Theory and practice of option pricing

Homework: Efficient delta hedging with transaction costs

Afet Ibadova – *453818*

Introduction

This project delves into the world of delta-hedging, a pivotal strategy in financial markets used to mitigate risks in options trading. It explores the theoretical foundations of delta-hedging, the challenges in its practical implementation, and the impact of transaction costs on its effectiveness. Through Monte-Carlo simulations and empirical findings, this project aims to provide a better understanding of how transaction costs influence delta-hedging performance in real-world market conditions.

Methodology

The methodology employs Monte-Carlo simulations to model stock price evolution, a well-established approach in financial risk assessment. This simulation framework helps understand how financial instruments react to different market conditions, crucial for delta-hedging, where market volatility is vital.

The process begins with calculating d_1 and d_2, critical variables in the Black-Scholes formula, used for option pricing in modern financial theory. These calculations are essential in the simulation as they determine the option's sensitivity to changes in the underlying asset's price, known as "delta". Table 1 shows used the parameters for simulations.

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

- S is the current price of the underlying asset.
- K is the strike price of the option.
- r is the risk-free interest rate.
- σ is the volatility of the underlying asset's returns.
- T is the time to expiration of the.

Stock Price	100
Volatility	0.23
Drift	0
Interest Rate	0
Strike Price	100
Maturity	1Y

Table 1: Assumptions

Task 1

- ***What is the expected value of your P/L and its standard deviation assuming portfolio rebalancing to net zero-delta occurs at every step;***

To investigate the impact of sample size on simulation accuracy and reliability, various scenarios were examined by altering the number of simulations. Initially, 10,000 simulations were conducted, each comprising 1,000 steps

in the path. This extensive approach was essential for capturing the wide range of possibilities in market behavior and hedging performance.

The histogram from the 1000-step simulation *(Figure 1)* shows a slightly right-skewed, normally distributed set of outcomes. The majority of results cluster around the peak, with the cumulative curve indicating nearly complete coverage within the displayed range. This distribution suggests a moderately consistent and predictable hedging strategy with a moderate level of risk, as indicated by the relatively compact frequency bars.
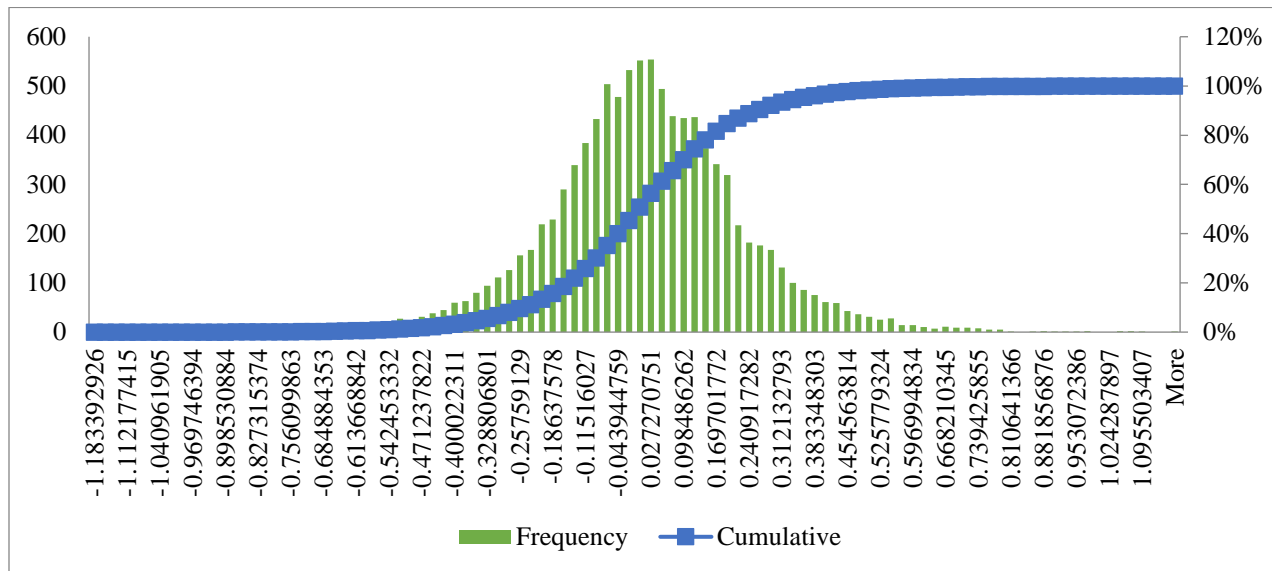

