EE C128 Lab 5

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1 Purpose

The purpose of this part A of lab 5 is to determine the physical parameters of the linearized model experimentally so that we can later design a controller to leviate the metal ball. In part B, we design and implement the controller to successfully levitate the ball.

2 Prelab A

2.1 Derivation of the System Transfer Function

2.1.1 Desired position / output offset circuitry

letting $y_{ref} = Y_0$

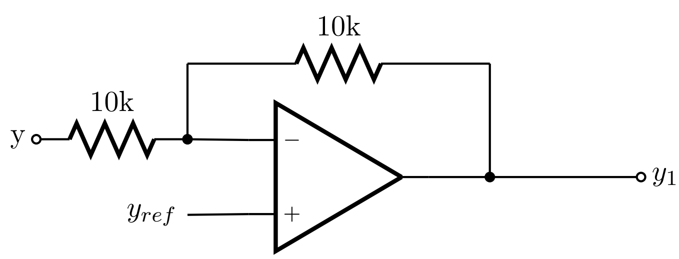

Figure 1: output offset circuit

$v_- = (y_1 - y)/2 + y = y_{ref}$

$y_1 - y + 2y = 2y_{ref}$

$\boxed{y_1 = 2y_{ref} - y}$

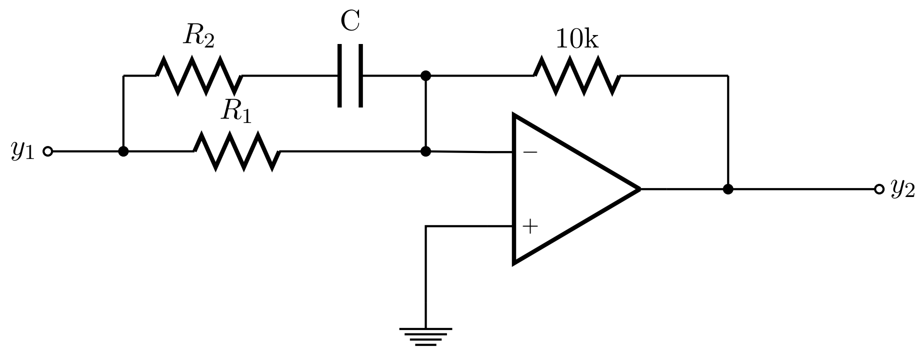

Figure 2: analog controller circuit

2.1.2 Transfer Function of the Analog Controller

$$\begin{aligned}
0 &= (y_2 - y_1)\frac{z_2}{z_2 + z_1} + y_1 \\
-y_1 &= (y_2 - y_1)\frac{z_2}{z_2 + z_1} \\
-y_1\frac{z_2 + z_1}{z_2} &= (y_2 - y_1) \\
y_1(1 - \frac{z_2 + z_1}{z_2}) &= y_2 \\
y_1(-\frac{z_1}{z_2}) &= y_2
\end{aligned}$$

with:

$$\begin{aligned}
z_1 &= (10k\Omega) \\
z_2 &= (R_2 + Z_C)||R_1 \\
&= \frac{(R_2 + Z_C) * R_1}{(R_2 + Z_C) + R_1} \\
&= \frac{(R_2 + 1/(sC)) * R_1}{(R_2 + 1/(sC)) + R_1} \\
&= \frac{R_1R_2Cs + R_1}{R_2Cs + 1 + R_1Cs}
\end{aligned}$$

now:

$$\begin{aligned}
Y_1/Y_2 &= -(10k\Omega) \cdot \frac{R_2Cs + 1 + R_1Cs}{R_1R_2Cs + R_1} \\
G_c(s) &= \frac{Y_2(s)}{Y_1(s)} = \frac{-(10k\Omega)}{R_1} \cdot \frac{(R_1 + R_2)Cs + 1}{R_2Cs + 1}
\end{aligned}$$

2.1.3 Current Offset Circuitry

letting y_i be the voltage drop across the potentiometer, and V_{out} be the output of the op amp. Observing that y_i will be negative,

