

**CHEG401 - Chemical Process Dynamics and Control**

**Lab 6 - Cascade Control and Feedforward Control**

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## 4.2 CASCADE CONTROL

### 4.2.1 Cascade Control Strategy for Temperature Control in Refrigerated Tank

#### 1. Block diagram and SIMULINK Implementation

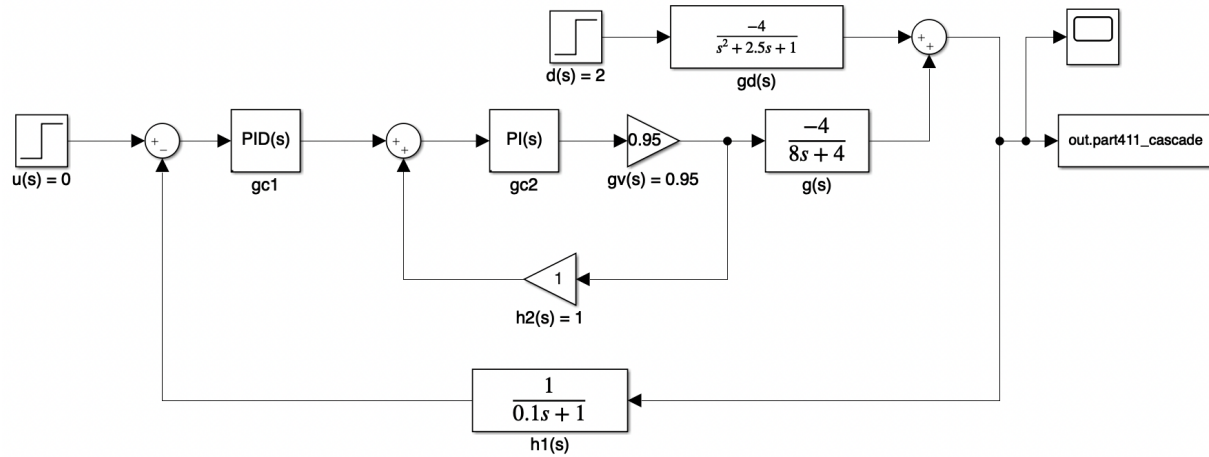


Figure 1. Simulink setup with cascade flow with a PI-controller.

#### 2. Cascade Controller Design and Implementation I. PI Control

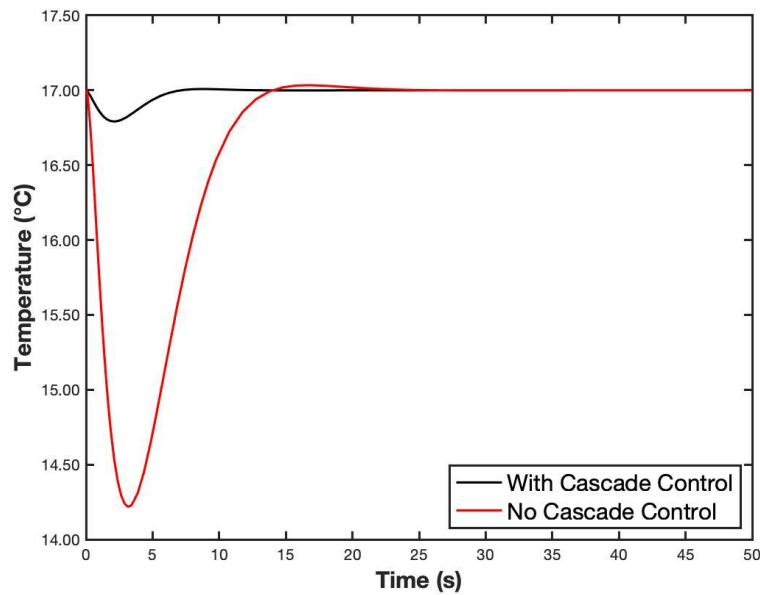


Figure 2. Comparison in temperature fluctuation around 17°C with and without cascade control.

For both system, the first controller,  $g_{c2}$ , is a PID controller with  $K_c = -1$ ,  $\tau_i = 2$ , and  $\tau_D = 1$ . To obtain the parameters for the second controller,  $g_{c1}$ , which operates as a PI controller, the overall system loop's characteristic equation should be determined first, as shown below.

$$g_{c_1} = -1 \left( 1 + \frac{1}{2s} + s \right), g_{c_2} = K_c, g = \frac{-4}{8s+4} = \frac{-1}{2s+1}, h_1 = \frac{1}{0.1s+1} = \frac{10}{s+10}, h_2 = 1$$

$$g_v = 0.95, g_{inner} = \frac{g_{c_2} g_v}{1 - g_{c_2} g_v h_2} \quad CLTF = \frac{g_{c_1} g_{inner} g}{1 + g_{c_1} g_{inner} g h_1}$$

$$\text{Characteristic equation} = (4 - 3.8K_c)s^3 + (42 - 20.9K_c)s^2 + 20s + 9.5K_c$$

By direct substitution of  $s = wj$ , and equating the characteristic equation to 0;

$$\left[ 20w - w^3(4 - 3.8K_c) \right]j + \left[ 9.5K_c - w^2(42 - 20.9K_c) \right] = 0$$

$$\text{Imaginary: } 20w - w^3(4 - 3.8K_c) = 0, \text{ Real: } 9.5K_c - w^2(42 - 20.9K_c) = 0$$

By simultaneous equation, combination of  $w$  and  $K_c$  are as shown in Table 1.

**Table 1.** Combination of values of  $w$  and  $K_c$ .

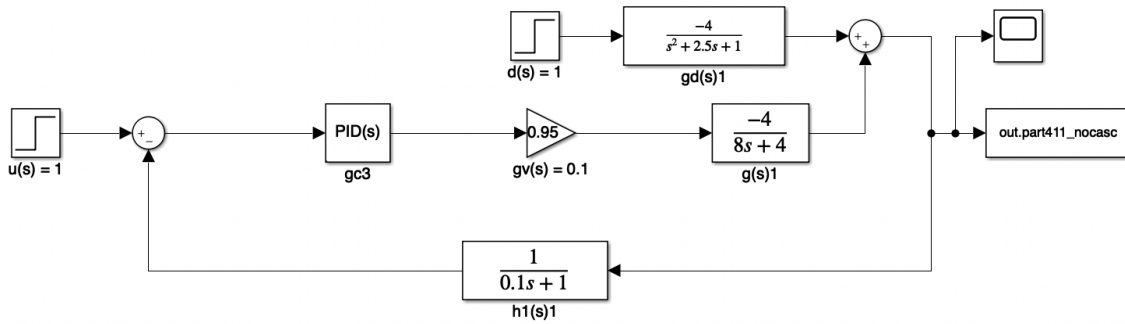
$w$	$K_c$
$0 + 0.0000i$	0.0000
$0 - 2.1063i$	2.2390
$0 + 2.1063i$	2.2390
$0 - 0.7507i$	10.3926
$0 + 0.7507i$	10.3926

Using the real part of  $w$  (0), and either 2.239 or 10.3926 for  $K_c$ , the optimum value for  $K_c$  and  $\tau_i$  of the PI controller can be found by using Ziegler-Nichols stability margin controller tuning parameters.

Starting with  $K_{cu} = 2.239$ ,  $K_c = 0.45K_{cu} = 0.45 \times 2.239 = 1.00755$

$$P_u = \frac{2\pi}{w_c} = \frac{2\pi}{0} \approx \infty, \tau_I = \frac{P_u}{1.2} = \infty$$

Using the parameters obtained here and inputting into Simulink, at the same time employing system simulation with no cascade control, the response in temperature obtained as shown in **Figure 2**.



