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COMMENT ON "AN EXCESS OF MASSIVE STARS IN THE LOCAL 30 DORADUS STARBURST"

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ABSTRACT

Schneider et al. (Reports, 5 January 2018, p. 69) found more stars above 30 solar masses than predicted by a standard Salpeter initial mass function (IMF) in the 30 Doradus star-forming region. They find an IMF power-law exponent of $1.90^{+0.37}_{-0.26}$ in contrast to the Salpeter exponent of 2.35; however, the significant statistical uncertainty in their result is based on a flawed analysis of the data. Correcting this error we infer an exponent of $2.05^{+0.14}_{-0.13}$. Although the mean of our posterior on the exponent suggests a steeper IMF, it is in fact more statistically significantly different (about 2-σ) from the Salpeter value. We discuss the impact of assumptions regarding data set completeness and the star formation history model; alternative assumptions can shift the inferred exponent to $2.11^{+0.19}_{-0.17}$ and $2.15^{+0.13}_{-0.13}$, respectively. We also consider an additional break in the IMF power law, but do not find convincing evidence for such a break.

~~The universality of the initial mass function of stars is a hot topic in modern astrophysics, with impact on galactic evolution, supernovae, and gravitational-wave sources (Kroupa 2002; Bastian et al. 2010; de Mink & Belczynski 2015).~~ Schneider et al. (2018) use spectroscopic observations of young massive stars in the 30 Doradus region of the Large Magellanic Cloud to infer a shallower-than-expected ~~IMF~~. They estimate the ages and masses of individual stars with the BONNSAI Bayesian code (Schneider et al. 2017)~~. They~~ then obtain an overall mass distribution by effectively adding together the posterior probability density functions of individual stars. There is no statistical meaning to a distribution obtained in this way, ~~and it~~ does not represent the posterior probability density function on the mass distribution.

Hierarchical Bayesian inference provides the statistically ~~correct~~ solution to this problem (Hogg et al. 2010). Mandel (2010) specifically considered inference on a mass distribution given a sample of uncertain measurements, and we use a similar methodology here. We interpret the Schneider et al. (2018) inference on individual masses and ages as independent Gaussian likelihoods for the ~~log~~ of the mass and the age, with parameters fixed by matching the mean parameter to the peak and the standard deviation parameter to the 68% width of the individual stellar distributions in the Schneider et al. (2018) data. ~~We have performed fits similar to those described here using a heavier-tailed, Student T likelihood with few degrees of freedom and find equivalent results, suggesting that heavy tails in the likelihood are not particularly influential in this analysis.~~

For our fiducial analysis we model the star formation history as a truncated Gaussian distribution, and generally find similar star formation history to Schneider et al. (2018), with the star formation rate in 30 Dor peaking about 4 Myr ago. We impose broad priors on the power–law exponent and the mean and standard deviation of the star formation Gaussian. We use the Hamiltonian Monte Carlo sampler STAN (Carpenter et al. 2017) to efficiently address the high-dimensional hierarchical problem with free parameters for each star's actual mass and birth age in addition to the IMF exponent and the mean and standard deviation of the star formation history.

Figure 1 shows the inferred power-law exponent of the IMF. ~~When using~~ the Schneider et al. (2018) fit to stellar lifetimes and ~~following the assumption~~ that their data set is complete above 15 solar masses (i.e. select~~ing~~ only those stars whose observed mass is above 15 $M_\odot$ (Loredo 2004; Abbott et al. 2016))~~, we~~ find an exponent of $2.05^{+0.14}_{-0.13}$ where the quoted value corresponds to the median of the posterior distribution and the range to the 16th and 84th percentiles (i.e. the symmetric 68% credible interval). ~~Correcting the erroneous statistical procedure used by Schneider et al. (2018)~~ steepens the preferred IMF slope and narrows the uncertainty interval; the preferred value from Schneider et al. (2018) lies about 1 σ below our median value.

The analysis above used the same assumptions as Schneider et al. (2018). Below, we consider the impact of three additional assumptions: the stellar lifetime fit; the choice of the completeness limit; and the model for the star formation history.