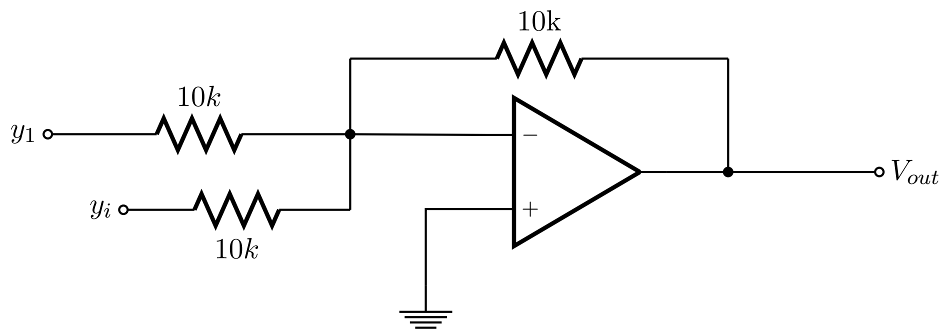

Figure 3: current offset circuit

$$v_- = 0$$
$$\frac{V_{out}}{(10k\Omega)} + \frac{y_1}{(10k\Omega)} + \frac{y_i}{(10k\Omega)} = 0$$
$$V_{out} = -(y_2 + y_i)$$

2.1.4 Linearization of the System

$$m\ddot{x} = f(I,x) - mg \approx f(I_o, x_0) + K_i\delta I + K_x\delta x - mg$$

linearizing yields:

$$m\ddot{x} = K_i\delta I + K_x\delta x$$
$$y = a\delta x$$

taking the laplace transform:

$$mX(s)s^2 = K_iI(s) + K_xX(s)$$
$$X(s)(ms^2 - K_x) = K_iI(s)$$
$$Y(s)(ms^2 - K_x) = aK_iI(s)$$
$$\frac{Y(s)}{I(s)} = \frac{aK_i}{(ms^2 - K_x)}$$
$$G(s) = \frac{Y(s)}{I(s)} = \frac{aK_i}{m(s^2 - \frac{K_x}{m})}$$

3 Lab : Part A

3.1 System Identification

1. The equilibrium height of the ball, positioned so that about half of the light going to the photo-resistor is covered is $x_0 \approx 1.6cm$.

2. Resistance of the photodetector under different levels of shadow.

conditions	resistance
no shadow	487Ω
halfway in shadow	$1.01k\Omega$
completely in shadow	$4.42k\Omega$

Any resistance will satisfy the power requirements (since $I = \frac{V}{R} = \frac{7V}{487\Omega} \approx 14mA$, and $P = IV = 14mA * 7V = 98mW < 250mW$), so we will choose $1k\Omega$ as the value for the other half of the voltage divider.

3. Linearizing h(x)

ball offset δx (mm)	outut voltage across photoresistor (volts)
-2	2.38
-1	2.7
0	3.49
1	4.43
2	5.60