**Figure 3.** Simulink setup with no cascade flow, using PID controller from Lab 4.

Observing the response in **Figure 2**, the temperature fluctuates less around 17°C for the system with cascade flow. It can be said that cascade flow to control the valve and temperature helps to regulate the temperature.

### 3. Cascade Controller Design and Implementation II. P Control

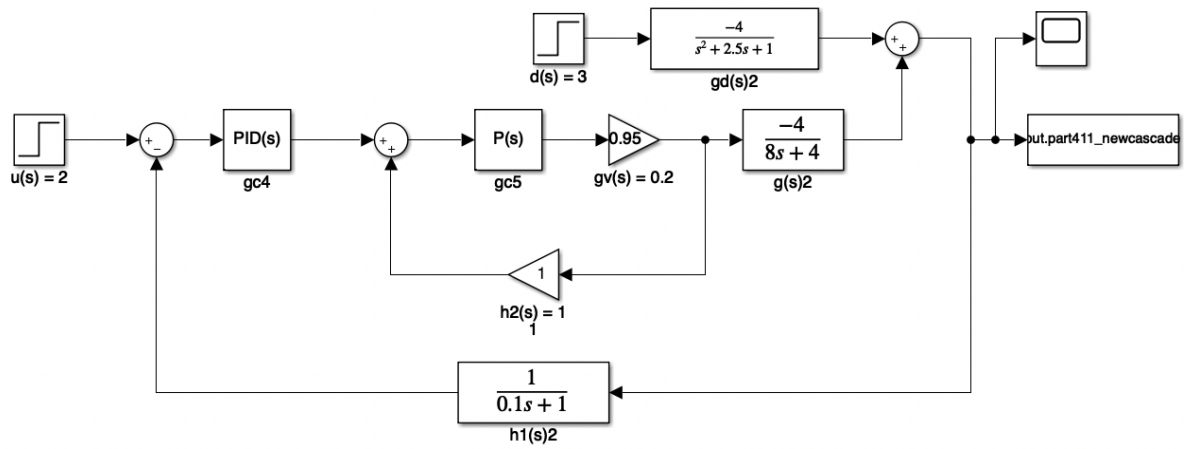


Figure 4. Simulink setup with cascade flow with a P-controller.

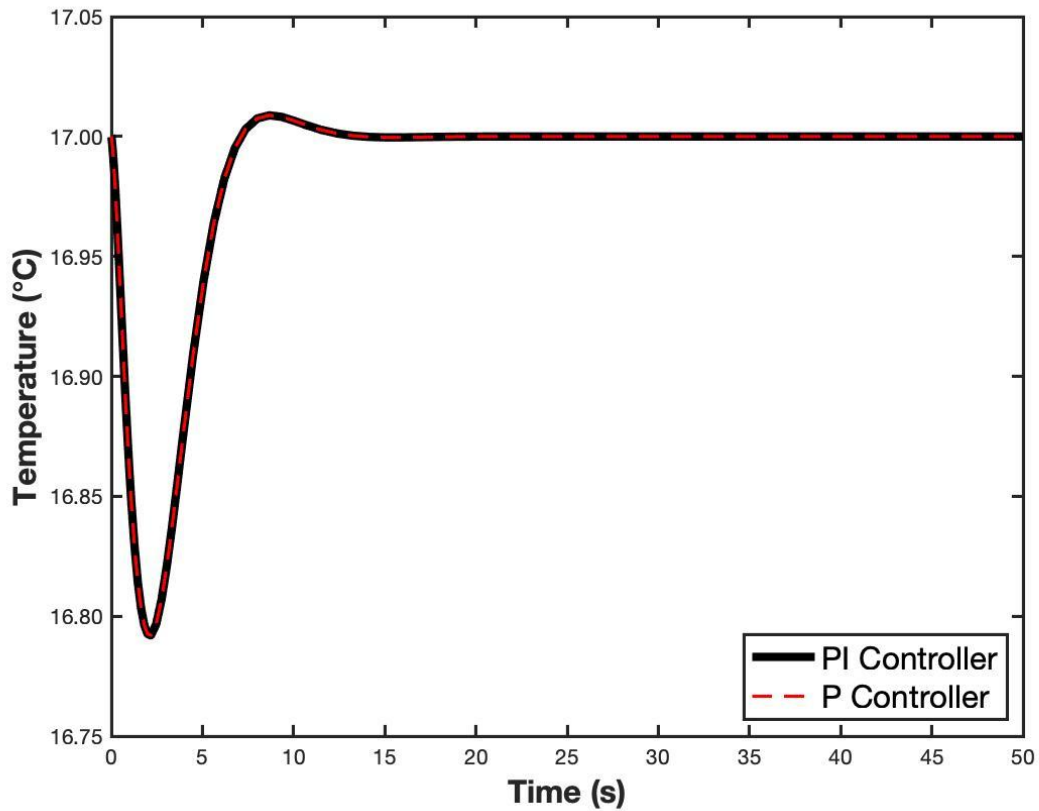
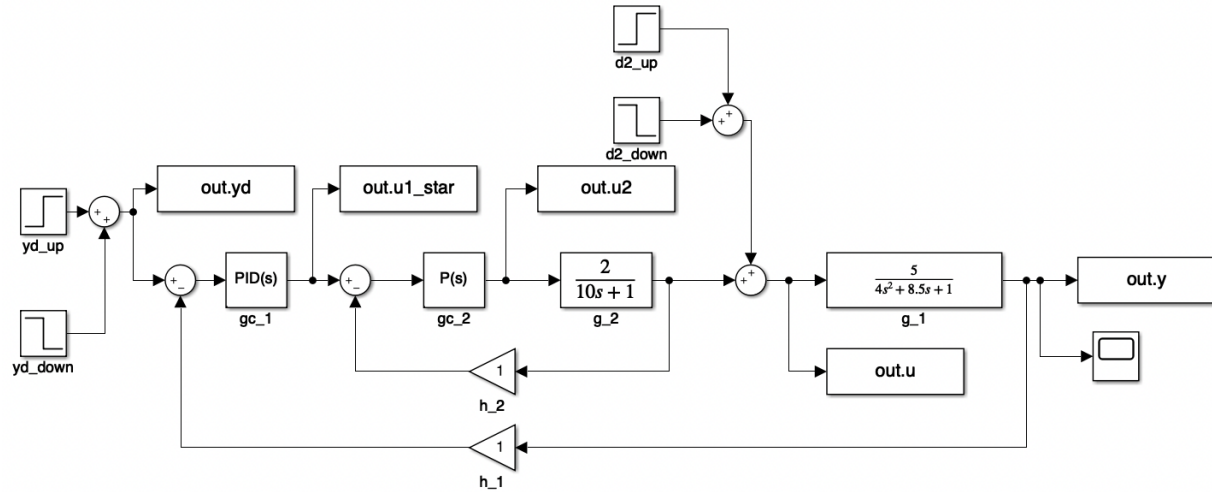


Figure 5. Comparison in temperature fluctuation around 17°C with a PI and P controller.

The response shows no difference in fluctuation when the controller used for the cascade control is a PI or P-type. In the cascade flow, there is no offset observed when using the P-controller. P controllers always result in offsets, however since it is used in an inner loop, it will get corrected by the controller at the outer loop. Since a PID controller is used, no offset will be observed.