We ~~make~~ an independent fit to the main sequence lifetimes τ_{MS} of non-rotating massive stars as modelled by Brott et al. (2011) and Köhler et al. (2015):

$$\ln \frac{\tau_{MS}(M)}{\mathrm{Myr}} = 9.1973 - 3.8955 \ln \frac{M}{M_\odot} + 0.6107 \left(\ln \frac{M}{M_\odot}\right)^2 - 0.0332 \left(\ln \frac{M}{M_\odot}\right)^3 . \quad (1)$$

~~In our models,~~ following Schneider et al. (2018), we increase the "observable" lifetime of a star by 10% beyond its main sequence lifetime to account for helium burning. We find that this alternative ~~fit~~ does not impact the inferred IMF, yielding ~~a~~ power-law exponent $2.05^{+0.14}_{-0.13}$.

The inferred power-law exponent is somewhat sensi-

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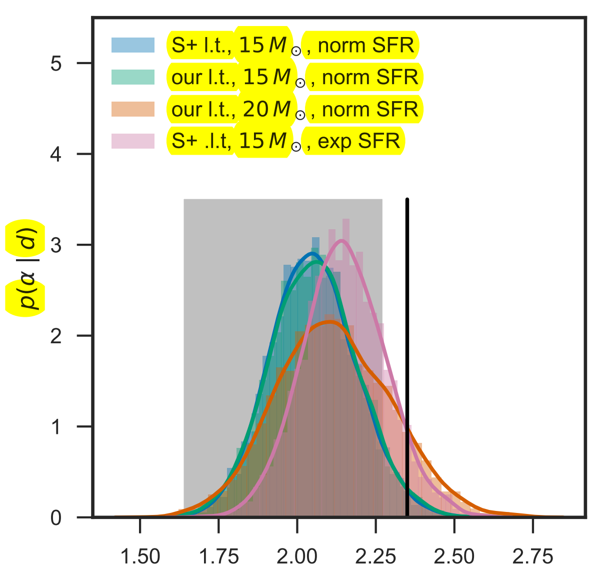

FIG. 1.— The posterior inferred on the power-law exponent under four models (see text for details): Schneider et al. (2018) stellar lifetimes, survey completeness for $M \geq 15\,M_\odot$, and Gaussian star formation history model (blue); as before, but with our lifetime fit (green); as before, but with completeness for $M \geq 20\,M_\odot$ (orange); as blue, but with a double-exponential star formation history model (pink). The Salpeter power-law exponent is $-\alpha = -2.35$ (Salpeter 1955), indicated by a black line. The 68.3% range of power-law exponents derived by Schneider et al. (2018) is shaded in grey.

tive to the choice of the cutoff mass for survey completeness. The data of Schneider et al. (2018) show a relative scarcity of stars between 15 and 20 solar masses; changing the mass cutoff from $15M_\odot$ to $20M_\odot$ further steepens the inferred exponent to $2.11^{+0.19}_{-0.17}$. However, these fluctuations are within the expected statistical variation based on the sample size, as confirmed with posterior predictive checking. In particular, ~~we do not claim~~ statistical evidence against the claim of Schneider et al. (2018) that the survey is complete for $M \geq 15\,M_\odot$.

Finally, we consider an alternative star formation history model: a double exponential, with three free parameters: the time of the peak of the star formation rate and the (possibly different) decay constants before and after the peak. This model allows for a sharper peak and longer tails than a Gaussian. This star formation rate history model is consistent with the data, as tested with posterior predictive checking (see below). However, it yields a power-law exponent $2.15^{+0.13}_{-0.13}$, almost $1\,\sigma$ steeper than for our fiducial analysis. This indicates that the inferred IMF is sensitive to the systematics of the assumed model.

We also considered the possibility that the IMF power law has an ~~extra~~ break at higher masses, allowing for three free parameters: the mass at which the break happens and the exponent below and above the break. However, we find that the data ~~are insufficient to~~ constrain the parameters of this more general model, and there is no preference for a broken power-law model.

