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### **Contents**

1	Introduction	
	1.1 Licensing	
	1.2 Reporting of Infeasible/Undbounded Models	
	1.3 Solving Problems in Parallel	
	1.3.1 Parallelized Optimizers	
	1.3.2 Concurrent Optimizer	
	1.4 The Infeasibility Report	
	1.5 Nonlinear Programs	
	1.6 Modeling Issues Involving Convex Programs	
2	Conic Programming	
	2.1 Introduction	
	2.2 Implemention of Conic Constraints in GAMS	
	2.3 Example	
3	The MOSEK Options	
	3.1 Memory Considerations for Nonlinear Problems	
4	Summary of MOSEK Options	
	4.1 General and Preprocessing Options	
	4.2 Problem Data Options	
	4.3 Output Options	
	4.4 Interior Point Optimizer Options	
	4.5 Simplex Optimizer and Basis Identification Options	
	4.6 Mixed Integer Optimizer Options	
5	Detailed Descriptions of MOSEK Options	
6	The MOSEK Log File	
	6.1 Log Using the Interior Point Optimizer	
	6.2 Log Using the Simplex Optimizer	
	6.3 Log Using the Mixed Integer Optimizer	

## 1 Introduction

MOSEK is a software package for the solution of linear, mixed-integer linear, quadratic, mixed-integer quadratic, quadratically constraint, and convex nonlinear mathematical optimization problems. MOSEK is particularly well suited for solving large-scale linear and convex quadratically constraint programs using an extremely efficient interior point algorithm. The interior point algorithm has many complex solver options which the user can specify to fine-tune the optimizer for a particular model.

Furthermore, MOSEK can solve generalized linear programs involving nonlinear conic constraints, convex quadratically constraint and general convex nonlinear programs.

These problem classes can be solved using an appropriate optimizer built into MOSEK. All the optimizers available in MOSEK are built for the solution of large-scale sparse problems. Current optimizers include:

- Interior-point optimizer for all continuous problems
- Conic interior-point optimizer for conic quadratic problems
- Simplex optimizer for linear problems
- Mixed-integer optimizer based on a branch and cut technology

#### 1.1 Licensing

Licensing of GAMS/MOSEK is similar to other GAMS solvers. MOSEK is licensed in three different ways:

- GAMS/MOSEK Base:
  - All continuous models
- GAMS/MOSEK Extended:

Same as GAMS/MOSEK Base, but also the solution of models involving discrete variables.

• GAMS/MOSEK Solver Link:

Users must have a seperate, licensed MOSEK system. For users who wish to use MOSEK within GAMS and also in other environments.

For more information contact sales@gams.com. For information regarding MOSEK standalone or interfacing MOSEK with other applications contact sales@mosek.com.

### 1.2 Reporting of Infeasible/Undbounded Models

MOSEK determines if either the primal or the dual problem is infeasible by means of a Farkas certificate. In such a case MOSEK returns a certificate indicating primal or dual infeasibility. A primal infeasibility certificate indicates a primal infeasible model and the certificate is reported in the marginals of the equations in the listing file. The primal infeasibility certificate for a minimization problem

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  
 $x \ge 0$ 

is the solution y satisfying:

$$A^T y \le 0, \quad b^T y > 0$$

A dual infeasibility certificate is reported in the levels of the variables in the listing file. The dual infeasibility certificate *x* for the same minimization problem is

$$Ax = 0$$
,  $c^T x < 0$ 

Since GAMS reports all model statuses in the primal space, the notion of dual infeasibility does not exist and GAMS reports a status of unbounded, which assumes the primal problem is feasible. Although GAMS reports the primal as unbounded, there is the possibility that both the primal *and* dual problem are infeasible. To check if this is the case, the user can set appropriate upper and lower bounds on the objective variable, using the (varible).LO and (variable).UP suffixes and resolve.

For more information on primal and dual infeasibility certificates see the MOSEK User's manual at www.mosek.com.

#### 1.3 Solving Problems in Parallel

If a computer has multiple CPUs (or a CPU with multiple cores), then it might be advantageous to use the multiple CPUs to solve the optimization problem. For instance if you have two CPUs you may want to exploit the two CPUs to solve the problem in the half time. MOSEK can exploit multiple CPUs.

#### 1.3.1 Parallelized Optimizers

Only the interior-point optimizer in MOSEK has been parallelized.

This implies that whenever the MOSEK interior-point optimizer should solve an optimization problem, then it will try to divide the work so each CPU gets a share of the work. The user decides how many CPUs MOSEK should exploit. Unfortunately, it is not always easy to divide the work. Also some of the coordination work must occur in sequential. Therefore, the speed-up obtained when using multiple CPUs is highly problem dependent. However, as a rule of thumb if the problem solves very quickly i.e. in less than 60 seconds, then it is not advantageous of using the parallel option.

The parameter MSK\_IPAR\_INTPNT\_NUM\_THREADS sets the number of threads (and therefore the number of CPU's) that the interior point optimizer will use.

#### 1.3.2 Concurrent Optimizer

An alternative to use a parallelized optimizer is the concurrent optimizer. The idea of the concurrent optimizer is to run multiple optimizers on the same problem concurrently. For instance the interior-point and the dual simplex optimizers may be applied to an linear optimization problem concurrently. The concurrent optimizer terminates when the first optimizer has completed and reports the solution of the fastest optimizer. That way a new optimizer has been created which essentially has the best performance of the interior-point and the dual simplex optimizer.

Hence, the concurrent optimizer is the best one to use if there multiple optimizers available in MOSEK for the problem and you cannot say beforehand which one is the best one. For more details inspect the MSK\_IPAR\_CONCURRENT\_\* options.

### 1.4 The Infeasibility Report

MOSEK has some facilities for diagnosing the cause of a primal or dual infeasibility. They can be turned on using the parameter setting MSK\_IPAR\_INFEAS\_REPORT\_AUTO. This causes MOSEK to print a report about an infeasible subset of the constraints, when an infeasibility is encountered. Moreover, the parameter MSK\_IPAR\_INFEAS\_REPORT\_LEVEL controls the amount info presented in the infeasibility report. We will use the trnsport.gms example from the GAMS Model Library with increased demand (b(j)=1.6\*b(j)) to make the model infeasible. MOSEK produces the following infeasibility report

MOSEK PRIMAL INFEASIBILITY REPORT.

Problem status: The problem is primal infeasible

The following constraints are involved in the primal infeasibility.

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
1	supply(seattle)	none	3.500000e+002	0.000000e+000	1.000000e+000
2	supply(san-diego)	none	6.000000e+002	0.000000e+000	1.000000e+000
3	demand(new-york)	5.200000e+002	none	1.000000e+000	0.000000e+000
4	demand(chicago)	4.800000e+002	none	1.000000e+000	0.000000e+000
5	demand(topeka)	4.400000e+002	none	1.000000e+000	0.000000e+000

The following bound constraints are involved in the infeasibility.

Index Name Lower bound Upper bound Dual lower Dual upper

which indicates which constraints and bounds that are important for the infeasibility i.e. causing the infeasibility. The infeasibility report is divided into two sections where the first section shows which constraints that are important for the infeasibility. In this case the important constraints are supply and demand. The values in the columns Dual lower and Dual upper are also useful, because if the dual lower value is different from zero for a constraint, then it implies that the lower bound on the constraint is important for the infeasibility. Similarly, if the dual upper value is different from zero on a constraint, then this implies the upper bound on the constraint is important for infeasibility.

### 1.5 Nonlinear Programs

MOSEK can efficiently solve convex programs, but is not intended for nonconvex optimization. For nonconvex programs, MOSEK can detect some nonconvexities and will print out a warning message and terminate. If MOSEK does not detect nonconvexities for a nonconvex model, the optimizer may continue but stagnate. Hence care must be taken when solving nonlinear programs if convexity is not immediately known.