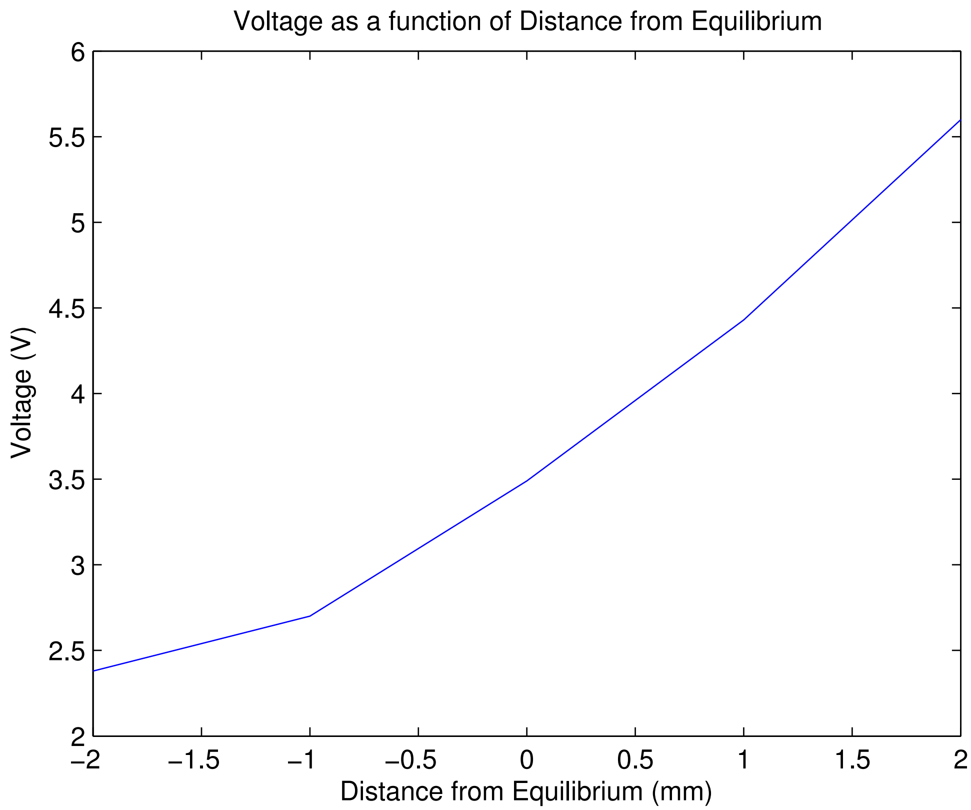

Figure 4: voltage across voltage divider for small displacements of the metal ball (See Listing 1)

Linearizing the portion of the graph near zero:

$h(x) \approx \frac{4.43V - 2.7V}{2mm} \delta x = \left(0.865 \frac{V}{mm}\right) \delta x$

$a = 865.0V/m$

4. Determining K_i (Newtons/Amps). We first position the ball at the position that half-shades the photo-resistor.

ball apparent mass (g)	ref+ pin (volts)	current magnitude $\|I\|$ (amps)
0.0	-0.4196	2.7973
0.3	-0.39768	2.6512
1.5	-0.3824	2.5493
4.0	-0.31030	2.0687
6.5	-0.26202	1.7467

Note scaling factor for ref+ is $(\frac{20}{3}\frac{A}{V})$.

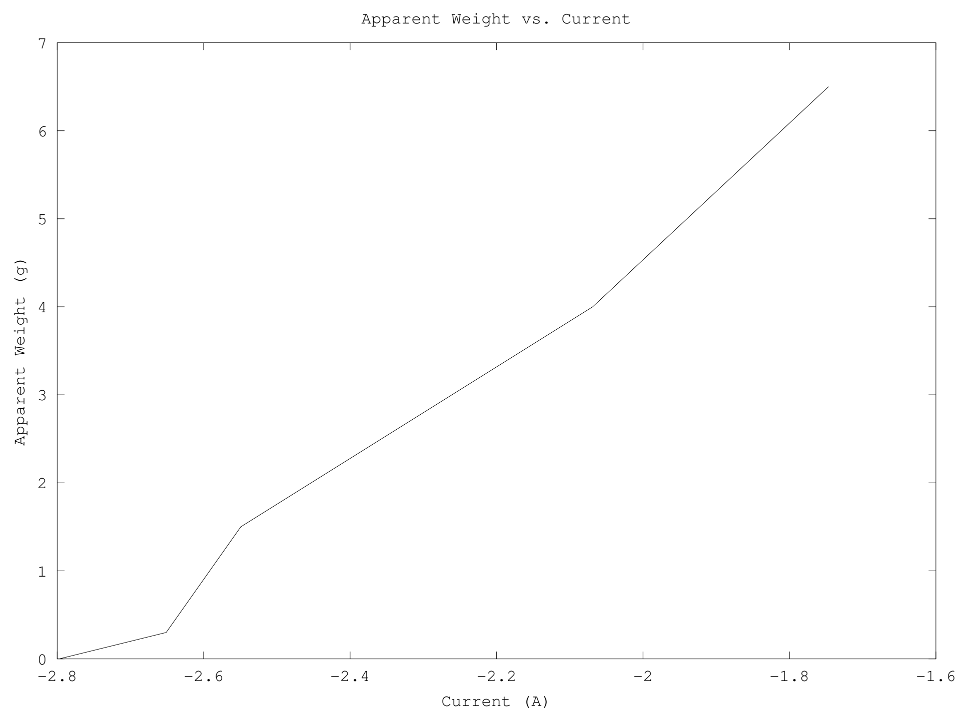

Figure 5: Measured weight on scale while varying current in coil manually (see listing 2)

