Univariate Statistics: Theoretical aspects and practical applications.

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Univariate Statistics: a reminder

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- whenever possible it should be visually inspected to obtain a better insight into the data.

Univariate Statistics: a reminder

Descriptive measures of a distribution are:

- Minimum
- mean
- median
- maximum
- quantiles

Quantiles

- A quantile is defined for a fraction α (between 0 and 1); it is the value when a fraction α of the data is below this value, and a fraction $\mathbf{1}$-α is above this value.

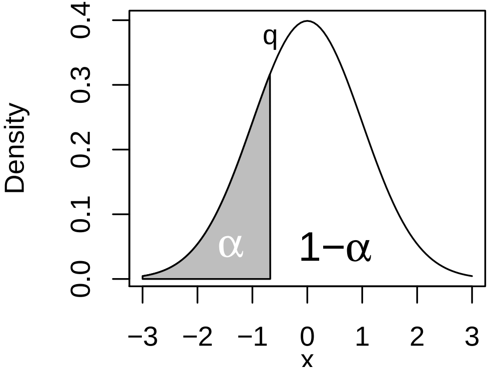

Figure 1: quantile graphical demonstration in standard normal distribution

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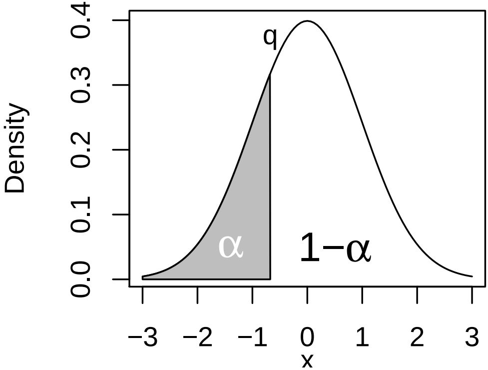
- For **precentiles**, α is expressed in percent (%).



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Quartiles

- **quartiles** divide the data distribution into four parts:

1. First quartile $\boldsymbol{(Q_1)} = 25\ \%$
2. Second quartile $\boldsymbol{(Q_2)} = 50\ \% = \text{median}$
3. Third quartile $\boldsymbol{(Q_3)} = 75\ \%$

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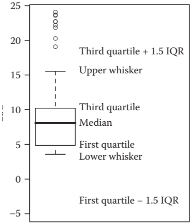
- 

Figure 2: Boxplot ilustration.

Boxplot

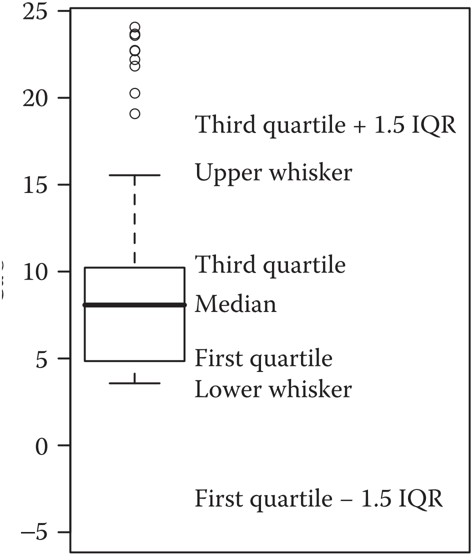


Figure 3: Boxplot ilustration. (zoom)

Theoretical distributions

- Theoretical distributions
  1. Normal distribution:

$$N(\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{1}$$

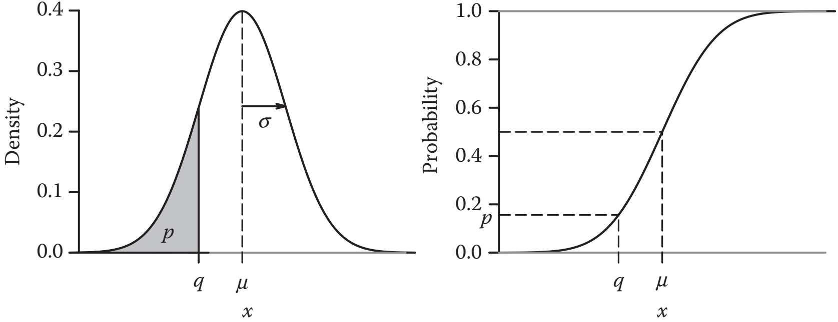


Figure 4: Probability density function (PDF) (left) and cumulative distribution function (right) of the normal distribution.

