Approximation methodology for determining adiabatic temperature profiles in boundary layer with heat conducting wall and inflow boundary layer enthalpy flux into a cavity volume through an orifice is developed.

1.0 <u>External inflow into a cavity through an orifice</u>

1.1 <u>Free stream properties</u>

Freestream properties are defined as

$$M_e = M_e(t) \qquad \text{flight Mach number} \tag{1.1}$$

$$T_e = T_e(t,h) \qquad \text{ambient static temperature (degR)} \tag{1.2}$$

$$p_e = p_e(t,h) \qquad \text{ambient static pressure (psfa)} \tag{1.3}$$

$$q_e = \frac{\gamma}{2} p_e M_e^2 \qquad \text{dynamics pressure (psf)} \tag{1.4}$$

$$\dot{m}_e = \rho_e u_e = k \frac{p_e}{\sqrt{T_e}} M_e \quad \text{external flow mass flux (lbm / s / ft}^2\text{)} \tag{1.5}$$

where

$t =$ flight time (s)

$h =$ altitude (ft)

$\gamma = c_p / c_v = 1.4$ ratio of specific heat

$k = g_e \sqrt{\gamma / R_g}$ mass flux constant ($\sqrt{\text{degR}}$ / s / ft^2)

$g_e = 32.17$ ft / s^2 gravity

$R_g = 1716$ ft^2 / s^2 / degR gas constant for air

<u>1.2 Local external environment</u>

Flow properties at the local venting orifice positions on the vehicle stage are defined as a function of time and xyz stage coordinate. The local external static pressures near the orifices which drive the inflow/outflow compartment venting process are computed by given local pressure coefficient (c_{pL}) histories.

$$p_L(t,xyz) = p_e + c_{pL} q_e; \qquad \text{local static pressure (psfa) near an orifice xyz} \tag{1.6}$$

where

$c_{pL} = c_{pl}(t, M_e, \alpha, xyz)$; local pressure coefficient at t and xyz

<u>1.3 Inflow mass into compartment through an orifice</u>

Inflow mass flux into a compartment through an orifice is computed if the external local pressure is greater than the compartment pressure.

For $p_L > p_c$,

let $p_r = p_L / p_c$ (1.7)

Flow Mach number through an orifice is computed as

$$M_h = \left\{ \frac{2}{\gamma - 1} \left[p_r^{(\gamma-1)/\gamma} - 1 \right] \right\}^{1/2} \tag{1.8}$$

If $M_h > 1.0$ then $M_h = 1.0$; choked flow at orifice

Because external total temperature is conserved across shock, etc., $T_{tL} = T_{te}$ is used, where

$$\frac{T_{te}}{T} = 1 + \frac{\gamma - 1}{2} M_e^2 \qquad (1.9)$$

Inflow mass flux through an orifice is then computed as

$$\dot{m}_h = k \frac{p_L}{\sqrt{T_{te}}} M_h \left\{ + \frac{\gamma\text{-}1}{2} M_h^2 \right\}^{(\gamma+1)/(2(\gamma\text{-}1))} \quad (\mathrm{lbm/s/ft^2}) \qquad (1.10)$$

1.4 Mass flux, enthalpy flux and mean T_{aw} profiles across boundary layer

Mass flux in the boundary layer (bl) approaching an orifice is determined by integration of bl mass flux profile. The flow properties of boundary layer is determined for laminar or turbulent layer depending upon the value of Reynolds number which will be discussed in Section 2.

Total mass flux in a boundary layer from the wall to the boundary layer edge is integrated as

$$\dot{m}_{bl} = \rho_e u_e \int_0^1 \frac{\rho(\eta) u(\eta)}{\rho_e u_e} d\eta = \dot{m}_e \int_0^1 \frac{\rho(\eta) u(\eta)}{\rho_e u_e} d\eta; \quad 0 \le \eta = (y/\delta_{bl}) \le 1.0 \qquad (1.11)$$

But $p(\eta) = p_e$ = constant for bl approximation, then $\dfrac{\rho(\eta)}{\rho_e} = \dfrac{T_e}{T(\eta)}$ (1.12)

$$\dot{m}_{bl}(\eta = 1) = \dot{m}_e \int_0^1 \frac{T_e}{T(\eta)} \frac{u(\eta)}{u_e} d\eta; \quad (\text{dimensionalized by } \dot{m}_e) \qquad (1.13)$$

For a boundary layer with non-adiabatic heat conducting wall, $T_{aw}(\eta) \ne T_{awe}$.

Then enthalpy flux across boundary layer is determined by

$$\dot{h}_{bl}(\eta = 1) = \dot{m}_e c_p T_{awe} \int_0^1 \frac{T_e}{T(\eta)} \frac{T_{aw}(\eta)}{T_{awe}} \frac{u(\eta)}{u_e} d\eta; \quad (\text{dimensionalized by } \dot{m}_e c_p T_{awe}) \qquad (1.14)$$

where c_p = specific heat at const pressure

1.5 mean $T_{aw}(\eta)$ profile

The ensemble average of mean adiabatic temperature from the wall up to the point in boundary layer can be determined by taking the ratio of Eqn 1.14 over Eqn. 1.1.3, except the integration is taken up to *η<1.0.*

$$mean \ \frac{T_{aw}(\eta)}{T_{awe}} = \frac{\dot{h}_{bl}(\eta)}{c_p T_{awe} \dot{m}_{bl}(\eta)} \qquad (1.15)$$

where $\dot{h}_{bl}(\eta)$ is by Eqn. 1.21 (limited version of Eqn. 1.14) and

$\dot{m}_{bl}(\eta)$ is by Eqn. 1.20 (Eqn. 1.13) both integrated up to $\xi = \eta$.

T_{awe} is computed by Eqn. 2.1

1.6 Determination of total ingested mass and enthalpy flows into compartment per orifice width

How much of the heated boundary layer air is ingested into the cavity to influence the cavity temperature will be determined by comparing between the required total mass flow rate (based on the local pressure ratio across the orifice) and the bl mass flow rate.

Consider a rectangular orifice with x_h, w_h dimension.

The external mass inflow rate per width $= \dot{m}_h x_h$ (1.16)

The boundary layer mass flow rate per width $= \dot{m}_{bl}\delta_{bl}$ (1.17)

where the boundary layer thickness (δ_{bl}) will be given by Eq 3.1a or 3.2a as will be discussed in Sect 3.

For $\dot{m}_h x_h > \dot{m}_{bl}\delta_{bl}$, all of boundary layer mass flow and a portion of external flow will be ingested into the cavity. In this case, a portion of external mass flow rate is approximated by

$$\Delta\dot{m}_h x_h = \dot{m}_h x_h - \dot{m}_{bl}\delta_{bl} . \qquad (1.18)$$

Total mass flow into compartment per width is

$$\dot{m}_{h\,\mathrm{inf}\,low} x_h = \Delta\dot{m}_h x_h + \dot{m}_{bl}\delta_{bl} \text{ . (total mass inflow per width)} \qquad (1.18a)$$

The total inflow enthalpy per width is approximated by

$$\dot{h}_{h\,\mathrm{inf}\,low} x_h = \dot{h}_{bl}\delta_{bl} + \Delta\dot{m}_h x_h c_p T_{awe} \qquad (1.19)$$

For a condition with $\dot{m}_h x_h < \dot{m}_{bl}\delta_{bl}$, all entrained mass within the boundary layer will be ingested but no external inviscid flow. In this case, a height (η_m) within boundary layer is approximated by equating the required mass flow rate and a bl mass flux integral.

$$\int_0^{\eta_m} \frac{T_e}{T(\xi)}\frac{u(\xi)}{u_e} d\xi = \frac{\dot{m}_h x_h}{\dot{m}_e \delta_{bl}}$$; mass flux between $0 \le \xi \le \eta_m$ where η_m is the matching point. (1.20)

The total mass flow into the compartment per width is

$$\dot{m}_{h\,\mathrm{inf}\,low} x_h = \dot{m}_e \delta_{bl} \int_0^{\eta_m} \frac{T_e}{T(\xi)}\frac{u(\xi)}{u_e} d\xi \qquad (1.20a)$$

Then the enthalpy flow to be ingested per width $\dot{m}_e\delta_{bl}$ will be integrated between $0 \le \xi \le \eta_m$.

$$\dot{h}_{h\,\mathrm{inf}\,low} x_h = \dot{m}_e c_p T_{awe} \delta_{bl} \int_0^{\eta_m} \frac{T_e}{T(\xi)}\frac{T_{aw}(\xi)}{T_{awe}}\frac{u(\xi)}{u_e} d\xi \qquad (1.21)$$

The ensemble average of total inflow adiabatic temperature into a compartment is given by either taking the ratio of Eqn. 1.19 over 1.18a or Eqn. 1.21 over 1.20a as shown in Eqn 1.22.

$$mean\ \frac{T_{awc}}{T_{awe}} = \frac{\dot{h}_{h\,\mathrm{inf}\,low}}{c_p T_{awe} \dot{m}_{h\,\mathrm{inf}\,low}} \qquad (1.22)$$

Normally a critical parameter is the ratio of boundary layer thickness and hole length (i.e., δ_{bl}/x_h).