Linear regression (see listing 2) yields:

$6.2276\frac{g}{A} \cdot 9.8\frac{m}{s^2} \cdot \frac{1kg}{1000g} = 0.06103048\frac{N}{A}$

$\boxed{K_i \approx 0.061\frac{N}{A}}$ This relationship contrasts with the voltage/displacement relationship in that they are opposite signs. So voltage correlates positively with displacement of the ball (as the ball moves up (positive x direction) the photoresistor sees less light, which increases its resistance, leading to a higher voltage across it), and force correlates negatively with the amplifier's output current (as more current flows through the electromagnet the apparent weight of the ball decreases). (Note that the value for K_i is positive because the voltage on the $ref+$ pin is negative, so the current measurements are negative, causing a sign flip in the value for K_i from negative to positive. In other words, force correlates positively with the negative current.) In both cases, the linear approximation is reasonably accurate. This is because the range of x-values we care about is very small in both cases. We only need to be concerned with small deviations (in distance and in current) from the equilibrium values because these values will not change much when the controller is levitating the ball. The idea is that the controller will "steer" the system to states very near the equilibrium and so we won't have to worry about other states, e.g. those where the ball is far below x_0 or where the current is very high.

5. Determining K_x (Newtons/meters)

   Ball weight versus distance:

ball offset δx (mm)	apparent weight (grams)
-0	0.3
-1	1.7
-2	3.7
-3	5.3
-4	6.5
-5	7.5
-6	8.6
-7	9.6

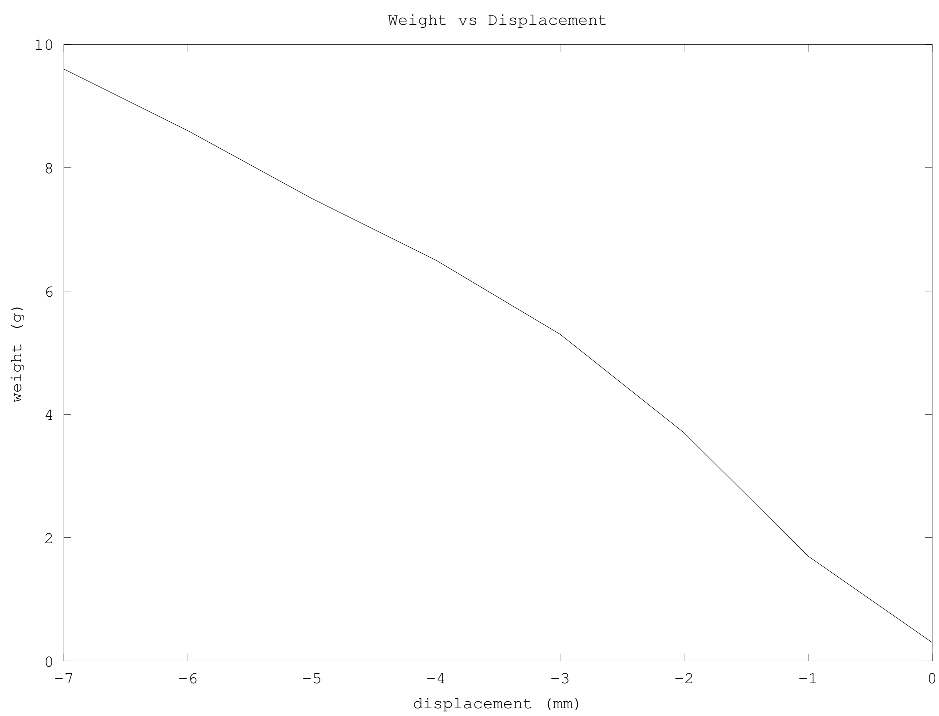

Figure 6: see listing 3

Linear regression yields:

$$1.33571\frac{g}{mm} \cdot 1000\frac{mm}{m} \cdot 9.8\frac{m}{s^2} \cdot \frac{1kg}{1000g} = -13.089958$$

$$\boxed{K_x \approx 13.089958\frac{N}{m}}$$

Again we consider the upwards direction to be positive. The apparent weight-distance relationship then is positive like the voltage-distance relationship, and it's magnitude is somewhere in between the magnitudes of the previous two relationships. It makes sense that the relationship is positive because as you move the ball closer to the electromagnet (positive x direction), the voltage increases (because a is positive), a voltage increase causes a current increase which causes an apparent weight decrease (because current and apparent weight are negatively correlated by K_i (ignoring the sign change caused by the negative $vref+$)), which means force on the ball increased. So this relationship makes sense in terms of the other relationships we have already derived. Another way to think about it is that as you move the ball closer to the electromagnet, the magnetic field strength (which is inversely proportional to distance squared) is increased and so the magnetic force on the ball increases.

