CHEG401 - Chemical Process Dynamics and Control

Lab 2 - Open Loop Process Dynamics

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2.1 PART I: LOW-ORDER SYSTEMS

2.1.1 General Characteristics

Table 1. Characteristics of three different transfer functions.

Transfer function	Steady state gain (K)	Time constant (τ)
$\frac{10}{5s+1}$	10	5
$\frac{5}{10s+1}$	5	10
$\frac{2}{s}$	2	1

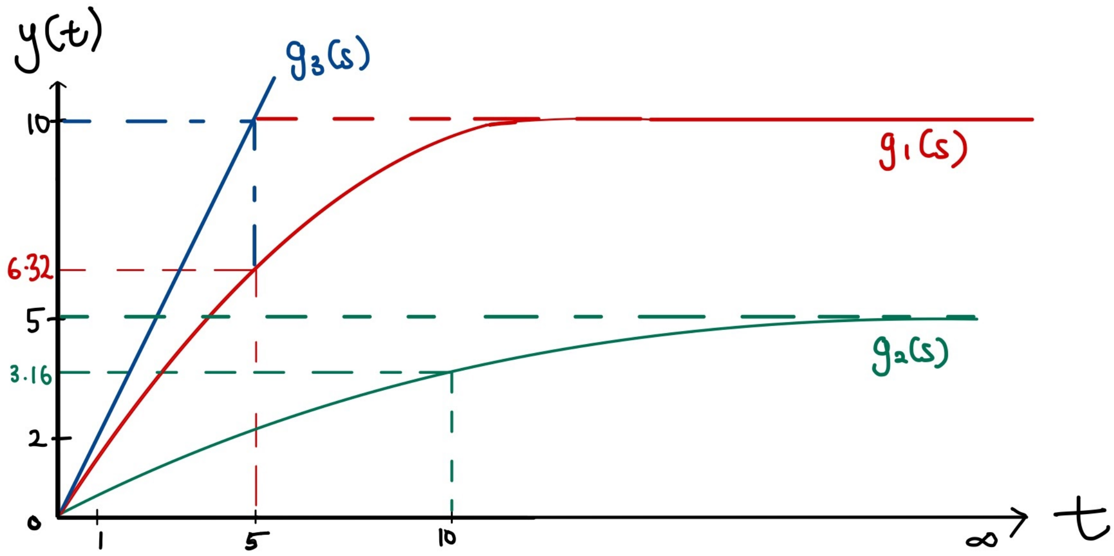

Figure 1. Sketch of step response to a unit change in the input.

2.1.2 Specific Systems I: A Chemical Reactor

1. ***Mathematical Model: Assuming constant volumetric flow rate, F.***

$$\frac{d(VC_A(t))}{dt} = FC_{Af} - FC_A - kVC_A \qquad (1)$$

$$Define:\ y(t) = C_A - C_A^*,\ u(t) = C_{Af} - C_{Af}^*$$

$$At\ steady\ state:\ 0 = FC_{Af}^* - FC_A^* - kVC_A^* \qquad (2)$$

$$(1) - (2):\ V\frac{dy(t)}{dt} = Fu(t) - Fy(t) - kVy(t)$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = V[sY(s) - y(t=0)] = FU(s) - FY(s) - kVY(s)$$

$$Y(s)\,[Vs + F + kV] = Vy(t=0) + FU(s)$$

$$Y(s) = \frac{y(t=0) + FU(s)}{Vs + F + kV},\ y(t=0) = 0$$

$$Y(s) = \frac{\frac{F}{F+kV}}{\frac{V}{F+kV}s + 1} \cdot U(s)$$

$$Y(s) = g(s) \cdot U(s)$$

$$g(s) = \frac{\frac{F}{F+kV}}{\frac{V}{F+kV}s + 1},\ K = \frac{F}{F+kV},\ \tau = \frac{V}{F+kV}$$

2. ***Dynamic Analysis in SIMULINK I***

$$V = 20\,m^3,\ k = 1.5 \times 10^{-3}\,ksec^{-1},\ F = 0.5\,m^3/ksec$$

$$Y(s) = \frac{\frac{50}{53}}{\frac{2000}{53}s + 1}U(s)$$

From Eq. 2 above, C^*_A from the supplied reactor = 84.91 mol/m^3

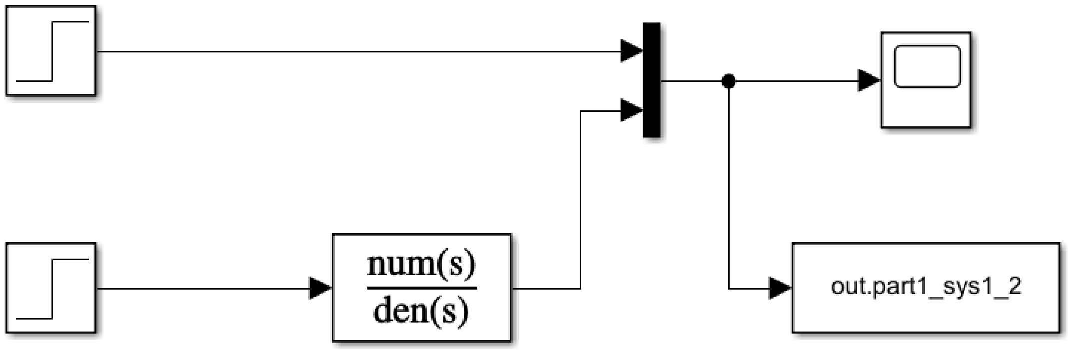

Figure 2. SIMULINK setup for response of C_A to a step change in C_{Af} from an initial steady state value of $C^*_{Af} = 90$ moles/m^3 to $C^*_{Af} = 100$ moles/m^3.

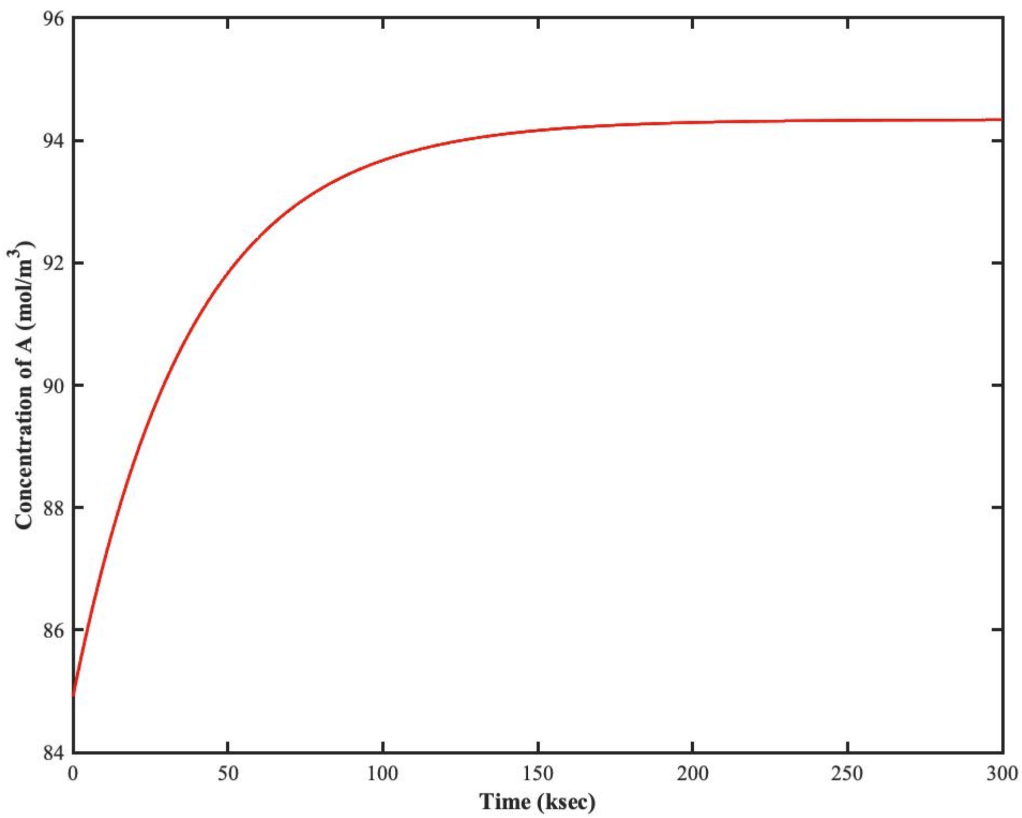

Figure 3. MATLAB plots for response of C_A to a step change in C_{Af} from an initial steady state value of $C^*_{Af} = 90$ moles/m^3 to $C_{Af} = 100$ moles/m^3.

