Ay190: Computational Astrophysics (Winter Term 2012)
HomeWork - 6

Jan 26, 2012

1 Exercise 1. Solving Large Linear Systems of Equation

Build-in : 10 S
Gaussian elimination : 182 S
Using NumPy : 7.52 S

2 Exercise 2. Root Finding: Eccentricity Anomality

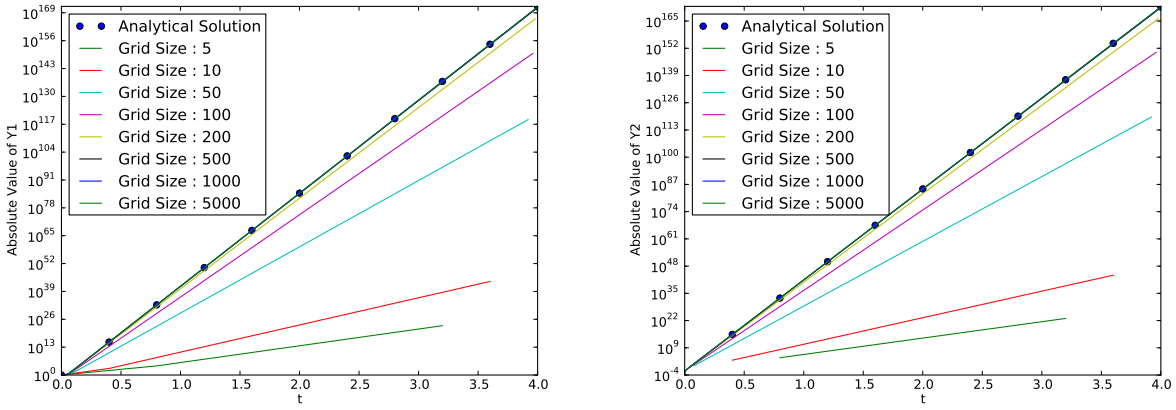
Part a :
The analytical answer for this problem is:

$$Y_1 = \frac{e^{100t}}{100} + \frac{99}{100} \tag{1}$$

and

$$Y_2 = -\frac{e^{100t}}{100} + \frac{1}{100} \tag{2}$$

Part b :



((a)) Y1 in different grid size

((b)) Y2 in different grid size

Figure 1: Plot of Y1 and Y2 in different grid size .

Figure 1(a) and 1(b) shows absolute value of Y_1 and Y_2 for some different grid sizes and analytical solution. Figure 2 showes how the relative error changing the grid size. All of this figures are the solution of out differential equation with RK4 method.

Part c :

In this part I have used backward Euler method for finding the numerical answer of our problem. the final equations for this part are:

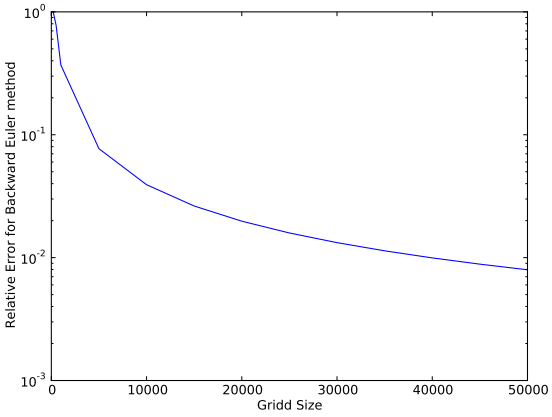


Figure 2: Plot of Relative error with increasing the grid size.

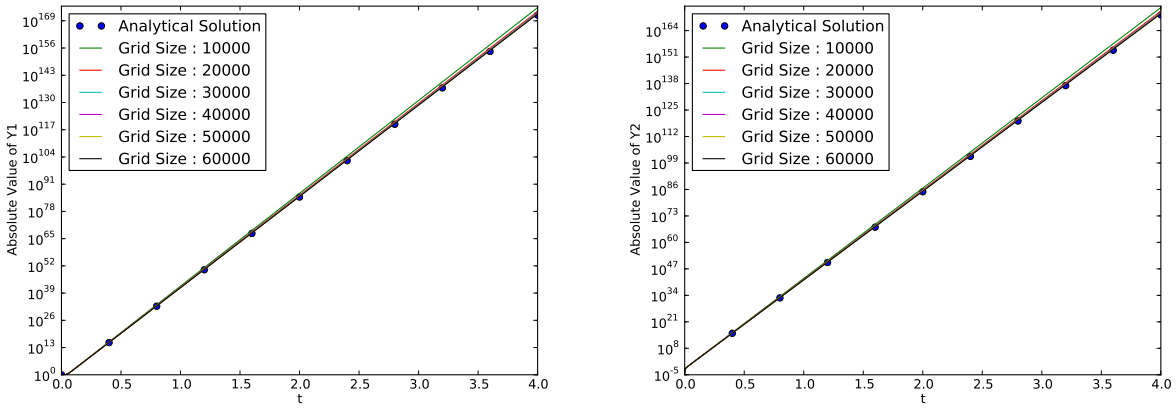
$$Y_1^{i+1} = -\frac{(1-99dt)Y_1^i - 99dt \times Y_2^i}{1-100dt} \tag{3}$$

and

$$Y_2^{i+1} = -\frac{-dt \times Y_1^i - (1-dt)Y_2^i}{1-100dt} \tag{4}$$

Figure 3(a) and 3(b) shows absolute value of Y_1 and Y_2 for some different grid sizes and analytical solution. Figure 2 showes how the relative error changing the grid size. All of this figures are the solution of out differential equation with back Euler method. Note that for grid size less that 400 our answer would be unstable so if we want to use this method we neet to use grid sizes more than 400.

Conclusion:
In conclusion based on the relative errors the KR4 method works better for this problem.



((a)) Y1 in different grid size

((b)) Y2 in different grid size

Figure 3: Plot of Y1 and Y2 in different grid size .

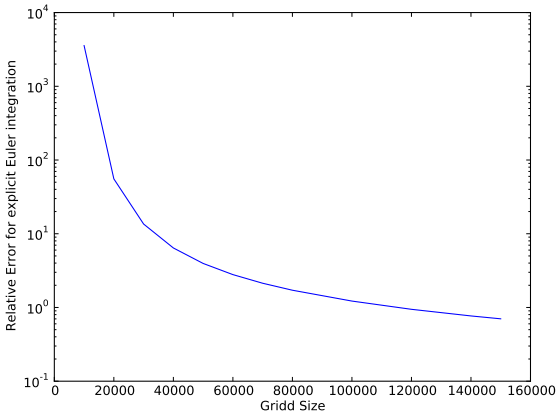


Figure 4: Plot of Relative error with increasing the grid size.