Université d'Ottawa
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École de science informatique
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School of Electrical Engineering
and Computer Science

Fall 2014

Lecture 6 – **Frequency response** (SS 9.1 & app F)

ELG3136
ELECTRONICS II

ELG3136 FALL 2014
ELECTRONICS II

Instructor: **J. Lostracco, P.Eng.**

Based on Course Material by:

Prof. Jianping Yao and material from Textbook:

Sedra and Smith, Microelectronic Circuits ,6th edition, Oxford

LECTURE 7 - OVERVIEW

- Recap of Lec 6 –multistage amplifiers
  - Requirements for multistage amplifiers
  - Functions of each stage
  - DC analysis, common-mode input range
  - Calculate input resistance, output resistance.
  - Calculate overall voltage gain
- Lec 7 – Frequency response
- s-domain Analysis: poles, zeros and Bode plots
- voltage transfer function
- Lab 1 discussion
- Quiz 1 overview

AMPLIFIER FREQUENCY RESPONSE

Amp gain is NOT constant and is dependent on the frequency of the input signal.

- bandwidth is finite

frequency range over which the gain remains almost constant is called the middle-frequency band or **midband.**

GAIN VS FREQUENCY OF BJT AMP

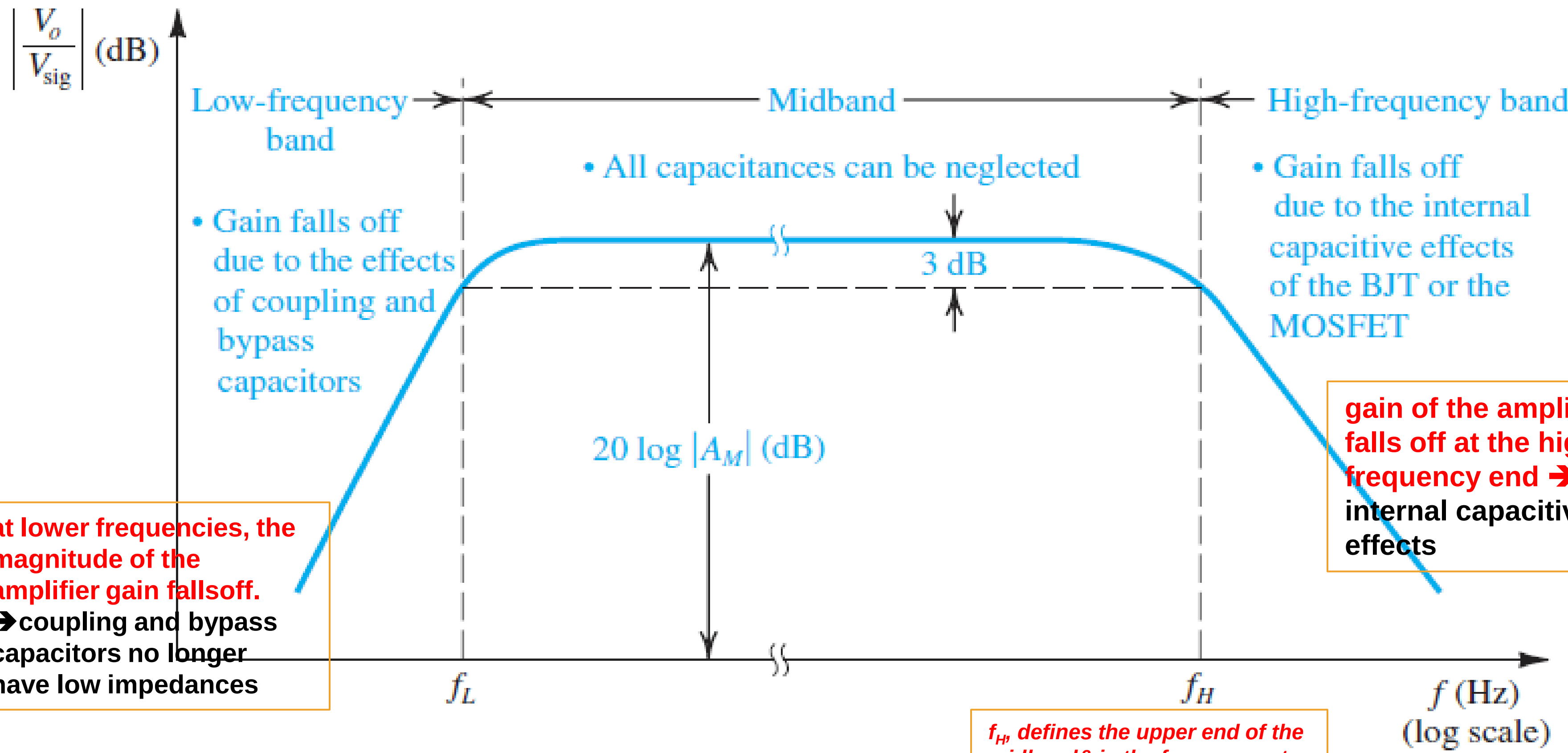


Figure 9.1 Sketch of the magnitude of the gain of a discrete-circuit BJT or MOS amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

at lower frequencies, the magnitude of the amplifier gain fallsoff.
➔coupling and bypass capacitors no longer have low impedances