We confirm the stability of our conclusions with posterior predictive checking. Figure 2 shows the distribution of observed masses and ages (i.e., the peak of the like-

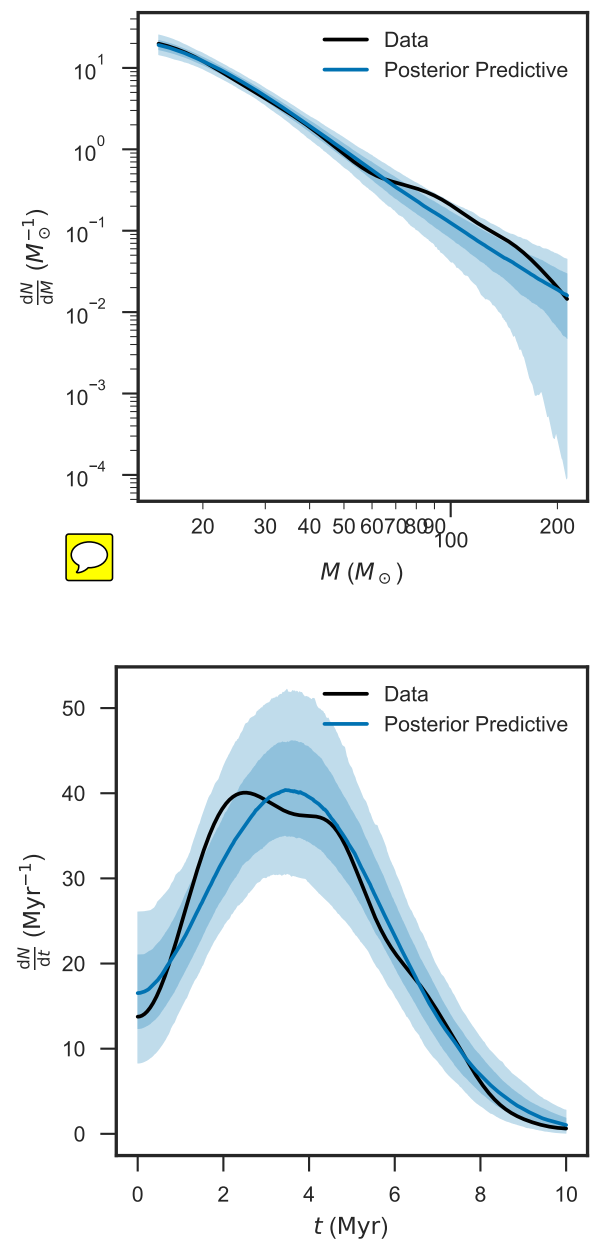

FIG. 2.— The observed distribution of (maximum likelihood) masses (top, black) and ages (bottom, black) and the range (median, 68%, and 95% credible intervals in blue) of distributions of mass and age from synthetic data drawn from our fitted model (i.e. the posterior predictive distribution). The observed data are consistent with being drawn from our model.

lihood) from the Schneider et al. (2018) data ~~on top of~~ the range of mass and age distributions that would be observed from a large number of data sets drawn according to our fitted IMF model. The data ~~can be seen to be~~ consistent with our IMF model. We have also confirmed that all of our models yield predictions for the numbers of stars heavier than $30M_\odot$ and $60M_\odot$ that are consistent with observations.

We find that we can ~~significantly~~ reduce the statistical uncertainty in the IMF by applying ~~the correct~~ statistical analysis to the observations of young massive stars in 30 Doradus. However, the systematics from modelling uncertainties, such as the assumed star formation history model, can potentially shift the inferred power-law exponent by more than the statistical uncertainty. Furthermore, we adopted the mass and age posteriors for individual stars directly from Schneider et al. (2018). Although ~~we do not discuss the possible~~ systematic uncertainty ~~on these due to~~ imperfect stellar models or the inclusion of other complicating factors described by Schneider et al. (2018) (rotation, mass transfer, mergers, etc.), any systematics could again shift the inferred IMF exponent. The combination of these factors makes it very challenging to infer the precise shape of the IMF even when a data set as good as that obtained by Schneider et al. (2018) is available.

We are grateful to Schneider et al. (2018) for making the data on which their conclusions are based available for further study and analysis, and to Fabian Schneider personally for very useful discussions. This analysis made use of `PySTAN` (Stan Development Team 2017), `astropy` (Astropy Collaboration et al. 2013), `numpy` (van der Walt et al. 2011), `scipy` (Jones et al. 2001–), `matplotlib` (Hunter 2007), and `seaborn` (Waskom et al. 2017) Python libraries. The code and LaTeXsource used to prepare this document are publicly available under an open-source MIT license at `https://github.com/farr/30DorIMF`. WMF and IM are partially supported by STFC.

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