The probability density, d, at value x is defined by

- For a standard normal distribution:

$$N(0.1) : d(x) = \frac{1}{\sqrt{2\pi}} exp\left(\frac{-x^2}{2}\right) \tag{2}$$

$\mathbb{R}$: `d <- dnorm(x, mean=0, sd=1)`

- For a normal distribution:

$$N(\mu, \sigma^2) : d(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{3}$$

$\mathbb{R}$: `d <- dnorm(x, mean=mu, sd=sigma)`

Standard normal distribution

- Data values x following a normal distribution $N(\mu, \sigma^2)$ can be transformed to a standard normal distribution by the so called z-transformation:

$$z = \frac{(x - \mu)}{\sigma} \tag{4}$$

Normal distribution

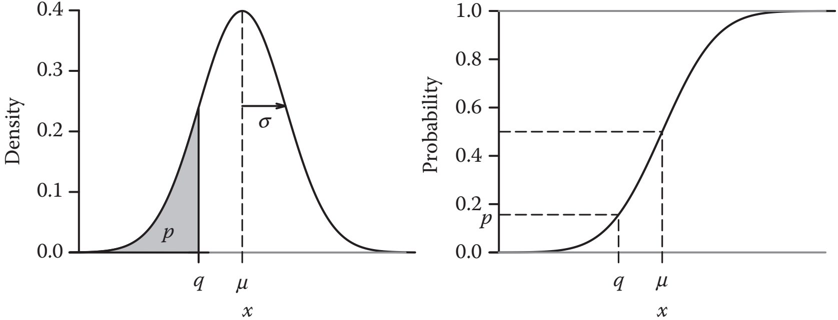


Figure 5: Probability density function (PDF) (left) and cumulative distribution function (right) of the normal distribution.

Other distributions

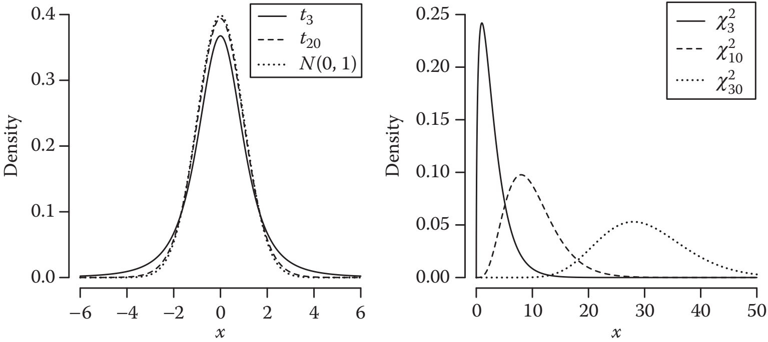


Figure 6: t- distribution and standard normal distribution with different degrees of freedom (left), chi-squared distribution with different degrees of freedom (right).

Other distributions

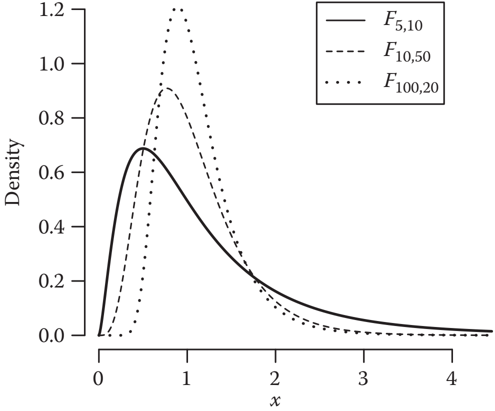


Figure 7: F distribution determined by two parameters.

Quantile Quantile plots

To corroborate that a theoretical distribution, for example the normal distribution, is in fact a good aproximation, we can use quantile quantile plots (qq-plots).

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- we can generate a vector of random numbers following a normal distribution by:
  $\mathbb{R}$:
  ```
  x.sim <- rnorm(50, mean=0, sd=1) %>% jitter
  ps <- (seq(0,99)+0.5)/100
  qs <- quantile(x.sim, ps)
  normalqs <- qnorm(ps, mean(x.sim), popsd(x.sim))
  ```

Bibliography I

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