- ***Discuss its key characteristics in terms of the relationship between the standard parameters in Eq. (2.5) and the reactor's physical characteristics***

$y(s) \;=\; \frac{K}{\tau s+1}\, u(s)$

$K \;= \frac{50}{53} = 0.9434$. The initial concentration of A in the outflow stream, C^*_A was 84.91 mol/m^3. With a steady state gain of 0.9434 and a step input of M = 10, the new steady state concentration should be 84.91 + 0.9434*10 = 94.34 mol/m^3, which could also be observed in **Figure 3**.

$\tau \;= \frac{2000}{53} \;=\; 37.74$. For a first-order system, the response at t = τ is at 0.632KM. For K = 0.9434 and M = 10, the response is found to be 5.962 mol/m^3. With C^*_A = 84.91 mol/m^3, C_A at t = τ will be 90.87 mol/m^3. From **Figure 3**, the system reached this concentration after approximately 40 ksecs, which is close to the actual τ value of 37.74 ksecs.

3. *Dynamic Analysis in SIMULINK II*

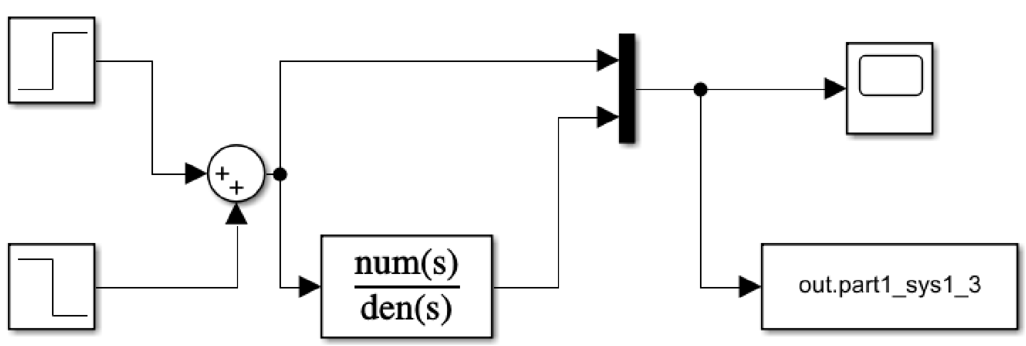

Figure 4. SIMULINK setup for response of C_A to a rectangular pulse input in the initial concentration from the same initial steady state value of C^*_{Af} = 90 moles/m^3 to C_{Af} = 100 moles/m^3.

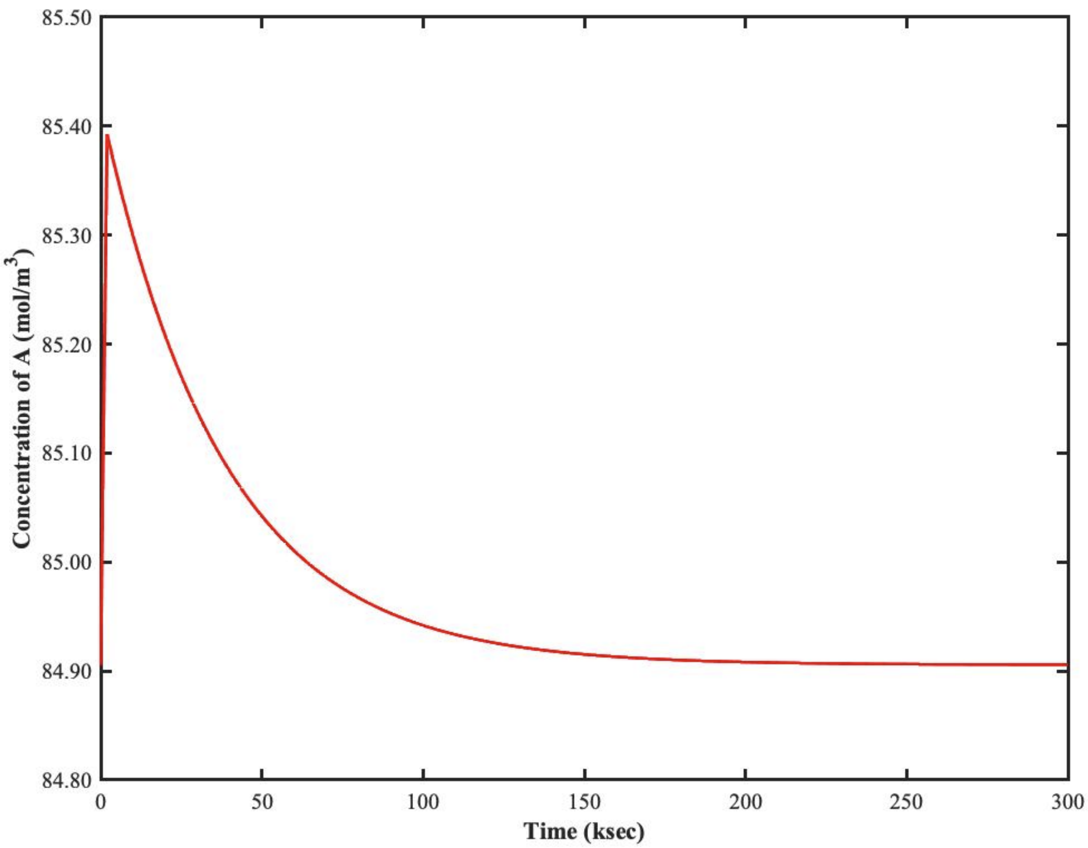

Figure 5. MATLAB plot for response of C_A to a rectangular pulse input in the initial concentration from the same initial steady state value of $C^*_{Af} = 90$ moles/m^3 to $C_{Af} = 100$ moles/m^3.

- ***Determine the maximum concentration of C_A^{max} attained by the reactant A in the reactor.***

  The maximum concentration $C_A^{max} = 85.39$ mol/m^3

2.1.3 Specific Systems II: Reflux Drum

1. ***Mathematical Model***

$$A_c \frac{dh(t)}{dt} = F_i(t) - F_R(t) - F_D(t) \qquad (1)$$

$Define$: $y(t) = h(t) - h^*$, $d(t) = F_i(t) - F_i^*$, $u_1(t) = F_R(t) - F_R^*$, $u_2(t) = F_D(t) - F_D^*$

$$At\ steady\ state:\ 0 = F_i^* - F_R^* - F_D^* \qquad (2)$$

$$(1) - (2):\ A_c \frac{dy(t)}{dt} = d(t) - u_1(t) - u_2(t)$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = A_c[sY(s) - y(t=0)] = d(s) - u_1(s) - u_2(s),\ y(t=0) = 0$$

$$Y(s)\,[A_c s] = d(s) - u_1(s) - u_2(s)$$

$$Y(s) = -\frac{1}{A_c s}u_1(s) - \frac{1}{A_c s}u_2(s) + \frac{1}{A_c s}d(s)$$

$$g_1(s) = -\frac{1}{A_c s},\ g_2(s) = -\frac{1}{A_c s},\ g_d(s) = \frac{1}{A_c s}$$

- ***What type of system is this reflux drum?***

  This system is a first-order, pure capacity system. Because the process is modeled by equation $\frac{dy(t)}{dt} = \frac{1}{A_c}d(t) - \frac{1}{A_c}u_1(t) - \frac{1}{A_c}u_2(t)$, this is the pure capacity system as the general form for a pure capacity system is: $\frac{dy(t)}{dt} = K \cdot u(t)$

2. ***Dynamic Analysis in SIMULINK***

$h^* = 0.5$ m, $F_i^* = 0.5\ m^3$/ksec, $F_R^* = 0.3\ m^3$/ksec, $F_D^* = 0.2\ m^3$/ksec

A step change in F_i from 0.5 to 0.6 m^3/ksec m → $d(s) = \frac{0.1}{s}$. $u_1(s) = u_2(s) = 0.$

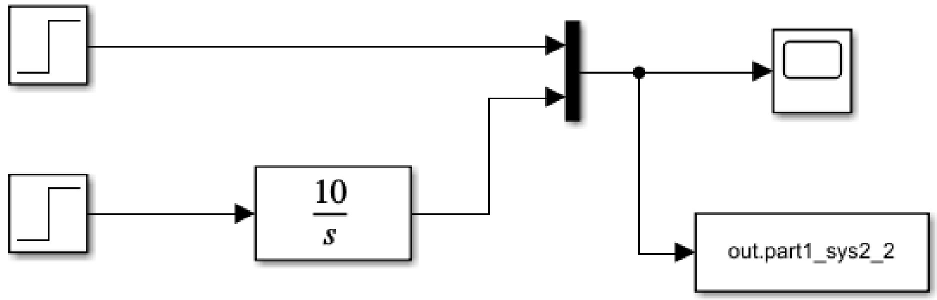

Figure 6. SIMULINK setup for a step change in F_i from 0.5 to 0.6 m^3/ksec for reflux drum.