Figure 1: The Histogram of Monte Carlo simulation of 10,000 paths with 1,000 steps

The asset price graph for the 1000-step simulation (*Figure 2*) shows a decline in value over time with a slight recovery towards the end. This trend suggests a downtrend in the market price of the asset, followed by a period of recovery, which is a common pattern in financial markets and can affect the P/L of a hedged position depending on the timing of the hedge adjustments.
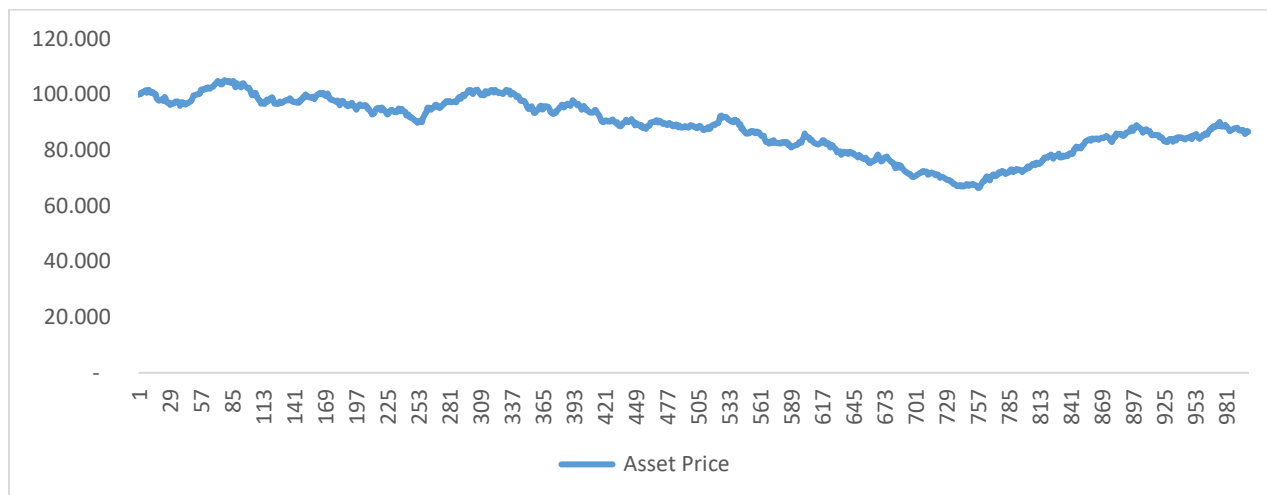

