

Name: \_\_\_\_\_

**Question 1:** Write out the joint probability for the following sentence using the chain rule:

$p(\text{There, is, only, one, person, who, is, not, ordinary})$

**Answer:**

$p(\text{There, is, only, one, person, who, is, not, ordinary})$   
 $= p(\text{There})p(\text{is}|\text{There})p(\text{only}|\text{There, is})p(\text{one}|\text{There, is, only})p(\text{person}|\text{There, is, only, one})p(\text{who}|\text{There, is, only, one, person})p(\text{is}|\text{There, is, only, one, person, who})p(\text{not}|\text{There, is, only, one, person, who, is})p(\text{ordinary}|\text{There, is, only, one, person, who, is, not})$

Write out the probability above using the second-order Markov assumption.

**Answer:**

$p(\text{There, is, only, one, person, who, is, not, ordinary})$   
 $= p(\text{There})p(\text{is}|\text{There})p(\text{only}|\text{There, is})p(\text{one}|\text{is, only})p(\text{person}|\text{only, one})p(\text{who}|\text{one, person})p(\text{is}|\text{person, who})p(\text{not}|\text{who, is})p(\text{ordinary}|\text{is, not})$

**Question 2:** Consider the following training corpus **T** of sentences:

- *START Karlsson is round STOP*
- *START He lives on the roof STOP*
- *START He is happy STOP*
- *START On the roof STOP*
- *START Karlsson lives happily STOP*

(b) Compute the following maximum likelihood parameters:

$p(\text{Karlsson}|\text{START}) = \text{count}(\text{START, Karlsson}) / \text{count}(\text{START}) = 2/5$

$p(\text{Karlsson}|\text{lives, happily}) = \text{count}(\text{lives, happily, Karlsson}) / \text{count}(\text{lives, happily}) = 0/1$

$p(\text{STOP}|\text{happy}) = \text{count}(\text{happy, STOP}) / \text{count}(\text{happy}) = 1/1$

(c) Compute the probability of the following sentences under the trigram model trained on **T**:

*START Karlsson is happy STOP*

**Answer:**

$$\begin{aligned} & p(\text{START}, \text{Karlsson}, \text{is}, \text{happy}, \text{STOP}) \\ &= p(\text{Karlsson} | \text{START}) p(\text{is} | \text{START}, \text{Karlsson}) p(\text{happy} | \text{Karlsson}, \text{is}) p(\text{STOP} | \text{is}, \text{happy}) \\ &= \text{count}(\text{START}, \text{Karlsson}) / \text{count}(\text{START}) * \text{count}(\text{START}, \text{Karlsson}, \text{is}) / \text{count}(\text{START}, \text{Karlsson}) * \\ & \quad \text{count}(\text{Karlsson}, \text{is}, \text{happy}) / \text{count}(\text{Karlsson}, \text{is}) * \text{count}(\text{is}, \text{happy}, \text{STOP}) / \text{count}(\text{is}, \text{happy}) \\ &= 2/5 * 1/2 * 0/1 * 1/1 = 0 \end{aligned}$$

*START Karlsson lives on the roof STOP*

**Answer:**

$$\begin{aligned} & p(\text{START}, \text{Karlsson}, \text{lives}, \text{on}, \text{the}, \text{roof}, \text{STOP}) \\ &= p(\text{Karlsson} | \text{START}) p(\text{lives} | \text{START}, \text{Karlsson}) p(\text{on} | \text{Karlsson}, \text{lives}) p(\text{the} | \text{lives}, \text{on}) p(\text{roof} | \text{on}, \text{the}) p(\text{STOP} | \text{the}, \text{roof}) \\ &= \text{count}(\text{START}, \text{Karlsson}) / \text{count}(\text{START}) * \text{count}(\text{START}, \text{Karlsson}, \text{lives}) / \text{count}(\text{START}, \text{Karlsson}) \\ & \quad * \text{count}(\text{Karlsson}, \text{lives}, \text{on}) / \text{count}(\text{Karlsson}, \text{lives}) * \text{count}(\text{lives}, \text{on}, \text{the}) / \text{count}(\text{lives}, \text{on}) * \text{count}(\text{on}, \text{the}, \text{roof}) / \text{count}(\text{on}, \text{the}) * \text{count}(\text{the}, \text{roof}, \text{STOP}) / \text{count}(\text{the}, \text{roof}) \\ &= 2/5 * 1/2 * 0/1 * 1/1 * 2/2 * 2/2 = 0 \end{aligned}$$

**Question 3:** We have the following training corpus:

The green book STOP  
My blue book STOP  
His green house STOP  
Book STOP

Assume we have a language model based on this corpus using linear interpolation with  $\lambda=1/3$  for all  $i$ . Compute the value of the parameter  $p(\text{book} | \text{the green})$  under this model. Assume STOP as part of your unigram model.

**Answer:**

$$\begin{aligned} & \text{total\_words} = 14 \\ & p(\text{book} | \text{the green}) \\ &= 1/3 * \text{count}(\text{the}, \text{green}, \text{book}) / \text{count}(\text{the}, \text{green}) + 1/3 * \text{count}(\text{green}, \text{book}) / \text{count}(\text{green}) + 1/3 * \text{count}(\text{book}) / \text{total\_words} \\ &= 1/3 * 1 + 1/3 * 1/2 + 1/3 * 3/14 = 1/3 + 1/6 + 1/14 \end{aligned}$$