

CHEG401 - Chemical Process Dynamics and Control

Lab 3 - Process Identification

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3.1 Part I: Step-Response Identification

3.1.1 System 1

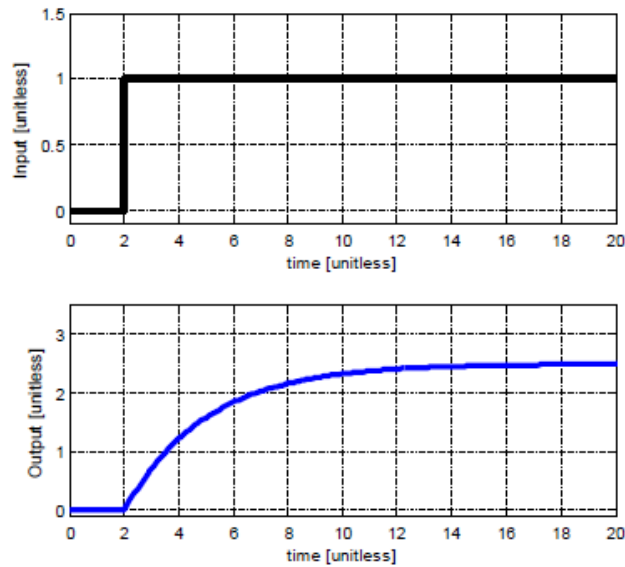


Figure 3.1: Unit step response for System 1.

1. There is no time delay in this system
 2. The initial slope of response is non-zero.
 3. The response doesn't show any distinguishing characteristics such as oscillations, overshoot and inverse response.
 4. $p - q = 1$
 5. $p = 1, q = 0$
 6. (1,0)-order system
- ***From your knowledge of process dynamics, provide your best “back-of-the-envelope” estimate of the process gain, K , a time constant, τ , and any other parameter you think may be relevant to this system. Justify your estimates clearly.***
 - The gain K is estimated to be 2.5 (Steady state output=SS gain*step, step (M) =1, SS output =2.5, so SS gain, $K=2.5$) .
 - With $KM= 2.5$, $0.632*KM=1.58=y(\tau)$, the time constant τ , is estimated to be $5-2=3$.

3.1.2 System 2

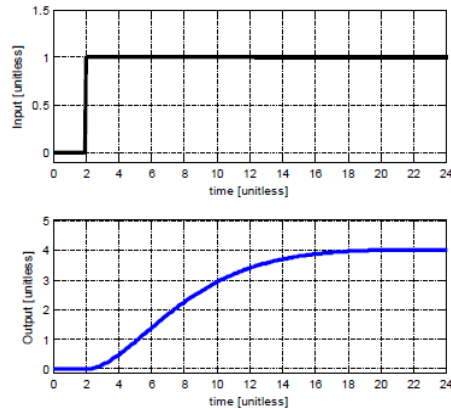


Figure 3.2: Unit step response for System 2.

1. There is no time delay in the system.
 2. The slope is zero at initial time $t = 0$.
 3. The response doesn't show any distinguishing characteristics such as oscillations, overshoot and inverse response. The shape of the output curve is slightly sigmoidal.
 4. $p - q > 1$
 5. $p = 2, q = 0$
 6. (2,0)-order system
- **Transfer function model postulation, and estimation of K**
 - The postulated transfer function: $\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$
 - The gain K is estimated to be 4 (Steady state output = SS gain * step, step (M) = 1, SS output = 4, so SS gain, $K = 4$).
 - **Estimation of τ and α for FOPTD and adequacy of FOPTD**
 - The first-order-time-delay model: $Y(s) = \left(\frac{K e^{-\alpha s}}{\tau s + 1} \right) u(s)$ has a maximum slope of $\frac{KM}{\tau}$.
 - With $KM = 4$, $0.632 * KM = 2.528 = y(\tau)$, the time constant τ , is estimated to be $8.8 - 2 = 6.8$.
 - The best estimate of effective time delay, $\alpha = 0.2$. Yes, a first-order-plus-time-delay model is adequate for this system.