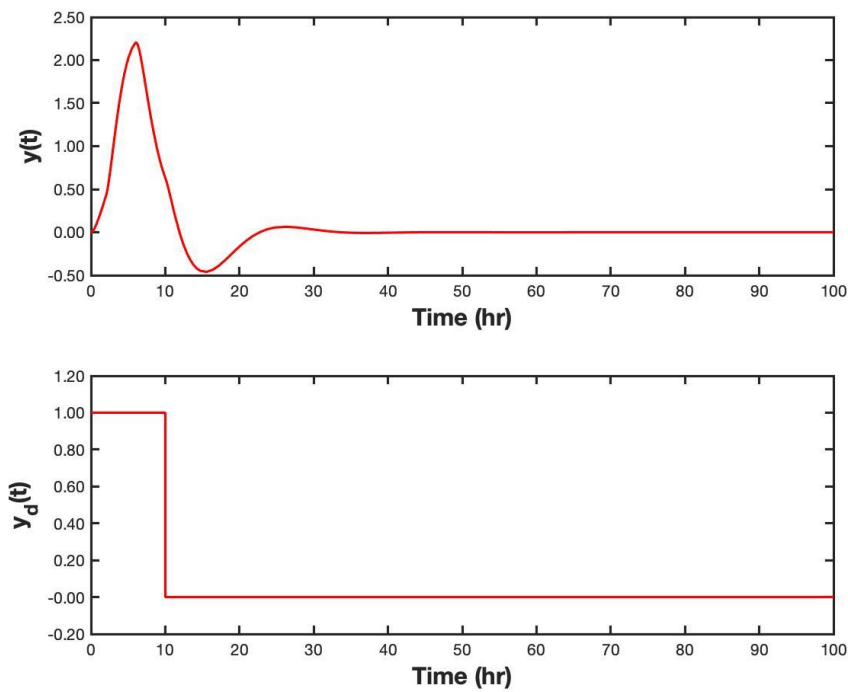
## 4.2.2 Cascade Control of Cortisol Production for Cell Metabolic Rate Control

### 1. Block diagram and SIMULINK Implementation

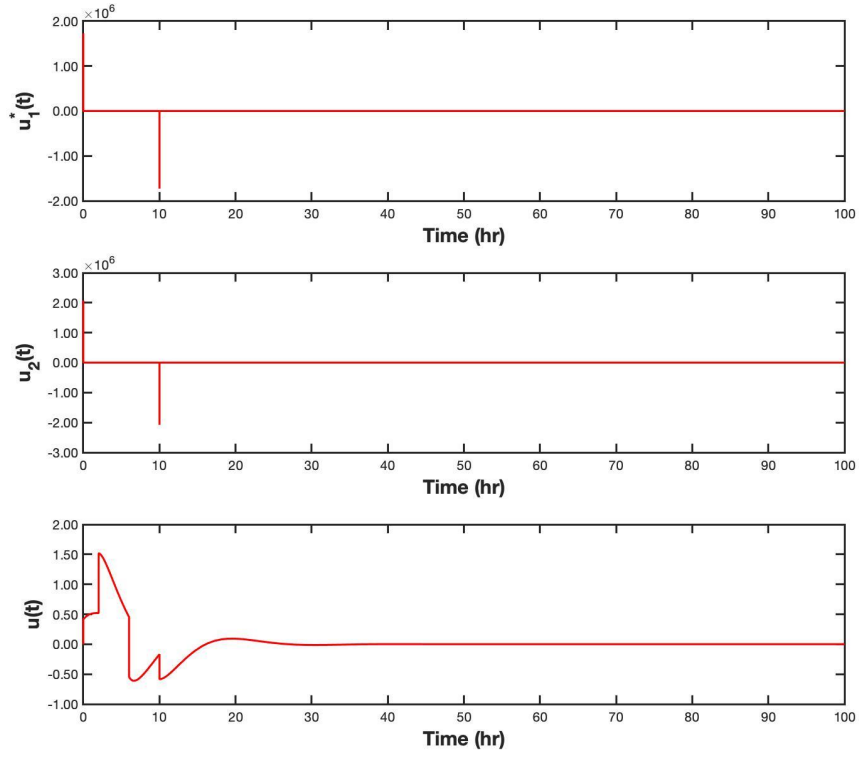


**Figure 6.** Simulink setup for cell metabolic rate control with cascade control.

### 2. Cascade Control System Design and Implementation



**Figure 7.** Output responses with time when employing cascade control.



**Figure 8.** Input changes with time when employing cascade control.

To design the controller in a systematic way, one should consider tuning the controller in the inner loop first (by ignoring the presence of the outer loop).

$$\text{CLTF of inner loop, } g_{in}: \frac{g_{c_2} g_2}{1 + g_{c_2} g_2 h_2} = \frac{2K_{c,in}}{10s + 1 + 2K_{c,in}}$$

$$g_{c_2} = K_{c,in}, g_2 = \frac{2}{10s + 1}, h_2 = 1 \quad \text{Characteristic equation: } 10s + 1 + 2K_{c,in} = 0$$

Without choosing the numbers for  $K_{c,in}$  just yet.

Proceed by considering the outer loop next.

$$\text{CLTF of overall system: } \frac{g_{c_1} g_{in} g_1}{1 + g_{c_1} g_{in} g_1 h_1}, g_{c_1} = K_{c,out}, g_1 = \frac{5}{4s^2 + 8.5s + 1}, h_1 = 1$$



Characteristic equation of the overall system's CLTF:

$$(4s^2 + 8.5s + 1)(10s + 1 + 2K_{c,in}) + 10K_{c,out}K_{c,in} = 0$$

$$40s^4 + (4 + 8K_{c,in})s^3 + 85s^2 + (18.5 + 17K_{c,in})s + (2K_{c,in} + 10K_{c,out}K_{c,in} + 1) = 0$$

By direct substitution of  $s = jw$  and rearranging the real and imaginary terms;

$$\text{Imaginary: } w(18.5 + 17K_{c,in}) - w^3(4 + 8K_{c,in}) = 0$$

$$w = 0; w = \pm \sqrt{\frac{18.5 + 17K_{c,in}}{4 + 8K_{c,in}}}$$

Since  $w$  cannot be zero and should be positive,  $K_{c,in}$  should be more than  $-1.088$

$$\text{Real: } 40w^4 - 85w^2 + 2K_{c,in} + 10K_{c,out}K_{c,in} + 1 = 0$$

$$K_{c,out} = \frac{85w^2 - 40w^4 - 1 - 2K_{c,in}}{10K_{c,in}}$$

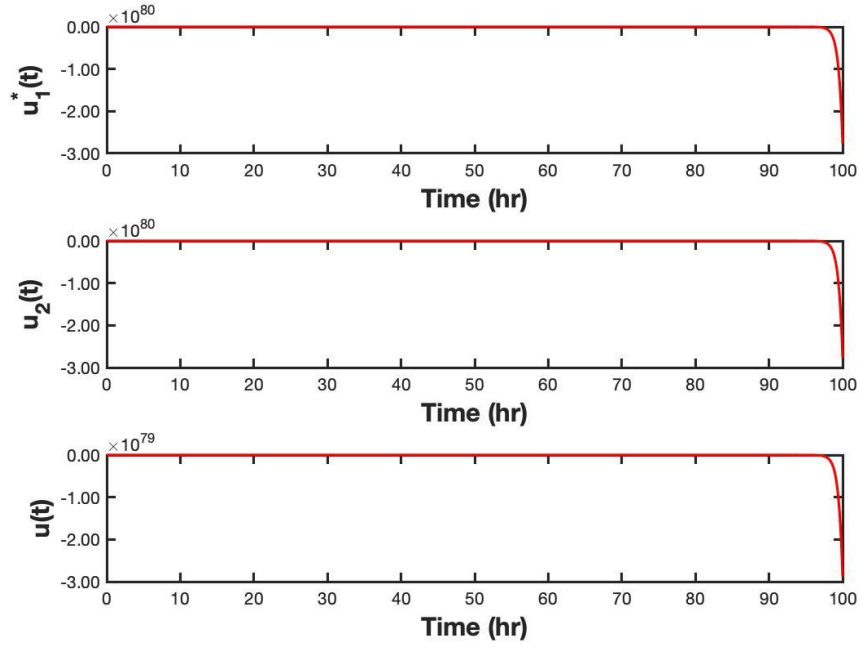
From Ziegler-Nichols stability margin controller tuning parameters, and using the analysis by only considering the inner loop,  $K_{cu,in} > -0.5$ ,  $K_{cu,in} > -0.25$ . Taking into account both constraints of  $K_{cu,in}$  that we have, choose  $K_{c,in} = 1.2$ .

