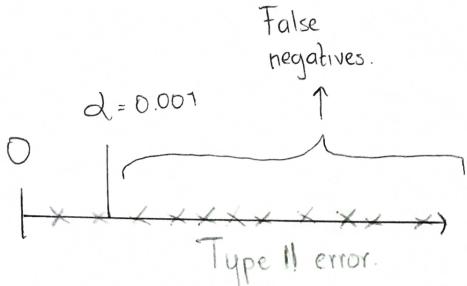


(A)

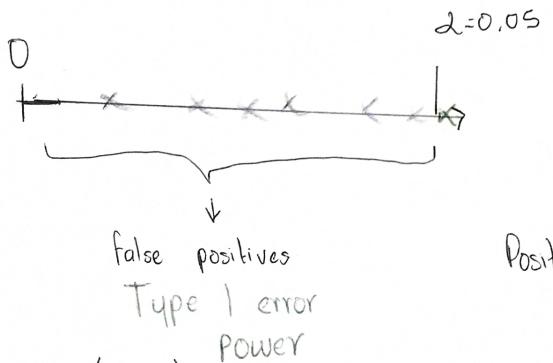
We set a small cutoff.



Result: we fail rejecting the null in cases that we should.

(B)

We set a relatively big cut-off.



Result: we reject the null in cases that we should not.

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_x^2}{M} + \frac{\sigma_y^2}{N}}} \sim N(0, 1)$$

Positive or negative: The act of rejecting the null hypothesis

Power = $f \left(\begin{array}{l} \text{Size effect: } \mu_x - \mu_y, \\ \text{Estimates standard error} = f \left(\frac{\text{sample size}}{\text{size}} \right) \otimes f \left(\frac{\text{Pop.}}{\text{s.d.}} \right) \end{array} \right)$

Type I error : the event of rejecting the null hypothesis when it is in fact true.

Type I error rate . Probability of making a Type I error.

Type II error rate. Probability that we fail rejecting the null hypothesis when the alternative is true

Power = probability of rejecting the null hypothesis when the alternative hypothesis is true.

Significance level (α) : threshold we set to know when we will reject the null hypothesis using the p-value.

The 0.05 and 0.01 cutoffs are arbitrary.

$$\text{Type II error rate} + \text{Power} = 1$$

$$\text{Power} = 1 - \text{Type II error rate}$$

The p-values get smaller and smaller with increasing the sample size because the numerator of the t-statistic has \sqrt{N} . Therefore, if

$\Delta = \mu_x - \mu_y \neq 0$, the t-statistic will increase with N.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i ; \sigma_x^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2$$

$$\bar{y} = \frac{1}{M} \sum_{i=1}^M y_i ; \sigma_y^2 = \frac{1}{M} \sum_{i=1}^M (y_i - \bar{y})^2$$

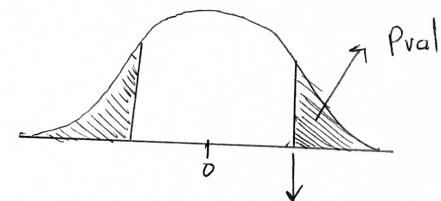
\bar{x} & \bar{y} are random variables:

$$\begin{cases} \bar{x} \sim N(\mu_x, \sigma_x) \\ \bar{y} \sim N(\mu_y, \sigma_y) \end{cases}$$

and, by CLT:

$$\frac{\bar{y} - \bar{x}}{\sqrt{\frac{\sigma_x^2}{N} + \frac{\sigma_y^2}{M}}} \sim N(0, 1) ; = \text{t-statistic.}$$

If $\sqrt{N} \uparrow\uparrow$ then t-statistic $\uparrow\uparrow$



$\text{abs}(t) = \text{quantile, defines } 1-p \text{ and therefore P-val}$

If t-stat $\uparrow\uparrow$, then P-val $\downarrow\downarrow$

$$\text{t-statistic} \propto \frac{1}{\sqrt{N}}$$