Applied Cryptography
Week 5: Hash Functions and Keyed Hashing

Bernardo Portela

M:ERSI, M:SI, M:CC - 25

What is a Hash Function?

Hash functions are everywhere

- Key derivation
- Digest for authentication
- Randomness extraction
- Password protection
- Proofs of work

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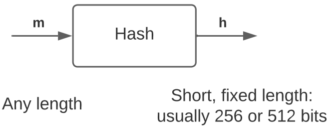
Not only in crypto:

- Indexing in version management
- Deduplication in cloud storage systems
- File integrity in intrusion detection



Describing Hash Functions

The hash output is short, aka hash, fingerprint or digest

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Use hash as an identifier

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Signing $H(m)$ is **as secure** as signing m

Hash functions need to be deterministic and public

- Everyone should be able to recompute hash/identifier
- … So what do we mean by security here?

Secure Cryptographic Hash Functions

Efficient algorithms with nice properties

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- Hard to find pre-images
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Hash functions are validated heuristically

- Similar to process for AES
- International competition for select designs
- Competitors are scrutinized wrt security and performance
- Several rounds, so more eyes on small number of proposals
- Most recent one: SHA-3

#1: Pre-image resistance

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Pre-image experiment

- Let $\mathcal{S}$ be the set of pre-images (domain)
- Let $\mathcal{R}$ be the set of images (range)
- Attacker is given a value $y \in \mathcal{R}$
- Attacker guesses $x \in \mathsf{S}$ and wins if $h(x) = y$

#2: Collision Resistance (CR)

- By definition, collisions **must exist**.
  - Recall that $|\mathcal{S}| >> |\mathcal{R}|$
- This can be argued from the *pidgeonhole principle*
  - If you have m holes and n pidgeons to put in these holes, if $n > m$, at least one hole will have more than one pidgeon!
- But can we find m_0 and m_1 s.t. $h(m_0) = h(m_1)$?

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Q1: What could that be?

- Lets think of the probability of collision
- Outputs are random, so $1/2^n$ where n is the output length
- Collision will be found if we check roughly 2^n pairs

Q2: Is CR harder or easier then pre-image resistance?

Breaking Hash Functions

Attack that finds a pre-image

- Search through all possible pre-images (brute-force)
- Consider a perfect hash function with output of n bits
- Cost: potentially more than 2^n operations!
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- Keep trying different values until you guess correctly

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But what if we only want to find a collision?

Finding Collisions

Collisions can be found with work $\sqrt{2^n}$, much better than 2^n!

Methodology

- Compute values like the brute-force attack
- Store them in a data structure *indexed by image value*
- Each new image value is searched in data structure
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- After n values, we checked $n * (n-1)/2$ pairs **Q: why?**
- Checking 2^n pairs takes roughly $\sqrt{2^n}$ values
- Overall complexity is that of finding the pre-image of a hash with $n/2$ bits of output (only half of the range)

The birthday paradox (not very paradoxical, just counterintuitive)

Implication of Birthday Attacks

For CR, hash outputs must be 2x security parameter

- 128-bit security $\rightarrow$ 256-bit hashes
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We can use security-parameter-sized hash outputs when:

- Security against arbitrary collisions is not required
- E.g. we might only need pre-image resistance
- Deriving a key from a secret input

Key Takeaways

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- For output of 2^n, collision can be found in $\approx 2^{\sqrt{n}}$
- So for 2^{128} resistance, output must be at least 2^{256}
- Birthday attack

Building Hash Functions

Two main approaches that use iterative processes

- **Merkle-Damgård construction:** Used for MD4, MD5, SHA-1, SHA-256, SHA-512. Relies on a $m + n$-to-n bits compression function to construct a hash function of output length n for arbitrary input lengths

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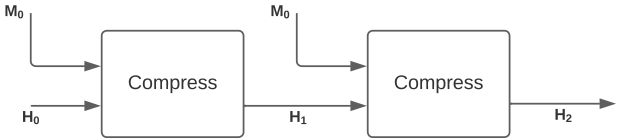
- **Sponge construction:** Used for SHA-3, uses a l-bit permutation to construct a hash function for arbitrary input and output lengths

Merkle-Damgård Construction

All prominent hash functions from 80s-2000s.