For $\delta_{bl} / x_h >> 1.0$,

all inflow mass into the cavity is entrained in boundary layer and no external inviscid flow enters the cavity. In this case, hot air near the wall to enter the cavity but amount of mass flow may be sufficiently small to cause insignificant cavity heating, provided, the boundary layer thickness is not too large.

For $\delta_{bl} / x_h << 1.0$,

most of the flow enters the cavity is external inviscid flow with ambient adiabatic wall temperature (T_{awe}). Of course, undesirable hot invscid flow may be ingested when the boundary layer edge Mach number is high supersonic to hypersonic.

2.0 Boundary layer analysis.

Direction of heat flux within boundary layer is determined by free stream adiabatic wall temperature and actual wall temperature. The adiabatic wall temperature is computed by a given free stream (or outer boundary layer edge) Mach number.

Compressible adiabatic heating is denoted by an adiabatic wall temperature (T_{awe}) as

$$\frac{T_{awe}}{T_e} = 1 + r_f \frac{\gamma - 1}{2} M_e^2 \quad \Rightarrow \quad \frac{T_{awe}}{T_e} - 1 = r_f \frac{\gamma - 1}{2} M_e^2 \tag{2.1}$$

where the recovery factor (r_f) is a function of Prandtl number (P_r) as

$r_f = P_r^{1/3}$ for turbulent flow

$r_f = P_r^{1/2}$ for laminar flow

$P_r \approx 0.7$ Prandtl number of air at approximately 600 deg R

An actual wall temperature (T_w as a function of frictional heating) normalized with respect to T_e is used to define the heat flux direction within the boundary layer.

$\frac{T_w}{T_e} - 1 > r_f \frac{\gamma - 1}{2} M_e^2$; heat flux from the wall to fluid

$\frac{T_w}{T_e} - 1 = r_f \frac{\gamma - 1}{2} M_e^2$; no heat flux, adiabatic wall

$\frac{T_w}{T_e} - 1 < r_f \frac{\gamma - 1}{2} M_e^2$; heat flux from fluid to the wall

2.1 Turbulent boundary layer profiles

Turbulent velocity profile is approximated by well accepted power law function of

$$\frac{u(y)}{u_e} = \left(\frac{y}{\delta_{bl}} \right)^{1/n} \quad \text{(see Schlicting, pp504-505)} \qquad (2.2)$$

Because turbulent velocity profile below $u / u_e \approx 0.5$ is concentrated in very thin sublayer near the wall, computation can be reversed to obtain more detail profiles as

$$\frac{y}{\delta_{bl}} = \left(\frac{u}{u_e} \right)^{n} \qquad (2.2a)$$

where $n = 6$ at $\mathrm{Re}_x = 4x10^3$

$n = 7$ at $\mathrm{Re}_x = 1.1x10^5$

$n = 10$ at $\mathrm{Re}_x = 3.24x10^6$

Static temperature distribution within boundary layer for a given velocity profile (*u/u*$_e$) and for a constant adiabatic wall temperature (T_{aw}=T_{awe}=const) can be derived as

$$\frac{T_{aw}}{T(y)} = 1 + r_f \tfrac{\gamma-1}{2} M(y)^2 = 1 + r_f \tfrac{\gamma-1}{2} \frac{u(y)^2}{\gamma R_g T_{aw}} \left(\frac{T_{aw}}{T(y)} \right) \qquad (2.3)$$

where one can show that

$$\frac{u(y)^2}{\gamma R_g T_{aw}} = \left(\frac{u(y)}{u_e} \right)^2 \frac{M_e^2}{1 + r_f \tfrac{\gamma-1}{2} M_e^2} \qquad (2.4)$$

By rearranging Eq 2.3, we get an expression for static temperature profile as

$$\frac{T(y)}{T_{aw}} = 1 - k_{aw} \left(\frac{u(y)}{u_e} \right)^2 \qquad (2.5)$$

where
$$k_{aw} = \frac{r_f \tfrac{\gamma-1}{2} M_e^2}{1 + r_f \tfrac{\gamma-1}{2} M_e^2} \qquad (2.6)$$

$$T_{aw} = T_{awe} \qquad (2.7)$$

For non-adiabatic, wall heating temperature profile for (flat plate, dp/dx=0) is given in Schlicting (page 343) as

$$\frac{T(y)}{T_w} = 1 + \frac{T_e}{T_w} \left\{ \left(1 - \frac{T_w}{T_e} \right) \frac{u(y)}{u_e} + \tfrac{\gamma-1}{2} M_e^2 \frac{u(y)}{u_e} \left(1 - \frac{u(y)}{u_e} \right) \right\} \qquad (2.8)$$

for boundary conditions of

$y = 0, \; u = 0, \; T = T_w$

$y = \infty, \; u = u_e, \; T = T_e$

Once u / u_e and T / T_e profiles are determined, then we can proceed to obtain M / M_e and T_{aw} / T_{awe} profiles, provided $T_w \neq T_{awe}$.

$$\frac{M(y)}{M_e} = \frac{u(y)}{u_e}\sqrt{\frac{T_e}{T(y)}} \qquad \text{(local bl Mach number ratio)} \qquad (2.9)$$

$$\frac{T_{aw}(y)}{T(y)} = 1 + r_f \tfrac{\gamma-1}{2} M(y)^2 \qquad \text{(local bl adiabatic temperature)} \qquad (2.10)$$

Finally, local bl adiabatic temperature normalized wrt to bl edge T_{awe} is expressed as

$$\frac{T_{aw}(y)}{T_{awe}} = \frac{T_e}{T_{awe}} \frac{T(y)}{T_e}\left[1 + r_f \tfrac{\gamma-1}{2} M_e^2 \left(\frac{M(y)}{M_e}\right)^2\right] \qquad (2.11)$$

2.2 Laminar boundary layer profiles

Laminar boundary layer velocity profile (u/u_e) is computed by Blasius tabulated profile for dp/dx=0 (Eshbach's Handbook of Engineering Fundamentals, p6.108) where tabulation is given below.

$\eta = y/\delta_{bl}$ = 0,.2,.4,.6,.8,1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7
$u(\eta)/u_e$ = 0,.0664,.1328,.1989,.2647,.3298,.4867,.6298,.7518,.8460,.9128,.9555,.9794,.9916, .9969,.9990,.9997,.9999

The laminar boundary layer edge is selected at η=5.0 where u/u_e=0.9916.

Once the velocity profile is defined, then other flow property relations for non-adiabatic, heat conducting wall can be estimated by the same equations given in Section 2.1.

3.0 Boundary layer thickness

Because LAP and OV diameters are large and Reynolds number based on the diameters or on the axial running length is an order of 10^5 to 10^8, the boundary layers on most of the vehicle are turbulent. Therefore, the boundary layer thickness computation is approximated by zero pressure gradient ($dp/dx=0$), flat plate approximations.

Turbulent boundary layer thickness for an 1/7th velocity profile is given as (Schlicting, p537)

$$\delta_{bl}(x) = 0.37x(\mathrm{Re}_x)^{-1/5} \qquad (3.1)$$

Laminar boundary layer thickness for a Blasius profile is given as

$$\delta_{bl}(x) = 5.0x(\mathrm{Re}_x)^{-1/2} \qquad (3.2)$$

Reynolds number is defined as

$$\mathrm{Re}_x = \frac{\rho_e u_e x}{\mu_e} \text{ ; Reynolds number defined by bl edge temperature} \qquad (3.3a)$$

$$\mathrm{Re}_{xw} = \frac{\rho_w u_e x}{\mu_w} \text{ ; Reynolds number defined by wall temperature} \qquad (3.3b)$$

where density and air viscosity are defined either by the bl edge temperature or by the wall temperature.

The boundary layer laminar/turbulent transition criteria is given by a transitional Reynolds number of approximately

$$\mathrm{Re}_{x-tr} \approx 10^5 \qquad (3.4)$$

Sutherland's formula (NACA 1135) is used to compute the viscosity for an appropriate air temperatures (i.e., $T = T_e$ or T_w).

$$\mu = 2.270 \frac{T^{3/2}}{T+198.6} \text{ x } 10^{-8} \quad (\mathrm{lbm\text{-}s/ft^2}) \qquad (3.5)$$

3.1 Boundary layer growth estimation due to wall temperature.

As can be seen in Eqn. 3.1 or 3.2, the thickness growth rate will be independent of wall temperature when the Reynolds number is based on the boundary layer edge value, provided altitude and Mach number are unchanged. However, wall heating or cooling affects the air composition near the wall to change the gas density and viscosity as a function of the wall temperature to change the boundary layer property profiles.

Since only a similarity boundary layer solution is used in the present analysis, it is appropriate to estimate the boundary layer thickness distribution as a function of the wall temperature (T_w) to account for the unknown internal boundary layer structure changes. As can be seen, the boundary layer thickness distribution is thicker than the ones estimated by T_e for $T_w>T_e$ and thinner for $T_w<T_e$. Therefore, the boundary layer thickness as defined by Eqn. 3.1a or 3.2a will be preferred for boundary layer mass and enthalpy flux estimation.