Figure 2: Asset price for 1,000-step simulation

Figure 3 shows a high initial delta for a call option, indicating sensitivity to the asset price. Over time, the delta drops, implying the option is moving out-of-the-money and becoming less affected by asset price movements, eventually stabilizing when further price changes have minimal impact on its value.
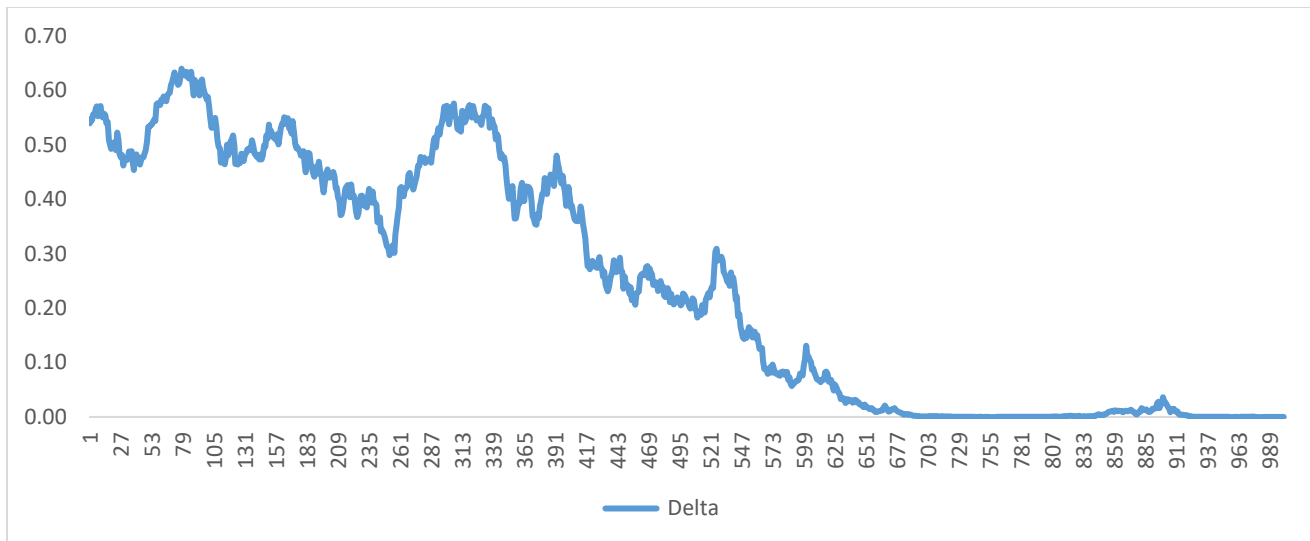
Figure 3: Delta graph from 1,000-step simulation

Task 2

- If each trade in the underlying stock costs 0.1%, which would be more efficient:

a) reducing the number of re-hedging from 1,000 to 100 (equally spaced); or

After the initial large-scale simulation, the number of steps was reduced to 100 in order to examine the stability and consistency of results with a more limited dataset. This comparison was essential for understanding how sample size influenced the reliability of simulation outcomes.