- H_0 is the initial value: constant and **public**
- M is broken into blocks of size m, $M_1, M_2, \ldots$



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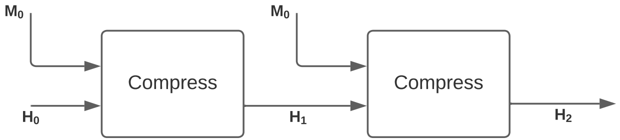

- SHA-256: block size 512, output size 256 bits
- SHA-512: block size 1024, output size 512 bits
- What if messages are not of the same size as the block?

Merkle-Damgård Construction – Padding

Padding is always added to the message

- Append the message with a 1 bit
- Fill with zeros up to 64/128 bits away from the block end
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Q: Can't we just pad by adding 0s?

Merkle-Damgård Construction – Security

Useful result

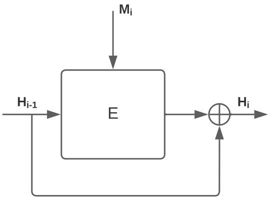
- Compression result is CR (for small inputs)
- Then the whole construction is CR (for arbitrary inputs)

To break the hash function you must break the compression function

So, does having a $2n$-to-n CR compression function solve all our problems?

Compression Functions: Davis-Meyer

All popular MD constructions use the Davis-Meyer construction:



Block ciphers used as compression functions!

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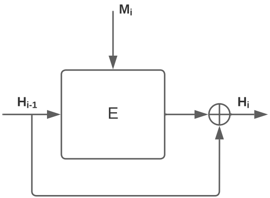

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$$H_i = E(M_i, H_{i-1}) \oplus H_{i-1}$$
$$H_i = E(M_i, D(M_i, 0)) \oplus D(M_i, 0)$$
$$H_i = H_{i-1}$$

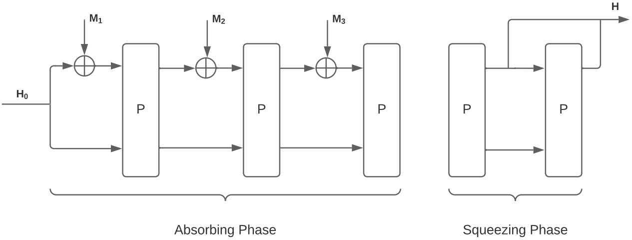
Sponge Construction

A more recent alternative to the MD is the sponge construction

It relies on a fixed (non-keyed) permutation

Very Versatile

- Varying input/output lengths
- PRGs and stream ciphers
- PRFs and keyed hashes



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Squeeze

- Dual process iteratively constructs output
- Output constructed block by block
- Permutation computed over the entire state
- Block-sized part of the state is accumulated in the output

MD5

- Broken! 128-bit output
- Most popular hash function until broken in 2005
- These days, it takes seconds to find collisions
- The SHA function family (next) uses a similar design

Secure Hash Function (SHA)

Standardized by NIST in the US. International *de facto* standard

SHA-0 published in 93', replaced with SHA-1 in 95'

- Both with 160-bit outputs
- Vulnerability not public at the time
- Later discovered collision attack in $2^{60} << 2^{80}$ operations
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SHA-1 remained unbroken until quite recently – (**2017**)

Most applications currently use SHA-2 (256 or 512 bits)

- Same design principles; larger parameters

Future applications adopting SHA-3 evolve to the Sponge

- Flexible output size is very useful!