Turbulent boundary layer thickness for an 1/7th velocity profile based on T_w is given by

$$\delta_{blw}(x) = 0.37x(\mathrm{Re}_{xw})^{-1/5} \qquad (3.1a)$$

Laminar boundary layer thickness for a Blasius profile based on T_w is given as

$$\delta_{blw}(x) = 5.0x(\mathrm{Re}_{xw})^{-1/2} \qquad (3.2a)$$

3.2 Axial flow boundary layer

The running length (x) is measured from the plate leading edge. In our application of axial flow over the LAP or OV, there is no finite sharp edge boundary layer starting point. Because unit Reynolds number usually exceeds 10^5 within most of the operational range (altitude and Mach number), the boundary layer is expected to be turbulent over most of the stages. Therefore, we can approximate the boundary layer starting point at the cylinder shoulder for axial flow calculations, except precisely at the shoulder.

3.3 Cross flow boundary layer over a cylinder

For cross flow over a near perpendicular cylinder, the characteristic length from the stagnation point will be defined in term of angular displacement angle. By knowing the large stage diameter (LAP or OV), the Reynolds number from the stagnation point is computed based on the running length that is defined by

$$x = R_{stage}\phi \qquad (3.6)$$

where R_{stage} = stage radius

ϕ = cylinder peripheral angle in radians.

For determining the critical separation points on the cross flow cylinder, Reynolds number based on a cylinder or sphere diameters is used. However, to evaluate the local boundary layer development over the cylinder periphery, Reynolds number on the running length from the stagnation point is used. Because the stage diameter is large with respect to boundary layer characteristic thickness, it is valid to assume the surface is locally flat. Of course, the boundary layer theory is singular and is invalid at the plate leading edge or at the cylinder stagnation point, the solutions must avoid these singular points.

3.3.1 Windward side cylinder surface

For cross flow calculations (i.e., during final landing approaches of the LAP or OV with AOA=90 deg,), there exists a negative (or favorable) pressure gradient ($dp/d\varphi<0$) around the cylinder peripherally from the stagnation point to φ=90 deg which produces attached boundary layer with little profile variations. The turbulent separation is delayed to approximately φ=120 deg in the leeward side. Also most of the turbulent boundary layer (TBL) changes are occurring within the thin sublayer, the total mass flux within TBL with pressure gradient may be small. Therefore, TBL interaction on the vent orifices exist on the windward side can be estimated with the zero pressure gradient solution with relatively small error. In fact, surface roughness uncertainties due to stringers, stage fabrication, etc. may contribute larger errors than our approximation. Therefore, the present approach is valid in the windward side application without introducing an appreciable error.

3.3.2. Leeward side cylinder surface

Positive (adverse) pressure gradient ($dp/d\varphi>0$) to occur beyond cylinder periphery of φ=90 deg so that boundary layer separation to appear in the cylinder leeward side. As quoted by Schlicting (p355) for laminar boundary layer with non-zero pressure gradient, quote

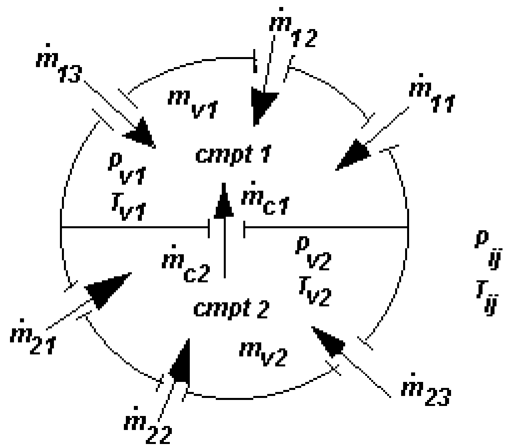
> "(1) when the wall is adiabatic, separation occurs for the same value of β as in incompressible flow ($\beta=-0.199$), however, (2) transfer of heat from the wall to the stream favors separation, or in other words, separation occurs with lower adverse $dp/dx>0$ than is the case with adiabatic wall."

Similar qualitative trend is expected to exist for the turbulent boundary layer with $dp/dx>0$.

Although laminar separation theory has been advanced to estimate the boundary layer development, no practical and easy method exist to predict turbulent boundary layer (TBL) in adverse pressure gradient regions. The vent orifices exist in the rear stagnation region of cylinder beyond φ=120 deg may be completely enveloped in the cylinder wake and the present approach may not predict realistic results. However, because momentum exchange near the wall in separated region is considerably weaker than in the attached boundary layer, the leeward side heating from the wall to fluid appears to be less critical than the windward side heating. And the wake with reverse flow in subsonic flow may be well mixed with ambient air at farfield to cause appreciable heating effect, provided the stage surface is sufficiently cooled. Therefore, ingestion of wake flow mass with ambient total temperature approximation is expected to be reasonable.

4.0 Two interconnecting compartment venting model

Two compartments are joined by interconnecting orifice or orifices. Each compartment is vented to local external environments by multiple venting orifices.



4.1 Mass/enthalpy exchange between two compartments

Across the compartments, a larger internal pressure *(p_{v1} or p_{v2})* will control the mass flow direction. In-flow is considered positive, i.e.,

For $p_{v1}>p_{v2}$, flow direction is from the volume 1 to volume2.

$$\begin{aligned} \text{Let}\quad p_r &= p_{v1} / p_{v2} \geq 1.0 \\ p_t &= p_{v1} \\ T_{tc} &= T_{v1} \\ \dot{m}_{c1} &= -\dot{m}_v \quad \text{outflow from cmptmt 1 to cmptmt 2} \\ T_{tc}\dot{m}_{c1} &= -T_{tc}\dot{m}_v \\ M_{c1} &= -M_v \\ \dot{m}_{c2} &= \dot{m}_v \quad \text{inflow into cmptmt 2 from cmptmt 1} \\ T_{tc}\dot{m}_{c2} &= T_{tc}\dot{m}_v \\ M_{c2} &= M_v \end{aligned}$$

For $p_{v2}>p_{v1}$, flow direction is from the volume 2 to volume1.

$$\begin{aligned} \text{Let}\quad p_r &= p_{v2} / p_{v1} \geq 1.0 \\ p_t &= p_{v2} \\ T_{tc} &= T_{v2} \\ \dot{m}_{c1} &= \dot{m}_v \quad \text{outflow from cmptmt 2 into cmptmt 1} \\ T_{tc}\dot{m}_{c1} &= T_{tc}\dot{m}_v \\ M_{c1} &= M_v \\ \dot{m}_{c2} &= -\dot{m}_v \quad \text{inflow into cmptmt 1 from cmptmt 2} \\ T_{tc}\dot{m}_{c2} &= -T_{tc}\dot{m}_v \\ M_{c2} &= -M_v \end{aligned}$$

where absolute value of mass flux across two compartments is determined by

$$\dot{m}_v = k \frac{p_t}{\sqrt{T_{tc}}} A_v M_v \left\{ 1 + \frac{\gamma - 1}{2} M_v^2 \right\}^{\frac{-(\gamma+1)}{2(\gamma-1)}} \qquad (4.1)$$

$$M_v^2 = \frac{2}{\gamma - 1} \left[p_r^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad (4.2)$$

if $M_v \geq 0$ then $M_v = 1$ (choked flow)

$A_v =$ cross sectional area of connecting orifices : ($A_v = x_h w_h$ rectangular holes) (4.3)

4.2 Mass/enthalpy flow exchanges between local external environments and compartment

In-flow/out-flow relationship between an *i*-th compartment to a local external environment is defined by following conditions.

4.2.1 Outfow of compartment mass/enthalpy

For $p_{vi} > p_{Lij}$, outflow of compartment mass and the controlling parameters are

$$\text{Let} \quad p_r = p_{vi} / p_{L_{ij}} \geq 1.0$$

$$p_t = p_{vi}$$

$$T_t = T_{vi}$$

$$\dot{m}_{ij} \geq 0 \quad \text{outflow from } i\text{ - th compartment}$$

$$c_p T_t \dot{m}_{ij} \geq 0$$

where the same mass flow rate equations (Eqn. 4.1-4.2) are used.

4.2.2 Inflow of external mass/enthalpy into compartment

For $p_{vi} < p_{Lij}$, inflow of local external mass into *i-th* compartment and the controlling parameters are

$$\text{Let} \quad p_r = p_{L_{ij}} / p_{vi} \geq 1.0$$

$$p_t = p_{L_{ij}}$$

$$T_t = T_{t_{ij}} \text{ or } T_{aw_{ij}} \text{ or some average value between wall temp and } T_{t_{ij}}$$

$$\dot{m}_{ij} \leq 0 \quad \text{inflow into } i\text{ - th compartment}$$

$$c_p T_t \dot{m}_{ij} \leq 0 \quad \text{inflow enthalpy into } i\text{ - th compartment}$$

where the same mass flow rate equations (Eqn. 4.1-4.2) are used.