the histogram for the 100-step simulation strategy, displays a right-skewed distribution, with more frequent losses than profits. The tall bars on the left signify common losses, while the extended right tail indicates occasional larger gains. The cumulative curve flattens early, covering most simulations. This strategy exhibits higher variability and risk compared to the 1000-step strategy but less than the trigger-based strategy.
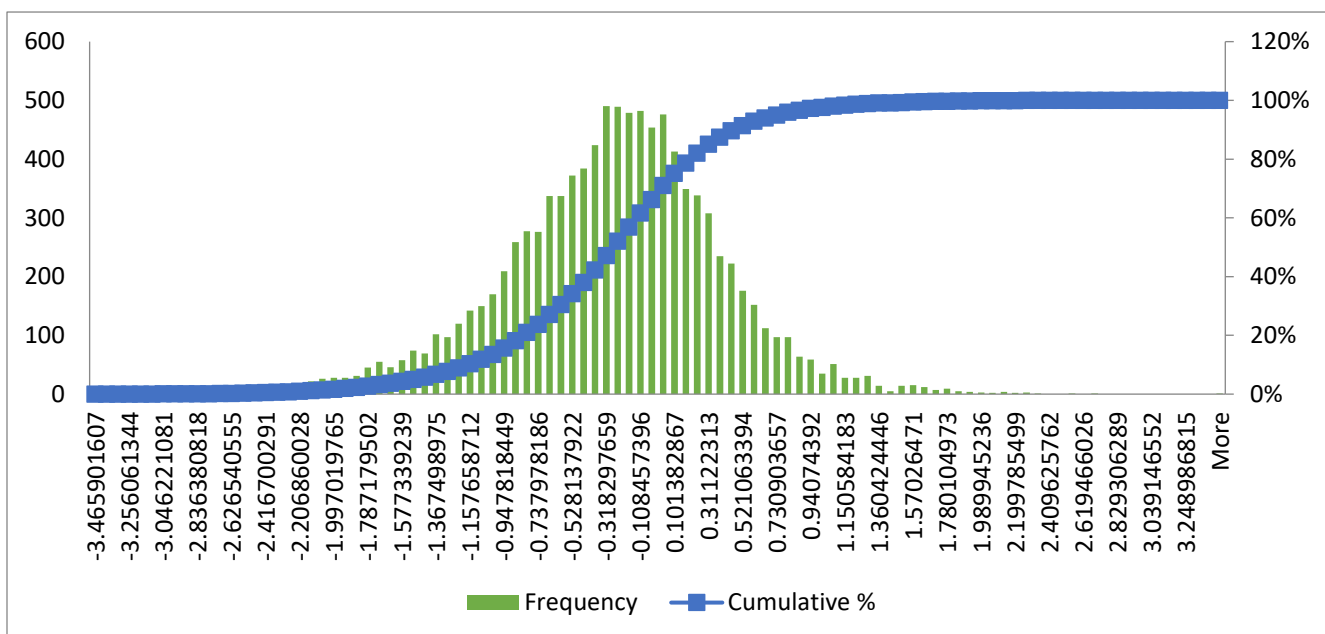

Figure 4: The Histogram of Monte Carlo simulation of 10,000 paths with 100 steps

In *Figure 5*, the delta graph from the 100-step simulation displays a stable trend with minor fluctuations, indicating consistent sensitivity of the option to underlying asset price changes. The slight dips and rises suggest minor adjustments in the delta, likely corresponding to small asset price movements relative to the option's strike price. Overall, the delta remains relatively steady, suggesting a lack of significant market events or volatility during the simulation.
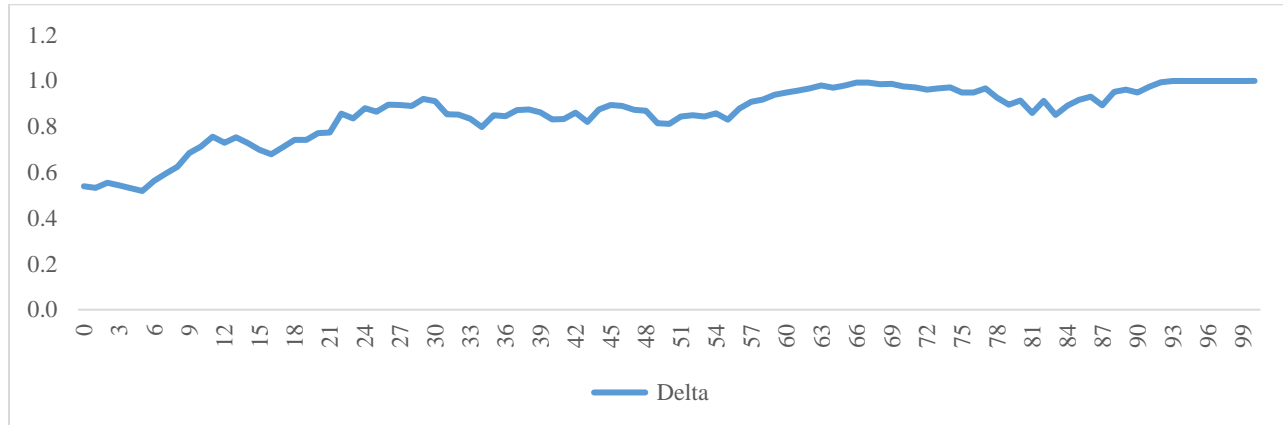

Figure 5: Delta from 100-step simulation

b) *reducing the number of re-hedging from 1,000 to 10 (equally spaced); or*

For this task, the number of simulations was further reduced to 10 to evaluate the model's performance in extreme conditions with minimal data points. While such a small sample size is not representative of typical market dynamics, it provided insights into the variability and potential outliers in delta-hedging effectiveness.