### 1.6 Modeling Issues Involving Convex Programs

It is often preferable to model convex programs in seperable form, if it is possible. Consider the following example of minizing an objective function f(x):

$$f(x) = \log(a' * x)$$

where  $a \in \Re^n$  is a parameter and  $x \in \Re^n$  the decision variable. The equation implies an implicit constraint of a' \* x > 0. Unfortunately, domain violations can still occur because no restrictions are set on a' \* x. A better approach is to introduce an intermediate variable y:

$$\begin{array}{rcl}
f(x) & = & \log(y) \\
y & = & a' * x \\
y & > & 0
\end{array}$$

This accomplishes two things. It implies an explicit bound on a' \* x, thereby reducing the risk of domain violations. Secondly, it speeds up computation since computations of gradients and Hessians in the first (non-seperable) form are more expensive. Finally, it reduces the amount of memory needed (see the section on "Memory Options")

## 2 Conic Programming

MOSEK is well suited for solving generalized linear programs involving nonlinear conic constraints. Conic programming is useful in a wide variety of application areas including engineering and financial management. Conic programming has been used, for example, in antenna array weight design, grasping force optimization, finite impulse response (FIR) filter design, and portfolio optimization.

This section gives an overview of conic programming and how conic constraints are implemented in GAMS.

### 2.1 Introduction

Conic programs can be thought of as generalized linear programs with the additional nonlinear constraint  $x \in C$ , where C is required to be a convex cone. The resulting class of problems is known as *conic optimization* and has the following form:

minimize 
$$c^T x$$
  
subject to  $Ax \le r^c$ ,  $x \in [l^x, u^x]$   
 $x \in C$ 

where  $A \in \Re^{m \times n}$  is the constraint matrix,  $x \in \Re^n$  the decision variable, and  $c \in \Re^n$  the objective function cost coefficients. The vector  $r^c \in \Re^m$  represents the right hand side and the vectors  $l^x, u^x \in \Re^n$  are lower and upper bounds on the decision variable x.

<sup>&</sup>lt;sup>1</sup>See M. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, *Applications of second-order cone programming* Linear Algebra and its Applications, 284:193-228, Special Issue on Linear Algebra in Control, Signals and Image Processing. November, 1998.

Now partition the set of decision variables x into sets  $S^t$ , t = 1, ..., k, such that each decision variables x is a member of at most one set  $S^t$ . For example, we could have

$$S^{1} = \begin{bmatrix} x_{1} \\ x_{4} \\ x_{7} \end{bmatrix} and S^{2} = \begin{bmatrix} x_{6} \\ x_{5} \\ x_{3} \\ x_{2} \end{bmatrix}.$$
 (2.1)

Let  $x_{S^t}$  denote the variables x belonging to set  $S^t$ . Then define

$$C := \{ x \in \Re^n : x_{S^t} \in C_t, t = 1, ..., k \}$$
(2.2)

where  $C_t$  must have one of the following forms:

• Quadratic cone: (also referred to as Lorentz or ice cream cone)

$$C_{t} = \left\{ x \in \Re^{n^{t}} : x_{1} \ge \sqrt{\sum_{j=2}^{n^{t}} x_{j}^{2}} \right\}.$$
 (2.3)

• Rotated quadratic cone: (also referred to as hyberbolic constraints)

$$C_t = \left\{ x \in \Re^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \ x_1, x_2 \ge 0 \right\}.$$
 (2.4)

These two types of cones allow the formulation of quadratic, quadratically constrained, and many other classes of nonlinear convex optimization problems.

### 2.2 Implemention of Conic Constraints in GAMS

GAMS handles conic equations using the =C= equation type. The conic cases are written as:

• Quadratic cone:

$$x('1') = C = sum(i[not sameas(i, '1')], x(i));$$
 (2.5)

• Rotated quadratic cone:

$$x('1') + x('2') = C = sum(i [not sameas(i, '1') and not sameas(i, '2')], x(i));$$
 (2.6)

Note that the resulting nonlinear conic constraints result in "linear" constraints in GAMS. Thus the original nonlinear formulation is in fact a linear model in GAMS. We remark that we could formulate conic problems as regular NLP using constraints:

• Quadratic cone:

$$x('1') = G = \operatorname{sqrt}[\operatorname{sum}(i\$[\operatorname{not} \operatorname{sameas}(i,'1')], \operatorname{sqr}[x(i)])]; \tag{2.7}$$

• Rotated quadratic cone: x('1') and x('2') are positive variables

$$2*x('1')*x('2') = \texttt{G} = \texttt{sum}(\texttt{i}\$[\texttt{not sameas}(\texttt{i}, '1') \texttt{ and not sameas}(\texttt{i}, '2')], \texttt{sqr}[\texttt{x}(\texttt{i})]); \tag{2.8}$$

The example below illustrates the different formulations for conic programming problems. Note that the conic optimizer in MOSEK usually outperforms a general NLP method for the reformulated (NLP) cone problems.

### 2.3 Example

Consider the following example (cone2.gms) which illustrates the use of rotated conic constraints. We will give reformulations of the original problem in regular NLP form using conic constraints and in conic form.

The original problem is:

minimize 
$$\sum_{i} \frac{d_{i}}{x_{i}}$$
  
subject to  $a^{t}x \leq b$   
 $x_{i} \in [l_{i}, u_{i}], l_{i} > 0, d_{i} \geq 0, i = 1, 2, ..., n$ 

$$(2.9)$$

where  $x \in \Re^n$  is the decision variable,  $d, a, l, u \in \Re^n$  parameters, and  $b \in \Re$  a scalar parameter. The original model (2.9) can be written in GAMS using the equations:

```
defobj.. sum(n, d(n)/x(n)) = E = obj;
e1.. sum(n, a(n)*x(n)) = L = b;
Model orig /defobj, e1/;
x.lo(n) = l(n);
x.up(n) = u(n);
```

We can write an equivalent NLP formulation, replacing the objective function and adding another constraint:

minimize 
$$\sum_{i} d_{i}t_{i}$$
  
subject to  $a^{t}x \leq b$   
 $2t_{i}x_{i} \geq 2, \quad i = 1,...,n$   
 $x \in [l,u], \quad l > 0, \quad d_{i} \geq 0$  (2.10)

where  $t \in \Re^n$  is a new decision variable. The GAMS formulation of this NLP (model cnlp) is:

```
defobjc.. sum(n, d(n)*t(n)) = E = obj;
e1.. sum(n, a(n)*x(n)) = L = b;
conenlp(n).. 2*t(n)*x(n) = G = 2;

Model cnlp /defobjc, e1, conenlp/;
x.lo(n) = l(n);
x.up(n) = u(n);
```

We can change the equality to an inequality since the parameter  $d_i \ge 0$  and we are dealing with a minimization problem. Also, note that the constraint conenlp(n) is almost in rotated conic form. If we introduce a variable  $z \in \Re^n, z_i = \sqrt{2}$ , then we can reformulate the problem using conic constraints as:

minimize 
$$\sum_{i} d_{i}t_{i}$$
  
subject to  $a^{l}x \leq b$   
 $z_{i} = \sqrt{2}$   
 $2t_{i}x_{i} \geq z_{i}^{2}, \quad i = 1,...,n$   
 $x \in [l,u], \quad l > 0, \quad d_{i} > 0$  (2.11)

The GAMS formulation using conic equations =C= is:

```
defobjc.. sum(n, d(n)*t(n)) = E = obj;
e1.. sum(n, a(n)*x(n)) = L = b;
e2(n).. z(n) = E = sqrt(2);
cone(n).. x(n) + t(n) = C = z(n);

Model clp /defobjc, e1, e2, cone/;
x.lo(n) = l(n);
x.up(n) = u(n);
```

Note that this formulation is a linear program in GAMS, although the constraints cone(n)... represent the nonlinear rotated quadratic cone constraint.