For a condition of inflow of external mass (with non-adiabatic, wall heat conducting effect) into a compartment, the definition of mass and enthalpy influx condition will be modified by the discussion of foregoing section from Sect. 1.0 to 3.0.

Specifically, mass flow rate and enthalpy flow rate per width as defined in Eqn. 1.16 to 1.19 are used for the case where all of the boundary layer flow and a portion of external inviscid flow enter the compartment, i.e.,

The inflow mass is

$$\dot{m}_{ij} = -(\dot{m}_{h\,\mathrm{inf}\,low} x_h)\, w_h = -(\Delta\dot{m}_h x_h + \dot{m}_{bl} \delta_{bl}) w_h \,. \qquad (1.18a)$$

The total inflow enthalpy is approximated by

$$c_p T_{aw} \dot{m}_{ij} = -(\dot{h}_{h\,\mathrm{inf}\,low} x_h) w_h = -(\dot{h}_{bl} \delta_{bl} + \Delta\dot{m}_h x_h c_p T_{awe}) w_h \qquad (1.19)$$

Eqn. 1.20a and 1.21 will be used for the case where only a portion of boundary layer flow enters the compartment but no external inviscid flow to enter.

The mass flow rate enters the compartment is

$$\dot{m}_{ij} = -(\dot{m}_{h\inf low} x_h) w_h = -(\dot{m}_e \delta_{bl}) w_h \int_0^{\eta_m} \frac{T_e}{T(\xi)} \frac{u(\xi)}{u_e} d\xi; \text{ mass flow rate between } 0 \le \xi \le \eta_m \quad (1.20a)$$

where η_m is the matching point.

Then the enthalpy flux to be ingested will be integrated between $0 \le \xi \le \eta_m$.

$$c_p T_{aw} \dot{m}_{ij} = -(\dot{h}_{h\inf low} x_h) w_h = -(\dot{m}_e c_p T_{awe} \delta_{bl}) w_h \int_0^{\eta_m} \frac{T_e}{T(\xi)} \frac{T_{aw}(\xi)}{T_{awe}} \frac{u(\xi)}{u_e} d\xi \quad (1.21)$$

4.3 Summation of i-th compartment properties

The inflow/outflow mass flow rates and enthalpy flow rates (i.e., products of temp*mass flow rates, T_{aw}*dm/dt*) are sum on each *i*-th compartment.. It is assumed the compartment volume is much larger than the orifice areas, therefore, the inflows are assumed to stagnated in the volume so that

$$T_{vi} \to T_{tci} \quad (4.4)$$

and inflow adiabatic temperature can be expressed in terms of total temperature by the relation

$$\frac{T_t}{T} - 1 = \frac{1}{r_f} \left(\frac{T_{aw}}{T} - 1 \right) \quad (4.5)$$

Then ensemble average of compartment temperature is taken and the rate of compartment temperature change is determined by following relation

$$c_p T_{vi} \dot{m}_{vi} = c_p T_{tci} \dot{m}_{ci} + \sum_j c_p T_{aw} \dot{m}_{ij} \quad \text{sum enthalpy flow on } j\text{-th orifices per } i\text{-th compartment} \quad (4.6)$$

$$\dot{m}_{vi} = \dot{m}_{ci} + \sum_j \dot{m}_{ij} \quad \text{sum of mass flow on } j\text{-th orifices per } i\text{-th compartment} \quad (4.7)$$

$$c_v \Delta T_{vi} = \frac{c_p T_{vi} \dot{m}_{vi} \Delta t + c_v T_{vi} m_{vi}}{\dot{m}_{vi} \Delta t + m_{vi}} - c_v T_{vi} \quad \text{change in well mixed } i\text{-th compartment temperature} \quad (4.8)$$

$$\Delta T_{vi} = \frac{\gamma T_{vi} \dot{m}_{vi} \Delta t + T_{vi} m_{vi}}{\dot{m}_{vi} \Delta t + m_{vi}} - T_{vi} \quad \text{for } \gamma = c_p / c_v \quad (4.8a)$$

The rate of change of well mixed i-th compartment temperature is approximated as

$$\frac{dT_{vi}}{dt} = \frac{\Delta T_{vi}}{\Delta t} \quad \text{per } i\text{-th compartment} \quad (4.9)$$

Once the rates of changes of mass and temperature within an i-th compartment are determined, then the rate of change of compartment pressure is determined by taking a logarithmic differential of the equation of state, $p=\rho RT$ as shown

$$p_{vi} = \rho_{vi} R_g T_{vi} = \frac{m_{vi}}{V_i} RT_{vi} \tag{4.10}$$

where a volume V and gas constant R_g are invariant.

The rate of compartment pressure change is expressed as

$$\frac{dp_{vi}}{p_{vi}} = \frac{dm_{vi}}{m_{vi}} + \frac{dT_{vi}}{T_{vi}} \tag{4.11}$$

or

$$\frac{dp_{vi}}{dt} = p_{vi} \left(\frac{\dot{m}_{vi}}{m_{vi}} + \frac{1}{T_{vi}} \frac{dT_{vi}}{dt} \right) \tag{4.12}$$

5.0 Appendix: Partial listings

5.1 Boundary layer properties

```
'compute velocity/temperature profiles
        ue       = Me*Vse      'bl edge flow velocity (fps)
        rhoe     = ge*rhoe     'bl edge density (lbm/ft3)
        masse  = rhoe*ue       'bl edge mass flux (lbm/s/ft2)
        mue    =2.27*Te^1.5/(Te+198.6)*1d-8    'sutherlands viscosity based on Te (lbm-s/ft2)
        Re_x   =masse/ge/Mue     'unit Reynolds number based on Te
        muw    =2.27*Tw^1.5/(Tw+198.6)*1d-8    'sutherlands viscosity based on Tw (lbm-s/ft2)
        rhow_rhoe =Te/Tw                       'density ratio at wall and bl edge
        rhow         =rhoe*rhow_rhoe       'density at bl wall based on Tw
        Re_xw       =Re_x*rhow_rhoe*mue/muw  'Reynolds number based on wall density

        f1          = .5*(gm-1)*Me^2
        Te_Tw  = Te/Tw
        Tw_Te  = Tw/Te
        Tte_Te  = 1+f1

        print       "     turbulent boundary layer profiles"
        pre    = .7         'Prandtl number at approx 600F
        rf      = pre^(1/3)  'turbulent recovery factor
        f2      = rf*f1
        Taw_Te   = 1+f2
        Te_Tawe = 1/Taw_Te
        Taw         = Taw_Te*Te
        Tw_Taw  = Tw/Taw
        kaw        = f2/Taw_Te
        dTw_Te  = (Tw-Te)/Te

        for i=1 to jp
           if i<=11 then
              u_ue=.001*(i-1)
           elseif i<=21 then
              u_ue=.01*(i-11)
           else
```

```
        u_ue=.05*(i-21)

      end if

      'bl velocity profiles by 1/7th power profile
      y_bl        =u_ue^7     ' u_ue^10
      y_bl(i)     =y_bl
      u_ue(i)     =u_ue

      'non-adiabatic wall heating profiles
      call non_adw_temp _
          (rf,gm,Tw_Te,u_ue(i),f1,Me,Te_Tawe,T_Te_w(i),M_w(i),Taw_Tawe_w(i))

     'adiabatic wall flow profiles
      call adw_temp(rf,gm,kaw,Taw_Te,u_ue,Me,Te_Tawe,T_Te(i),M,Taw_Tawe)

      if i/2<>i\2 then
        print using "   ###   ##.####  ##.####  ##.####  ##.####  ##.####   ##.####";_
                 i,y_bl(i),u_ue(i),T_Te_w(i),T_Te(i),M_w(i),Taw_Tawe_w(i)
      end if
    next

    print      "      compute integrated properties"
    print      "      turbulent boundary layer"
    print      "      i     y/bl    sum mass    sum massTaw    mean Taw/Tawe"

    'non-adiabatic wall heating
    for i=1 to jp
      'compute local properties within bl
      rho_rhoe  =1/T_Te_w(i)                  'density ratio
      mass(i)     =rho_rhoe*u_ue(i)            'mass ratio
      mass_T(i) =mass(i)*Taw_Tawe_w(i)  'mass*Taw ratio
    next
    s_mass    =0
    s_mass_T=0
    mean_Taw(1)=Tw_Taw                        'at wall
    for i=2 to jp
      'mass, mass*Taw integration
      d_mass     =.5*(mass(i)+mass(i-1))*(y_bl(i)-y_bl(i-1))
      d_mass_T =.5*(mass_T(i)+mass_T(i-1))*(y_bl(i)-y_bl(i-1))
      incr s_mass,d_mass
      incr s_mass_T,d_mass_T
      sum_mass(i)     =s_mass
      sum_mass_T(i) =s_mass_T
      mean_Taw(i)    =s_mass_T/s_mass    'mean Taw up to integrated point in bl

      if i/2<>i\2 then
        print using "    ###  ##.####     ##.####       ##.####      ##.####";_
                 i,y_bl(i),s_mass,s_mass_T,mean_Taw(i)
      end if
    next

sub blasius(jp_l,y_bl_l(),u_ue_l())
    'laminar bl profile with dp/dx=0
```