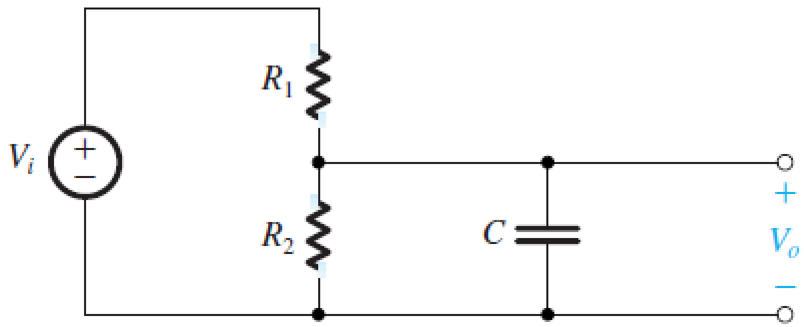
gain of the amplifier falls off at the high-frequency end ➔due to internal capacitive effects

***f_H, defines the upper end of the midband& is the frequency at which the gain* drops by 3 dB**

S-DOMAIN ANALYSIS: POLES, ZEROS, AND BODE PLOTS

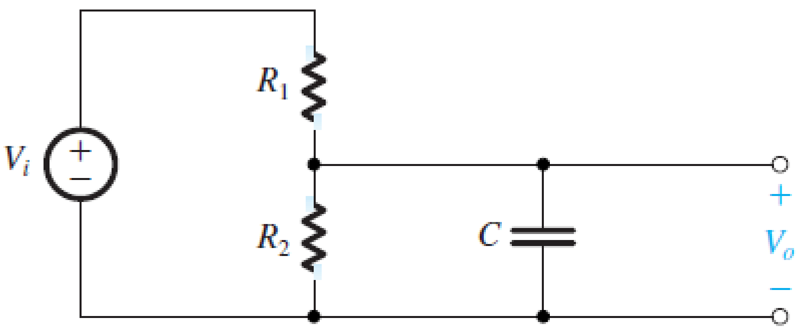
In analyzing the frequency response of an amplifier, most of the work involves finding the amplifier voltage gain as a function of the complex frequency s. In this s-domain analysis, a capacitance C is replaced by an admittance sC, or equivalently an impedance $1/sC$, and an inductance L is replaced by an impedance sL. Then, using usual circuit-analysis techniques, one derives the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$.

Find the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$ for the STC network shown



TRANSFER FUNCTION

Find the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$ for the STC network shown



Ans. $T(s) = \dfrac{1/CR_1}{s + 1/C(R_1 \,//\, R_2)}$

TRANSFER FUNCTION

Once the transfer function is obtained, it can be evaluated for physical frequencies by replacing s by $j\omega$. The resulting transfer function $T(j\omega)$ is in general a complex quantity whose magnitude gives magnitude response and whose angle gives the phase response.

In many cases, $T(s)$ will reveals, many useful facts about the circuit. In general, $T(s)$ has the form:

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_0}$$

where a and b are real numbers and the order m of the numerator is smaller than or equal to the order n of the denominator. n is called the ***order of the network***.

POLES AND ZEROS

An alternate form of $T(s)$ is

$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_n)}$$

a_m is multiplicative constant. $Z_1, Z_2, \ldots, Z_m$ are the roots of the numerator polynomial, called **transfer function zeros or transmission zeros**, and $P_1, P_2, \ldots, P_m$ are the roots of the denominator polynomial, called **transfer function poles.**

The poles and zeros can be either real or complex numbers.

FIRST-ORDER FUNCTIONS

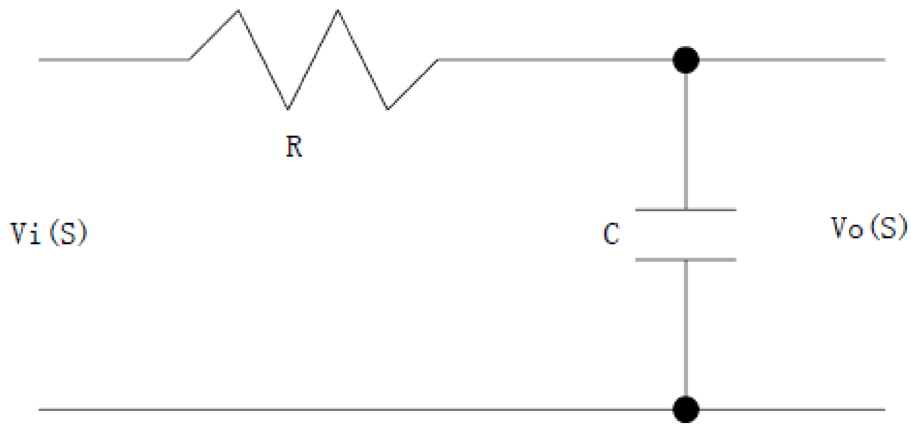
Many of the transfer functions encountered in this book have real poles and zeros and can therefore be written as the product of first-order transfer functions of the general form

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \tag{F.3}$$

where $-\omega_0$ is the location of the real pole. The quantity ω_0, called the **pole frequency**, is equal to the inverse of the time constant of this single-time-constant (STC) network (see Appendix E). The constants a_0 and a_1 determine the type of STC network. Specifically, we