The complete model is listed below:

```
Set n / n1*n10 /;
Parameter d(n), a(n), l(n), u(n);
Scalar b:
d(n) = uniform(1,2);
a(n) = uniform (10,50);
1(n) = uniform(0.1,10);
u(n) = l(n) + uniform(0,12-l(n));
Variables x(n);
x.l(n) = uniform(l(n), u(n));
b = sum(n, x.l(n)*a(n));
Variables t(n), z(n), obj;
Equations defobjc, defobj, e1, e2(n), cone(n), conenlp(n);
              sum(n, d(n)*t(n)) = E = obj;
defobjc..
defobj..
              sum(n, d(n)/x(n)) = E = obj;
e1..
              sum(n, a(n)*x(n)) = L = b;
e2(n)..
              z(n) = E = sqrt(2);
cone(n)..
              x(n) + t(n) = C = z(n);
conenlp(n).. 2*t(n)*x(n) = G= 2;
Model clp /defobjc, e1, e2, cone/;
Model cnlp /defobjc, e1, conenlp/;
Model orig /defobj, e1/;
x.lo(n) = l(n);
x.up(n) = u(n);
Solve clp min obj using lp;
Solve cnlp min obj using nlp;
Solve orig min obj using nlp;
```

## 3 The MOSEK Options

MOSEK works like other GAMS solvers, and many options can be set in the GAMS model. The most relevant GAMS options are reslim, nodlim, optca, optcr, and optfile. The option iterlim works only for the simplex optimizer. A description of all available GAMS options can be found in Chapter "Using Solver Specific Options".

We remark that MOSEK contains many complex solver options, many of which require a deep understanding of the algorithms used. For a complete description of the more than 175 MOSEK options, consult the MOSEK User's Guide, available online at www.mosek.com.

If you specify "<modelname>.optfile = 1;" before the SOLVE statement in your GAMS model, MOSEK will then look for and read an option file with the name *mosek.opt* (see "Using Solver Specific Options" for general use of solver option files). The syntax for the MOSEK option file is

```
optname value
```

with one option on each line.

For example,

```
MSK_IPAR_INTPNT_MAX_ITERATIONS 20
MSK_IPAR_INTPNT_SCALING 1
```

The first option specifies the maximum number of interior-point iterations, in this case 20. The seond option indicates a

scaling option of 1, which is no scaling.

We remark that users can also use symbolic constants in place of numerical values. For example, for the scaling option users could use MSK\_SCALING\_NONE in place of the value 1. For a complete list of applicable symbolic constants, consult the MOSEK parameter list available online at www.mosek.com.

### 3.1 Memory Considerations for Nonlinear Problems

The GAMS *workfactor* option can be used to increase the amount of memory available to MOSEK. The general syntax is *(modelname).workfactor = (value)* 

with a default value of 1. See the section on "Using Solver Specific Options" for details. If GAMS/MOSEK runs out of memory, an error message is printed out:

\*\*\* GAMS/MOSEK interface error.

The size estimate for Hessian of Lagrangian is too small. Try to increase workfactor option from 1 to a larger value.

GAMS/MOSEK estimates the size of the Hessian as  $5 * (number\ of\ nonlinear\ variables) * (workfactor)$ . Because of symmetry, the size of the Hessian is bounded by

 $H_d * (number of nonlinear variables)^2/2$ 

where  $H_d$  detotes the density of the Hessian and  $H_d \in [0,1]$ . Therefore, one can choose the workfactor as:

 $workfactor = H_d * (number of nonlinear variables) * /(5 * 2)$ 

Note that for a seperable model (see "Modeling Issues Involving Convex Programs"), the workfactor can in fact be reduced to 1/5.

