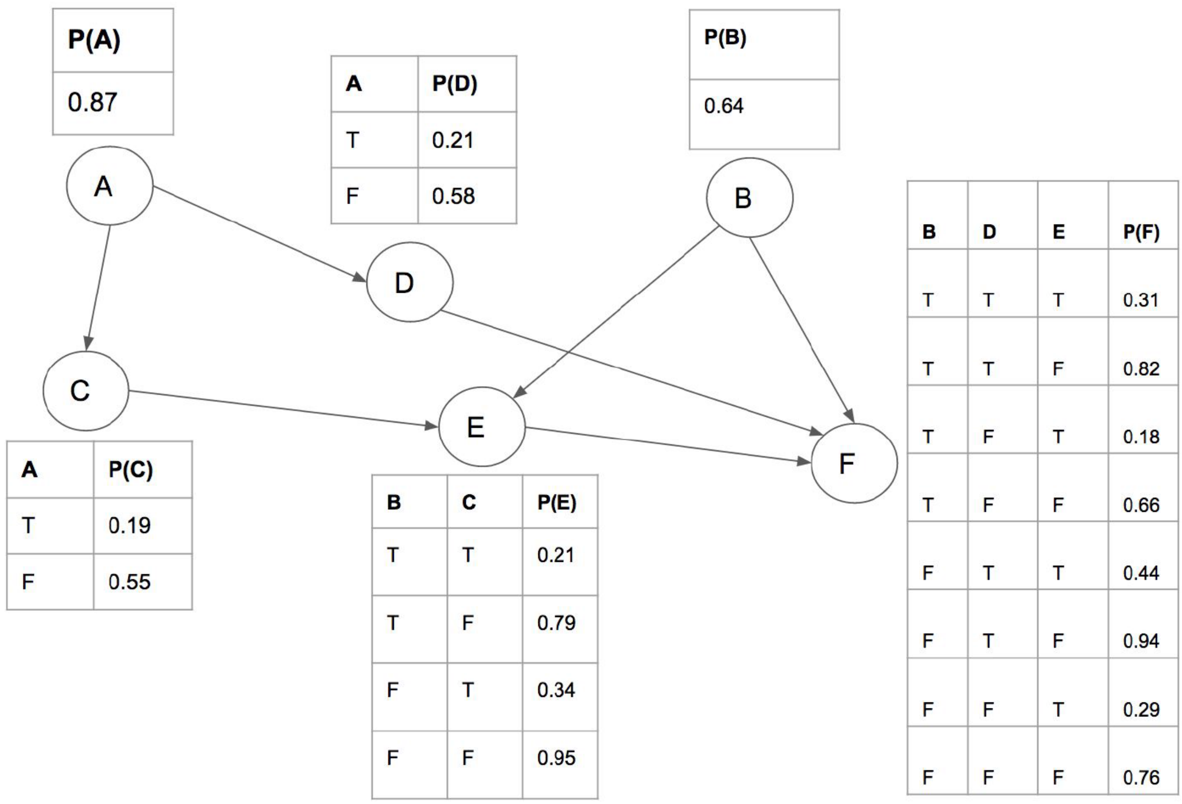
There are a few things to keep in mind:

– There will be two applet windows; make sure you can see both of them because the text window gives useful feedback.

– To specify CPTs, click on the "Edit Function" button, and make sure that your specified values sum to 1 where appropriate. Also make sure that you use the pull-down lists to specify all values of parent nodes.

– When specifying evidence (click observe), the nodes will change color when observed.

Encode the following network in JavaBayes, and use it to answer the following questions. For each question, include in your submission the output generated by JavaBayes for the query/queries relevant to that question.



P(A)
0.87

A	P(D)
T	0.21
F	0.58

P(B)
0.64

A	P(C)
T	0.19
F	0.55

B	C	P(E)
T	T	0.21
T	F	0.79
F	T	0.34
F	F	0.95

B	D	E	P(F)
T	T	T	0.31
T	T	F	0.82
T	F	T	0.18
T	F	F	0.66
F	T	T	0.44
F	T	F	0.94
F	F	T	0.29
F	F	F	0.76

Figure 3: JavaBayes Network

A. What is the unconditional probability that F = true?

0.397

B. What is the probability that D = true and E = true, given that A = false? (Note that this cannot be computed directly by JavaBayes; you will have to put the query into an appropriate form, performing multiple queries if necessary, and combining them algebraically.)

$P(E,D|\neg A) = P(E|D,\neg A)\ P(D|\neg A) = 0.523 \times 0.580 = 0.303$

C. The properties of conditional independence can be demonstrated using probability calculations (using the equation on page 498 of R&N).

1. **Are D and E conditionally independent given C?**

   Yes they are independent. If we fix C then the probability of E is independent of other variables.

2. **Demonstrate your claim by selecting a set of truth assignments for the random variables D, E, A, and C and showing whether or not the equation holds.**

   $P(D,E|A,C) = P(D|E,A,C) \times P(E|A,C) = 0.21 \times 0.2568 = 0.053928$

   $P(D|A,C) \times P(E|A,C) = 0.21 \times 0.2568 = 0.053928$

   which is consistent with our expectation.