EXAMPLE1: LOW-PASS FIRST-ORDER NETWORK

$$T(s) = \frac{a_0}{s + \omega_0}$$



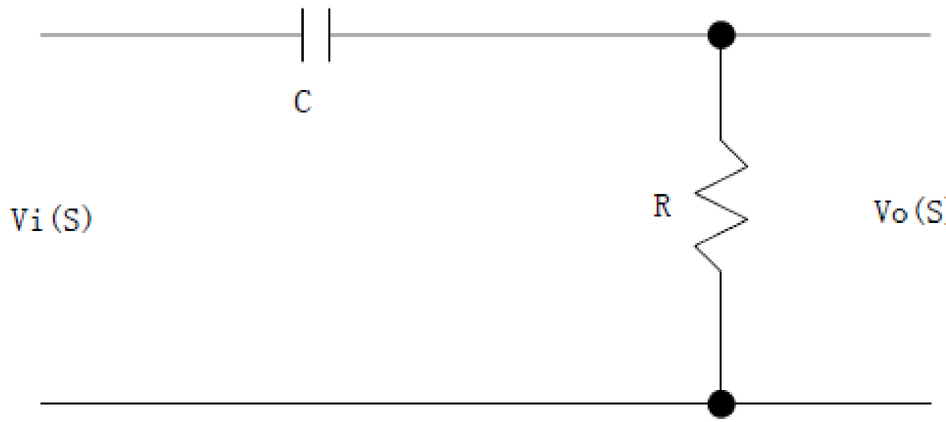
The transfer function from the RC circuit is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

The dc gain is $a_0 / \omega_0 = 1$ and $\omega_0 = \frac{1}{RC}$ is the corner or 3-dB frequency. $T(s) \to 0$, when $s \to \infty$.

EXAMPLE2: HIGH-PASS FIRST-ORDER NETWORK

$$T(s) = \frac{a_1 s}{s + \omega_0}$$



The transfer function from RC circuit is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{s}{s + \frac{1}{RC}}$$

FREQUENCY RESPONSE OF LOW-PASS CIRCUIT

$$T(s) = \frac{a_0}{s+\omega_0} = \frac{a_0/\omega_0}{1+s/\omega_0} = \frac{K}{1+s/\omega_0}$$

For physical frequencies,

$$T(j\omega) = \frac{K}{1+j(\omega/\omega_0)}$$

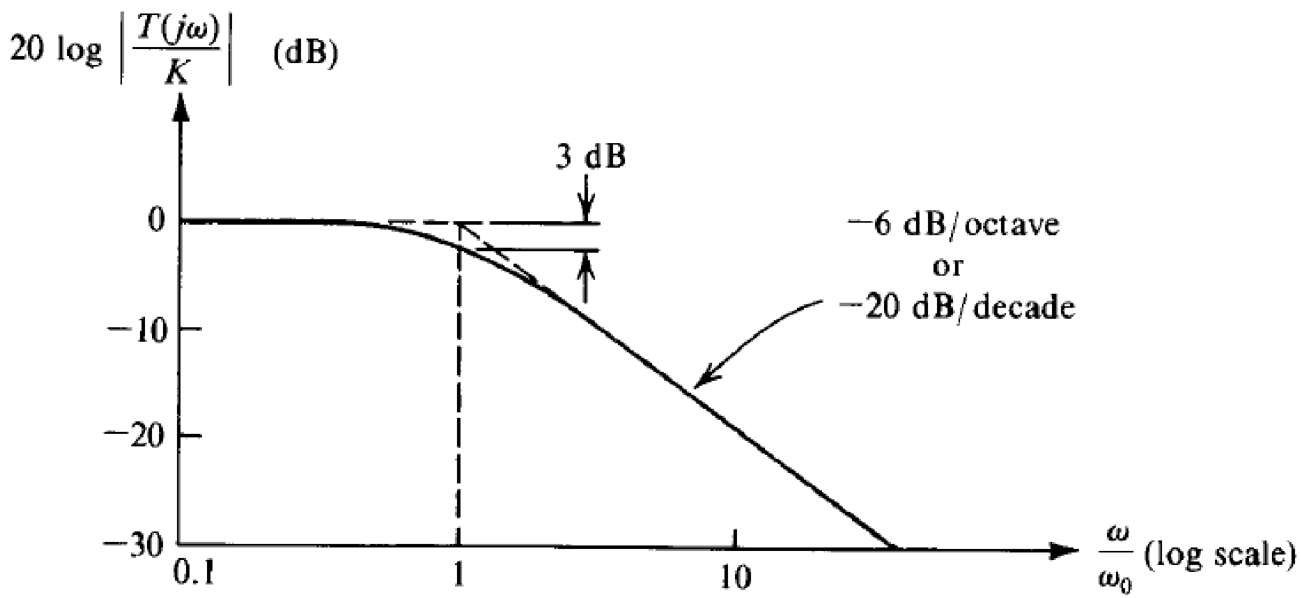
where K is the magnitude of the transfer function at $\omega = 0$ (dc) and $\omega_0 = 1/\tau$.