## 4 Summary of MOSEK Options

### 4.1 General and Preprocessing Options

MSK\_IPAR\_CACHE\_SIZE\_L1

L1 cache size used

MSK\_IPAR\_CACHE\_SIZE\_L2

L2 cache size used

MSK\_IPAR\_CONCURRENT\_NUM\_OPTIMIZERS

maximum number of optimizers during concurrent run

MSK\_IPAR\_CONCURRENT\_PRIORITY\_DUAL\_SIMPLEX

priority of dual simplex algorithm in concurrent run

MSK\_IPAR\_CONCURRENT\_PRIORITY\_FREE\_SIMPLEX

priority of free simplex algorithm in concurrent run

MSK\_IPAR\_CONCURRENT\_PRIORITY\_INTPNT

priority of interior point algorithm in concurrent run

MSK\_IPAR\_CONCURRENT\_PRIORITY\_PRIMAL\_SIMPLEX

priority of primal simplex algorithm in concurrent run

MSK\_IPAR\_CPU\_TYPE

specifies the CPU type

MSK\_IPAR\_INFEAS\_REPORT\_AUTO

switch for infeasibility report

MSK\_IPAR\_INFEAS\_REPORT\_LEVEL

output level for infeasibility report

MSK\_IPAR\_OPTIMIZER

optimizer selection

MSK\_DPAR\_OPTIMIZER\_MAX\_TIME

time limit

MSK\_SPAR\_PARAM\_READ\_FILE\_NAME

name of a secondary MOSEK option file

MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_USE

switch for free variable elimination

MSK\_IPAR\_PRESOLVE\_ELIM\_FILL

fill-in control during presolve

MSK\_IPAR\_PRESOLVE\_LINDEP\_USE

linear dependency check

MSK\_IPAR\_PRESOLVE\_LINDEP\_WORK\_LIM

maximum work for finding linear dependencies

MSK\_IPAR\_PRESOLVE\_USE

switch for presolve

### **4.2 Problem Data Options**

MSK\_IPAR\_CHECK\_CONVEXITY

level of convexity check for quadratic problems

MSK\_DPAR\_DATA\_TOL\_AIJ

zero tolerance for matrix coefficients

MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE

error for large coefficients in matrix

MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE

warning for large coefficients in matrix

MSK\_DPAR\_DATA\_TOL\_BOUND\_INF

bound value for infinity

MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN

warning for large bounds

MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE

warning for large coefficients in objective

MSK\_DPAR\_DATA\_TOL\_C\_HUGE

error for huge coefficients in objective

MSK\_DPAR\_DATA\_TOL\_QIJ

zero tolerance for Q matrix coefficients

MSK\_DPAR\_DATA\_TOL\_X

tolerance for fixed variables

MSK\_DPAR\_LOWER\_OBJ\_CUT

lower objective limit

MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

upper objective limit threashold

MSK\_DPAR\_UPPER\_OBJ\_CUT

upper objective limit

MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

lower objective limit threashold

### 4.3 Output Options

MSK\_IPAR\_LOG\_BI

output control for basis identification

MSK\_IPAR\_LOG\_BI\_FREQ

frequency of log output of basis identification

MSK\_IPAR\_LOG\_INTPNT

output level of the interior-point optimizer

MSK\_IPAR\_LOG\_MIO

output level for mixed integer optimizer

MSK\_IPAR\_LOG\_MIO\_FREQ

frequency of log output of mixed integer optimizer

MSK\_IPAR\_LOG\_PRESOLVE

output level for presolve

MSK\_IPAR\_LOG\_SIM

output level for simplex

MSK\_IPAR\_LOG\_SIM\_FREQ

frequency of log output of simplex optimizer

MSK\_IPAR\_MAX\_NUM\_WARNINGS

maximum number of warnings

MSK\_IPAR\_WARNING\_LEVEL

warning level

### 4.4 Interior Point Optimizer Options

MSK\_IPAR\_INTPNT\_BASIS

switch for basis identification

MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS

dual feasibility tolerance for the conic interior-point optimizer

MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS

infeasibility control for the conic interior-point optimizer

MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED

relative complementarity tolerance for the conic interior-point optimizer

MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

termination tolerances for near optimal for the conic interior-point optimizer

MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

primal feasibility tolerance for the conic interior-point optimizer

MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

relative optimality tolerance for the conic interior-point optimizer

MSK\_IPAR\_INTPNT\_DIFF\_STEP

switch for different step sizes

MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS

iteration limit for the interior-point optimizer

MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR

maximum number of correctors

MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS

number of steps to be used by the iterative refinement

MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL

balance for complementarity and infeasibility

MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS

dual feasibility tolerance for nonlinear problems

MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED

relative complementarity tolerance for nonlinear problems

MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS

primal feasibility tolerance for nonlinear problems

MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP

relative optimality tolerance for nonlinear problems

MSK\_IPAR\_INTPNT\_NUM\_THREADS

number of threads for interior-point optimizer

MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH

offending column selection

MSK\_IPAR\_INTPNT\_ORDER\_METHOD

ordering strategy selection

MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE

switch for regularization

MSK\_IPAR\_INTPNT\_SCALING

scaling selection for interior-point optimizer

MSK\_IPAR\_INTPNT\_SOLVE\_FORM

solve primal or the dual problem with interior-point optimizer

MSK\_IPAR\_INTPNT\_STARTING\_POINT

starting point for interior-point optimizer

MSK\_DPAR\_INTPNT\_TOL\_DFEAS

dual feasibility tolerance

MSK\_DPAR\_INTPNT\_TOL\_DSAFE

initial dual starting control

MSK\_DPAR\_INTPNT\_TOL\_INFEAS

infeasibility control

MSK\_DPAR\_INTPNT\_TOL\_MU\_RED

relative complementarity tolerance

MSK\_DPAR\_INTPNT\_TOL\_PATH

central path following for interior-point optimizer

MSK\_DPAR\_INTPNT\_TOL\_PFEAS

primal feasibility tolerance

MSK\_DPAR\_INTPNT\_TOL\_PSAFE

initial primal starting control

MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP

relative optimality tolerance

 $MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP$ 

relative step size to boundary

USE\_BASIS\_EST

use MOSEK basis estimation in case of an interior solution

### 4.5 Simplex Optimizer and Basis Identification Options

MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

simplex optimizer section after basis identification

MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER

continues BI in case of iteration limit

MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

continues BI in case of numerical error

MSK\_IPAR\_BI\_MAX\_ITERATIONS

maximum number of simplex iterations after basis identification

MSK\_IPAR\_SIM\_DUAL\_CRASH

dual simplex crash

MSK\_IPAR\_SIM\_DUAL\_SELECTION

dual simplex pricing selection

MSK\_IPAR\_SIM\_HOTSTART

controls simplex hotstart

MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV

relative pivot tolerance for simplex and basis identification

MSK\_IPAR\_SIM\_MAX\_ITERATIONS

simplex iteration limit

MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS

maximum number of setbacks

MSK\_IPAR\_SIM\_PRIMAL\_CRASH

primal simplex crash

MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

primal simplex pricing selection

MSK\_IPAR\_SIM\_REFACTOR\_FREQ

refactorization frequency

MSK\_IPAR\_SIM\_REFORMULATION

controls if the simplex optimizers are allowed to reformulate

MSK\_IPAR\_SIM\_SCALING

scaling selection for simplex optimizer

MSK\_IPAR\_SIM\_SCALING\_METHOD

controls how the problem is scaled before a simplex optimizer is used

MSK\_IPAR\_SIM\_SOLVE\_FORM

solve primal or the dual problem with simplex optimizer

### 4.6 Mixed Integer Optimizer Options

MSK\_IPAR\_MIO\_BRANCH\_DIR

control branching directions

MSK\_IPAR\_MIO\_CONSTRUCT\_SOL

switch for mip start

MSK\_IPAR\_MIO\_CUT\_LEVEL\_ROOT

cut level control at root for mixed integer optimizer

MSK\_IPAR\_MIO\_CUT\_LEVEL\_TREE

cut level control in tree for mixed integer optimizer

MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL

heuristic control for mixed integer optimizer

MSK\_DPAR\_MIO\_HEURISTIC\_TIME

time limit for heuristic search

MSK\_IPAR\_MIO\_KEEP\_BASIS

switch for basis saving

MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

maximum number of branches

MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS

maximum number of relaxations solved

MSK\_DPAR\_MIO\_MAX\_TIME

time limit for mixed integer optimizer

MSK\_DPAR\_MIO\_MAX\_TIME\_APRX\_OPT

time limit before some relaxation

MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP

termination criterion on absolute optimality tolerance

MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP

termination criterion on relative optimality tolerance

MSK\_IPAR\_MIO\_NODE\_OPTIMIZER

solver for the sub problems

MSK\_IPAR\_MIO\_NODE\_SELECTION

node selection strategy

#### MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATE

switch for aggregation during mixed integer presolve

MSK\_IPAR\_MIO\_PRESOLVE\_PROBING

switch for probing

MSK\_IPAR\_MIO\_PRESOLVE\_USE

switch for mixed integer presolve

MSK\_DPAR\_MIO\_REL\_ADD\_CUT\_LIMITED

cuts factor

MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER

solver for the root problem

MSK\_IPAR\_MIO\_STRONG\_BRANCH

strong branching control

MSK\_DPAR\_MIO\_TOL\_ABS\_GAP

absolute optimality tolerance in the mixed integer optimizer

MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT

absolute integrality tolerance

MSK\_DPAR\_MIO\_TOL\_FEAS

feasibility tolerance for mixed integer solver

MSK\_DPAR\_MIO\_TOL\_REL\_GAP

relative optimality tolerance in the mixed integer optimizer

MSK\_DPAR\_MIO\_TOL\_REL\_RELAX\_INT

relative integrality tolerance

**MIPSTART** 

use integer solution provided by user

## 5 Detailed Descriptions of MOSEK Options

#### MSK\_IPAR\_CACHE\_SIZE\_L1 (integer)

Controls the size of the L1 cache used by MOSEK.

(default = -1)

### MSK\_IPAR\_CACHE\_SIZE\_L2 (integer)

Controls the size of the L2 cache used by MOSEK.

(default = -1)

### MSK\_IPAR\_CONCURRENT\_NUM\_OPTIMIZERS (integer)

(default = 2)

### MSK\_IPAR\_CONCURRENT\_PRIORITY\_DUAL\_SIMPLEX (integer)

(default = 2)

### MSK\_IPAR\_CONCURRENT\_PRIORITY\_FREE\_SIMPLEX (integer)

(default = 3)

### MSK\_IPAR\_CONCURRENT\_PRIORITY\_INTPNT (integer)

(default = 4)

### MSK\_IPAR\_CONCURRENT\_PRIORITY\_PRIMAL\_SIMPLEX (integer)

(default = 1)

#### MSK\_IPAR\_CPU\_TYPE (string)

This option specifies the CPU type.

- MSK\_CPU\_AMD\_ATHLON
- MSK\_CPU\_AMD\_OPTERON
- MSK\_CPU\_GENERIC
- MSK\_CPU\_INTEL\_CORE2
- MSK\_CPU\_INTEL\_P3
- MSK\_CPU\_INTEL\_P4
- MSK\_CPU\_INTEL\_PM
- MSK\_CPU\_POWERPC\_G5
- MSK\_CPU\_UNKNOWN

### MSK\_IPAR\_INFEAS\_REPORT\_AUTO (integer)

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

(default = 0)

#### MSK\_IPAR\_INFEAS\_REPORT\_LEVEL (integer)

Controls the amount info presented in an infeasibility report. Higher values implies more information.

(default = 1)

### MSK\_IPAR\_OPTIMIZER (string)

Controls which optimizer is used to optimize the task.

- MSK\_OPTIMIZER\_CONCURRENT
- MSK\_OPTIMIZER\_CONIC
- MSK\_OPTIMIZER\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_FREE
- MSK\_OPTIMIZER\_FREE\_SIMPLEX
- MSK\_OPTIMIZER\_INTPNT
- MSK\_OPTIMIZER\_MIXED\_INT
- MSK\_OPTIMIZER\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX
- MSK\_OPTIMIZER\_QCONE

#### MSK\_DPAR\_OPTIMIZER\_MAX\_TIME (real)

Maximum amount of time the optimizer is allowed to spend on the optimization. A negative number means infinity. (default = GAMS ResLim)

### MSK\_SPAR\_PARAM\_READ\_FILE\_NAME (string)

The name of a secondary MOSEK option file that by the MOSEK option reader.

#### MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_USE (integer)

Controls whether free or implied free variables are eliminated from the problem.