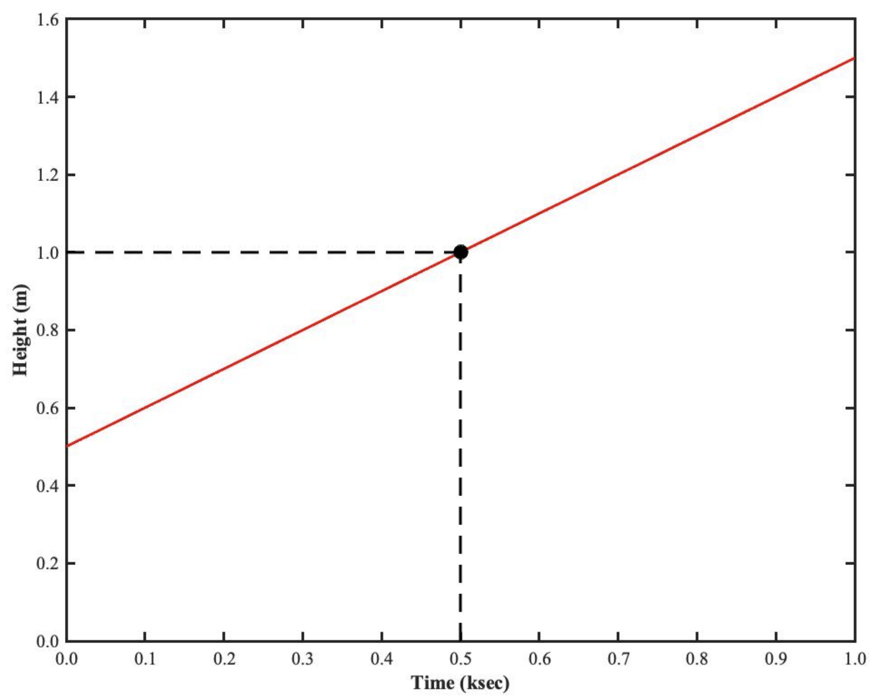

Figure 7. MATLAB plot for a step change in F_i from 0.5 to 0.6 m^3/ksec for reflux drum.

- ***From the plot, determine how long before the liquid level in the reflux drum reaches the maximum limit of H = 1 m.***

  After 0.5 ksec, the liquid level in the reflux drum reaches H = 1 m.

- ***Next, return to the transfer function model in Eq. (4.15) and obtain from it the theoretical response of h(t) to the indicated step change in d(t). From the theoretical response, predict by what time the liquid level in the reflux drum will reach its maximum limit. Compare your theoretical prediction with the result obtained from the SIMULINK simulation plot.***

$$Y(s) \;=\; \frac{1}{A_c s}\left[d(s) \;-\; u_1(s) \;-\; u_2(s)\right]$$

$$d(s) \;=\frac{0.1}{s};\; u_1(s) \;= u_2(s) \;=\; 0.\;;\; A_c \;=\; 0.1$$

$$Y(s) \;=\frac{1}{s^2};\; \mathcal{L}\{Y(s)\} = y(t) \;=\; t \qquad (3)$$

From the previous part, using the interpretation from the plot, the time when the liquid level reaches H = 1 m is at t = 0.5 ksec. Using t = 0.5 ksec and plugging into y(t) = t, y(t) = 0.5 m. As y(t) = H - H* and H* = 0.5 m, H (t = 0.5 ksec) = 1 m. This theoretical prediction is consistent with the results obtained from the SIMULINK plot.

3. *Exploring an Alternative Operation Strategy*

$$F_R(t) = F_R^*(t) + K_c[h(t) - h^*],\ K_c = 1$$

$$u_1(t) = K_c y(t);\ A_c \frac{dy(t)}{dt} = d(t) - K_c y(t) - u_2(t)$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = A_c sY(s) = d(s) - K_c y(s) - u_2(s)$$

$$Y(s)\,[A_c s + K_c] = d(s) - u_2(s)$$

$$Y(s) = \frac{1}{A_c s + K_c}[d(s) - u_2(s)]$$

$$u_2(s) = 0;\ Y(s) = \frac{1}{A_c s + K_c} d(s)$$

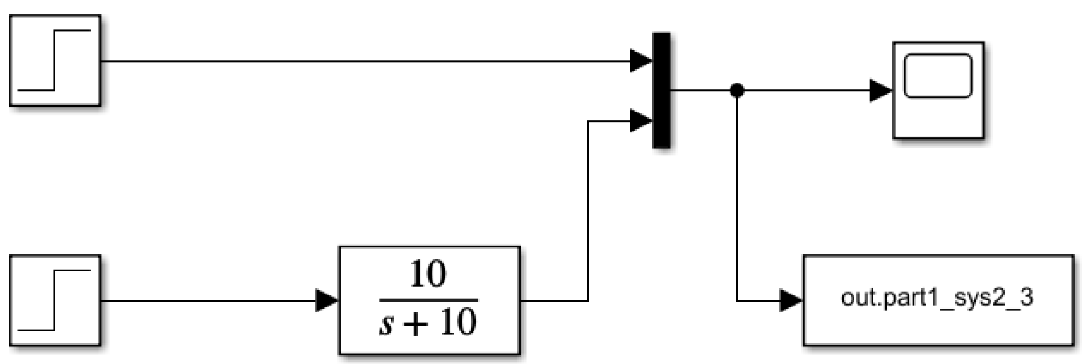

Figure 8. SIMULINK setup for dynamic response of the liquid level and of the reflux flow rate.

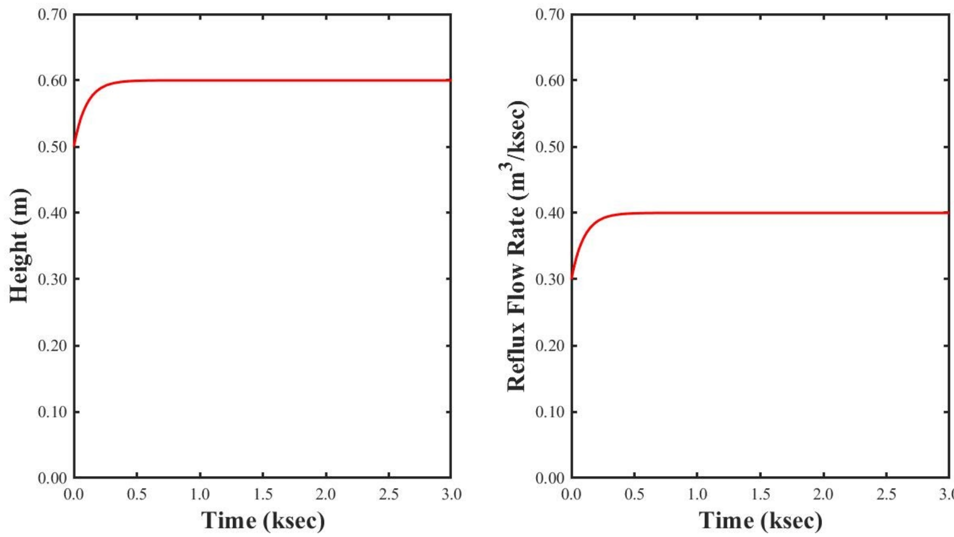

Figure 9. MATLAB plots for dynamic response of the liquid level and of the reflux flow rate.

- ***Determine the new steady state value for F_R; compare the indicated change in F_R with the change in F_i that stimulated the response. Interpret the results clearly but succinctly.***

  The new steady state value for F_R is 0.4 m^3/ksec. When F_i changes from 0.5 to 0.6 m^3/ksec, F_R also increases by 0.1 m^3/ksec. The system is said to be bounded as the change in F_R depends on the change in F_i.