THE MAGNITUDE RESPONSE OF T:

$$|T(j\omega)| = \frac{K}{\sqrt{1+(\omega/\omega_0)^2}}$$

$$20\log\left|\frac{T(j\omega)}{K}\right| = 20\log(1) - 20\log\left(\sqrt{1+(\omega/\omega_0)^2}\right)$$

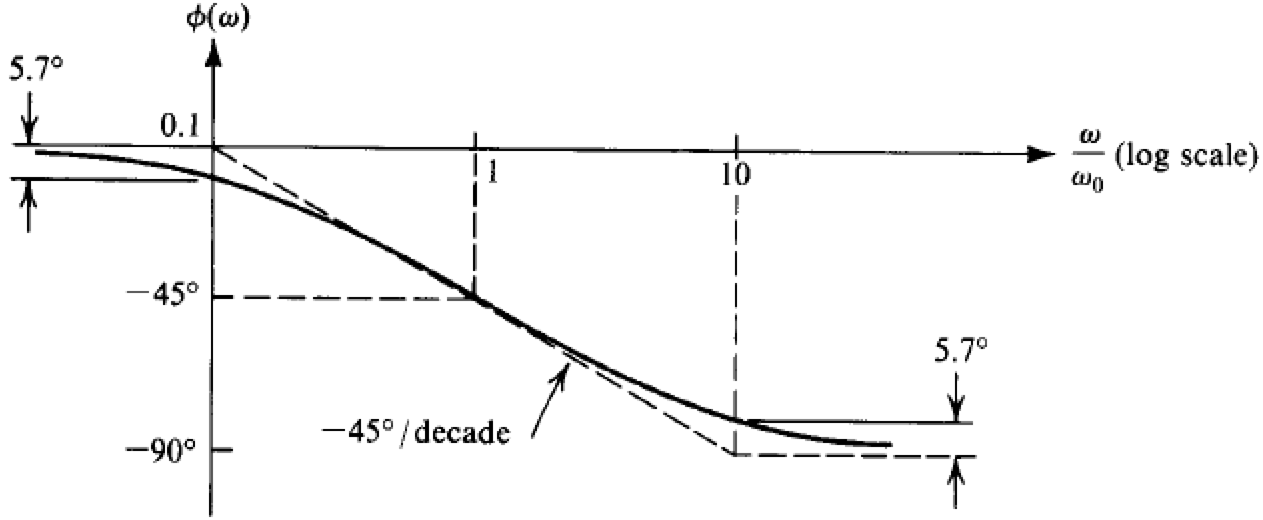
$$= \begin{cases} 0, & for\ \omega << \omega_0 \\ -20\log(\omega/\omega_0), & for\ \omega >> \omega_0 \end{cases}$$



THE PHASE RESPONSE OF T:

$$\phi(\omega) = -\tan^{-1}(\omega/\omega_0)$$

$$= \begin{cases} -45^0, & when \quad \omega = \omega_0 \\ 0, & when \quad \omega << \omega_0 \\ -90^0, & when \quad \omega >> \omega_0 \end{cases}$$



BODE PLOTS

- To obtain the Bode plot for the magnitude of a transfer function, the asymptotic plot for each pole and zero is first drawn.
- The slope of the high-frequency of the curve corresponding to a zero is +20 dB/decade, while for the poles is -20 dB/decade.
- The various plots are then added together
- The overall curve is shifted vertically by an amount determined by the multiplicative constant of the transfer function.

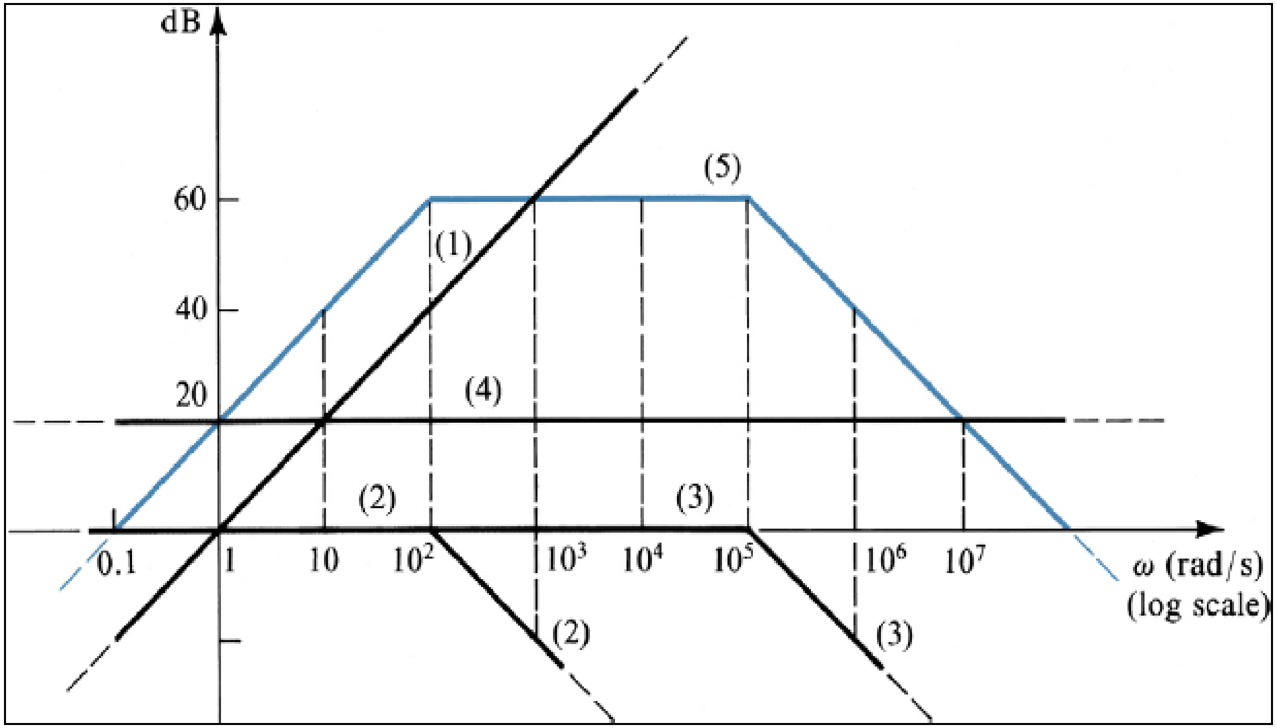
Example3: $T(s) = \dfrac{10s}{(1+s/10^2)(1+s/10^5)}$, find the magnitude BP