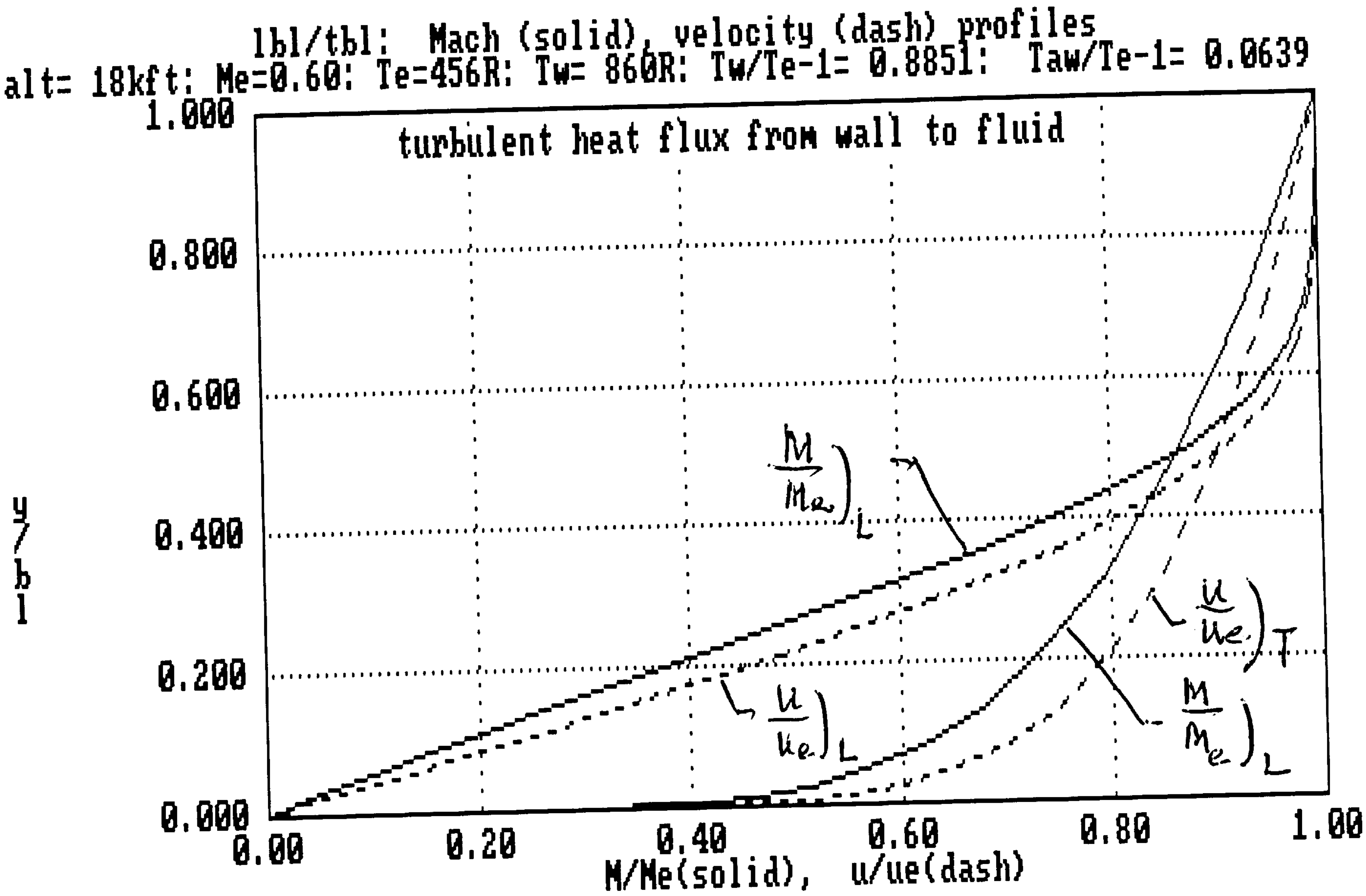
```
  blasius_velocity_profile:
  data 0,.2,.4,.6,.8,1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7
  data  0,.0664,.1328,.1989,.2647,.3298,.4867,.6298,.7518,.8460,.9128,.9555,.9794,.9916
  data .9969,.9990,.9997,.9999
  redim eta(jp_l)
  restore  blasius_velocity_profile:
  for i=1 to jp_l
    read eta(i)
  next
  for i=1 to jp_l
    y_bl_l(i) = eta(i)/eta(jp_l)
    read u_ue_l(i)
  next
 end sub

 sub non_adw_temp(rf,gm,Tw_Te,u_ue,f1,Me,Te_Tawe,T_Te_w,M_w,Taw_Tawe_w)
  'non-adiabatic wall heating profiles
      T_Te_w     =Tw_Te+(1-Tw_Te)*u_ue + f1*u_ue*(1-u_ue) 'T-profile due to Tw normalized wrt
Te
      M_Me_w  =u_ue/sqr(T_Te_w)          'Mach ratio profile for non-adiabatic wall heating
      M_w        =M_Me_w*Me              'Mach profile for non-adiabatic wall heating
      Taw_Tawe_w=Te_Tawe*T_Te_w*(1+.5*rf*(gm-1)*M_w^2)  'adiabatic wall tem profile wrt
Tawe
 end sub

 sub adw_temp(rf,gm,kaw,Taw_Te,u_ue,Me,Te_Tawe,T_Te,M,Taw_Tawe)
  'adiabatic wall flow profiles
       T_Te       =Taw_Te*(1-kaw*u_ue^2)     'adiabatic wall T-profile due to Me normalized wrt Te
       M_Me    =u_ue/sqr(T_Te)            'Mach ratio profile for adiabatic wall
       M          =M_Me*Me                  'Mach profile for adiabatic wall
       Taw_Tawe  =Te_Tawe*T_Te*(1+.5*rf*(gm-1)*M^2) 'adiabatic wall temp profile
 end sub


 print "    compute laminar/turbulent bl height and bl edge mass flow rate"
   x=0
   if i_veh_attit=0 then
     print i_veh_attit_axial$
     print       "      i       x(ft)              Rex          bl_e(ft)  rhoe*ue*ble(lbm/s/ft)"
     delx=2.5    'axial running length (ft)
   else
     print i_veh_attit_cross$
     print       "      i    phi(deg)              Rex          bl_e(ft)  rhoe*ue*ble(lbm/s/ft)"
     delx=r_stage/12               'lap/ov cylinder radius (ft)
   end if
   for i=1 to jx
      if i_veh_attit=0 then
        x    =delx*i                  'axial running length (ft)
      else
        phi(i)=2.5*i                  'cylinder peripheral angle (deg) from stagnation point
        x    =delx*phi(i)/57.3     'cylinder peripheral running length (ft)
      end if
```