$$w = 0.5825, K_{cu,out} = 2.1362$$

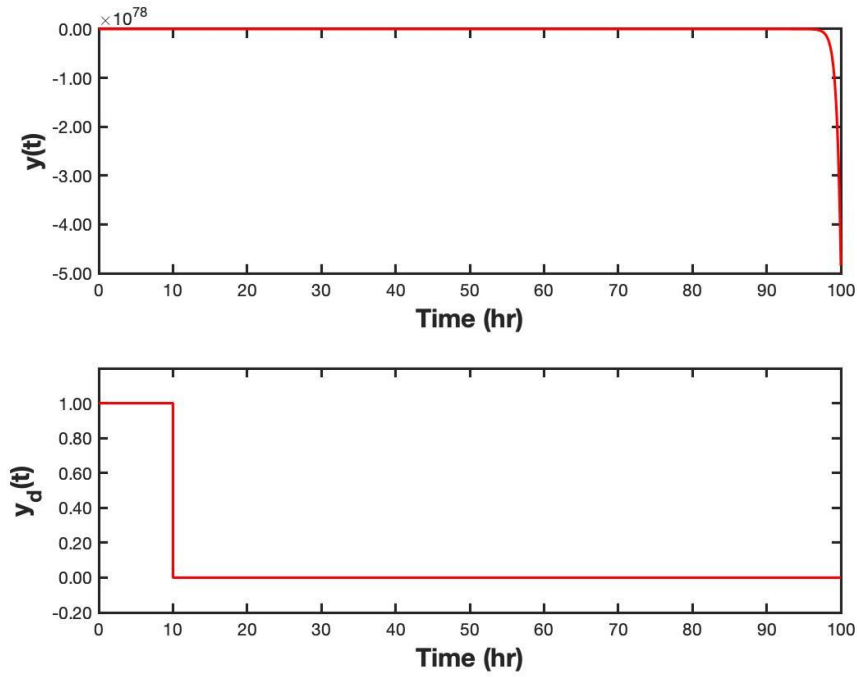
After tuning for PID controller for the outer controller,  $K_{c,out} = 0.6K_{cu,out} = 1.2871$

$$P_u = \frac{2\pi}{w_c} = 10.79, \tau_I = \frac{P_u}{2} = 5.39, \tau_D = \frac{P_u}{8} = 1.35$$

### 3. Feedback-Only Control with No Inner Cortisol Control Loop



**Figure 9.** Output changes with time with  $h_2 = 0$  and  $g_{c2} = 1$ .



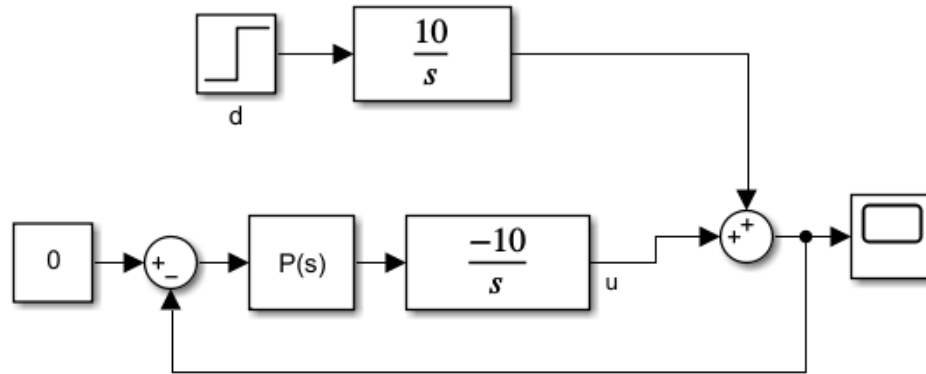
**Figure 10.** Input changes with time with  $h_2 = 0$  and  $g_{c2} = 1$ .

It was hypothesized that without the presence of cascade control i.e. removing the inner loop from the previous part, the system will have an unstable response. It was proven as shown in **Figure 9 and 10**, where both responses and inputs were declining dramatically with time. From **Figure 7 and 8** on the other hand, the system was stable with small fluctuations in output. The inner loop helps to maintain the stability of the system and achieve the primary goal of having such a system.