6. DC gain Analysis:

   $(DCgain) = aK_cK_a$

   $\frac{(DCgain)}{aK_a} = K_c$

   $K_c = \frac{\left(1000\frac{A}{m}\right)}{\left(0.865 \cdot 10^3 \frac{V}{m}\right)\left(2\frac{A}{V}\right)} = 0.578$

Summary of important quantities:

$a = 865V/m$

$K_i = 0.061$

$K_x = 13.09N/m$

$m = 16.3g$

4 Prelab B

1. root locus and frequency response of system:

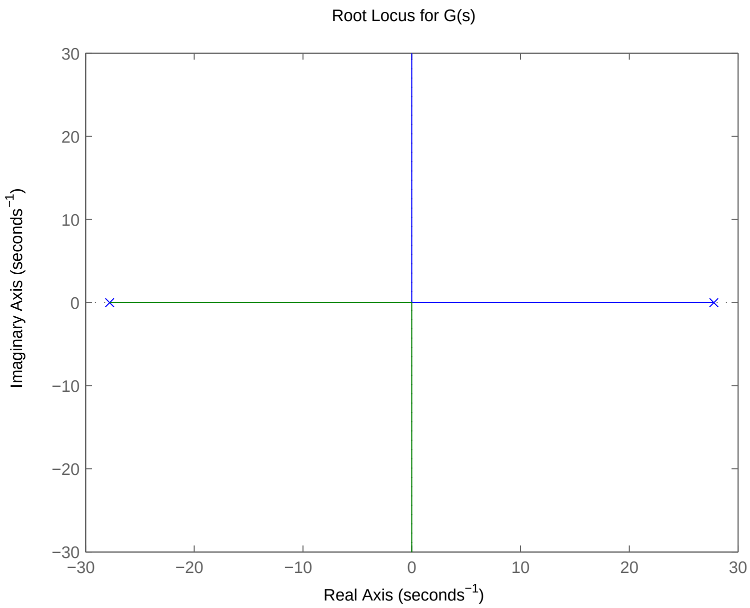

Figure 7: root locus of the system (See listing 4)

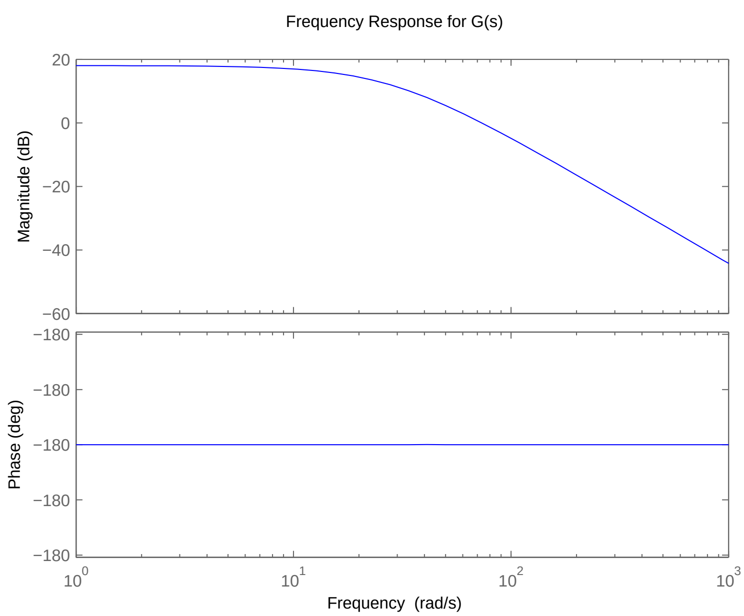

Figure 8: frequency response of the system (See listing 4)

We can see from the root locus that the system is not stable. The frequency response shows that the system has zero phase margin–the phase is 180 where the gain is unity.