3.1.3 System 3

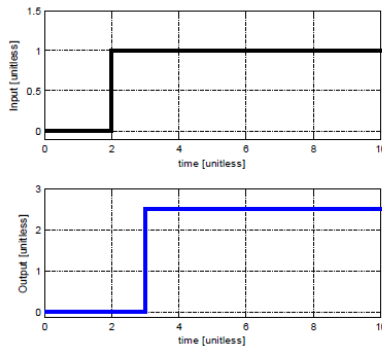


Figure 3.3: Unit step response for System 3.

1. There is a time delay in the system
 2. The initial slope of response is non-zero.
 3. The response is an instant increment at $t=3$, then it reaches steady state, and no other distinguishing characteristic such as oscillations, overshoot and inverse response.
 4. $p = 0, q=0$, this is a pure gain system.
 5. $p = 0, q = 0$
 6. (0,0)-order system
- **Transfer function model postulation, and estimation of K**
 - Most likely transfer function: $g(s)=2.5 \times e^{-s}$
 - With steady state output=SS gain*step, step (M) =1, SS output =2.5, so the gain, K is estimated to be 2.5 .
 - **Estimation of τ and α for FOPTD and adequacy of FOPTD**
 - The best estimate of the effective time constant, $\tau \approx 0$ because it is a pure gain system which respond immediately.
 - Based on the graph of system 3, our best estimate of effective time delay, α is 1. Thus, $Y(s)=2.5e^{-s}u(s)$ is the function for FOPTD.
 - Because $Y(s)=2.5e^{-s}u(s)$, this is a special case in first order system where $\tau = 0$. With $\alpha = 1$ and $\tau = 0$, the model is adequate to predict the response of the system 3.

3.1.4 System 4

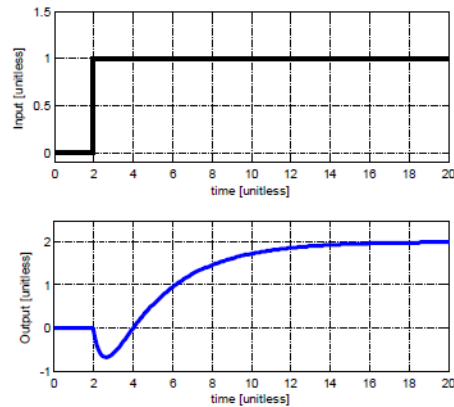


Figure 3.4: Unit step response for System 4.

1. There is no time delay in the system.
 2. The initial slope of response is non-zero.
 3. The response shows an inverse response from $t = 2$ to $t = 4$. No oscillations and overshoot in the system.
 4. $p - q = 1$
 5. $p = 2, q = 1$
 6. (2,1)-order system
- **Transfer model postulation, and estimation of K**

Most likely transfer function: $\frac{K(\xi s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}$. With steady state output = SS gain * step, step

(M) = 1, SS output = 2, so the gain, K is estimated to be 2.

- **Estimation of τ and α for FOPTD and adequacy of FOPTD**

- The first-order-time-delay model: $Y(s) = \left(\frac{K e^{-\alpha s}}{\tau s + 1}\right) u(s)$ has a maximum slope of $\frac{KM}{\tau}$.
- With $KM=2$, $y(\tau)=0.632 \cdot KM=0.632 \cdot 2=1.264$, the best estimate for $\tau = 7-4=3$. Based on the graph of system 4, there is no time delay, so our best estimate of effective time delay, α is 2.

- No, because the first-order-time-delay system is adequate only after $t = 4$. The inverse response of the output (when $t < 4$) would not be captured by the first-order-time-delay system.
- Use *SIMULINK* to recreate a plot with similar characteristics to Figure 3.4. Obtain a new FOPTD model for the system. Superimpose on your original plot the response from your estimated FOPTD model.