Solution: **Curve 1**, a straight line with +20 dB/decade slope corresponding to s term (the zero at s = 0) in the numerator.

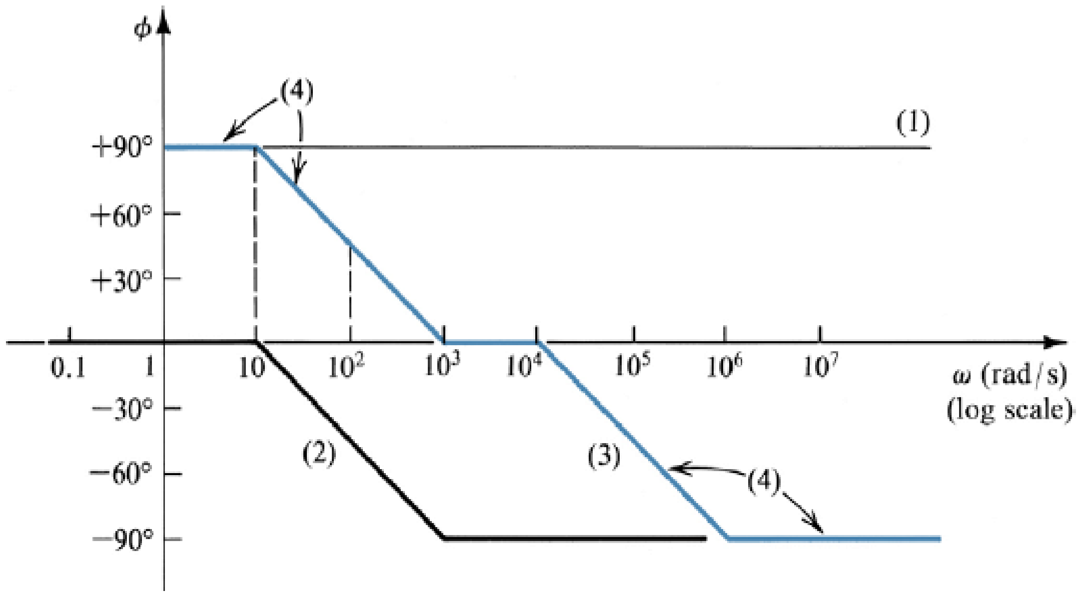
Curve 2: resulted from the pole $s = -10^2$, consisting of two asymptotes intersecting at $\omega = 10^2$.

Curve 3: resulted from the pole $s = -10^5$, consisting of two asymptotes intersecting at $\omega = 10^5$.

Curve 4: multiplicative constant: $20\log(10) = 20$ dB.



Example 4: The same transfer function in Example 3, find the phase BP.



Solution: **Curve 1**: the zero at s = 0 gives rise to a constant $+90^0$ phase function

$(\phi(\omega) = \tan^{-1}(\omega / 0) = 90^0)$

Curve 2: The pole at s = - 10^2 give rise to the phase function $\phi_1 = -\tan\dfrac{\omega}{10^2}$. When $\omega = 10^2$, $\phi_1 = -45^0$, when $\omega = 10\ (\omega << 10^2)$, $\phi_1 = 0^0$, when $\omega = 10^3\ (\omega >> 10^2)$, $\phi_1 = -90^0$.