2.2 PART II: HIGHER-ORDER SYSTEMS

2.2.1 Two first-order systems in series

1. ***Mathematical Model****

Reactor 1: $\frac{dV_1C_{A1}}{dt} = FC_{Af} - FC_{A1} - k_1V_1C_{A1}$ (1)

Reactor 2: $\frac{dV_2C_{A2}}{dt} = F(C_{A1} - C_{A2}) - k_2V_2C_{A2}$ (2)

2. ***Transfer Function Model****

$Define$: $y_1(t) = C_{A1} - C^*_{A1}$, $y_2(t) = C_{A2} - C^*_{A2}$, $u(t) = C_{Af} - C^*_{Af}$

$At\ steady\ state$: $0 = FC^*_{Af} - FC^*_{A1} - k_1V_1C^*_{A1}$ (3)

$0 = F(C^*_{A1} - C^*_{A2}) - k_2V_2C^*_{A2}$ (4)

$(1) - (3)$: $\frac{dy_1(t)}{dt} = \frac{F}{V_1}u(t) - \frac{F}{V_1}y_1(t) - k_1y_1(t)$ (5)

$(2) - (4)$: $\frac{dy_2(t)}{dt} = \frac{F}{V_2}y_1(t) - \frac{F}{V_2}y_2(t) - k_2y_2(t)$ (6)

$\mathcal{L}\{(5)\}$: $sY_1(s) + \frac{F}{V_1}\cdot Y_1(s) + k_1Y_1(s) = \frac{F}{V_1}\cdot u(s)$

$$Y_1(s) = \frac{\frac{F}{V_1}u(s)}{s+\frac{F}{V_1}+k_1},\ K_1 = \frac{F}{F+k_1V_1}$$

$\mathcal{L}\{(6)\}\ and\ substitute$: $sY_2(s) + \frac{F}{V_2}Y_2(s) + k_2Y_2(s) = \frac{F}{V_2}\left(\frac{\frac{F}{V_1}u(s)}{s+\frac{F}{V_1}+k_1}\right)$

$$Y_2(s) = \frac{\frac{F}{V_2}}{s+\frac{F}{V_2}+k_2}\cdot\frac{\frac{F}{V_1}}{s+\frac{F}{V_1}+k_1}\cdot u(s)$$

$$K = K_1K_2 = \frac{F^2}{(F+k_1V_1)(F+k_2V_2)}$$

$\tau_1 = \frac{V_1}{F+k_1V_1}$, $\tau_2 = \frac{V_2}{F+k_2V_2}$

$$g(s) = \frac{\frac{F}{V_2}}{s+\frac{F}{V_2}+k_2} \cdot \frac{\frac{F}{V_1}}{s+\frac{F}{V_1}+k_1}$$

By calculating equation (3) and (4) above, we found that C^*_{A1}= 84.9057 mol/m^3 and C^*_{A2}= 82.4327 mol/m^3.

3. *Dynamic Analysis in SIMULINK I*

$$Y_1(s) = \frac{\frac{F}{V_1}u(s)}{s+\frac{F}{V_1}+k_1} = \frac{0.025}{s+0.0265} = \frac{0.943}{37.7s+1} = \frac{K_1}{\tau_1 s+1}$$

$$Y_2(s) = \frac{\frac{F}{V_2}}{s+\frac{F}{V_2}+k_2} \cdot \frac{\frac{F}{V_1}u(s)}{s+\frac{F}{V_1}+k_1} = \frac{1.25\times10^{-3}u(s)}{(s+0.0515)(s+0.0265)} = \frac{1.25\times10^{-3}u(s)}{(s+0.0515)(s+0.0265)} = \frac{1.25\times10^{-3}u(s)}{s^2+0.078s+0.0014}$$

In standard general equation, $Y_2(s) = \frac{0.9709}{19.417s+1} \cdot \frac{0.943}{37.7s+1} \cdot u(s)$

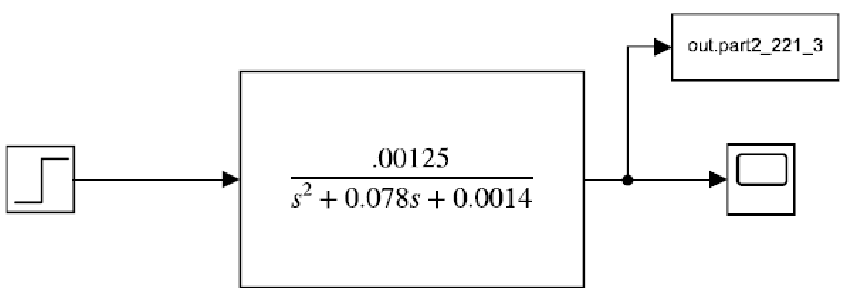

Figure 10. SIMULINK setup for two first-order systems in series.

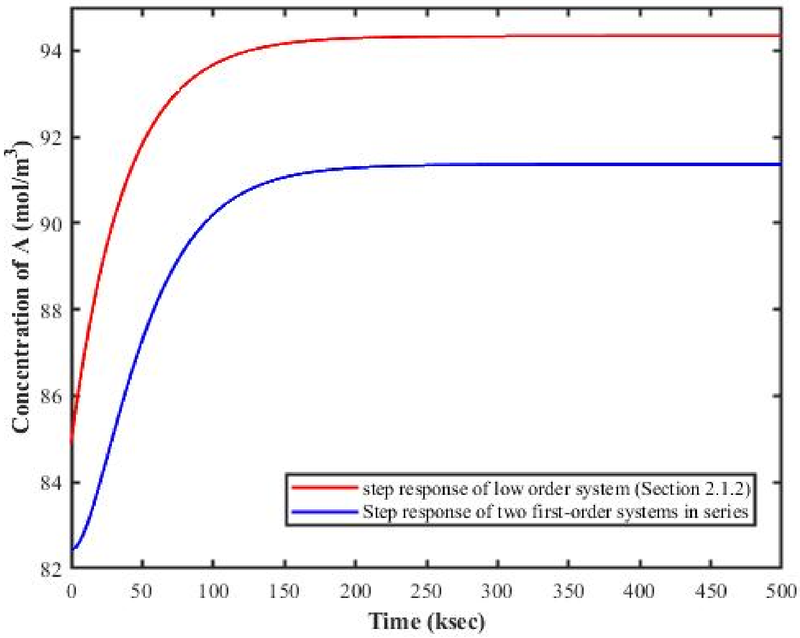

Figure 11. MATLAB plots for a response of two first-order systems in series compared to Section 2.1.2

- ***Compare it with the response in Section 2.1.2. Discuss the key characteristics of this two-reactor-in-series response in terms of what you know about general second-order system step response characteristics.***

By comparing the **Figure 3** from Section 2.1.2 and the response obtained for C_A in a single reactor, the response of two first-order systems in series shows more sigmoidal behavior. Putting the equation into $Y_2(s) = \frac{K_1 K_2 u(s)}{(\tau_1 s+1)(\tau_2 s+1)}$, we found that τ_1= 37.7 ksec and τ_2=19.4 ksec, and $\tau_1 + \tau_2 = \tau = 57.1\ ksec$; K_1=0.9434, K_2 =0.9709, and K_1*K_2=K=0.9159**,** and thus, the maximum concentration C_{A2}^{max} attained by the reactant A in the second reactor is estimated to be K*M=(0.9159*10) + 82.4327 = 91.5917 mol/m^3.This system has a larger time constant than that in Section 2.1.2, thus has slower response.

4. Dynamic Analysis in SIMULINK II

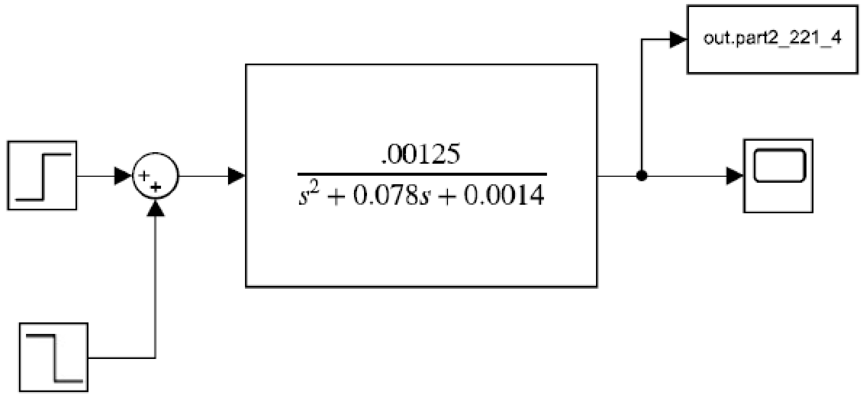

Figure 12. SIMULINK setup for a rectangular pulse input for two first-order systems in series

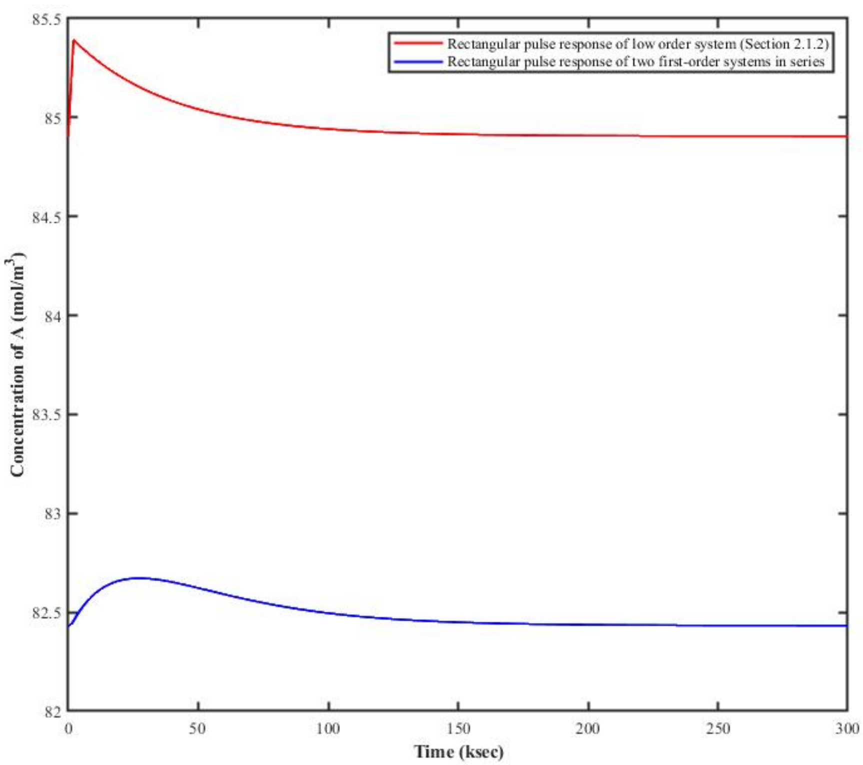

Figure 13. MATLAB plots for a rectangular pulse input for two first-order systems in series compared to Section 2.1.2