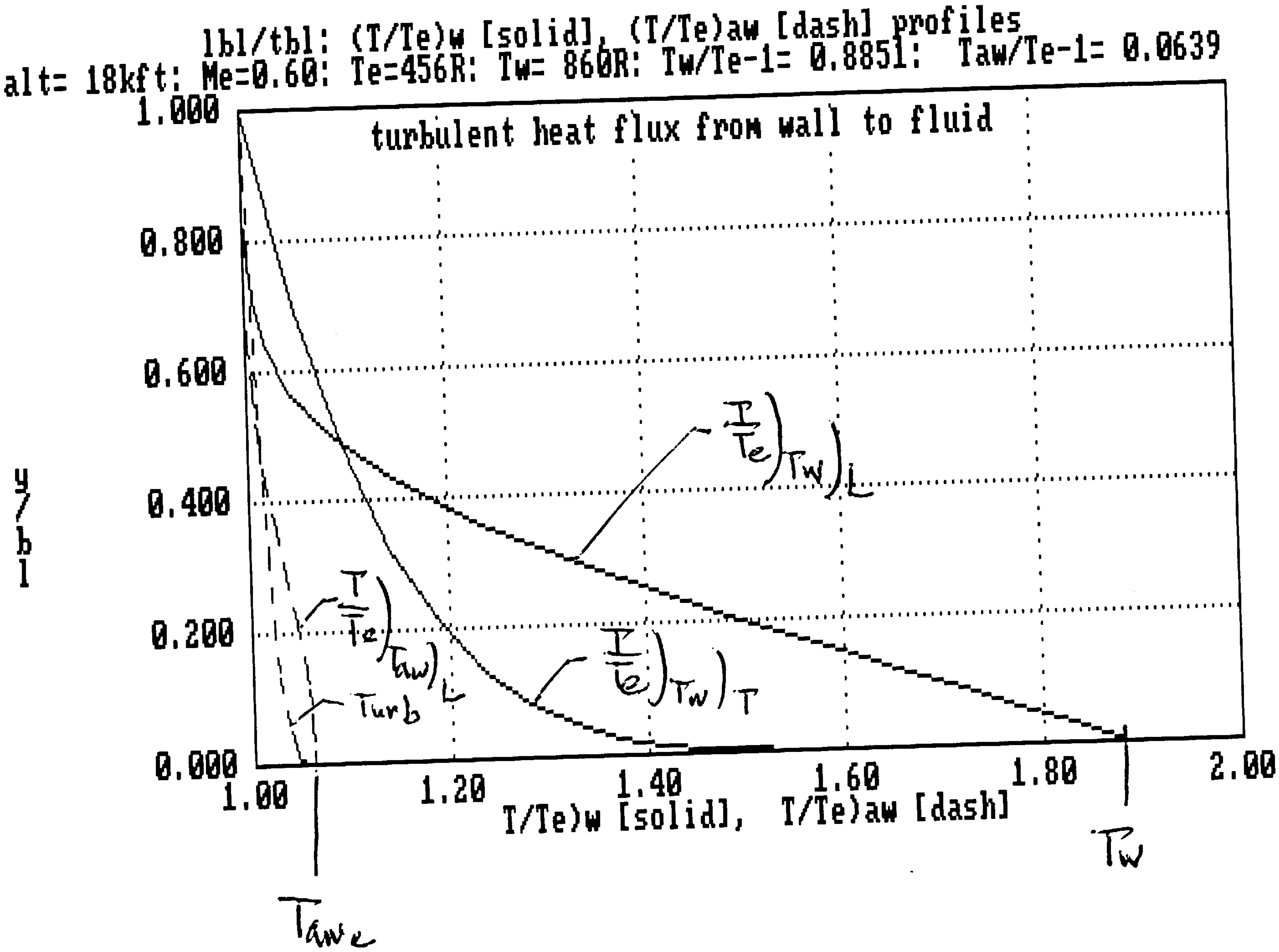
```
    Rex  =Re_x*x                 'Reynolds number at x

    'bl thickness (ft)
    if Rex>5d5 then              'transitional Reynolds number = 5x10^5
      bl_th = 0.37*x/(Rex^.2)   'turbulent 1/7th power law profile (Schlicting p537)
    else
      bl_th = 5*x/sqr(Rex)       'laminar Blasius flow (dp/dx=0)
    end if

    mass_e = masse*bl_th         'bl edge mass flow rate per width (lbm/s/ft)

    x(i)          =x
    Rex(i)      =Rex
    bl_th(i)    = bl_th
    mass_e(i) =mass_e
  next

  'bl thickness based on wall temp
  for i=1 to jx
    x       =x(i)
    Rex   =Re_xw*x           'Reynolds number at x based on

    'bl thickness (ft)
    if Rex>5d5 then
      bl_th = 0.37*x/(Rex^.2)   'turbulent 1/7th power law profile (Schlicting p537)
    else
      bl_th = 5*x/sqr(Rex)       'laminar Blasius flow (dp/dx=0)
    end if
    mass_e  =masse*bl_th       'rhoe*ue*bl_th: bl edge mass flow rate per width (lbm/s/ft)

    if i=1 or i\2=i/2 then
      if i_veh_attit=0 then

        print using "    ###    ###.#  bl edge ->:  ##.####^^^^     ###.####       ####.####";_
                  i,x(i),Rex(i),bl_th(i),mass_e(i)
      else
        print using "    ###    ###.#  bl edge ->:  ##.####^^^^     ###.####       ####.####";_
                  i,phi(i),Rex(i),bl_th(i),mass_e(i)
      end if
      print using    "    based on wall density ->:  ##.####^^^^     ###.####       ####.####";_
                  Rex,bl_th,mass_e
    end if

    Rex_w(i)   =Rex
    bl_th_w(i) =bl_th
    mass_e_w(i)=mass_e
  next
```

5.2 compartment venting properties

```
   Tt_vol =500#
   T_ext  =550
   wt_vol =1000#
   dw_in  =.8#
```

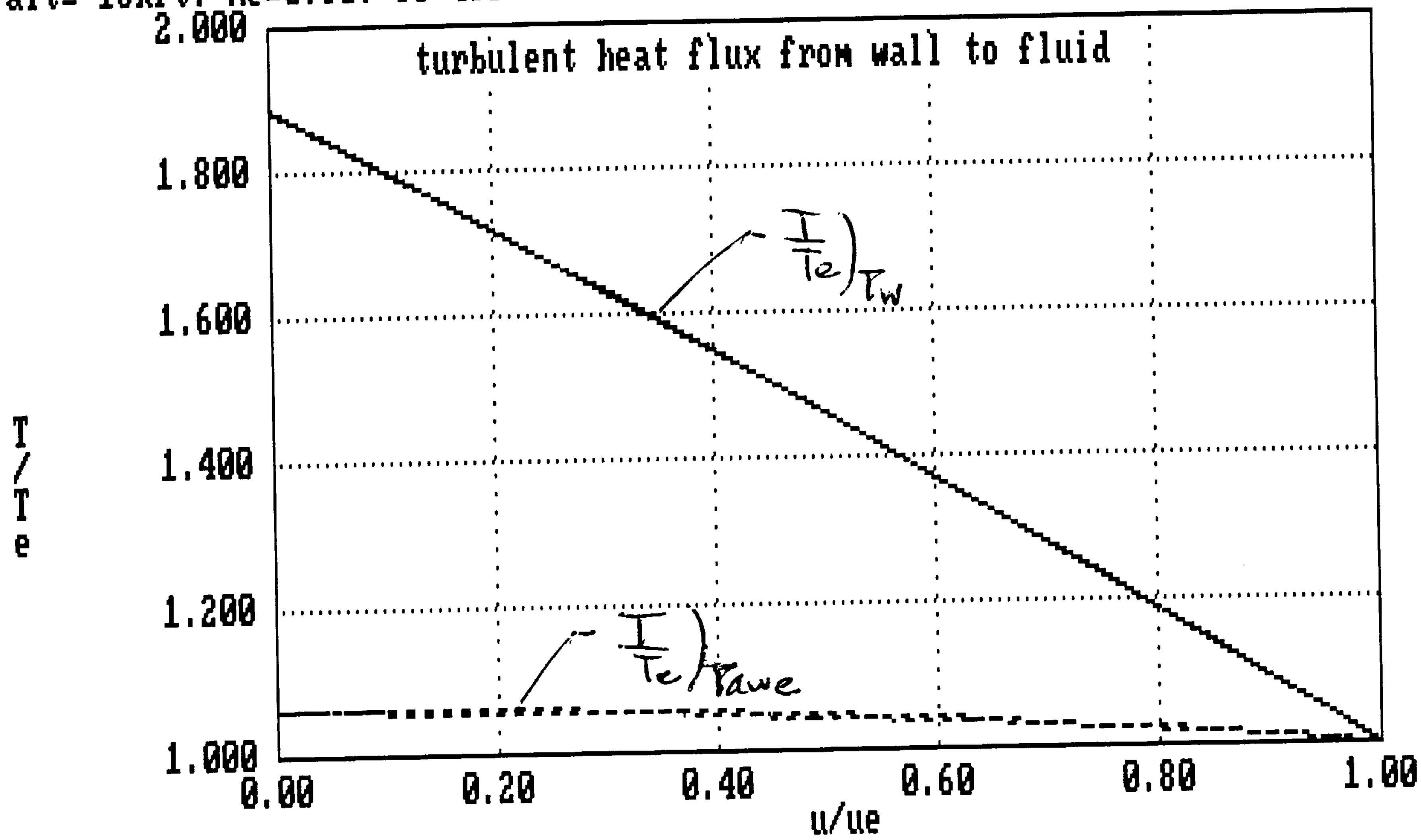
```
        dw_ou t=-1#
        mf_T   =-1
        dt        =10
        gm       =1.4

for i=1 to imx
        if wt_vol<800# then
           dw_in=1.2
        end if
        if T_ext<400# then
           mf_T=1
        elseif T_ext>600# then
           mf_T=0
        end if
        dw_net      =dw_in+dw_out                          net inflow/outflow mass into compartment
        T_ext       =T_ext+mf_t                            decreasing external temp history
        dTw_wdt  =T_ext*dw_in+Tt_vol*dw_out         net inflow/outflow enthalpy
        Tw_sum   =gm*dTw_Wdt*dt+Tt_vol*Wt_vol    total mixed temp*mass in compartment at t
        Wt_sum   =Wt_vol+dW_net*dt                     total mixed mass in compartment
        dTv         =(Tw_sum/Wt_sum-Tt_vol)/dt          emsemble averaged temp change rate in cmpt

        incr Tt_vol,dTv*dt                                 sum of compartment temp change
        incr wt_vol,dw_net*dt                              sum of compartment mass change
        print using "  Text=####.##   dTv=##.####   Ttv=#####.##   wtv=#####.##";_
                    T_ext,dTv,Tt_vol,wt_vol
        dTv_out(i)     = dTv
        T_ext_out(i)  =T_ext
        Tt_vol_out(i) =Tt_vol
        wt_vol_out(i)=wt_vol
    next
```