(default = 1)

#### MSK\_IPAR\_PRESOLVE\_ELIM\_FILL (integer)

Controls the maximum amount of fill-in that can be created during the eliminations phase of the presolve. This parameter times the number of variables plus the number of constraints denotes the amount of fill in.

```
(default = 1)
```

#### MSK\_IPAR\_PRESOLVE\_LINDEP\_USE (integer)

Controls whether the linear constraints is checked for linear dependencies.

(default = 1)

#### MSK\_IPAR\_PRESOLVE\_LINDEP\_WORK\_LIM (integer)

Is used to limit the work that can used to locate the linear dependencies. In general the higher value this parameter is given the less work can be used. However, a value of 0 means no limit on the amount work that can be used.

(default = 1)

#### MSK\_IPAR\_PRESOLVE\_USE (string)

Controls whether presolve is performed.

- MSK\_PRESOLVE\_MODE\_FREE
- MSK\_PRESOLVE\_MODE\_OFF
- MSK\_PRESOLVE\_MODE\_ON

#### MSK\_IPAR\_CHECK\_CONVEXITY (string)

Specify the level of convexity check on quadratic problems.

- MSK\_CHECK\_CONVEXITY\_FULL
- MSK\_CHECK\_CONVEXITY\_NONE
- MSK\_CHECK\_CONVEXITY\_SIMPLE

#### MSK\_DPAR\_DATA\_TOL\_AIJ (real)

Absolute zero tolerance for coefficients in the constraint matrix.

Range: [1.0e-16,1.0e-6] (default = 1.0e-12)

#### MSK\_DPAR\_DATA\_TOL\_ALJ\_HUGE (real)

An element in the constraint matrix which is larger than this value in absolute size causes an error.

DATA\_TOL\_BOUND\_INF Any bound which in absolute value is greater than this parameter is considered infinite.

(default = 1.0e20)

#### MSK\_DPAR\_DATA\_TOL\_ALJ\_LARGE (real)

A coefficient in the constraint matrix which is larger than this value in absolute size causes a warning message to be printed.

(default = 1.0e10)

### MSK\_DPAR\_DATA\_TOL\_BOUND\_INF (real)

(default = 1.0e16)

#### MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN (real)

If a bound value is larger than this value in absolute size, then a warning message is issued.

(default = 1.0e8)

### MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE (real)

A coefficient in the objective which is larger than this value in absolute terms causes a warning message to be printed. (default = 1.0e8)

#### MSK\_DPAR\_DATA\_TOL\_C\_HUGE (real)

A coefficient in the objective which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

(default = 1.0e16)

#### MSK\_DPAR\_DATA\_TOL\_QIJ (real)

Absolute zero tolerance for coefficients in the Q matrices.

(default = 1.0e-16)

#### MSK\_DPAR\_DATA\_TOL\_X (real)

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and lower bound is considered identical.

(default = 1.0e-8)

#### MSK\_DPAR\_LOWER\_OBJ\_CUT (real)

If a feasible solution having and objective value outside, the interval LOWER\_OBJ\_CUT, UPPER\_OBJ\_CUT, then MOSEK is terminated.

(default = -1.0e30)

#### MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH (real)

If the lower objective cut (LOWER\_OBJ\_CUT) is less than LOWER\_OBJ\_CUT\_FINITE\_TRH, then the lower objective cut LOWER\_OBJ\_CUT is treated as infinity.

(default = -0.5e30)

#### MSK\_DPAR\_UPPER\_OBJ\_CUT (real)

If a feasible solution having and objective value outside, the interval LOWER\_OBJ\_CUT, UPPER\_OBJ\_CUT, then MOSEK is terminated.

(default = 1.0e30)

### MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH (real)

If the upper objective cut (UPPER\_OBJ\_CUT) is greater than UPPER\_OBJ\_CUT\_FINITE\_TRH, then the upper objective cut UPPER\_OBJ\_CUT is treated as infinity.

(default = 0.5e30)

### MSK\_IPAR\_LOG\_BI (integer)

Controls the amount of output printed by the basis identification procedure.

(default = 4)

#### MSK\_IPAR\_LOG\_BI\_FREQ (integer)

Controls how frequent the optimizer outputs information about the basis identification is called.

(default = 2500)

### MSK\_IPAR\_LOG\_INTPNT (integer)

Controls the amount of output printed by the interior-point optimizer.

(default = 4)

#### MSK\_IPAR\_LOG\_MIO (integer)

Controls the print level for the mixed integer optimizer.

(default = 4)

#### MSK\_IPAR\_LOG\_MIO\_FREQ (integer)

Controls how frequent the mixed integer optimizer prints the log line. It will print a line every time MSK\_INTPAR\_LOG\_MIO\_FREQ relaxations have been solved.

(default = 1000)

#### MSK\_IPAR\_LOG\_PRESOLVE (integer)

Controls amount of output printed by the presolve procedure.

(default = 1)

#### MSK\_IPAR\_LOG\_SIM (integer)

Controls amount of output printed by the simplex optimizer.

(default = 4)

#### MSK\_IPAR\_LOG\_SIM\_FREQ (integer)

Controls how frequent the simplex optimizer outputs information about the optimization.

(default = 500)

#### MSK\_IPAR\_MAX\_NUM\_WARNINGS (integer)

Sets the maximum number of warnings.

(default = 10)

#### MSK\_IPAR\_WARNING\_LEVEL (integer)

Warning level. A higher value implies more warnings.

(default = 1)

#### MSK\_IPAR\_INTPNT\_BASIS (string)

Controls whether the interior-point optimizer also computes an optimal basis.

- MSK\_BI\_ALWAYS
- MSK\_BI\_IF\_FEASIBLE
- MSK\_BI\_NEVER
- MSK\_BI\_NO\_ERROR
- MSK\_BI\_OTHER

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS (real)

Dual feasibility tolerance used by the conic interior-point optimizer.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS (real)

Controls when the conic interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

Range: [0.0,1.0](default = 1.0e-8)

### MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED (real)

Relative complementarity gap tolerance feasibility tolerance used by the conic interior-point optimizer.

Range: [0.0,1.0] (default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL (real)

If MOSEK cannot compute a solution that has the prescribed accuracy, then it will multiply the termination tolerances with value of this parameter. If the solution then satisfies the termination criteria, then the solution is denoted near optimal, near feasible and so forth.

(default = 100)

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS (real)

Primal feasibility tolerance used by the conic interior-point optimizer.

Range: [0.0,1.0](default = 1.0e-8)

### MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP (real)

Relative gap termination tolerance used by the conic interior-point optimizer.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_IPAR\_INTPNT\_DIFF\_STEP (integer)

Controls whether different step sizes are allowed in the primal and dual space.

(default = 1)

### MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS (integer)

Sets the maximum number of iterations allowed in the interior-point optimizer.

(default = 400)

#### MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR (integer)

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that Mosek is making the choice.

(default = -1)

#### MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS (integer)

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

(default = -1)

### MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL (real)

Controls if the complementarity and infeasibility is converging to zero at about equal rates.

Range: [0.0,0.99] (default = 1.0e-4)

### MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS (real)

Dual feasibility tolerance used when a nonlinear model is solved.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED (real)

Relative complementarity gap tolerance used when a nonlinear model is solved..

Range: [0.0,1.0] (default = 1.0e-12)

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS (real)

Primal feasibility tolerance used when a nonlinear model is solved.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP (real)

Relative gap termination tolerance for nonlinear problems.

(default = 1.0e-6)

### MSK\_IPAR\_INTPNT\_NUM\_THREADS (integer)

Controls the number of threads employed by the interior-point optimizer.

(default = 1)

### MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH (integer)

Controls how many offending columns there are located in the Jacobian the constraint matrix. 0 means no offending columns will be detected. In general by increasing the number fewer offending columns will be detected.

(default = 40)

#### MSK\_IPAR\_INTPNT\_ORDER\_METHOD (string)

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

- MSK\_ORDER\_METHOD\_APPMINLOC1
- MSK\_ORDER\_METHOD\_APPMINLOC2
- MSK\_ORDER\_METHOD\_FREE
- MSK\_ORDER\_METHOD\_GRAPHPAR1
- MSK\_ORDER\_METHOD\_GRAPHPAR2
- MSK\_ORDER\_METHOD\_NONE

### MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE (integer)

Controls whether regularization is allowed.