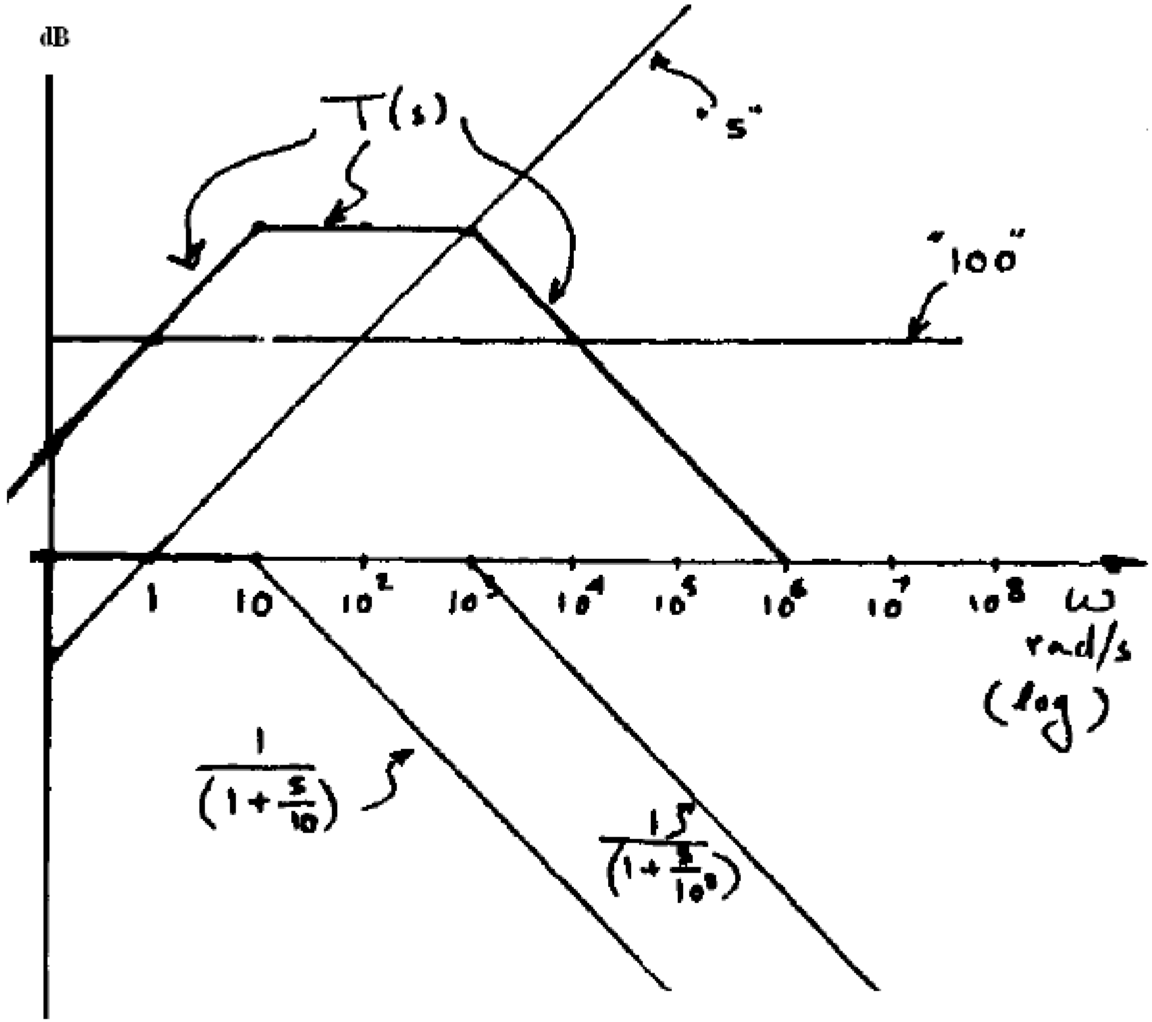
Curve 3: $\phi_2 = -\tan\dfrac{\omega}{10^5}$, when $\omega = 10^4\ (\omega << 10^5)$, $\phi_2 = 0^0$, when $\omega = 10^6\ (\omega >> 10^5)$, $\phi_2 = -90^0$.

Exercise: An amplifier has a voltage transfer function

$$T(s) = \frac{10^6 s}{(s+10)(s+10^3)}.$$

Convert this to the form convenient for constructing Bode plots (that is, place the denominator factors in the form $\left(1+\frac{s}{a}\right)$).

Find the BP for the magnitude response.



EX: FREQ RESPONSE OF CE WITH ACTIVE LOAD

In the circuit of Fig. 10.11, as in most IC amplifier stages, the output impedance is very high compared to the discrete stage.

For this circuit,the output impedance of the amplifier consists of the output impedance of *Q2 in parallel* with that of *Q1. This value will generally be several tens of k.*

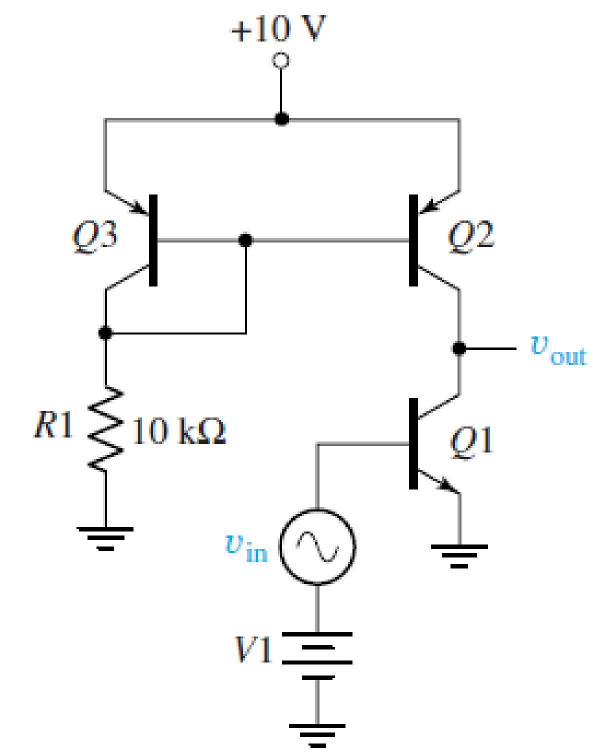
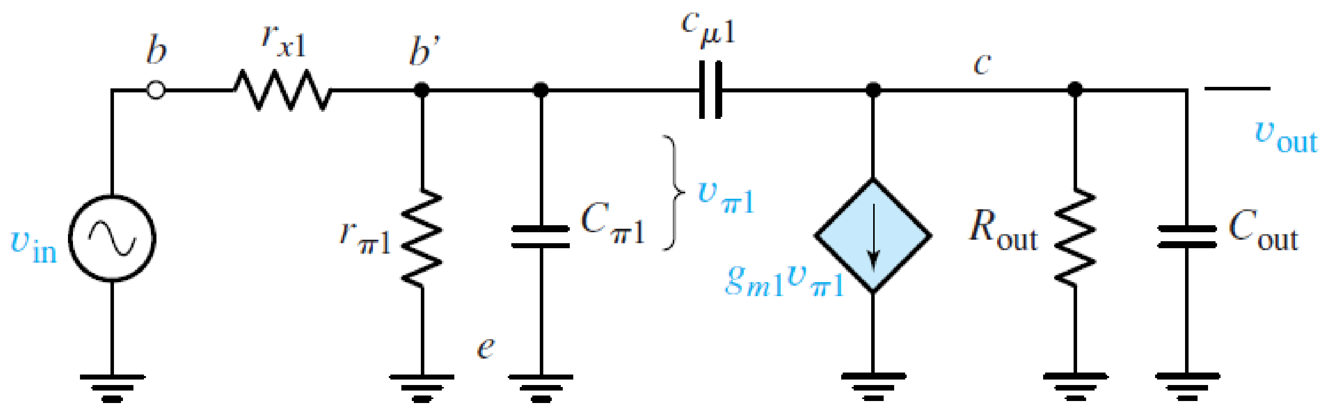