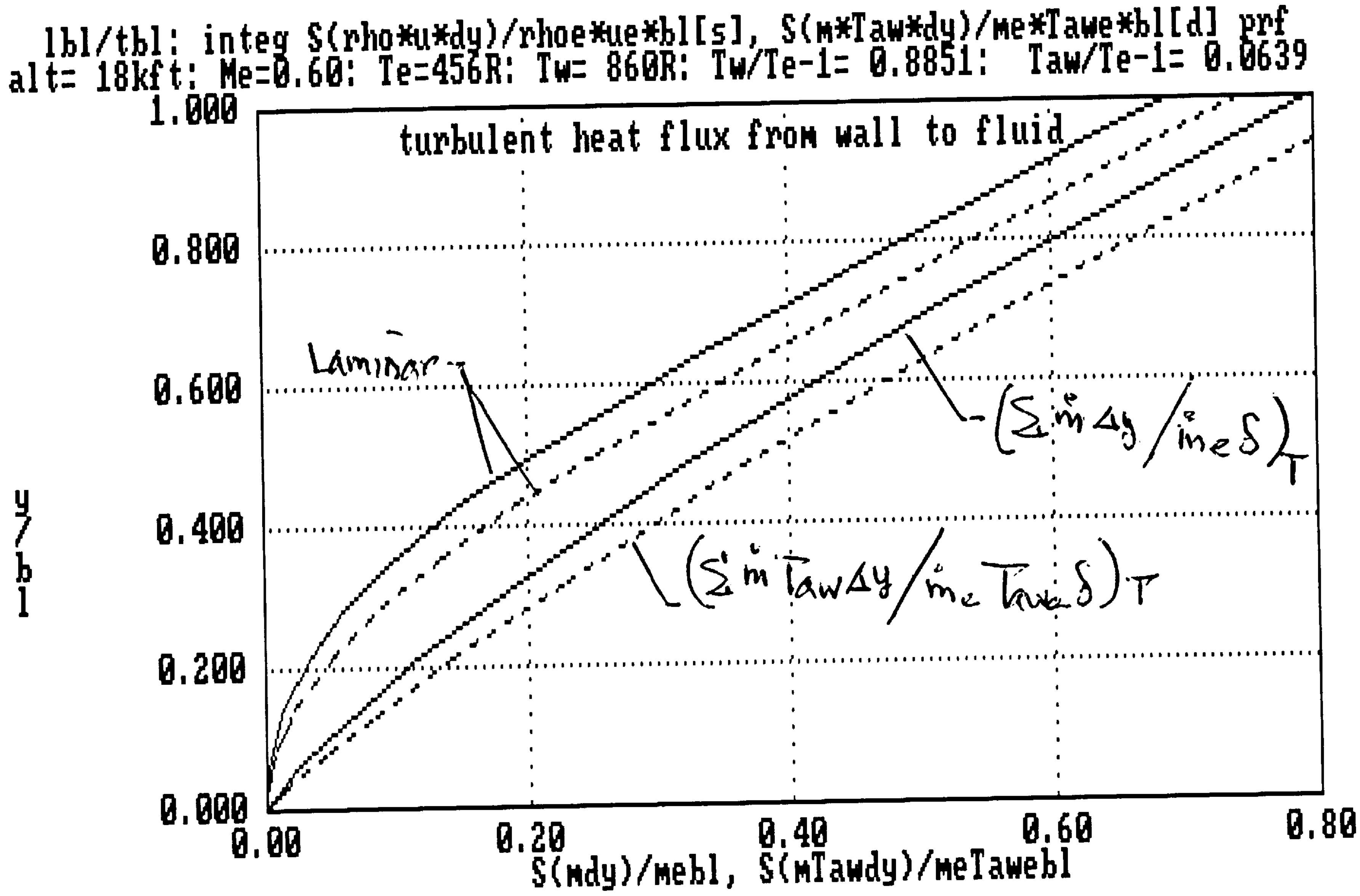

lbl/tbl: Mach (solid), velocity (dash) profiles
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
1.000
0.800
0.600
0.400
0.200
0.000
y/bl
0.00
0.20
0.40
0.60
0.80
1.00
M/Me(solid), u/ue(dash)
M/Me)L
u/ue)L
u/ue)T
M/Me)L