## 4.3 FEEDFORWARD CONTROL

### 4.3.1 Feedback and Feedforward Control in a Reflux Drum

#### 1. SIMULINK Implementation

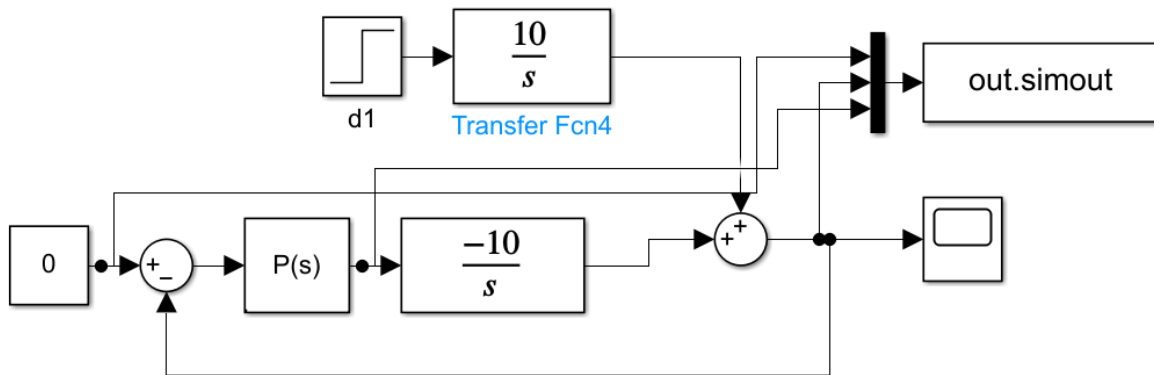


**Figure 11.** System configuration with feedback controller using proportional-only controller.

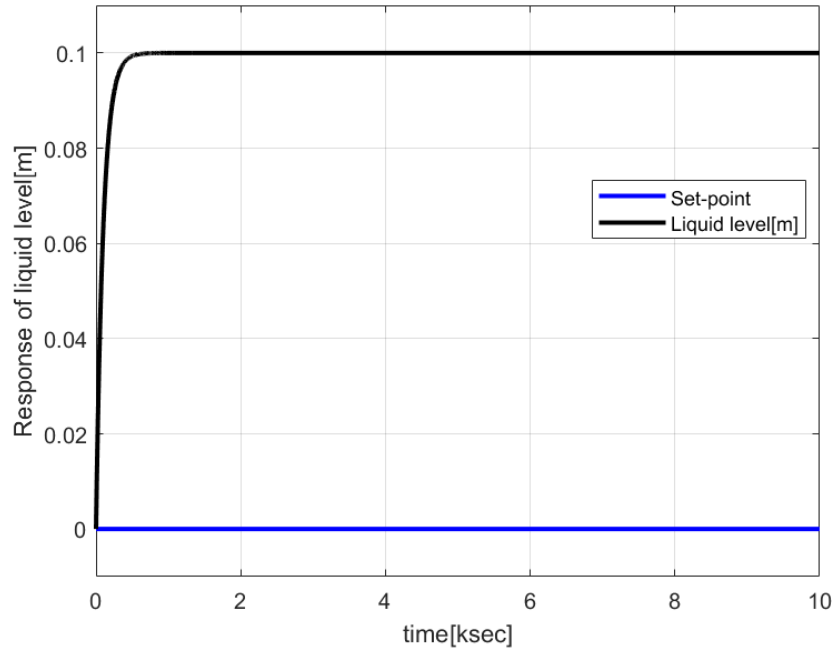
#### 2. Feedback Control

The characteristic equation:  $1 + gg_c h = 0$  which gives  $s - 10K_c = 0$

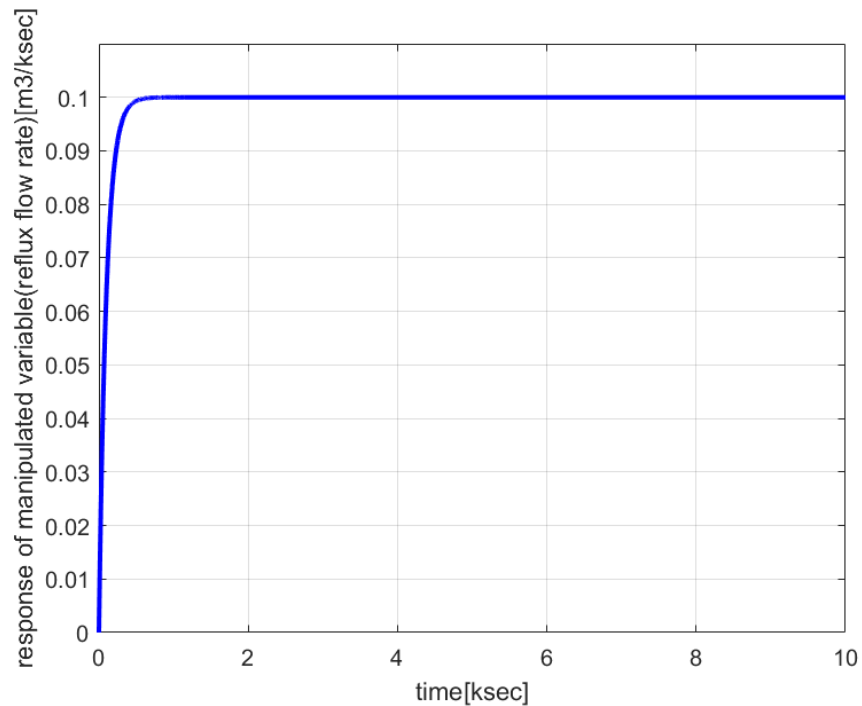
By using the Routh stability test,  $-10K_c > 0$ ,  $K_c < 0$ . Thus, we picked  $K_c = -1$  and got  $g_c = K_c = -1$ .



**Figure 12.** Simulink system configuration with feedback controller and a step change of 0.1 m<sup>3</sup>/ksec in the liquid inflow rate( $F_i$ ) using proportional-only controller.



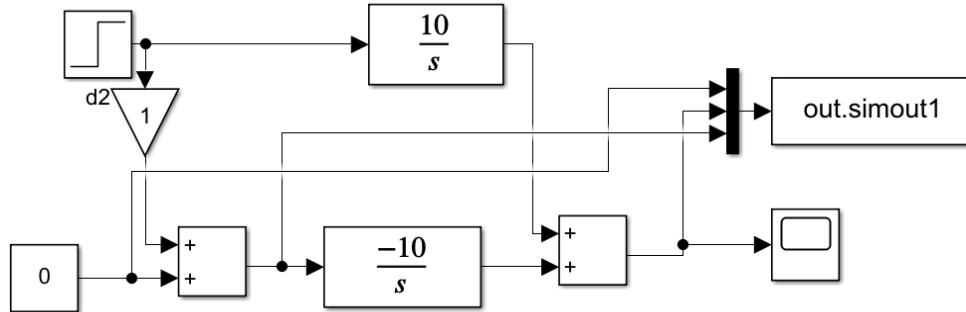
**Figure 13.** Response of the liquid level and the liquid set-point in the reflux drum when inlet flow rate has a step change of  $0.1\text{m}^3/\text{ksec}$  (with proportional-only feedback controller,  $K_c=-1$ ).



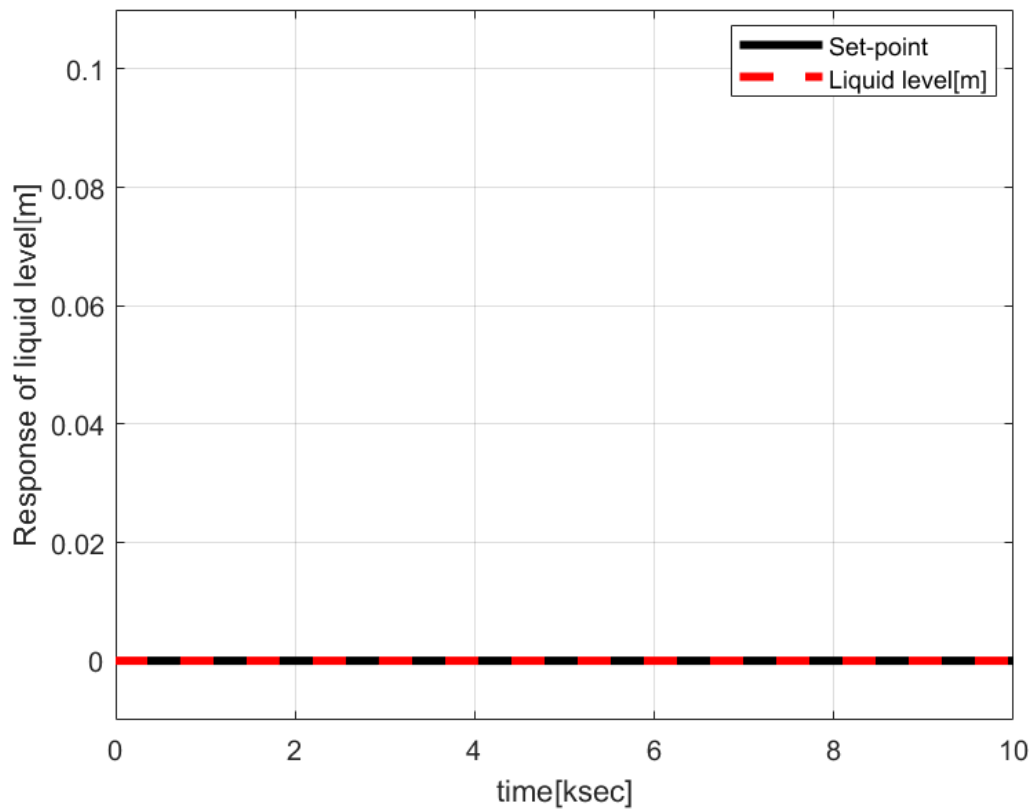
**Figure 14.** Response of the reflux flow rate when inlet flow rate has a step change of  $0.1\text{m}^3/\text{ksec}$  (with proportional-only feedback controller,  $K_c=-1$ ).