(default = 1)

### MSK\_IPAR\_INTPNT\_SCALING (string)

Controls how the problem is scaled before the interior-point optimizer is used.

- MSK\_SCALING\_AGGRESSIVE
- MSK\_SCALING\_FREE
- MSK\_SCALING\_MODERATE
- MSK\_SCALING\_NONE

### MSK\_IPAR\_INTPNT\_SOLVE\_FORM (string)

Controls whether the primal or the dual problem is solved.

- MSK SOLVE DUAL
- MSK\_SOLVE\_FREE
- MSK\_SOLVE\_PRIMAL

#### MSK\_IPAR\_INTPNT\_STARTING\_POINT (string)

Selection of starting point used by the interior-point optimizer.

- MSK\_STARTING\_POINT\_CONSTANT
- MSK\_STARTING\_POINT\_FREE
- MSK\_STARTING\_POINT\_GUESS
- MSK\_STARTING\_POINT\_SATISFY\_BOUNDS

### MSK\_DPAR\_INTPNT\_TOL\_DFEAS (real)

Dual feasibility tolerance used for linear and quadratic optimization problems.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_TOL\_DSAFE (real)

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

(default = 1.0)

#### MSK\_DPAR\_INTPNT\_TOL\_INFEAS (real)

Controls when the optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_TOL\_MU\_RED (real)

Relative complementarity gap tolerance

Range: [0.0,1.0] (default = 1.0e-16)

### MSK\_DPAR\_INTPNT\_TOL\_PATH (real)

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central is followed very closely. On numerical unstable problems it might worthwhile to increase this parameter.

Range: [0.0,0.9999] (default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_TOL\_PFEAS (real)

Primal feasibility tolerance used for linear and quadratic optimization problems.

Range: [0.0,1.0](default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_TOL\_PSAFE (real)

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

(default = 1.0)

#### MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP (real)

Relative gap termination tolerance.

(default = 1.0e-8)

#### MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP (real)

Relative step size to the boundary for linear and quadratic optimization problems.

Range: [1.0e-4,0.999999] (default = 0.9999)

## USE\_BASIS\_EST (integer)

(default = 0)

### MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER (string)

Controls which simplex optimizer that is used in the clean up phase.

- MSK\_OPTIMIZER\_CONCURRENT
- MSK\_OPTIMIZER\_CONIC
- MSK\_OPTIMIZER\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_FREE
- MSK\_OPTIMIZER\_FREE\_SIMPLEX
- MSK\_OPTIMIZER\_INTPNT
- MSK\_OPTIMIZER\_MIXED\_INT
- MSK\_OPTIMIZER\_PRIMAL\_DUAL\_SIMPLEX

- MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX
- MSK\_OPTIMIZER\_QCONE

### MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER (integer)

If the parameter MSK\_IPAR\_INTPNT\_BASIS has the value 2 and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value 1.

(default = 0)

### MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR (integer)

If the parameter MSK\_IPAR\_INTPNT\_BASIS has the value 2 and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value 1.

(default = 0)

### MSK\_IPAR\_BI\_MAX\_ITERATIONS (integer)

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

(default = 1000000)

### MSK\_IPAR\_SIM\_DUAL\_CRASH (integer)

Controls whether crashing is performed in the dual simplex optimizer. In general if a basis consists of more than (100\*SIM\_DUAL\_CRASH) percent fixed variables, then a crash will be performed.

(default = GAMS BRatio)

#### MSK\_IPAR\_SIM\_DUAL\_SELECTION (string)

Controls the choice of the incoming variable known as the selection strategy in the dual simplex optimizer.

- MSK\_SIM\_SELECTION\_ASE
- MSK\_SIM\_SELECTION\_DEVEX
- MSK\_SIM\_SELECTION\_FREE
- MSK\_SIM\_SELECTION\_FULL
- MSK SIM SELECTION PARTIAL
- MSK\_SIM\_SELECTION\_SE

#### MSK\_IPAR\_SIM\_HOTSTART (string)

Controls whether the simplex optimizer will do hotstart if possible.

- MSK\_SIM\_HOTSTART\_FREE
- MSK\_SIM\_HOTSTART\_NONE
- MSK\_SIM\_HOTSTART\_STATUS\_KEYS

### MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV (real)

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure. A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Range: [1.0e-6,0.999999]

(default = 0.01)

#### MSK\_IPAR\_SIM\_MAX\_ITERATIONS (integer)

Maximum number of iterations that can used by a simplex optimizer.

(default = GAMS IterLim)

#### MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS (integer)

Controls how many setbacks that are allowed within a simplex optimizer. A setback is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

(default = 250)

#### MSK\_IPAR\_SIM\_PRIMAL\_CRASH (integer)

Controls whether crashing is performed in the primal simplex optimizer. In general if a basis consists of more than (100\*SIM\_PRIMAL\_CRASH) percent fixed variables, then a crash will be performed.

(default = GAMS BRatio)

### MSK\_IPAR\_SIM\_PRIMAL\_SELECTION (string)

Controls the choice of the incoming variable known as the selection strategy in the primal simplex optimizer.

- MSK\_SIM\_SELECTION\_ASE
- MSK\_SIM\_SELECTION\_DEVEX
- MSK\_SIM\_SELECTION\_FREE
- MSK\_SIM\_SELECTION\_FULL
- MSK\_SIM\_SELECTION\_PARTIAL
- MSK\_SIM\_SELECTION\_SE

#### MSK\_IPAR\_SIM\_REFACTOR\_FREQ (integer)

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines when the best point of refactorization is.

(default = 0)

#### MSK\_IPAR\_SIM\_REFORMULATION (string)

- MSK\_SIM\_REFORMULATION\_AGGRESSIVE
- MSK\_SIM\_REFORMULATION\_FREE
- MSK\_SIM\_REFORMULATION\_OFF
- MSK\_SIM\_REFORMULATION\_ON

#### MSK\_IPAR\_SIM\_SCALING (string)

Controls how the problem is scaled before a simplex optimizer is used.

- MSK\_SCALING\_AGGRESSIVE
- MSK\_SCALING\_FREE
- MSK\_SCALING\_MODERATE
- MSK\_SCALING\_NONE

#### MSK\_IPAR\_SIM\_SCALING\_METHOD (string)

- MSK\_SCALING\_METHOD\_FREE
- MSK\_SCALING\_METHOD\_POW2

#### MSK\_IPAR\_SIM\_SOLVE\_FORM (string)

Controls whether the primal or the dual problem is solved by the simplex optimizers.

- MSK\_SOLVE\_DUAL
- MSK\_SOLVE\_FREE
- MSK SOLVE PRIMAL

#### MSK\_IPAR\_MIO\_BRANCH\_DIR (string)

Controls whether the mixed integer optimizer is branching up or down by default.

- MSK\_BRANCH\_DIR\_DOWN
- MSK\_BRANCH\_DIR\_FREE
- MSK\_BRANCH\_DIR\_UP

#### MSK\_IPAR\_MIO\_CONSTRUCT\_SOL (integer)

If set to 1 and all integer variables has been given a value for which a feasible MIP solution exists, then MOSEK generates an initial solution to the MIP by fixing all integer values and solving for the continues variables.

```
(default = 0)
```

#### MSK\_IPAR\_MIO\_CUT\_LEVEL\_ROOT (integer)

Controls the cut level employed by the mixed integer optimizer. A negative value means a default value determined by the mixed integer optimizer is used. By adding the appropriate values from the following table the employed cut types can be controlled.