3. **Does the equality/inequality for this single set of truth assignments prove your claim? Why or why not?**

   NO it does not. Because there are other potential configuration which we need to show the equation holds. Here we just showed for only one configuration.

5 Markov Decision Processes

Consider a Markov Decision Process (MDP) that describes a (much simplified) environment for an autonomous Mars rover designed to investigate Martian terrain. Because the landing spot for the rover cannot be precisely determined ahead of time, it is necessary to know the best course of action for the rover from any position within the grid shown below. That is, we would like to find the optimal policy π^* for the environment. Satellite imaging has already determined the type of terrain present at each location (F - Flat, R - Rocky, C - Crater) as shown in the grid below.

The rover move in any direction (W - West, E - East, S - South, N - North) as long as the move would not take it outside the grid. The rover may also choose to remain (R) in its current location. The results of an action depend on the terrain at the roverâĂŹs current location:

F - Flat Moves from flat locations transition to the adjacent location in the chosen direction.

R - Rocky Moves from a rocky location are dangerous and the rover may become damaged. They work correctly with probability Pr, but otherwise transition to BROKEN.

C - Crater Only the Remain action is allowed - the rover is not able to climb out of a crater.

BROKEN No actions allowed (terminal state).

Note that the success of a move action depends only on the location from which the agent is moving and not on the location into which it is moving. There are 17 possible states the rover can be in: each of the 16 locations and BROKEN. The only terminal state is BROKEN. Because the ultimate goal is being in craters in order to investigate them, states of type Crater have a large positive reward of +5. As rocks are also somewhat interesting, each Rock state has a small positive reward of +1. The BROKEN state and Flat states have no reward. To review:

Available actions: W, E, S, N, R

Rewards:

Table 8: P(P|A,G)

Type of State	F-Flat	R-Rocky	C-Crater	BROKEN
Reward	0	1	5	0

A. Using a programming language of your choice, implement value iteration (R&N Figure 17.4) for this problem. You should design your implementation such that changing the rewards for each type of state and changing the discount factor are easy. Your implementation should update the values for Ui(s) in a batch fashion (i.e. don't use values just calculated previously in the same iteration). You should initialize the expected utilities to 0 and assume $\gamma = 0.95$ and $P_r = 0.20$. Please turn in your code and a readme.txt **file explaining how to run your code. If you were only able to implement part of the algorithm, please specify in your readme exactly what parts you were able to implement.**

The README.md file is provides. Here is a brief explanation as well.

The user need to run the code

Python main.py HW5

and the parameters of the model are specified in a JSON file `./inputs/hw5_parameters.json` and the grid is defined in a TXT file `./inputs/grid.txt`. By changing the parameters and running the code on can easily reproduce the tables here.

B. Run value iteration until convergence, with an epsilon value of 0.0001. What are the expected utilities for each location and the resulting policy? Write your answer in the format shown below, where for each location (x,y), you report the expected utility for the location (shown in the grid as U(x,y)) along with the action determined for that cell by the policy (shown in the grid as [action]).

Table 9.

Table 9: Expected utilities and the resulting policy

90.25 [S]	20.00 [S]	90.25 [S]	18.15 [W]
95.00 [E]	100.00 [R]	95.00 [W]	90.25 [W]
19.05 [N]	95.00 [N]	19.05 [N]	20.00 [S]
18.15 [E]	90.25 [N]	95.00 [E]	100.00 [R]

C. Our rover is pretty fragile, given the 80% chance of breaking in rocky locations. Let's say we replace our rover with a more robust rover with only a 1% chance of breaking in a rocky location. Run your algorithm again with $P_r = 0.99$ and $\gamma = 0.95$. Describe in a few sentences how the rover's behavior changes compared with part b) as a result of this change. (You don't have to show the expected utilities and the complete policy for this part).

Table 10.

Table 10: Expected utilities and the resulting policy

90.30 [E]	95.05 [S]	90.30 [W]	85.92 [W]
95.00 [E]	100.00 [R]	95.00 [W]	90.30 [S]
90.35 [N]	95.00 [N]	90.39 [E]	95.05 [S]
85.97 [N]	90.25 [N]	95.00 [E]	100.00 [R]

Because the likelihood of breaking down goes down it worth exploring the Rocky side of the planet. So the Rover on its way to the Crater passes through the Rocky locations. The utility of Crater does not change because the Rover ends up there for ever and does not broke down, though the utility of the Rocky locations changes.

D. Now run your code again with $P_r = 0.99$ and $\gamma = 0.1$. What change in rover's behavior do you notice compared to the previous parts as a result of this change? (Again, you don't have to show the expected utilities and the complete policy for this part).

Table 11.

Table 11: Expected utilities and the resulting policy

0.15 [E]	1.55 [S]	0.15 [W]	1.11 [R]
0.56 [E]	5.56 [R]	0.56 [W]	0.15 [S]
1.11 [R]	0.56 [N]	1.15 [E]	1.55 [S]
1.11 [R]	0.11 [W]	0.56 [E]	5.56 [R]

Now because of the γ the Rover forget very easily and breaks down rarely enough to worth staying in a Rocky location, unless the Rocky location be next to a Crater. If the Rocky spot is not next to a Crater, the Rover which goes into a Rocky spot stays there until it breaks down. Though the utility of Crater is much larger than a Rocky spot as expected (by around order of 5 it is larger).