### 3. Feedforward Control

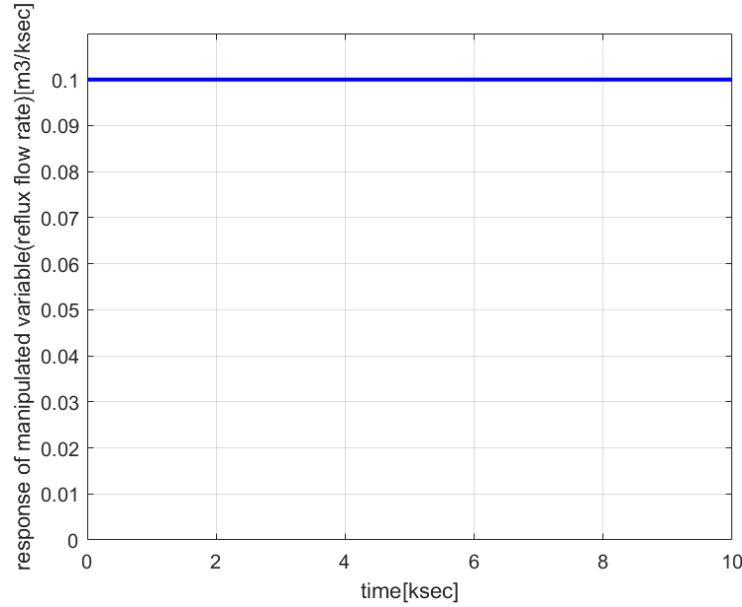
Design  $g_{ff}(s) = -g_d(s)/g(s) = 1$



**Figure 15.** System configuration with feedforward controller.



**Figure 16.** Response of the liquid level and the liquid set-point in the reflux drum when inlet flow rate has a step change of  $0.1 \text{ m}^3/\text{ksec}$  (with feedforward controller,  $g_{ff}=1$ ).



**Figure 17.** Response of the reflux flow rate when inlet flow rate has a step change of  $0.1 \text{ m}^3/\text{ksec}$  (with feedforward controller,  $g_{ff}=1$ ).

When using a P-controller in a feedback control system with  $K_c=-1$ , the liquid level increases in response to the step change of disturbance, then converges to a steady state under the influence of the feedback controller. An offset of  $0.1 \text{ m}$  for the liquid level, away from the setpoint due to the P-controller. While in a feedforward control system, we already know the exact amount of changes in disturbance, thus we designed a very good feedforward control, with no offset away from setpoint in steady state. The liquid level response is not affected by the changes in disturbance because the feedforward transfer function has removed the effect since the beginning. In both feedforward and feedback control systems, there is  $0.1 \text{ m}^3/\text{ksec}$  in the manipulated variable to account for the changes in the system brought by the disturbance changes. The feedback augmentation is not necessary here, because the step change in disturbance given in the prompt is a fixed value and it is the only change in the system. With the information given, we can tune the system to remove the effect caused by the step change completely.

### 4.3.2 Feedforward Control in a Refrigerated Tank I

From Lab 4:  $y(s) = \frac{A}{Bs+C}u(s)$ ,  $A = -4$ ,  $B = 8$ ,  $C = 4$

$$g_d(s) = \frac{2.0}{(5s+1)(0.5s+1)}$$

$$y(s) = \frac{A}{Bs+C}u(s) + \frac{2.0}{(5s+1)(0.5s+1)}d(s)$$

#### 1. SIMULINK Implementation

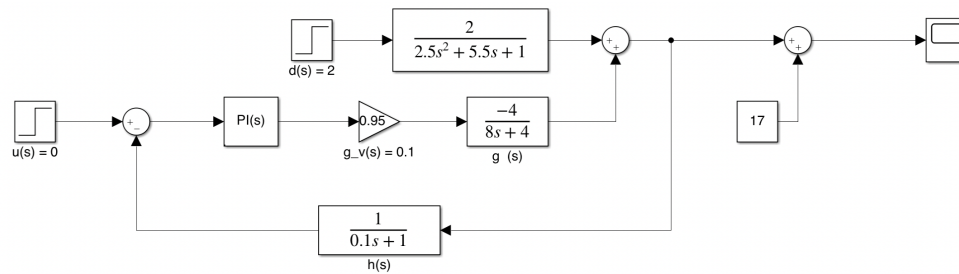


Figure 18. SIMULINK for PI controller with  $K_c = -6$ ,  $\tau_1 = 1$  s (from Lab 4)

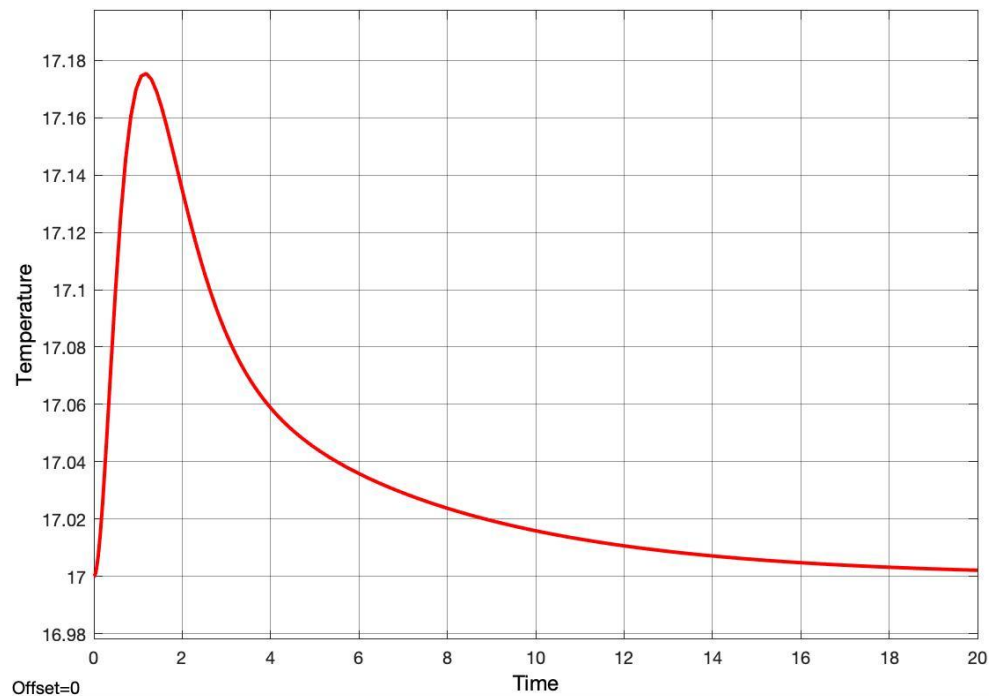


Figure 19. MATLAB output for PI controller



When  $K_c = -6$ ,  $\tau_I = 1$ s, the temperature increase to about 17.17 °C, and then decrease to reach steady state after 1.14s.