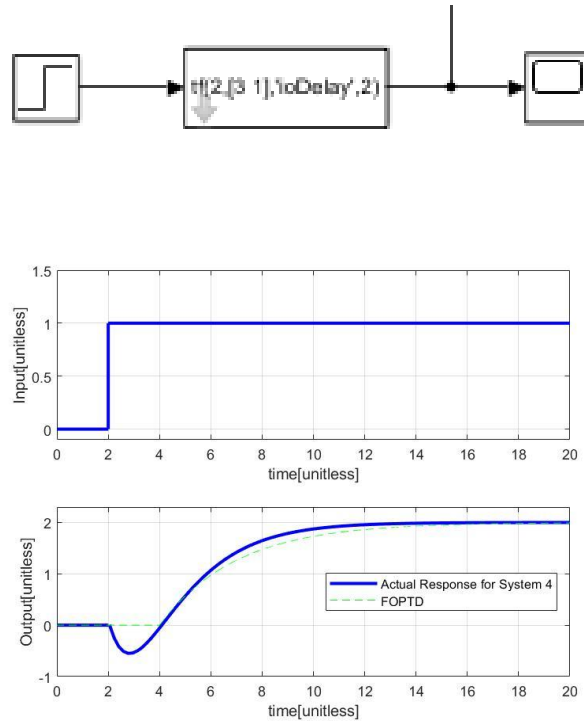


Figure 1. System 4's SIMULINK simulation model

(a) What important characteristics of your original response (if any) are lost in the FOPTD representation?

The negative net effect of response with negative initial slope is missing in FOPTD representation.

(b) Is a first-order-plus-time-delay model adequate for this system?

No. It is adequate to predict the system's behavior when it is after $t = 4$. FOPTD fails to capture the inverse response ($t < 4$).

3.1.5 System 5

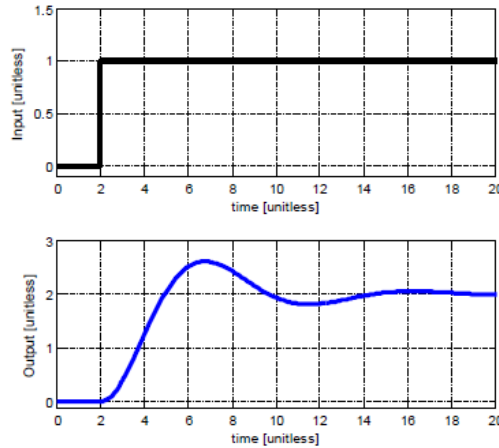


Figure 3.5: Unit step response for System 5.

1. There is no time delay in the system.
2. The initial slope of response is zero.
3. The response shows overshoot at $t = 5$. The response also shows oscillations. No inverse response in the system.
4. $p - q > 1$
5. $p = 2, q = 0$
6. (2,0)-order system.

- **Transfer model postulation, and estimation of K**

- Most likely transfer function: $\frac{2}{(\tau_1 s + 1)(\tau_2 s + 1)}$.
- With steady state output = SS gain * step, step, $M = 1$, SS output = 2, so the gain, K is estimated to be 2.

- **Estimation of τ and α for FOPTD and adequacy of FOPTD**

- The first-order-time-delay model: $Y(s) = \left(\frac{K e^{-\alpha s}}{\tau s + 1}\right) u(s)$ has a maximum slope of $\frac{KM}{\tau}$.
- With $KM = 2$, $y(\tau) = 0.632 * KM = 0.632 * 2 = 1.264$, the best estimate for $\tau = 4 - 2 = 2$.
- Based on the graph of system 4, our best estimate of effective time delay, α is 0.2. No, a first-order-plus-time-delay model is adequate for this system.

- Use *SIMULINK* to recreate a plot with similar characteristics to Figure 3.5. Obtain a new *FOPTD* model for the system. Superimpose on your original plot the response from your estimated *FOPTD* model.