Figure 10.11
A common-emitter stage with current mirror active load.

FREQ RESPONSE OF CE WITH ACTIVE LOAD

Figure 10.12
Equivalent circuit of the amplifier in Fig. 10.11.



$$R_{out} = r_{o1} \parallel r_{o2} = r_{ce1} \parallel r_{ce2}$$

The capacitance in parallel with R_{out} is approximately

$$C_{out} = C_{\mu 1} + C_{\mu 2} + C_{cs1} + C_{cs2}$$

MIDBAND GAIN

In this equation, $C_{\mu 1}$ and $C_{\mu 2}$ are the collector-to-base junction capacitances, and C_{cs1} and C_{cs2} are the collector-to-substrate capacitances of the respective transistors. If no generator resistance is present, $C_{\mu 1}$ will also appear in parallel with the output terminal and ground. When R_g is present, we will still approximate the output capacitance with the same

The midband gain is easy to evaluate as

$$A_{MB} = \frac{-\beta_1 R_{\text{out}}}{r_{x1} + r_{\pi 1}}$$

The calculation of upper corner frequency begins by reflecting the bridging capacitance, C_{μ}, to both the input and the output. The value reflected to the input side, across terminals b' and e, is

$$(1 - A_{b'c1})C_{\mu 1} \tag{10.21}$$

INPUT CAPACITANCE

as in the discrete circuit amplifier. Thus, the total input capacitance in parallel with $r_{\pi 1}$ is

$$C_{\text{in}} = C_{\pi 1} + (1 - A_{b'c1})C_{\mu 1} \tag{10.22}$$

The upper corner frequency resulting from the input circuit of this stage is

$$f_{\text{in-high}} = \frac{1}{2\pi C_{\text{in}} R_{\text{eq}}} \tag{10.23}$$

where $R_{\text{eq}} = r_{x1} \parallel r_{\pi 1}$.

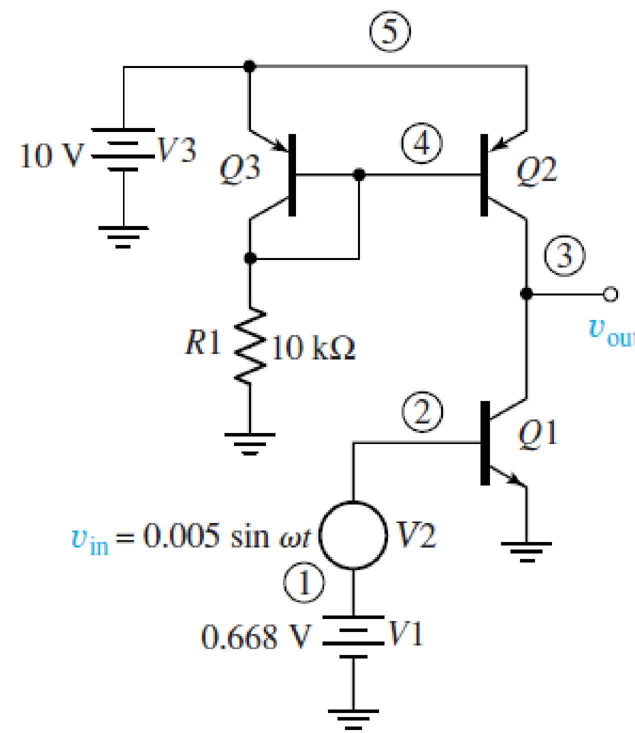
The upper corner frequency resulting from the output side of the stage is

$$f_{\text{out-high}} = \frac{1}{2\pi C_{\text{out}} R_{\text{out}}} \tag{10.24}$$

EXAMPLE 2

Assume that the circuit of Fig. 10.11 is biased so that the collector currents of $Q1$ and $Q2$ have a magnitude of 1.14 mA. The parameters for $Q1$ are $\beta = 160$, $r_{x1} = 10\ \Omega$, $r_{ce1} = 68\ \text{k}\Omega$, $C_{\pi 1} = 20$ pF, and $C_{\mu 1} = 2.1$ pF. For device $Q2$, the necessary parameters are $r_{ce2} = 21\ \text{k}\Omega$ and $C_{\mu 2} = 3.1$ pF. Each device has a value of $C_{cs1} = C_{cs2} = 2.5$ pF. In this circuit, the power supply is 10 V and $R1 = 10\ \text{k}\Omega$.