## 2. Feedforward Controller Design

$$y(s) = g(s) u(s) + g_d(s) d(s)$$

$$g(s) = -\frac{4}{8s+4}, \quad g_d(s) = \frac{2}{2.5s^2+5.5s+1}$$

$$u(s) = \frac{1}{g(s)} y_d - \frac{g_d(s)}{g(s)} d(s)$$

$$u(s) = g_{st}(s) y_d + g_{ff}(s) d(s)$$

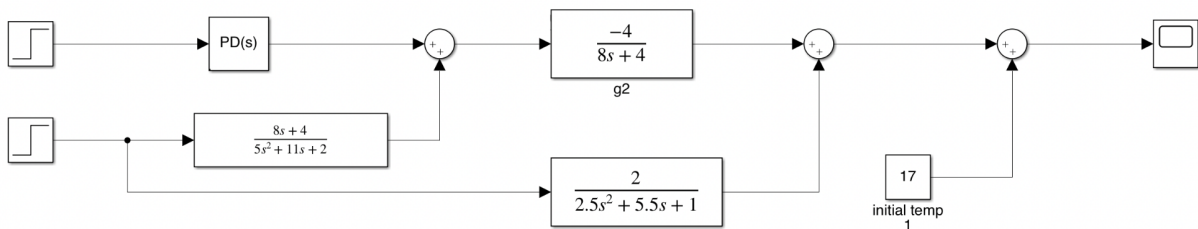
$$g_{st} = \frac{1}{g(s)} = \frac{8s+4}{-4} = -2s-1$$

$$g_{ff} = -\frac{g_d(s)}{g(s)} = \frac{-2}{2.5s^2+5.5s+1} \left( \frac{8s+4}{-4} \right) = \frac{8s+4}{5s^2+11s+2}$$

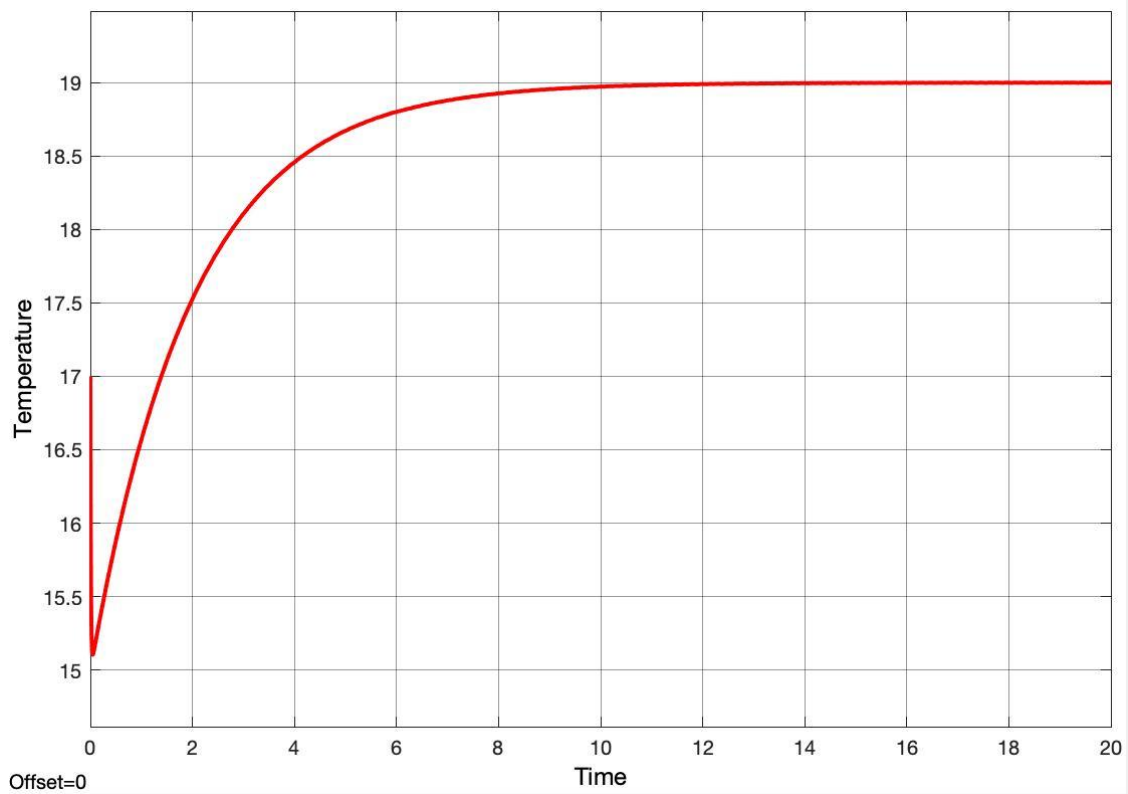
$$u(s) = (-2s-1) y_d + \left( \frac{8s+4}{5s^2+11s+2} \right) d(s)$$

The feedforward controller is realizable because  $g(s)$  and  $g_d(s)$  have no time delay elements. The  $g_{st}$  and  $g_{ff}$  are not complicated. All poles are on LHP, the system is stable.

## 3. Feedforward Controller Implementation



**Figure 20.** SIMULINK setup for feedforward controller



**Figure 21.** MATLAB output of feedforward controller

*Discuss the accept-ability of the control system performance under these conditions*

When the ambient temperature increases by 2 °C, the temperature sharply decreases to 15.1015 °C and then increases back and reaches a steady state at 19 °C. The feedforward controller enables perfect rejection of disturbance.