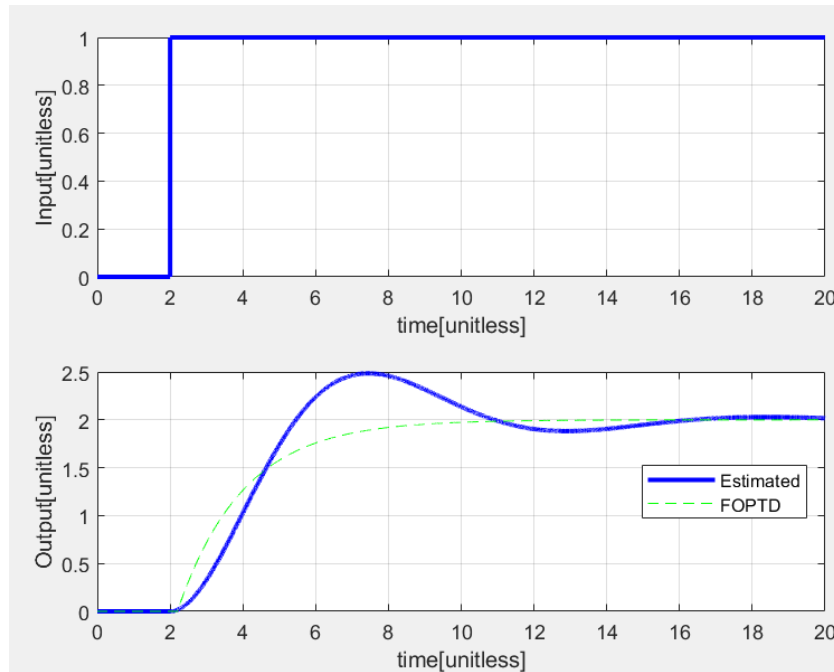


Figure 2. System 5's SIMULINK simulation model

(a) What important characteristics of your original response (if any) are lost in the FOPTD representation?

The oscillatory pattern is lost in the FOPTD presentation.

(b) Is a first-order-plus-time-delay model adequate for this system?

No, because FOPTD does not show the oscillatory pattern of the system.

3.1.6 System 6

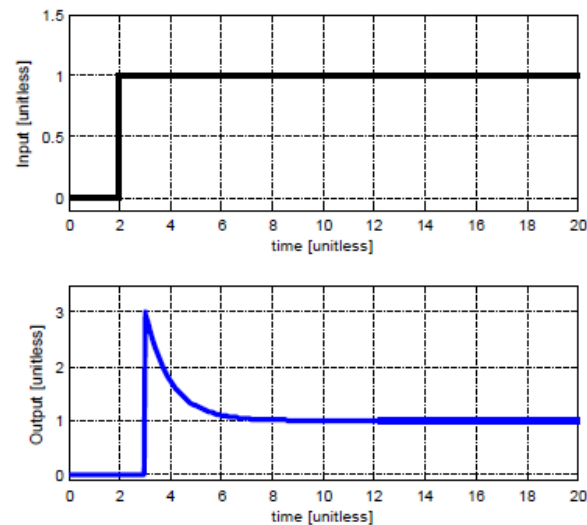


Figure 3.6: Unit step response for System 6.

1. There is a time delay in the system.
2. The slope is non-zero at initial time $t = 0$.
3. The system shows an overshoot characteristic at $t=3$: the output reaches 3, exceeding the steady state ($=1$) by 2 units.
4. The presence of overshoot indicates that the zero at numerator, $\xi > \text{time constant}, \tau_{1,2}$.

With $p - q = 1$, the initial slope of the response is non-zero.