GUB cover	+2
Flow cover	+4
Lifting	+8
Plant location	+16
Disaggregation	+32
Knapsack cover	+64
Lattice	+128
Gomory	+256
Coefficient reduction	+512
GCD	+1024
Obj. integrality	+2048

### $MSK\_IPAR\_MIO\_CUT\_LEVEL\_TREE\ (integer)$

Controls the cut level employed by the mixed integer optimizer at the tree. See MSK\_IPAR\_MIO\_CUT\_LEVEL\_ROOT. (default = -1)

#### MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL (integer)

Controls the heuristic employed by the mixed integer optimizer to locate an integer feasible solution. A value of zero means no heuristic is used. A large value than 0 means a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic to be used.

```
(default = -1)
```

(default = -1)

#### MSK\_DPAR\_MIO\_HEURISTIC\_TIME (real)

Maximum time allowed to be used in the heuristic search for an optimal integer solution. A negative values implies that the optimizer decides the amount of time to be spend in the heuristic.

```
(default = -1.0)
```

#### MSK\_IPAR\_MIO\_KEEP\_BASIS (integer)

Controls whether the integer presolve keeps bases in memory. This speeds on the solution process at cost of bigger memory consumption.

```
(default = 1)
```

#### MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES (integer)

Maximum number branches allowed during the branch and bound search. A negative value means infinite.

```
(default = -1)
```

#### MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS (integer)

Maximum number relaxations allowed during the branch and bound search. A negative value means infinite.

(default = -1)

#### MSK\_DPAR\_MIO\_MAX\_TIME (real)

This parameter limits the maximum time spend by the mixed integer optimizer. A negative number means infinity. (default = -1.0)

#### MSK\_DPAR\_MIO\_MAX\_TIME\_APRX\_OPT (real)

Number of seconds spend by the mixed integer optimizer before the MIO\_TOL\_REL\_RELAX\_INT is applied.

(default = 60)

#### MSK\_DPAR\_MIO\_NEAR\_TOL\_ABS\_GAP (real)

Relaxed absolute optimality tolerance employed by the mixed integer optimizer. The mixed integer optimizer is terminated when this tolerance is satisfied.

 $(default = GAMS \ OptCa)$ 

#### MSK\_DPAR\_MIO\_NEAR\_TOL\_REL\_GAP (real)

Relaxed relative optimality tolerance employed by the mixed integer optimizer. The mixed integer optimizer is terminated when this tolerance is satisfied.

 $(default = GAMS \ OptCr)$ 

### MSK\_IPAR\_MIO\_NODE\_OPTIMIZER (string)

Controls which optimizer is employed at non root nodes in the mixed integer optimizer.

- MSK\_OPTIMIZER\_CONCURRENT
- MSK\_OPTIMIZER\_CONIC
- MSK\_OPTIMIZER\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_FREE
- MSK\_OPTIMIZER\_FREE\_SIMPLEX
- MSK\_OPTIMIZER\_INTPNT
- MSK\_OPTIMIZER\_MIXED\_INT
- MSK\_OPTIMIZER\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX
- MSK\_OPTIMIZER\_QCONE

#### MSK\_IPAR\_MIO\_NODE\_SELECTION (string)

Controls the node selection strategy employed by the mixed integer optimizer.

- MSK\_MIO\_NODE\_SELECTION\_BEST
- MSK\_MIO\_NODE\_SELECTION\_FIRST
- MSK\_MIO\_NODE\_SELECTION\_FREE
- MSK\_MIO\_NODE\_SELECTION\_HYBRID
- MSK\_MIO\_NODE\_SELECTION\_PSEUDO
- MSK\_MIO\_NODE\_SELECTION\_WORST

### MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATE (integer)

Controls whether the presolve used by the mixed integer optimizer tries to aggregate the constraints.

(default = 1)

#### MSK\_IPAR\_MIO\_PRESOLVE\_PROBING (integer)

Controls whether the mixed integer presolve performs probing. Probing can be very time consuming.

(default = 1)

#### MSK\_IPAR\_MIO\_PRESOLVE\_USE (integer)

Controls whether presolve is performed by the mixed integer optimizer.

(default = 1)

#### MSK\_DPAR\_MIO\_REL\_ADD\_CUT\_LIMITED (real)

Controls how many cuts the mixed integer optimizer is allowed to add to the problem. The mixed integer optimizer is allowed to MIO\_REL\_ADD\_CUT\_LIMITED\**m* w cuts, where *m* is the number constraints in the problem.

Range: [0.0,2.0](default = 0.75)

#### MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER (string)

Controls which optimizer is employed at the root node in the mixed integer optimizer.

- MSK\_OPTIMIZER\_CONCURRENT
- MSK\_OPTIMIZER\_CONIC
- MSK\_OPTIMIZER\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_FREE
- MSK\_OPTIMIZER\_FREE\_SIMPLEX
- MSK\_OPTIMIZER\_INTPNT
- MSK\_OPTIMIZER\_MIXED\_INT
- MSK\_OPTIMIZER\_PRIMAL\_DUAL\_SIMPLEX
- MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX
- MSK\_OPTIMIZER\_QCONE

#### MSK\_IPAR\_MIO\_STRONG\_BRANCH (integer)

The value specifies the depth from the root in which strong branching is used. A negative value means the optimizer chooses a default value automatically.

(default = -1)

#### MSK\_DPAR\_MIO\_TOL\_ABS\_GAP (real)

Absolute optimality tolerance employed by the mixed integer optimizer.

(default = 0.0)

#### MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT (real)

Absolute relaxation tolerance of the integer constraints, i.e. if the fractional part of a discrete variable is less than the tolerance, the integer restrictions assumed to be satisfied.

(default = 1.0e-5)

#### MSK\_DPAR\_MIO\_TOL\_FEAS (real)

Feasibility tolerance for mixed integer solver. Any solution with maximum infeasibility below this value will be considered feasible.

(default = 1.0e-7)

#### MSK\_DPAR\_MIO\_TOL\_REL\_GAP (real)

Relative optimality tolerance employed by the mixed integer optimizer.

(default = 1.0e-4)

#### MSK\_DPAR\_MIO\_TOL\_REL\_RELAX\_INT (real)

Relative relaxation tolerance of the integer constraints, i.e. if the fractional part of a discrete variable is less than the tolerance times the level of that variable, the integer restrictions assumed to be satisfied.

```
(default = 1.0e-6)
```

#### MIPSTART (integer)

(default = 0)

- 0 No mipstart
- 1 Mipstart with discrete variables only. Solve fixed problem first
- 2 Mipstart with all variables, including continuous

## **6** The MOSEK Log File

The MOSEK log output gives much useful information about the current solver progress and individual phases.

### 6.1 Log Using the Interior Point Optimizer

The following is a MOSEK log output from running the transportation model trnsport.gms from the GAMS Model Library:

```
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Presolve
          - time
                                     : 0.00
                                    : 0
Presolve
         - Stk. size (kb)
                                                                                 : 0.00
Eliminator - tries
                                    : 0
                                                         time
                                    : 0
Eliminator - elim's
Lin. dep. - tries
                                    : 1
                                                         time
                                                                                 : 0.00
                                    : 0
Lin. dep. - number
Presolve terminated.
Matrix reordering started.
Local matrix reordering started.
Local matrix reordering terminated.
Matrix reordering terminated.
Optimizer - threads
                                    : 1
Optimizer - solved problem
                                    : the primal
Optimizer - constraints
                                    : 5
                                                         variables
                                                                                 : 11
                                    : 0.00
                                                                                 : 0.00
Factor
           - setup time
                                                         order time
Factor
           - GP order used
                                                         GP order time
                                                                                 : 0.00
                                    : no
Factor
                                                         after factor
                                                                                 : 13
           - nonzeros before factor : 11
Factor
           - offending columns
                                                         flops
                                                                                 : 2.60e+01
```

The first part gives information about the presolve (if used). The main log follows:

ITE	PFEAS	DFEAS	KAP/TAU	POBJ	DOBJ	MU	TIME
0	6.0e+02	1.0e+00	1.0e+00	1.053000000e+00	0.00000000e+00	1.2e+01	0.00
1	5.9e+02	1.1e+00	1.0e+00	3.063646498e+00	5.682895191e+00	3.0e+01	0.00
2	4.6e+01	8.6e-02	9.8e+00	3.641071165e+01	4.750801284e+01	2.3e+00	0.00
3	8.7e-01	1.6e-03	1.7e+01	1.545771936e+02	1.719072826e+02	4.4e-02	0.00
4	8.1e-02	1.5e-04	8.8e-01	1.543678291e+02	1.552521470e+02	4.1e-03	0.00

```
1.3e-02 2.4e-05 1.3e-01 1.537617961e+02
                                                 1.538941635e+02
                                                                   6.4e-04 0.00
    1.3e-03 2.4e-06 1.1e-02 1.536766256e+02
6
                                                 1.536876562e+02
                                                                   6.6e-05 0.00
    1.6e-07 3.1e-10 1.2e-06 1.536750013e+02
                                                 1.536750025e+02
                                                                   8.4e-09 0.00
Basis identification started.
Primal basis identification phase started.
ITER
          TIME
          0.00
1
Primal basis identification phase terminated. Time: 0.00
Dual basis identification phase started.
ITER
          TIME
0
          0.00
Dual basis identification phase terminated. Time: 0.00
Basis identification terminated. Time: 0.00
Interior-point optimizer terminated. CPU Time: 0.00. Real Time: 0.00.
Interior-point solution
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal - objective: 1.5367500132e+02
                                       eq. infeas.: 5.61e-06 max bound infeas.: 0.00e+00 cone infeas.:
       - objective: 1.5367500249e+02
                                       eq. infeas.: 1.06e-08 max bound infeas.: 0.00e+00 cone infeas.:
Dual
Basic solution
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal - objective: 1.5367500000e+02
                                       eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00
      - objective: 1.5367500000e+02
                                       eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00
Optimizer
                          - real time : 0.00
                                                    cpu time: 0.00
  Interior-point
                          - iterations : 7
                                                    cpu time: 0.00
    Basis identification -
                                                    cpu time: 0.00
      Primal
                          - iterations : 1
                                                    cpu time: 0.00
      Dual
                          - iterations : 0
                                                    cpu time: 0.00
      Clean
                                                    cpu time: 0.00
                          - iterations : 0
  Simplex
                                                    cpu time: 0.00
    Primal simplex
                          - iterations : 0
                                                    cpu time: 0.00
    Dual simplex
                          - iterations : 0
                                                    cpu time: 0.00
  Mixed integer
                          - relaxations: 0
                                                    cpu time: 0.00
```

The last section gives details about the model and solver status, primal and dual feasibilities, as well as solver resource times. Furthermore, the log gives information about the basis identification phase. Some of this information is listed in the GAMS solve summary in the model listing (.LST) file as well.

The fields in the main MOSEK log output are:

Field	Description
ITE	The number of the current iteration.
PFEAS	Primal feasibility.
DFEAS	Dual feasibility.
KAP/TAU	This measure should converge to zero if the problem has a primal/dual optimal solution. Whereas
	it should converge to infinity when the problem is (strictly) primal or dual infeasible. In the case
	the measure is converging towards a positive but bounded constant then the problem is usually
	ill-posed.
POBJ	Current objective function value of primal problem.
DOBJ	Current objective function value of dual problem.
MU	Relative complementary gap.
TIME	Current elapsed resource time in seconds.

### **Log Using the Simplex Optimizer**

Below is a log output running the model trnsport.gms from the GAMS model library using the MOSEK simplex optimizer.

Reading parameter(s) from "mosek.opt" >> MSK\_IPAR\_OPTIMIZER MSK\_OPTIMIZER\_DUAL\_SIMPLEX

Simplex optimizer started.

Presolve started.

Linear dependency checker started.

Linear dependency checker terminated.

- time Presolve : 0.00 Presolve - Stk. size (kb) : 0 : 0

Eliminator - tries time : 0.00 Eliminator - elim's : 0 Lin. dep. - tries : 0.00 : 1

time

Lin. dep. - number : 0

Presolve terminated.

Dual simplex optimizer started.

Dual simplex optimizer setup started. Dual simplex optimizer setup terminated.

Optimizer - solved problem : the primal

Optimizer - constraints : 5 variables : 6

Optimizer - hotstart : no

ITER DEGITER% FEAS DOBJ TIME(s) 0 0.00 0.000000000e+00 0.000000000e+00 0.00 3 0.00 0.000000000e+00 1.5367500000e+02 0.00

Dual simplex optimizer terminated.

Simplex optimizer terminated.

Basic solution

Problem status : PRIMAL\_AND\_DUAL\_FEASIBLE

Solution status : OPTIMAL

Primal - objective: 1.5367500000e+02 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 Dual - objective: 1.5367500000e+02 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00

The fields in the main MOSEK log output are:

Field	Description
ITER	Current number of iterations.
DEGITER%	Current percentage of degenerate iterations.
FEAS	Current (primal or dual) infeasibility.
D/POBJ	Current dual or primal objective
TIME	Current elapsed resource time in seconds.

### **Log Using the Mixed Integer Optimizer**

Below is a log output running the model cube gms from the GAMS model library using the MOSEK mixed-integer optimizer.

Mixed integer optimizer started.

BRANCHES	RELAXS	ACT_NDS	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(%)	TIME
0	1	0	1.6000000000e+01	0.000000000e+00	100.00	0.1
0	1	0	4.0000000000e+00	0.000000000e+00	100.00	0.1
128	250	5	4.000000000e+00	0.000000000e+00	100.00	0.3

167	502	6	4.000000000e+00	0.000000000e+00	100.00	0.7
241	758	65	4.0000000000e+00	0.000000000e+00	100.00	0.9
200	809	83	4.0000000000e+00	1.333333333e-01	96.67	0.9

A near optimal solution satisfying the absolute gap tolerance of 3.90e+00 has been located.

Objective of best integer solution: 4.00000000e+00

Number of branches : 267

Number of relaxations solved : 810

Number of interior point iterations: 0

Number of simplex iterations : 10521

Mixed integer optimizer terminated. Time: 0.95

The fields in the main MOSEK log output are:

Field	Description
BRANCHES	Current number of branches in tree.
RELAXS	Current number of nodes in branch and bound tree.
ACT_NDS	Current number of active nodes.
BEST_INT_OBJ.	Current best integer solution
BEST_RELAX_OBJ	Current best relaxed solution.
<pre>REL_GAP(%)</pre>	Relative gap between current BEST_INT_OBJ. and BEST_RELAX_OBJ.
TIME	Current elapsed resource time in seconds.

The log then gives information about solving the model with discrete variables fixed in order to determine marginals. We also get information about crossover to determine a basic solution, and finally MOSEK provides information about using the Simplex Method to determine an optimal basic solution.

```
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Presolve
                                    : 0.00
          - time
Presolve - Stk. size (kb)
                                    : 12
Eliminator - tries
                                    : 0
                                                        time
                                                                               : 0.00
Eliminator - elim's
                                    : 0
Lin. dep. - tries
                                    : 1
                                                        time
                                                                               : 0.00
Lin. dep. - number
                                    : 0
Presolve terminated.
```

Interior-point optimizer terminated. CPU Time: 0.00. Real Time: 0.00.

```
Interior-point solution
```

Problem status : PRIMAL\_FEASIBLE Solution status : PRIMAL\_FEASIBLE

Primal - objective: 4.0000000000e+00 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 cone infeas.: Dual - objective: -8.0000000000e+00 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 cone infeas.:

#### Basic solution

Problem status : PRIMAL\_FEASIBLE Solution status : PRIMAL\_FEASIBLE

Primal - objective: 4.00000000000e+00 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00 Dual - objective: -8.0000000000e+00 eq. infeas.: 0.00e+00 max bound infeas.: 0.00e+00

Optimizer - real time : 1.35 cpu time: 0.95
Interior-point - iterations : 0 cpu time: 0.00

Basis identification	_		cpu time: 0.00
Primal	- iterations :	0	cpu time: 0.00
Dual	- iterations :	0	cpu time: 0.00
Clean	- iterations :	0	cpu time: 0.00
Simplex	-		cpu time: 0.00
Primal simplex	- iterations :	0	cpu time: 0.00
Dual simplex	- iterations :	0	cpu time: 0.00
Mixed integer	- relaxations:	810	cpu time: 0.95