- ***Determine the maximum concentration C_{A2}^{max} attained by the reactant A in the second reactor. How does this compare with the results in Section 2.1.2?***

  By observing **Figure 13,** the maximum concentration C_{A2}^{max} attained by the reactant A in the second reactor is 82.6717 mol/m^3; this result is less than that of the single rector in Section 2.1.2 (=85.30 mol/m^3). After 2 ksec, the input of 10 mol/m^3 is removed, the C_{A2} decreases to its original steady state value (82.4327 mol/m^3). The rectangular pulse input of 2 ksec causes a smaller increase of C_{A2} (~0.2 mol/m^3) for two first order systems in series, compared to that of a first-order system (~0.5 mol/m^3). This is reasonable as the former has longer time constants, thus slower and less sensitive to response, compared to the latter.

2.2.2 Multiple first-order systems in series

1. ***Transfer Function Model****

$g(s) = \frac{K}{\tau s + 1}$, $K = \frac{50}{53}$, $\tau = \frac{2000}{53}$ from Section 2.1.2

Reactor 1: $Y_1(s) = g_1(s) \cdot u(s)$; $g_1(s) = \frac{K}{\tau s + 1}$

Reactor 2: $Y_2(s) = g_1(s) \cdot u_2(s)$, $u_2(s) = Y_1(s)$

$$Y_2(s) = g_1(s) \cdot g_1(s) \cdot u(s); g_2(s) = \frac{K^2}{(\tau s + 1)^2}$$

Reactor 3: $Y_3(s) = g_1(s) \cdot u_3(s)$, $u_3(s) = Y_2(s)$

$$Y_3(s) = g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot u(s); g_3(s) = \frac{K^3}{(\tau s + 1)^3}$$

Reactor 4: $Y_4(s) = g_1(s) \cdot u_4(s)$, $u_4(s) = Y_3(s)$

$$Y_4(s) = g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot u(s); g_4(s) = \frac{K^4}{(\tau s + 1)^4}$$

Reactor 5: $Y_5(s) = g_1(s) \cdot u_5(s)$, $u_5(s) = Y_4(s)$

$$Y_5(s) = g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot g_1(s) \cdot u(s); g_5(s) = \frac{K^5}{(\tau s + 1)^5}$$

In general:

$$Y_n(s) = \left[\prod_{i=1}^{n} g_i(s)\right] \cdot u(s) ; Y_5(s) = \left[\prod_{i=1}^{5} g_i(s)\right] \cdot u(s)$$

$Since\ g_1(s) = g_2(s) = g_3(s) = g_4(s) = g_5(s)$, $Y_5(s) = [g(s)]^5 \cdot u(s)$

2. *SIMULINK Implementation*

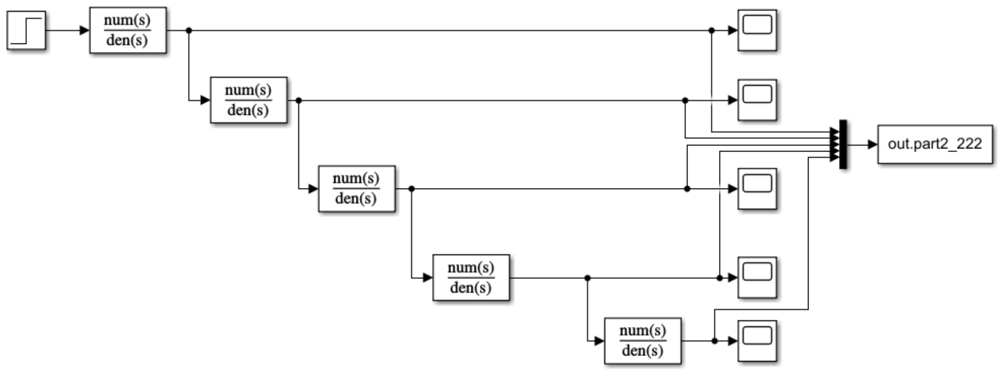

Figure 14. SIMULINK setup connecting five individual CSTR transfer functions.

3. *Dynamic Analysis in SIMULINK*

V = 20 m^3, rate constant k = 1.5 x 10^{-3} $ksec^{-1}$, flow rate F = 0.5 m^3/ksec.

Initial steady state value of C^*_{Af} = 90 moles/m^3 to C_{Af} = 100 moles/m^3.

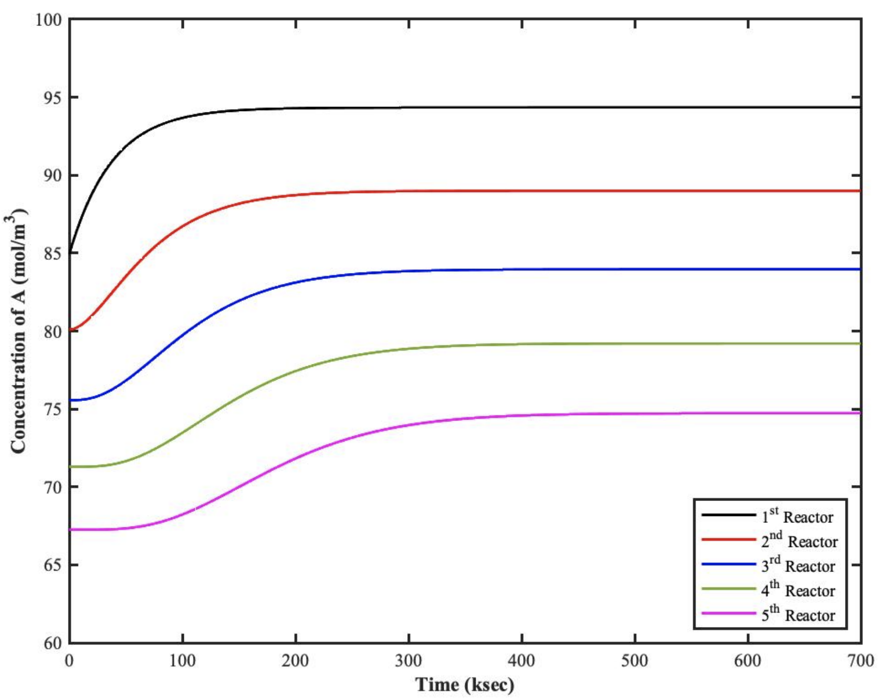

Figure 15. The concentration profile of A in each reactor.

- ***Discuss the characteristics of each response, noting especially the effect of each additional reactor on the responses.***

  As the feed containing species A enters the first reactor, the steady-state concentration of A is decreasing, as it goes from reactor 1 to 5. As the number of reactors increases, there is a noticeable delay in change of concentration A, which is expected as the reaction is running from one reactor at a time before entering the next. The initial concentration of A in the successive reactor is always lower than the previous reactor, indicating the rate of consumption is greater than the incoming flow rate of A into each reactor.

