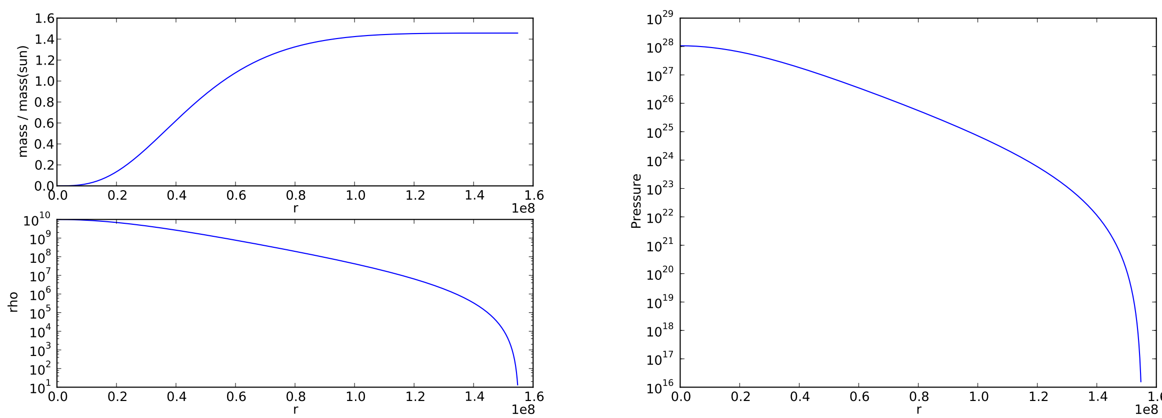
Ay190: Computational Astrophysics (Winter Term 2012)
HomeWork - 4 (ODE and Root finding)

Jan 22, 2012

1 Exercise 1. ODE Integration: Simplified Stellar Structure

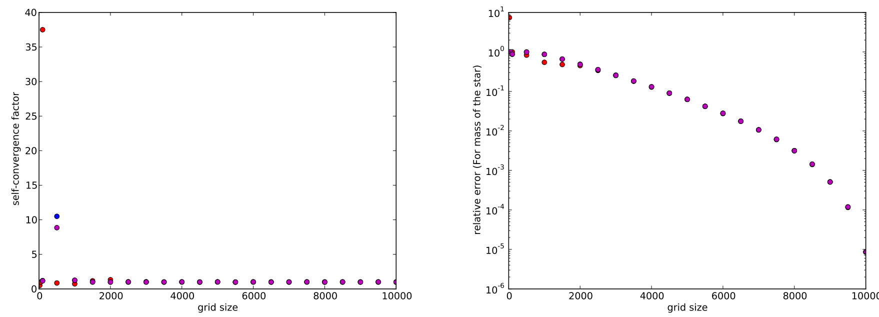


((a)) Mass Density

((b)) Pressure

Figure 1: Plot of the mass, density, and pressure of the star with changing radius.

Figure 1(a) and 1(b) showes the plot of the mass of the star, its density, and its pressure with changing radius. Also the after the convergence it can be shown that mass of the star would be equal to $1.4574 \times M_{\odot}$and its radius would be equal to $154850km$. Figure 2(a) and 2(b) shows that our solution is converging. Figure 2(a) is the Self Convergence factor which is changing with grid size and Figure 2(b) is the Relative error of the mass of star which is changing with grid size. It is obvious that for all three methods the numerical solution converge to real answer.



((a)) Self Convergence factor

((b)) Relative error

Figure 2: Self Convergence factor with changing grid size and Relative error of the mass of star with changing grid size.

2 Exercise 2. Root Finding: Eccentricity Anomality

Part a :
In this problem I used Newtons Method for finding the root of equation $f(E) = E - \omega t - e \sin E$. So we need to find its derivitive respect to E. We find its derivitive analyticaly which is $F'(E) = 1 - e \cos E$. And for the initial trial I used $E = 0$ and with just 4 trial the answer converge. Then for $t = 91$ days, $t = 182$ days,and $t = 273$ days E would be equal to 1.58, 3.13, and 4.67 respectively.

Part b :

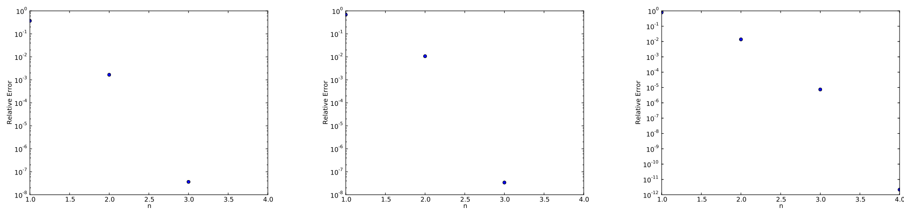
I used the same method for the second part of the problem. But if we want to use $E = 0$ as our initial rtial then the method would not converge to the root of the function. But if we change the initial trial with small number of trial we are able to find the answer, just with 7 tial. And for $t = 91$ days, $t = 182$ days,and $t = 273$ days E would be equal to 2.30, 3.13, and 3.967 respectively.

The density of electrons in this environment by useing Gauss-Laguerre Quadrature and Trapezoidal Rule method for finding the numerical integral is same and it is equal to : 1.90216×10^{41}. We used 50 pionts for finding the answer of integral.

Figure 3 and 4 shows the the relative error with changing the number of trial for part a and part b of the problem.

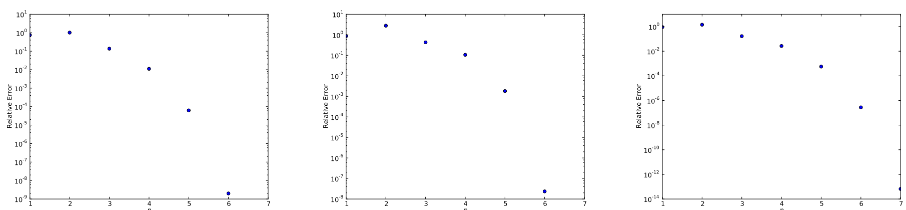
$$n_{e^{\pm}} = \frac{8\pi}{(2\pi\hbar c)^3} \int_0^\infty \frac{E^2 dE}{e^{\beta E} + 1} = \frac{8\pi}{(2\pi\hbar c)^3} \sum \int_{E_i}^{E_i + \Delta E} \frac{E^2 dE}{e^{\beta E} + 1} \tag{1}$$

For each δE range we calculate the integral by Gaussian-Legendre Quadrature method. We used 10 points for finding the integral by Gaussian-Legendre Quadrature method. And we choose



((a)) 91 days ((b)) 182 days ((c)) 273 days

Figure 3: Plot of Relative error of E with changing the number of tiral for $e = 0.0167$.



((a)) 91 days ((b)) 182 days ((c)) 273 days

Figure 4: Plot of Relative error of E with changing the number of tiral for $e = 0.99999$.

$\Delta E = 3MeV$ and $E_{max} = 450MeV$ because for E more than $450MeV$ the integral is negligible. And the final answer is the same as the answer of equation **??**.

Figure **??** shows the spectral distribution of density of electrons in this environment. It is obvoius that after $450Mev$ it is negiligble and we can ignore that for calculating the integral.

3 Exercise 3.