### 4.3.3 Feedforward Control in a Refrigerated Tank II

$$\widehat{g}_d(s) = \frac{1.8}{(5.3s+1)(0.3s+1)}$$

#### 1. Feedforward Controller Implementation

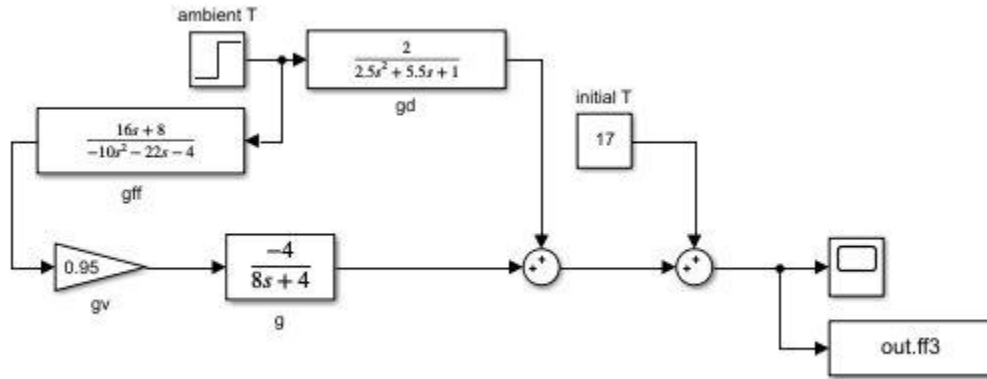


Figure 22. SIMULINK for actual disturbance

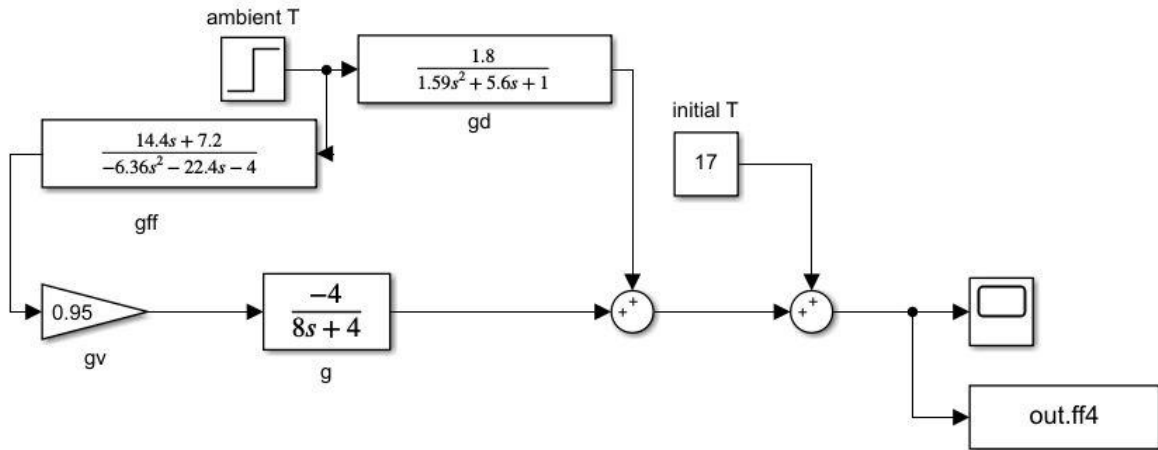
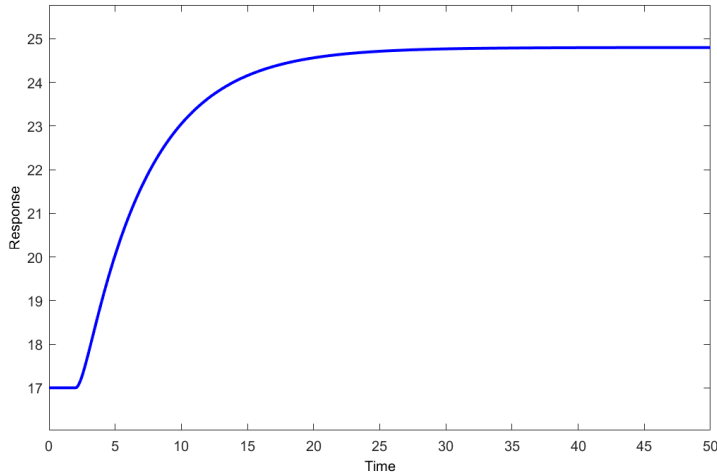
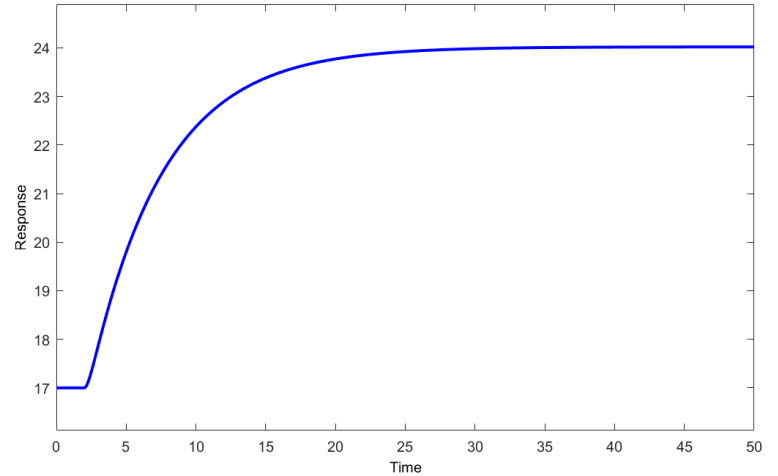


Figure 23. SIMULINK for implementing  $\widehat{g}_d(s)$



(a) Modeled by actual disturbance

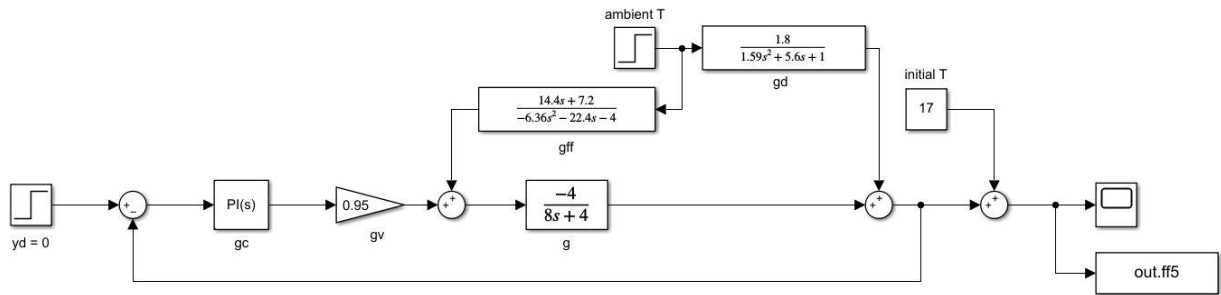


(b) Modeled by  $\hat{g}_d(s)$

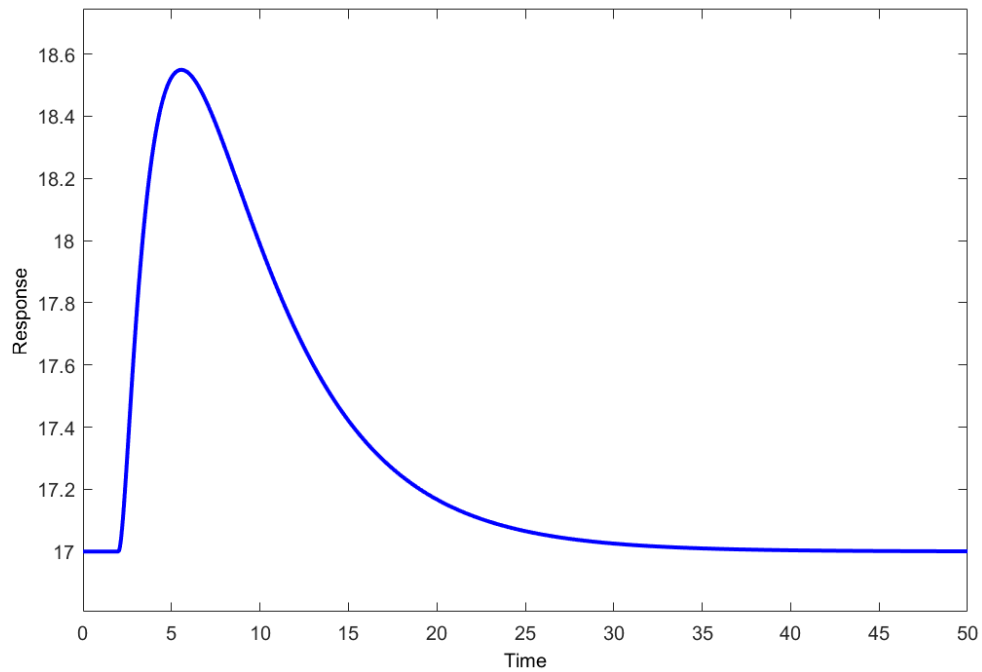
**Figure 24. Temperature of the Refrigerated Tank II**

The two responses do not show much differentiation. The steady state temperature modeled by actual disturbance is 25 degree C, and by  $\hat{g}_d(s)$  is 24 degree C. There is only 1 °C difference between the two models. Furthermore, comparing this result with part 4.3.2, the behavior of the responses do not show any overshoot, or inverse response, or any odd behavior like the inverse response in 4.3.2. However, the steady state temperatures are much higher than the previous part's and are not within the limit of operation (15-20 ° C - lab 4), therefore, this might pose a threat of explosion.

## 2. Feedback Augmentation



**Figure 25.** SIMULINK for Feedback Augmentation



**Figure 26.** Temperature versus Time for Feedback Augmentation (PI parameters:  $K = -1$ ,  $\tau_I = 2$ , from lab 4)

The response has a sharp increase from 17 to 18.6 °C and then gradually decreases to the initial temperature of 17 degree C. There is no steady state offset for this response. In this case, it is beneficial to use the feedback control to augment the feed forward control.