SHA-1 Internals

- Merkle-Damgård, with Davis-Meyer compression function
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```
SHA1-blockcipher(a, b, c, d, e, M) {
  W = expand(M);
  for i = 0 to 79 { // K are constants
    new = (a <<< 5) + f(i, b, c, d) + e + K[i] + W[i]
    (a, b, c, d, e) = (new, a, b >>> 2, c, d)
  }
  return (a, b, c, d, e)
}
```

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No non-generic attacks exist on these hash functions

- Still SHA-3 was (prudently) developed with different design
- Also has the benefit of varying sized outputs
- Good to generate keys!

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Keccack is very different and very flexible

- Sponge based with 1600-bits permutation (in SHA-3)
- Blocks can be 1152, 1088, 832 or 576 bits
- Corresponding to 224, 256, 384 or 512 bit outputs
- As a bonus we get the SHAKE functions
  - SHAKE128 and SHAKE256
  - eXtendable Output Functions (XOFs)
  - You can specify output length

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  - Absorb phase permutes message
  - Squeeze phase retrieves the output
- SHA-2 and SHA-3 currently the *de facto* standards

MACs as Keyed Hashes

Short Summaries of Potentially Large Messages

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Message Authentication Codes – MACs

- Symmetric Authentication $t \leftarrow \mathrm{MAC}(k, m)$
- t guarantees that m was produced by someone that knows k
- Implies message m was not changed since its creation
- Digital signatures in the symmetric paradigm!

Message Authentication Codes

Typical use of MACs – SSH, IPSec, TLS

- Two parties was message authentication and integrity
- Some form of set-up/agreement to establish common key k
- Sender computes $t \leftarrow \mathrm{MAC}(k, m)$ and sends (m, t)
- Receiver gets (m, t), recomputes $t' \leftarrow \mathrm{MAC}(k, m)$
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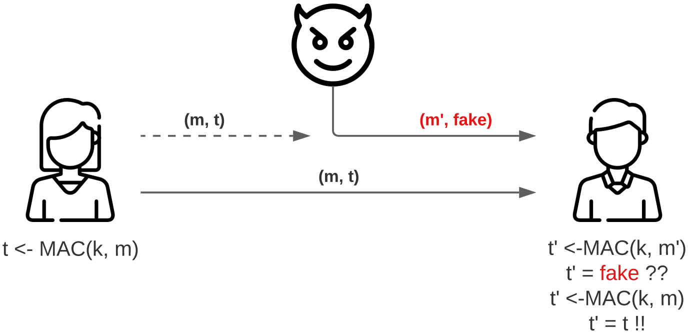
Acceptance means m was produced while knowing k

In this process, message is public!

MACs do not give confidentiality. They provide integrity

Its orthogonal to encryption. In real-world applications, we will need to combine these

Authentication and Message Integrity



- No possibility of computing t without k implies
- Adversary cannot change the message
- Adversary cannot conjure new messages

MAC Security

Standard notion is UF-CMA

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Security Experiment

- Experiment generates a key k
- Adversary (adaptively) sends m to get $t \leftarrow \text{MAC}(k, m)$
- Eventually, attacker outputs (m^*, t^*)

Attacker wins if $t^* = \text{MAC}(k, m^*)$, and if t^* was not produced by the experiment. Contrary to IND-CPA, a victory here implies a broken MAC scheme.

MAC Security Nuances

- MAC on its own does not protect against replay attacks
- Suppose a network scenario
  - Attacker sees authenticated message (m, t)
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- Suppose a network scenario
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  - MAC will verify every time!
- Simple technique: impose message never repeats in network
- Sequence numbers
  - Prepend counter and keep counter as state in both sides
  - Prepend timestamp (local clock reading)
  - How should the receiver operate in both cases?

Some Context

MACs constructed from hash functions and block ciphers

Simplest construction: prefix key

$$\mathrm{MAC}(k, m) = \mathrm{H}(K||M) \text{ or } \mathrm{PRF}(k, m) = \mathrm{H}(K||M)$$

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- This can be computed just from t' and m'
- **Length extension attack**