The below histogram for the 10-step simulation strategy displays a distribution with a somewhat normal but slightly right-skewed pattern, suggesting that there are more frequent small losses and occasional larger gains. The frequency bars show the outcomes of the simulation across different profit and loss bins, while the cumulative percentage curve provides a running total of the observations. This strategy seems to have less risk compared to the trigger-based strategy, as indicated by a lower standard deviation, yet it still presents a significant spread in outcomes, suggesting variability in performance.
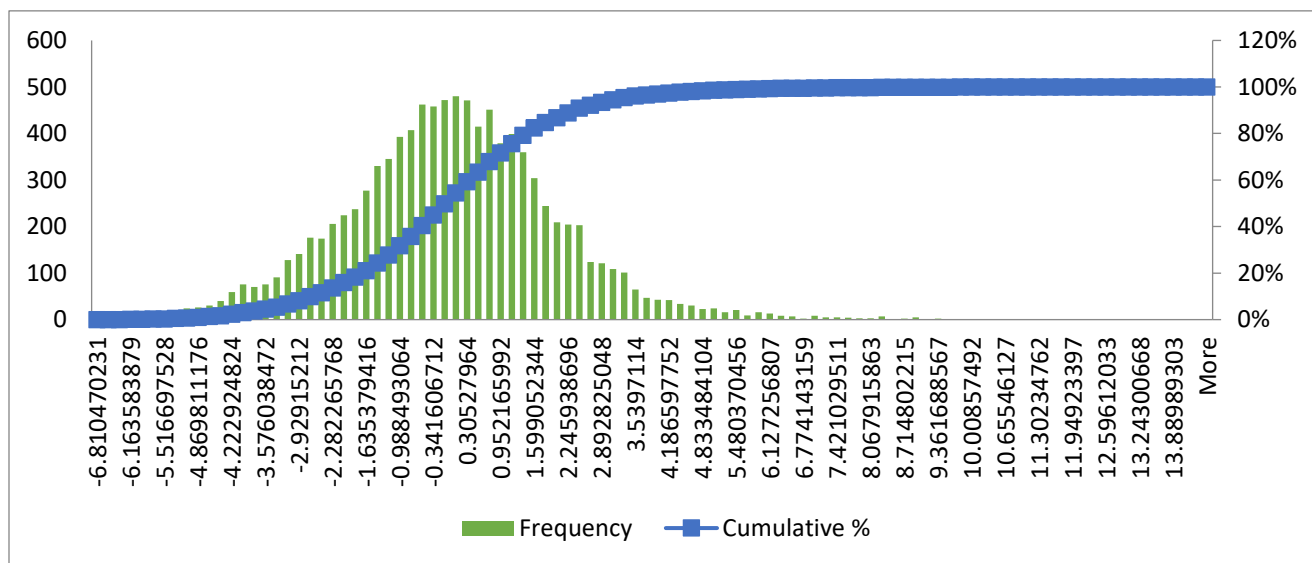

Figure 6: The Histogram of Monte Carlo simulation of 10,000 paths with 10 steps

c) *adopting a trigger-based re-hedging strategy whereby the portfolio is rebalanced only if delta changes by 5 percentage points.*