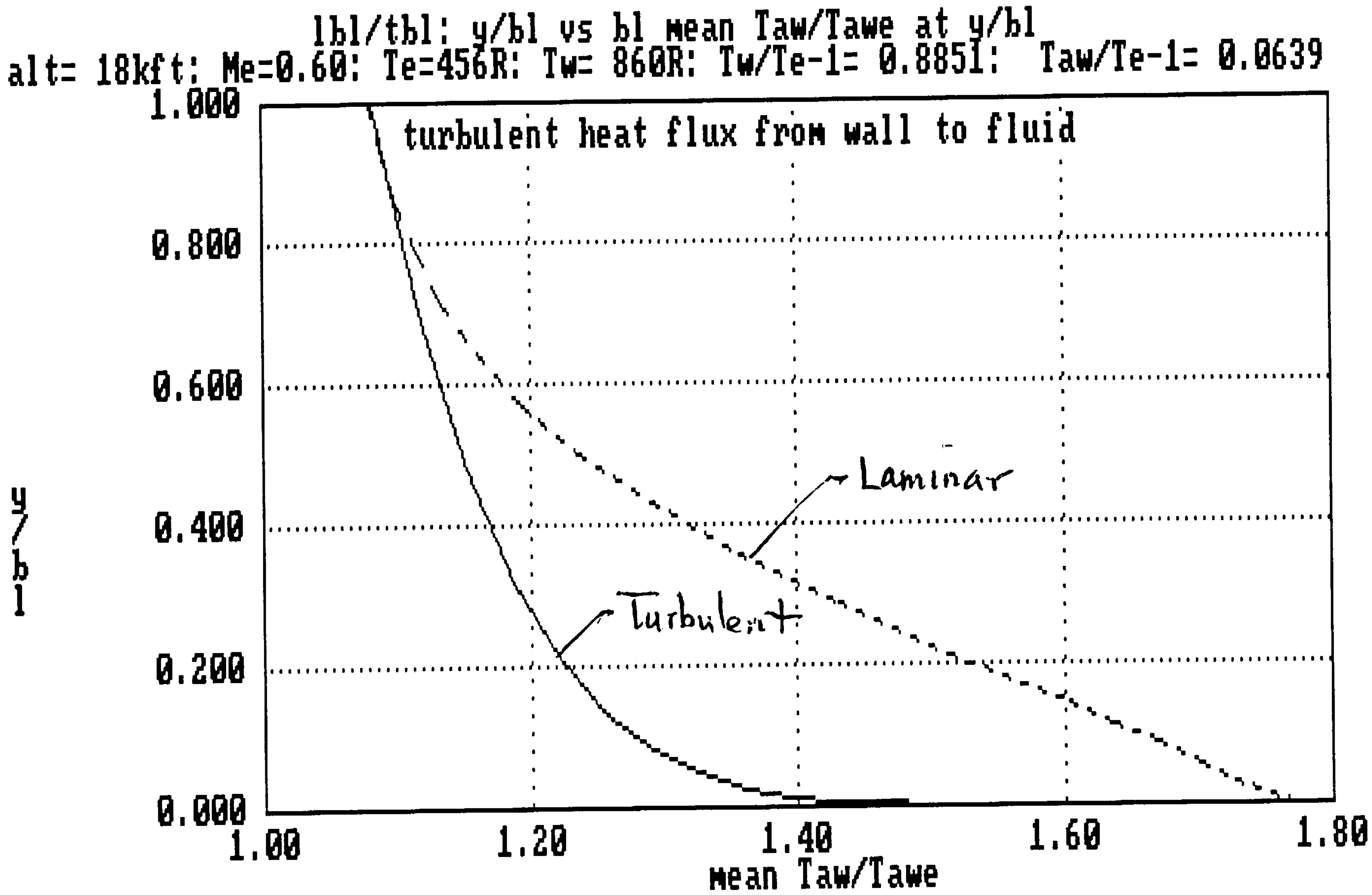


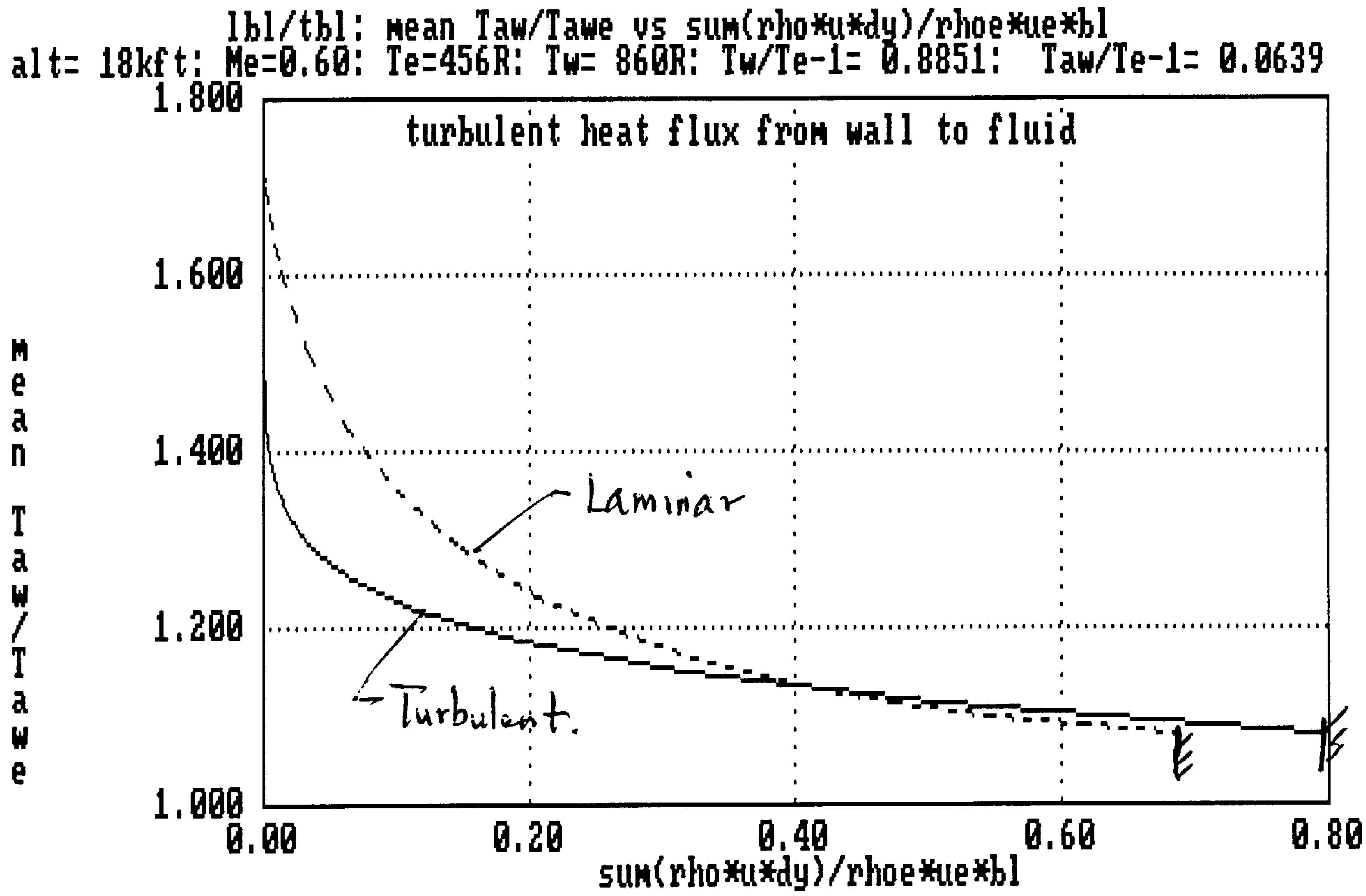
lbl/tbl: (T/Te)w [solid], (T/Te)aw [dash] profiles
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
1.000
0.800
0.600
0.400
0.200
0.000
y/bl
1.00
1.20
1.40
1.60
1.80
2.00
T/Te)w [solid], T/Te)aw [dash]
T/Te)Tw)L
T/Te)Taw)L
Turb
T/Te)Tw)T
Tw
Tawe

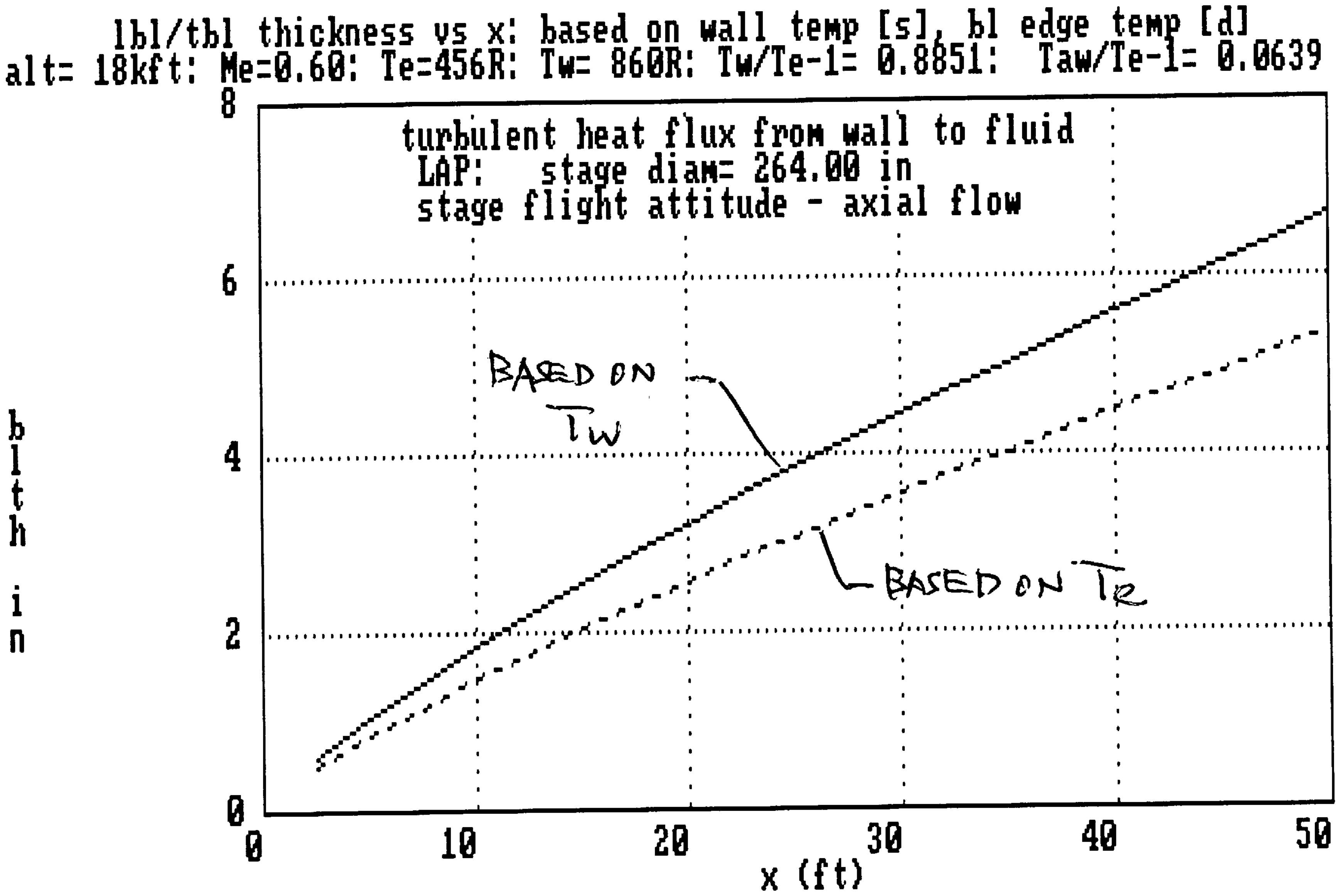
lbl/tbl: (T/Te)w [solid], (T/Te)aw [dash] vs u/ue profile
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639




lbl/tbl: integ S(rho*u*dy)/rhoe*ue*bl[s], S(m*Taw*dy)/me*Tawe*bl[d] prf
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
Laminar
$(\Sigma \dot{m} \Delta y / \dot{m}_e \delta)_T$
$(\Sigma \dot{m} T_{aw} \Delta y / \dot{m}_e T_{awe} \delta)_T$
y/bl
S(mdy)/mebl, S(mTawdy)/meTawebl
1.000
0.800
0.600
0.400
0.200
0.000
0.00
0.20
0.40
0.60
0.80


lbl/tbl: y/bl vs bl mean Taw/Tawe at y/bl
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
Laminar
Turbulent
1.000
0.800
0.600
0.400
0.200
0.000
y/bl
1.00
1.20
1.40
1.60
1.80
mean Taw/Tawe


lbl/tbl: mean Taw/Tawe vs sum(rho*u*dy)/rhoe*ue*bl
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
Laminar
Turbulent.
1.800
1.600
1.400
1.200
1.000
0.00
0.20
0.40
0.60
0.80
Mean Taw/Tawe
sum(rho*u*dy)/rhoe*ue*bl


lbl/tbl thickness vs x: based on wall temp [s], bl edge temp [d]
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639
turbulent heat flux from wall to fluid
LAP: stage diam= 264.00 in
stage flight attitude - axial flow
BASED ON Tw
BASED ON TR
bl th in
x (ft)
0
2
4
6
8
0
10
20
30
40
50

ref mass flow rate (rhoe*ue*bl_th) per width vs x: wall [s], fs [d]
alt= 18kft: Me=0.60: Te=456R: Tw= 860R: Tw/Te-1= 0.8851: Taw/Te-1= 0.0639