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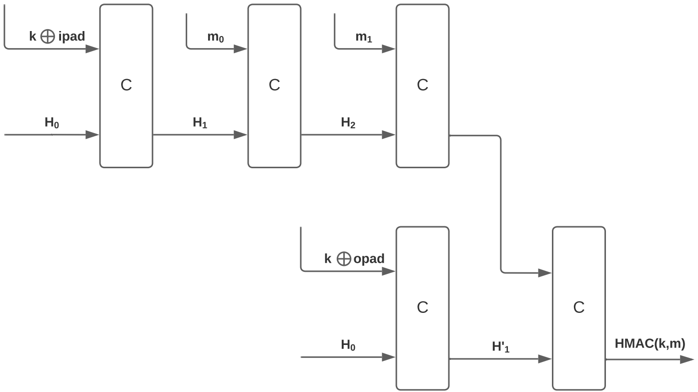
A consideration in SHA-3 construction

- Abandon MD construction
- Include explicit keyed hash

HMAC Construction

When instantiated with MD construction

- Compression function is PRF $\rightarrow$ Secure MAC
- HMAC is simply $\mathrm{H}((K \oplus opad)||\mathrm{H}((k \oplus ipad)||m))$
- *ipad* and *opad* are constraints: align to block size



On Collision-based Forgeries

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Collisions in MAC also yield forgeries

- True for **any** MAC
- Collision occur when $\sqrt{2^n}$ MACs are issued

Building MACs from Block Ciphers

We have seen block ciphers $\rightarrow$ hash functions $\rightarrow$ MACs

But there are also direct constructions: block ciphers $\rightarrow$ MACs

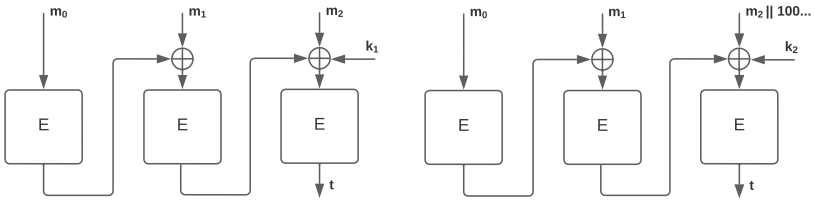
CMAC

- Used in IPSec
- CMAC improves on CBC-MAC (which was broken!)
- Use CBC mode of operation
- Fix IV to all zero blocks
- Take the last ciphertext block as a tag

CMAC Internals

CMAC fixes CBC-MAC by processing last block differently

- All blocks except last are processed like CBC-MAC
- Keys k_1 and k_2 derived from k
  - $I \leftarrow E(k, 0)$
  - $k_1 = (I << 1) \oplus (0x00..0087 * LSB(I)))$
  - $k_2 = (k_1 << 1) \oplus (0x00..0087 * LSB(k_1)))$



Custom MAC Constructions

More efficient MAC constructions are designed from scratch

Poly1305 is one such construction by D. J. Bernstein

Based on

- Universal Hash Functions
- Wegman-Carter construction

Universal Hash Functions

UHF are a Weak form of Hashing

- Don't need to be collision resistant
- Parametrised by a key $\mathrm{UH}(k, m)$
- Guarantee that, for two fixed messages $m_0 \neq m_1$:

$$\Pr[\mathrm{UH}(k, m_0) = \mathrm{UH}(k, m_1)] \leq \epsilon$$

- Considering random k and very small ϵ

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UHF are a Weak form of Hashing

- Don't need to be collision resistant
- Parametrised by a key $\mathrm{UH}(k, m)$
- Guarantee that, for two fixed messages $m_0 \neq m_1$:

$$\Pr[\mathrm{UH}(k, m_0) = \mathrm{UH}(k, m_1)] \leq \epsilon$$

- Considering random k and very small ϵ

No other security requirements $\rightarrow$ easy to construct

We can use a universal hash function as a MAC

Provided that we only authenticate **one message!**

Wegman-Carter Construction

How to circumvent this limitation?

- Use a PRF to strengthen the UH
- Converts a UH into a fully secure MAC
- AES can fill the PRF role!

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Intuition: Encrypt Universal Hash Value

$$\mathsf{UH}(k_1, m) \oplus \mathsf{PRF}(k_2, n)$$

- The full MAC key is (k_1, k_2)
- n is a public value that must never repeat
  - A.k.a. a nonce
- This can be kept as a counter, or generated at random

Poly1305-AES: Wegman-Carter in Practice

- Initial proposal used AES as the Wegman-Carter PRF
- The universal hash function uses prime $p^{130} - 5$

$$\text{Poly1305}((k_1, k_2), m) = (m_1 k + \ldots + m_n k^n \pmod{p}) + \text{AES}(k_2, n)$$

- Blocks are 128 bits and last block is padded with 100
- All blocks set bit 129, so MSB is 1
- The final addition is performed modulo 2^{128}
- TLS recommends Poly1305 with ChaCha20, rather than AES

Key Takeaways

- Keyed hashing allows for message authentication
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- Provides integrity; ciphers give confidentiality

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- Wegman-Carter
  - Use a UHF for a unique message
  - XOR it with an encryption of a nonce
  - Used in AES-GCM (next class)

Applied Cryptography
Week 5: Hash Functions and Keyed Hashing

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