The histogram for the trigger-based strategy shows a wide distribution of frequency across different P/L bins, indicating a significant spread in outcomes. The tall spikes in frequency, particularly at the extreme ends, suggest large outliers in performance. The cumulative percentage line shows that most outcomes are concentrated in the middle range, but there's a long tail to the right, indicating some occurrences of very high profits. This suggests a high-risk, high-reward strategy, with potential for both substantial gains and losses. The majority of the outcomes, however, result in a loss as the average P/L is negative. The high standard deviation confirms the high variability and risk of this strategy.
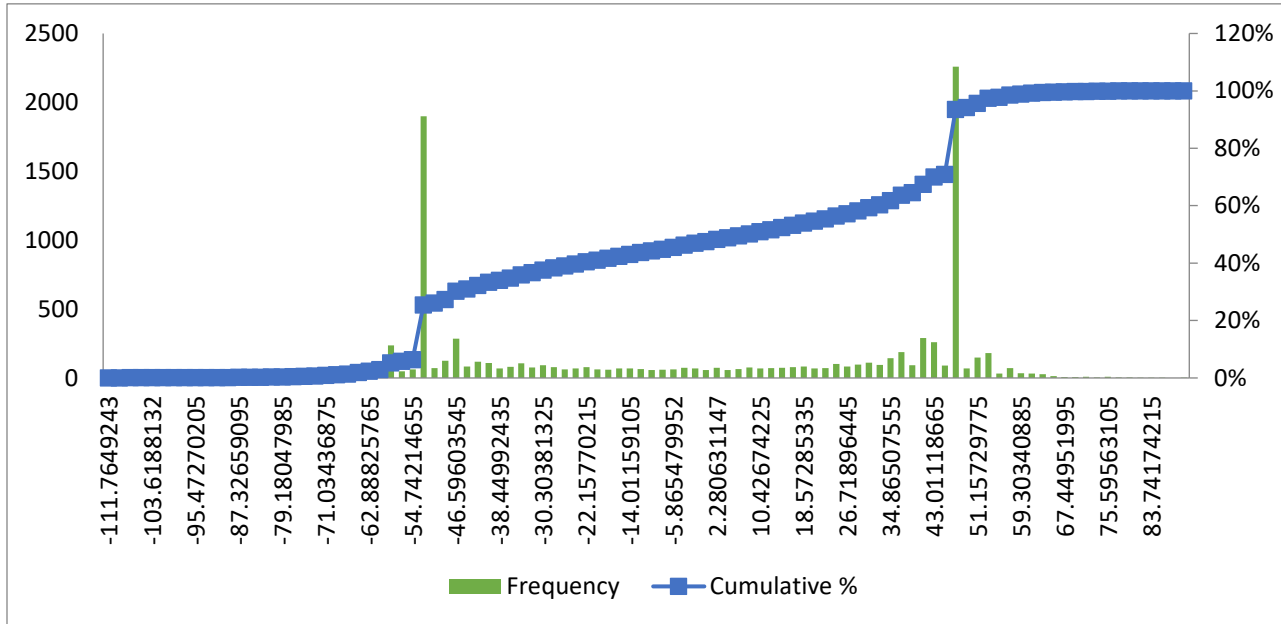

Figure 7: *The Histogram of Monte Carlo simulation of 10,000 paths with trigger-based strategy*

Result

Table 2 compares different delta-hedging scenarios by their average P/L and associated standard deviation. The strategy with 1000 steps showed a modest average profit and the lowest risk, as indicated by its standard deviation. The strategies with 100 and 10 steps resulted in losses, and an increased standard deviation pointed to heightened risk. The trigger-based strategy, although aiming to reduce transaction frequency, incurred the largest average loss and carried an extremely high level of risk, with the standard deviation being significantly larger than in other scenarios. These findings suggest that more frequent rebalancing tends to stabilize the portfolio against market volatility.

Scenarios	Average P/L	Standard deviation
1000 steps	0.001919	0.218985
100 steps	(0.317027)	0.672296
10 steps	(0.070657)	2.085996
Trigger-based strategy	(0.759136)	44.529708

Table 2: Result for all scenarios

References

- Source for working file - https://github.com/afatirium/OptionPricing_HW