4. Approximate Transfer Function Model

$$y_5(s) = g(s)e^{-\alpha s}u(s)$$

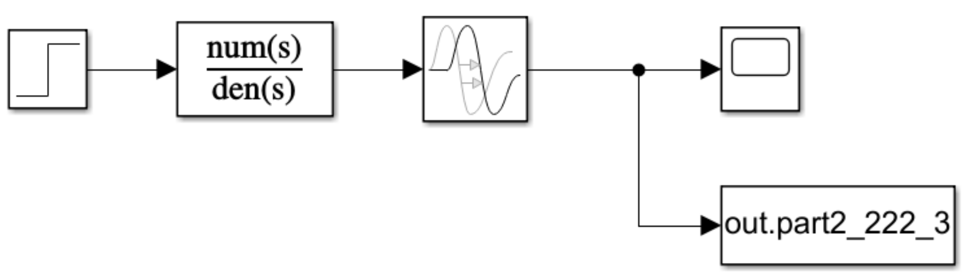

Figure 16. SIMULINK setup for approximate transfer function model

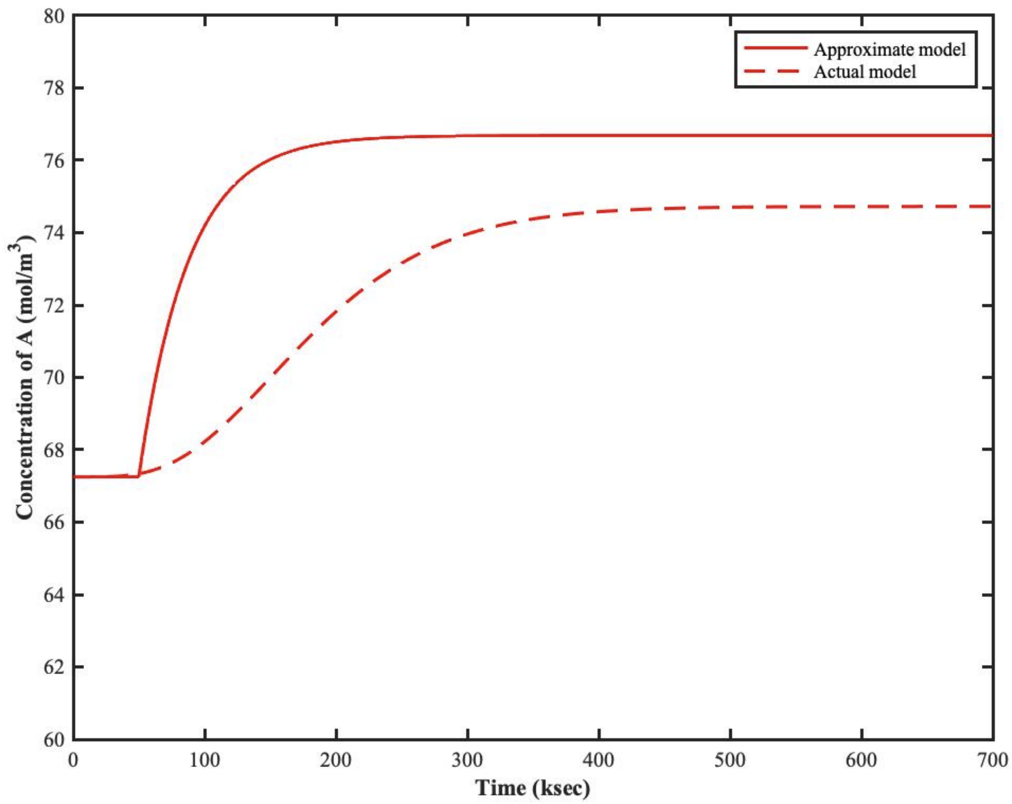

Figure 17. Matlab plot of approximate transfer function model compared to actual model

- ***How good is this approximation?***

  The approximation is poor. The value of alpha used was 50 ksecs. After much trial and error, it was found that neither of the α values resulted in the same steady-state gain as that of the actual model used in the previous part. The approximate model reached a steady-state concentration at 76.69 mol/m^3, while the actual model reached a steady-state concentration of 74.73 mol/m^3.

2.2.3 Human Balance Keeping

$$y(s) = \left(\frac{K}{\gamma s^2 - 1}\right)u(s)$$

1. ***Dynamic Analysis in SIMULINK***

Given K = 3.15 (rad/N-m), γ = 4.84 (sec^2)

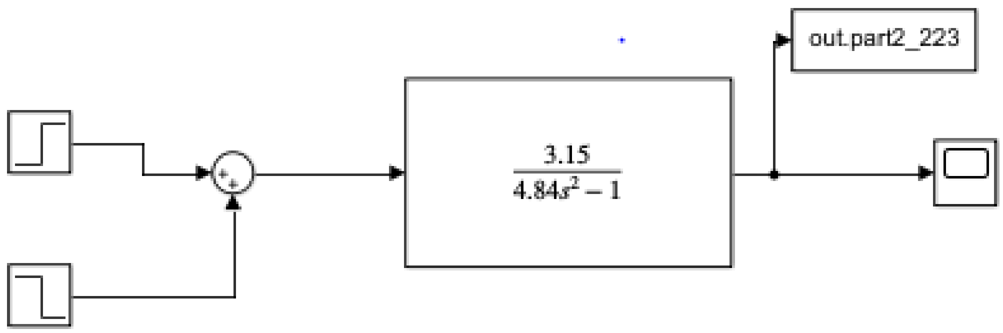

Figure 18. SIMULINK setup for sway angle response

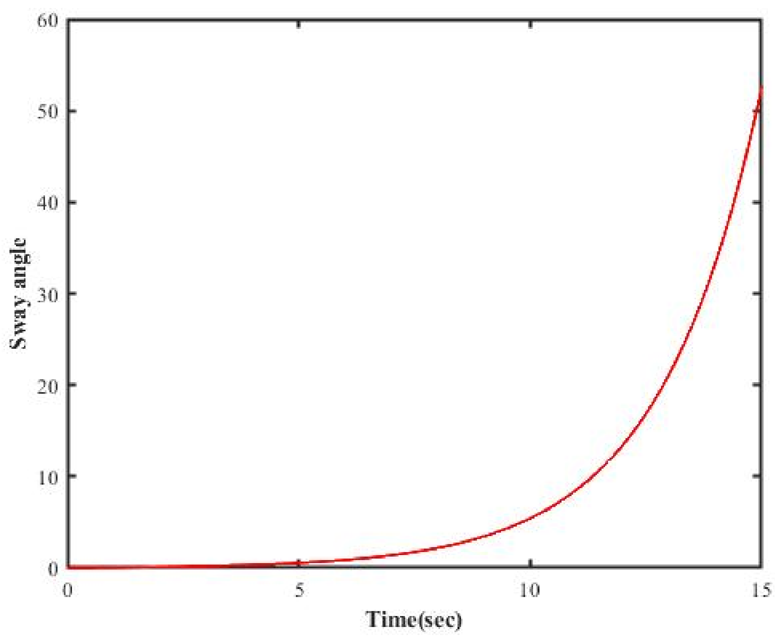

Figure 19. MATLAB plot for sway angle response to 1-second rectangular pulse of 0.1 n-m in moment of rotation.

2. ***Physical Interpretations of Results***

- ***Discuss what these results indicate about the subject's ability to recover from stumbles on its own, without the assistance of a balance-keeping neural controller whose characteristics have not been included in the mode.***

  The subject is not able to recover from stumbles without the assistance of a balance-keeping neural controller. We can see from **Figure 19** that the graph went all the way up after the step input, and does not return to a steady state; the system steady state is unstable. Furthermore, the transfer function $g(s) = \frac{K}{\gamma s^2 - 1} = \frac{3.15}{4.84s^2 - 1}$ has 2 poles; one is positive, and one is negative. As the transfer function has a positive pole, the system will have unstable behavior and would not be able to recover from stumbles on its own.