2. We use a lead compensator, because we need a zero closer to the origin than the pole in order to increase gain and phase margin. A smaller zero than pole will lift part of the bode plot up, while a smaller pole would push part of it down.

3. system design:

   Transfer function for a lead compensator: $G_c = \frac{s+z}{s+p}$ with $z < p$.

   Full transfer function: $H = K \cdot G_c \cdot G = K\left(\frac{s+z}{s+p}\right)\left(\frac{aK_aK_i}{m(s^2 - \frac{K_x}{m})}\right)$

   Setting $s = 0$, we get:

   $(DCgain) = K\frac{z}{p}aK_aK_i$.

   Requiring that this be 2 A/m results in:

   $\boxed{K = 0.2375 \cdot \frac{p}{z}}$

   Now we try values of p and z until we get the desired phase margin and DC gain. With $p = 300$ and $z = 20$, we get a phase margin of 60.265 and a DC gain of 2.0059 as is demonstrated by the code in Listing 4

   K becomes 3.5625

4. Circuit values:

$$\begin{aligned} H(s) = \frac{Y_2(s)}{Y_1(s)} &= \frac{-(10k\Omega)}{R_1} \cdot \frac{(R_1 + R_2)Cs + 1}{R_2Cs + 1} \\ &= \frac{s+z}{s+p} 0.2375 \cdot p/z \end{aligned}$$

$\frac{(10k\Omega)}{R_1} \cdot \frac{(R_1+R_2)Cs+1}{R_2Cs+1} = \frac{(s+20)\cdot 3.5625}{(s+300)}$

$\frac{(10k\Omega)}{R_1} \frac{(R_1+R_2)C}{R_2C} \frac{s+\frac{1}{(R_1+R_2)C}}{s+\frac{1}{R_2C}} = \frac{(s+20)\cdot 3.5625}{(s+300)}$

   System of equations:

$$\begin{aligned} 3.5625 &= 10000(R_1 + R_2)/(R_1R_2) \\ 1/((R_1 + R_2)C) &= 20 \\ 1/(R_2C) &= 300 \end{aligned}$$

   Solving for component values:

$$\begin{aligned} R_1 &= 42.1k\Omega \\ R_2 &= 3.007k\Omega \\ C &= 1.11\mu F \end{aligned}$$

5 Lab Part B

5.1 Final Component Values:

Our general procedure to get the ball to levitate was to hold the ball near the coil and feel for where the equilibrium spot would be. We decided not to use the scale to determine when the ball was weightless because the magnetic field altered the scale readings too much for it to be reliable. Our initial value of Kc that we calculated in the prelab was 0.2375. When we tried this out, the ball would almost levitate but the field was too weak to prevent the ball from dropping. When we checked to see how much covering the light sensor affected the output voltage, we noticed that the voltage only changed by 100 millivolts or so, which was not a strong enough signal to keep the ball levitated. Thus, we decided to raise our Kc to 1.0 with the reasoning that the higher gain would increase the strength of the magnet and better hold the ball in place. This increase in Kc meant that R1 and R2 values decreased and C value increased. With this increase in gain, we were able to achieve a controller that was robust to disturbances and could keep the ball afloat for a long period of time.

Gain was increased to $K_c = 1.0$. In order to prevent the phase margin from decreasing too much, we also moved our controller pole to $p = 400$. We re-solved the system of equations given in the previous section to find the following component values:

$$R_1 = 20k\Omega$$
$$R_2 = 1.4k\Omega$$
$$C = 2.3\mu F$$

We used Matlab to check the new phase margin and DC gain of our system. We find a phase margin of 52.47 degrees and a DC gain of 7.98.