5. 1 pole and 1 zero.
 6. (1,1)-order system
- **Transfer model postulation, estimation of K , and the relationship between the poles and zeros of the system.**
 - Transfer function model : $\frac{K(\xi s + 1)}{\tau s + 1}$. The gain K is estimated to be 1 (Steady state output = SS gain * step, step = 1, SS output = 1, so SS gain, $K=1$).
 - Zero at numerator, $\xi > \text{time constant}, \tau_{1,2}$, because an overshoot is observed.
 - $\rho = \frac{\xi}{\tau} = 3$

3.2 Part II: Parameter Estimation for a FOPTD Model

$$y(t) = y_{\infty}(1 - e^{-\frac{t-\alpha}{\tau}}); \text{ for } t > \alpha$$

1. Steady-state Gain Estimation

$$y_{\infty} = 15.56 \text{ ft (average of the last 10 data points for } y(t))$$

$$K = \frac{y_{\infty}}{M} = \frac{15.56 \text{ ft}}{5 \frac{\text{gal}}{\text{min}}} \cdot 7.4805 \frac{\text{gal}}{\text{ft}^3} = 23.28 \frac{\text{min}}{\text{ft}^2}$$

2. Dynamic Parameter Estimation

$$y(t) = y_{\infty}(1 - e^{-\frac{t-\alpha}{\tau}})$$

$$\frac{y(t)}{y_{\infty}} = 1 - e^{-\frac{t-\alpha}{\tau}}$$

$$1 - \frac{y(t)}{y_{\infty}} = \frac{y_{\infty} - y(t)}{y_{\infty}} = e^{-\frac{t-\alpha}{\tau}}$$

$$\ln\left[\frac{y_{\infty} - y(t)}{y_{\infty}}\right] = -\frac{t-\alpha}{\tau}$$

$$\ln\left[\frac{y_{\infty} - y(t)}{y_{\infty}}\right] = \frac{\alpha}{\tau} - \frac{t}{\tau}$$

After linearizing the data by only considering the data points from 2 minutes and beyond,