2.3 PART III. MORE COMPLEX SYSTEMS

2.3.1 Two First-Order Systems in Parallel

- ***Dynamic Analysis in SIMULINK***

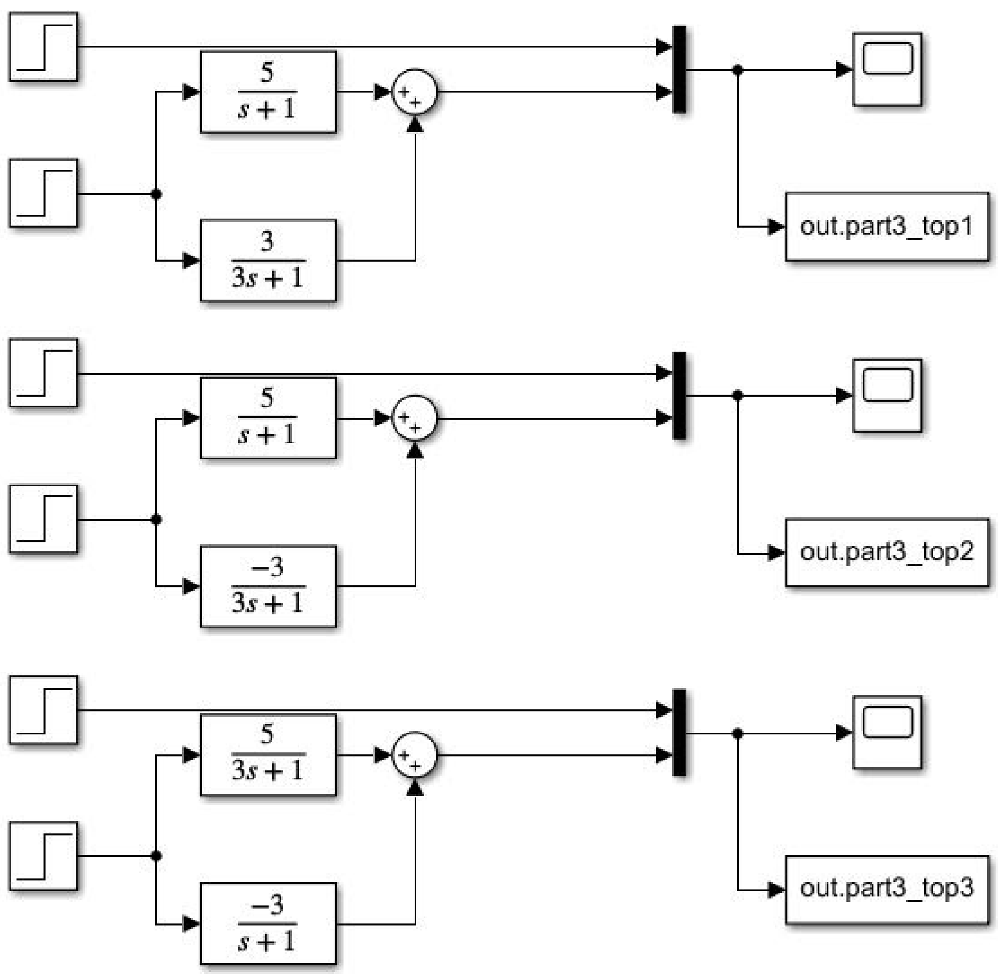

Figure 20. SIMULINK setup for 3 different two systems in parallel

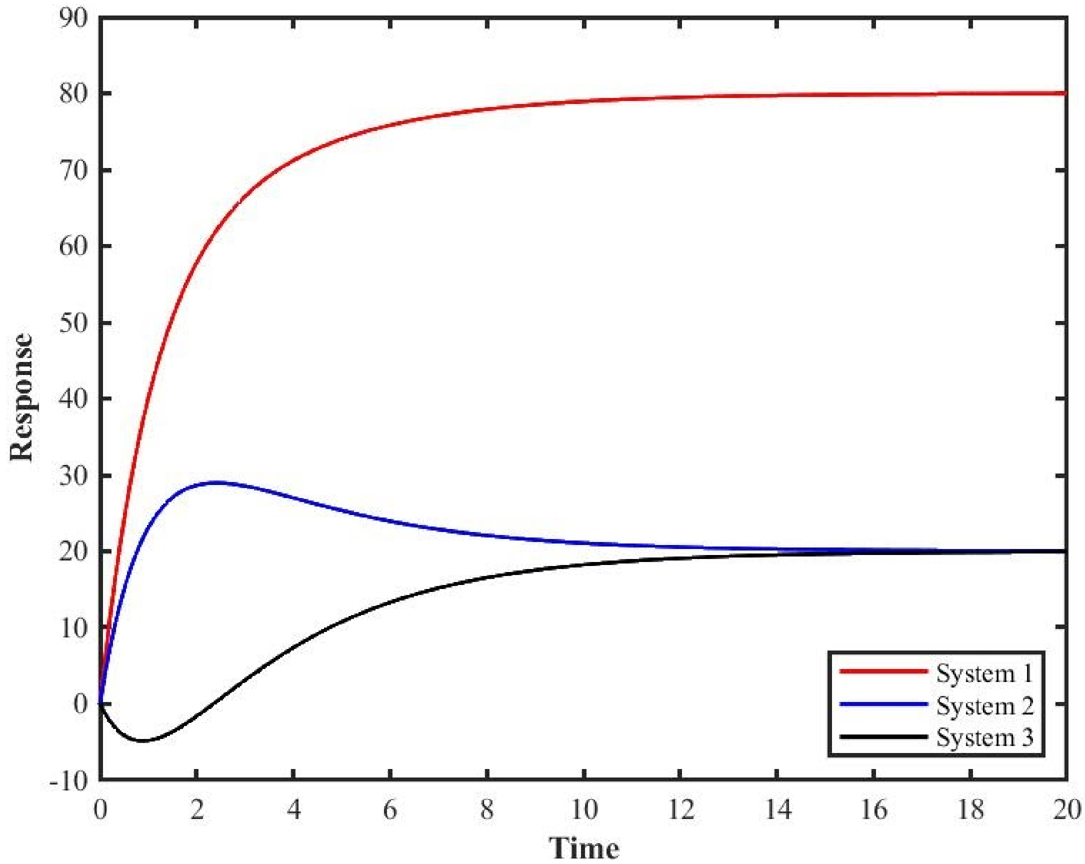

Figure 21. MATLAB plot for 3 different two systems in parallel

- ***Compare and contrast these three systems and their unit step responses***

  For system 1, the steady state gain is around 80, which is quadruple the gain of system 2 and 3. For system 1, it has an exponential response and reaches 95% of the steady state value around t = 6. System 2 has an overshoot response and reaches the steady state gain value at 20. System 3 has an inverse response before regaining response to the steady state value of 20.

2.3.2 Immunization Dynamics

1. ***Dynamic Analysis in SIMULINK***

k_1 = 4.0/day, V_0 = 27,000 (antigen particles[1]/mL), $k_2 = 4.875 \times 10^{-4}$/day$\cdot\xi_2$, $k_3 = 5 \times 10^{-1}$/day.

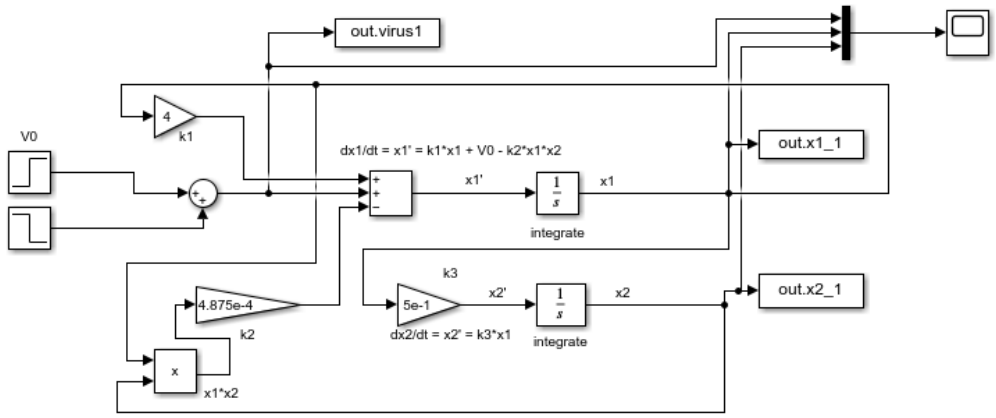

Figure 22. SIMULINK setup for the responses for antigen level, antibody level and the net immune response respectively.

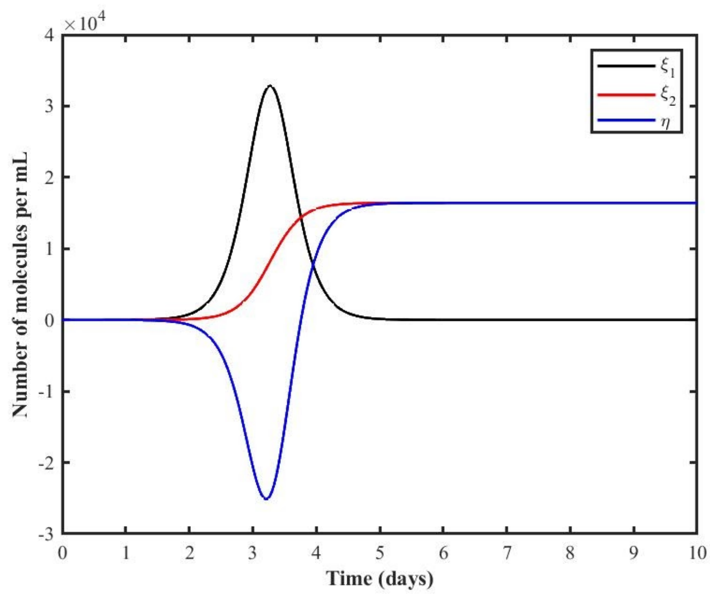

Figure 23. MATLAB plots for the responses for antigen level, antibody level and the net immune response.

2. Response Analysis

(a) How long (in days) after the vaccination will it take for the immune response to reach its worst level?

It took about 3 days after the vaccination for the immune response to reach its worst level.

(b) What is the value of the net residual antibody level 10 days after vaccination?

About 1.641×10^4 antibody molecules per mL.

(c) How many days elapse after the vaccination before the subject achieves this steady state value?

After 5 days.