Calculate the midband voltage gain and the upper corner frequency for this amplifier stage. Do a Spice simulation using 2N3904 (*npn*) and 2N3905 (*pnp*) transistors.



SOLUTION The midband voltage gain can be calculated from Eq. (10.20) after evaluating $r_{\pi 1}$ and R_{out}. These resistances are

$$r_{\pi 1} = (\beta + 1)r_{e1} = 161 \times \frac{26}{1.14} = 3672\ \Omega$$

and

$$R_{\text{out}} = r_{ce1} \,\|\, r_{ce2} = 68 \,\|\, 21 = 16\ \text{k}\Omega$$

The midband gain is then

$$A_{MB} = -\frac{\beta R_{\text{out}}}{r_{x1} + r_{\pi 1}} = -\frac{160 \times 16{,}000}{10 + 3672} = -695\ \text{V/V}$$

The upper corner frequency is found from a consideration of the two poles caused by the input circuit and the output circuit. The corner frequency of the input circuit is

$$f_{\text{in-high}} = \frac{1}{2\pi C_{\text{in}} R_{\text{eq}}} = \frac{1}{2\pi(C_{\pi 1} + [1 - A_{b'c}]C_{\mu 1})(r_{x1} \,\|\, r_{\pi 1})}$$

$$= \frac{1}{2\pi \times 1481 \times 10^{-12} \times 10} = 10.7\ \text{MHz}$$

Since $r_{\pi 1} \gg r_{x1}$, the value of 10 Ω was used for R_{eq}. In addition, the midband gain was used to approximate $A_{b'c}$.

SOL'N 2

The corner frequency of the output circuit is

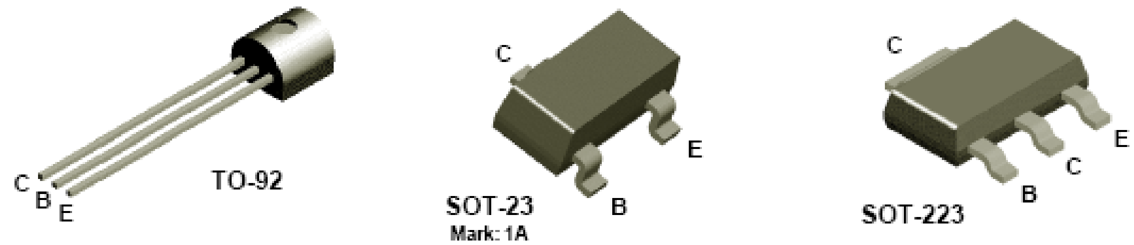
$$f_{\text{out-high}} = \frac{1}{2\pi C_{\text{out}} R_{\text{out}}} = \frac{1}{2\pi \times (2.1 + 3.1 + 2.5 + 2.5) \times 10^{-12} \times 16{,}000} = 975 \text{ kHz}$$

The input corner frequency is much higher than the output corner frequency; consequently, the latter value approximates the overall corner frequency. The result is a value of $f_{2o} = 975$ kHz.

LAB 1 DISCUSSION

V_{BE} max = 6v

2N3904 **MMBT3904** **PZT3904**



NPN General Purpose Amplifier

This device is designed as a general purpose amplifier and switch. The useful dynamic range extends to 100 mA as a switch and to 100 MHz as an amplifier.

Absolute Maximum Ratings*

T_A = 25°C unless otherwise noted

Symbol	Parameter	Value	Units
V_{CEO}	Collector-Emitter Voltage	40	V
V_{CBO}	Collector-Base Voltage	60	V
V_{EBO}	Emitter-Base Voltage	6.0	V
I_C	Collector Current - Continuous	200	mA
T_J, T_{stg}	Operating and Storage Junction Temperature Range	-55 to +150	°C

*These ratings are limiting values above which the serviceability of any semiconductor device may be impaired.

LAB 1 DISCUSSION

1. Current gain varies with *Ic* and with temperature
2. V_{BE} causes saturation if > 0.65

ON CHARACTERISTICS*

h_{FE}	DC Current Gain	I_C = 0.1 mA, V_{CE} = 1.0 V	40		
		I_C = 1.0 mA, V_{CE} = 1.0 V	70		
		I_C = 10 mA, V_{CE} = 1.0 V	100	300	
		I_C = 50 mA, V_{CE} = 1.0 V	60		
		I_C = 100 mA, V_{CE} = 1.0 V	30		
$V_{CE(sat)}$	Collector-Emitter Saturation Voltage	I_C = 10 mA, I_B = 1.0 mA		0.2	V
		I_C = 50 mA, I_B = 5.0 mA		0.3	V
$V_{BE(sat)}$	Base-Emitter Saturation Voltage	I_C = 10 mA, I_B = 1.0 mA	0.65	0.85	V
		I_C = 50 mA, I_B = 5.0 mA		0.95	V


QS
Tell me and I'll forget; show me and I may remember; involve me and I'll understand.
Chinese proverb
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