$$\alpha = 4.24 \text{ min and } \tau = 12.20 \text{ min.}$$

3. Model Fit Evaluation

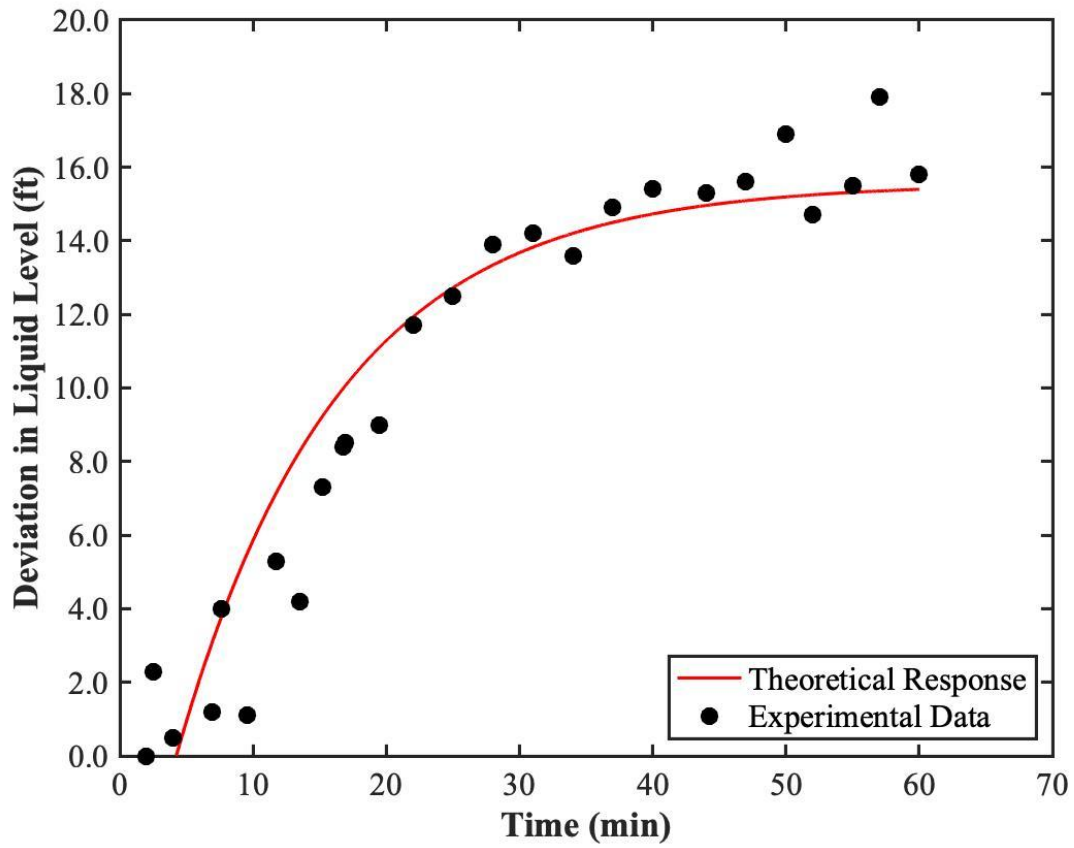


Figure 3. Parameter Estimation for FOPTD Model versus Experimental Data

The model fits the data relatively well. It does a good job of predicting the trends and trajectory of the data collected. However, the model fails to reflect the fluctuation of the data. As can be seen from **Figure 3**, the red line crosses very few of the data points.

3.3 Part III: Discrete-Time Process Identification

3.3.1 Background

$y(k + 1) = \phi y(k) + \beta u(k - m)$, where $k = 0, 1, 2, \dots$

$$t_{k+1} = t_k + \Delta t, \phi = e^{-\Delta t/\tau}, \beta = K(1 - \phi), m = \frac{\alpha}{\Delta t}$$

3.3.2 Discrete-Time Model Identification for the Water Tank

1. Data Generation and Pre-Processing

Table 1. Data for Identification of Discrete-Time Model

Time (min)	k	F _i (k) (gal/min)	L(k) (ft)	y(k) (ft)	u(k) (gal/min)
0	0	50	95.00	0.00	0
2	1	55	94.70	-0.30	5
4	2	55	95.20	0.20	5
6	3	55	94.95	-0.05	5
8	4	55	98.12	3.12	5
10	5	55	96.60	1.60	5
12	6	55	99.82	4.82	5
14	7	55	99.81	4.81	5
16	8	55	102.55	7.55	5
18	9	55	103.41	8.41	5
20	10	55	104.24	9.24	5
22	11	55	106.40	11.40	5
24	12	55	106.93	11.93	5
26	13	55	107.67	12.67	5
28	14	55	108.60	13.60	5
30	15	55	108.80	13.80	5
32	16	55	108.70	13.70	5
34	17	55	108.30	13.30	5
36	18	55	109.17	14.17	5
38	19	55	109.77	14.77	5
40	20	55	110.10	15.10	5
42	21	55	110.05	15.05	5
44	22	55	110.00	15.00	5
46	23	55	110.20	15.20	5
48	24	55	110.73	15.73	5
50	25	55	111.60	16.60	5
52	26	55	109.40	14.40	5
54	27	55	109.93	14.93	5
56	28	55	111.40	16.40	5
58	29	55	111.90	16.90	5
60	30	55	110.50	15.50	5

2. Discrete-Model Parameter Estimation

Data were recast in Table 1 in terms of deviations from initial steady state.

By using MATLAB to determine the estimates for the parameters, it was found that

$$\phi = 0.81, \beta = 0.58 \frac{\text{min}}{ft^2}, \text{ and } m = 4.$$

3. Recovery of Continuous Model Parameter Estimates

$K = 3.053 \frac{\text{min}}{ft^2}$, $\tau = 9.49 \text{ min}$, and $\alpha = 8 \text{ min}$. Comparing both direct and indirect approach to determine K , τ , and α , K predicted from the discrete-model approach (indirect) is smaller than the direct method. τ also shows the same trend, where the value predicted from the indirect method is slightly smaller than the direct method. However, α obtained from the indirect method is approximately two times greater than what was obtained from the direct method. These two methods are not in agreement for any of the parameters.