3. Analysis of Post-Vaccination Exposure

- ***Obtain responses for antigen level, antibody level and the net immune response***

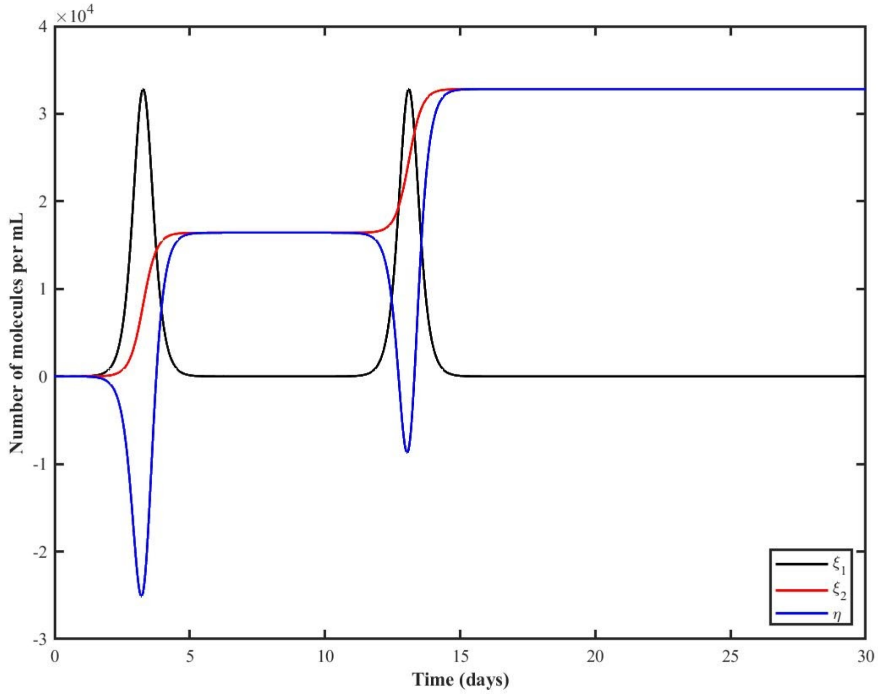

Figure 24. MATLAB plots for the responses for antigen level, antibody level and the net immune response after vaccination.

- ***Compare the post-immunization response to the initial immune response and comment on the effectiveness of the vaccination***

  The simulation was performed after about 10 days of receiving the vaccine. From the final antigen and antibody level in the body, setting them as the initial concentration level for the disease exposure simulation, it could be seen that the immune response is getting better, reaching about 3.282×10^4 molecules per mL. Same goes for the antibody level. The vaccine is effective to improve the immune response.

4. ***Approximate Transfer Function Model***

- ***Identify the gain, poles and zeros of $g_d(s)$, and discuss clearly but succinctly, what this approximate transfer function indicates about fundamental dynamic behavior of the net immune response to vaccination.***

$$\frac{d\xi_1}{dt} = k_1\xi_1 - k_2\xi_1\xi_2 + V_0$$

$$\frac{d\xi_2}{dt} = k_3\xi_1$$

$$Define:\ x_1 = \xi_1 - \xi_{1s},\ x_2 = \xi_2 - \xi_{2s},\ d = V_0 - V_{0s}$$

$$\xi_1\xi_2 = \xi_{1s}\xi_{2s} + \frac{\partial(\xi_1\xi_2)}{\partial\xi_1}\Big|_{\xi_{1s},\xi_{2s}}(\xi_1 - \xi_{1s}) + \frac{\partial(\xi_1\xi_2)}{\partial\xi_2}\Big|_{\xi_{1s},\xi_{2s}}(\xi_2 - \xi_{2s})$$

$$\xi_1\xi_2 = \xi_{1s}\xi_{2s} + \xi_{2s}(\xi_1 - \xi_{1s}) + \xi_{1s}(\xi_2 - \xi_{2s})$$

$$\frac{d\xi_1}{dt} = k_1\xi_1 - k_2[\xi_{1s}\xi_{2s} + \xi_{2s}(\xi_1 - \xi_{1s}) + \xi_{1s}(\xi_2 - \xi_{2s})] + V_0 \quad (1)$$

$$@steady\ state:\ 0 = k_1\xi_{1s} - k_2\xi_{1s}\xi_{2s} + V_{0s} \quad (2)$$

$$(1) - (2):\ \frac{d(\xi_1 - \xi_{1s})}{dt} = k_1(\xi_1 - \xi_{1s}) - k_2[\xi_{2s}(\xi_1 - \xi_{1s}) + \xi_{1s}(\xi_2 - \xi_{2s})] + (V_0 - V_{0s})$$

$$\frac{dx_1}{dt} = k_1x_1 - k_2[\xi_{2s}x_1 + \xi_{1s}x_2] + d$$

$$\frac{d\xi_2}{dt} = k_3\xi_1 \quad (3)$$

$$@\ steady\ state:\ 0 = k_3\xi_{1s} \quad (4)$$

$$(3) - (4):\ \frac{d(\xi_2 - \xi_{2s})}{dt} = k_3(\xi_1 - \xi_{1s})$$

$$\frac{dx_2}{dt} = k_3x_1$$

$$\mathcal{L}\{\frac{dx_2}{dt}\} = sX_2(s) = k_3 X_1(s)$$

$$X_2(s) = \frac{k_3}{s} X_1(s) \quad (5)$$

$$\mathcal{L}\{\frac{dx_1}{dt}\} = sX_1(s) = k_1 X_1(s) - k_2 \xi_{2s} X_1(s) - k_2 \xi_{1s} X_2(s) + d(s) \qquad (6)$$

$$substitute\ (5)\ into\ (6): sX_1(s) = k_1 X_1(s) - k_2 \xi_{2s} X_1(s) - k_2 \xi_{1s} \frac{k_3}{s} X_1(s) + d(s)$$

$$X_1(s) = \frac{1}{s - k_1 + k_2 \xi_{2s} + \frac{k_2 k_3 \xi_{1s}}{s}} \cdot d(s)$$

$$X_2(s) = \frac{k_3}{s} \cdot \frac{1}{s - k_1 + k_2 \xi_{2s} + \frac{k_2 k_3 \xi_{1s}}{s}} \cdot d(s)$$

$$Define: y = \eta - \eta_s, \ \eta = \xi_2 - \xi_1, \eta_s = \xi_{2s} - \xi_{1s}$$

$$y = \xi_2 - \xi_1 - (\xi_{2s} - \xi_{1s})$$

$$y = \xi_2 - \xi_{2s} - (\xi_1 - \xi_{1s})$$

$$y = x_2 - x_1$$

$$\mathcal{L}\{y\}: Y(s) = X_2(s) - X_1(s)$$

$$Y(s) = \frac{k_3}{s} \cdot \frac{1}{s - k_1 + k_2 \xi_{2s} + \frac{k_2 k_3 \xi_{1s}}{s}} \cdot d(s) - \frac{1}{s - k_1 + k_2 \xi_{2s} + \frac{k_2 k_3 \xi_{1s}}{s}} \cdot d(s)$$

$$Y(s) = \frac{1}{s - k_1 + k_2 \xi_{2s} + \frac{k_2 k_3 \xi_{1s}}{s}} \cdot \left[\frac{k_3}{s} - 1\right] \cdot d(s)$$

By multiplying with $\frac{s}{s}$,

$$Y(s) = \frac{1}{s^2 - (k_1 + k_2 \xi_{2s}) s + k_2 k_3 \xi_{1s}} \cdot \left[k_3 - s\right] \cdot d(s)$$

$$Y(s) = \frac{\frac{1}{k_2 k_3 \xi_{1s}} \left[-s + k_3\right]}{\frac{1}{k_2 k_3 \xi_{1s}} s^2 - \frac{k_1 + k_2 \xi_{2s}}{k_2 k_3 \xi_{1s}} s + 1} \cdot d(s)$$

$$Y(s) \;=\; \frac{\frac{1}{k_2\xi_{1s}}\left[-\frac{1}{k_3}s+1\right]}{\frac{1}{k_2k_3\xi_{1s}}s^2 - \frac{k_1+k_2\xi_{2s}}{k_2k_3\xi_{1s}}s + 1} \cdot d(s)$$

By plugging in the given parameter values,

$$g(s) \;=\; \frac{k_2k_3^2\xi_{1s}\left[-\frac{1}{k_3}s+1\right]}{\frac{1}{k_2k_3\xi_{1s}}s^2 - \frac{k_1+k_2\xi_{2s}}{k_2k_3\xi_{1s}}s + 1}$$

Gain: K = $k_2k_3^2\xi_{1s}$

Zeros: $\frac{1}{k_2\xi_{1s}}\left[-\frac{1}{k_3}s + 1\right] = 0 \rightarrow s = k_3$

Poles: $s = \frac{k_1+k_2\xi_{2s}}{2k_2k_3\xi_{1s}} \pm \sqrt{\frac{(k_1+k_2\xi_{2s})^2-16}{(k_2k_3\xi_{1s})^2}}$