5.2 YouTube Levitation Video

6 Appendix

6.1 Lab A

Listing 1: used to generate plot for linearizing $h(x)$

```
%Lab 5a
%5.2.3: Linearizing h(x)
x=[-2,-1,0,1,2];
h=[2.38,2.7,3.49,4.43,5.6];
plot(x,h);
title('Voltage as a function of Distance from Equilibrium');
xlabel('Distance from Equilibrium (mm)');
ylabel('Voltage (V)');
p=polyfit(x,h,1) %output is [0.817 3.72]
```

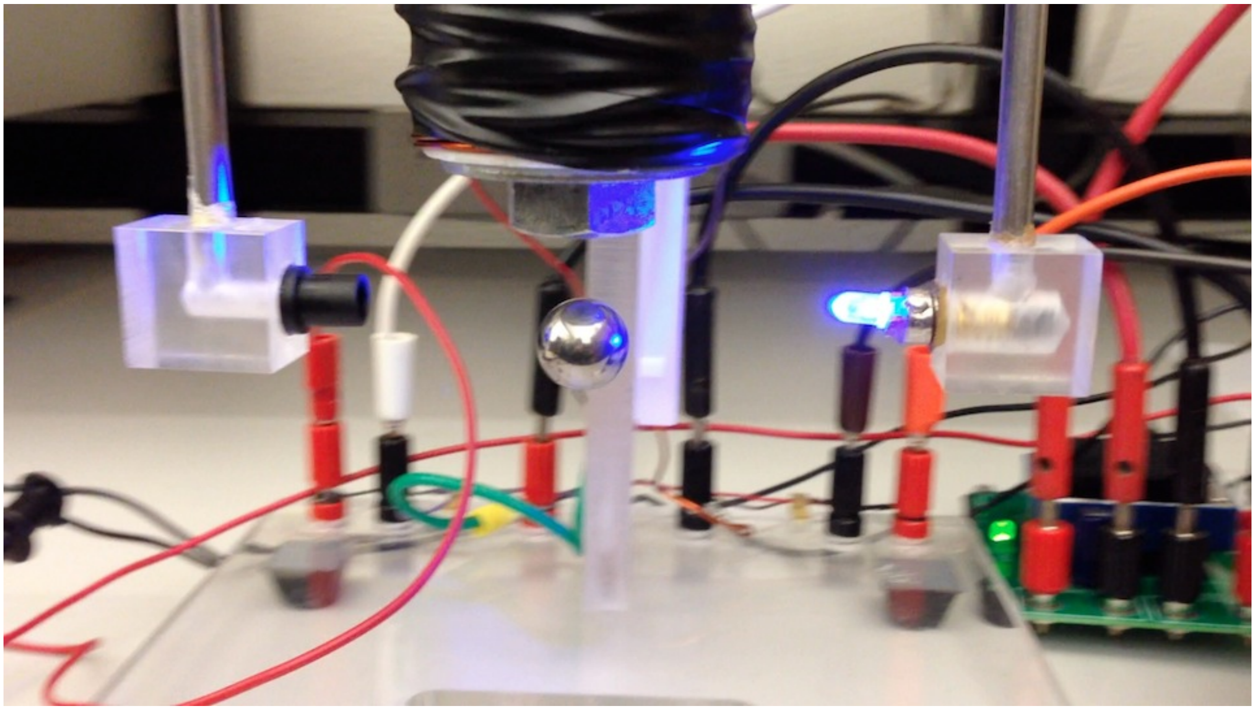

Figure 9: Screenshot of the levitation in action. See `http://www.youtube.com/watch?v=hf-9Wg1Nqxw` for the full video

Listing 2: below

```
weight=[0.0 0.3 1.5 4.0 6.5];
current=[-2.7973 -2.6512 -2.5493 -2.0687 -1.7467];
plot(current,weight);
title('Apparent Weight vs. Current');
xlabel('Current (A)');
ylabel('Apparent Weight (g)');
print Lab5a_2_4.eps
p = polyfit(current,weight,1)
display p[0]
```

Listing 3: below

```

x=[0 -1 -2 -3 -4 -5 -6 -7];
weight=[0.3 1.7 3.7 5.3 6.5 7.5 8.6 9.6];
plot(x,weight);
title('Weight vs Displacement');
xlabel('displacement (mm)');
ylabel('weight (g)');
print Lab5a_2_5.eps
p = polyfit(x,weight,1)
```

6.2 Prelab B

Listing 4: code for prelab b

```
%%%%  PRELAB 5B  %%%%

a = 865.0; %V/m
Ki = 0.0610; %N/A
Kx = 13.089958; %N/m
m=0.0162; %kg
Ka = 2;
plant = tf([a*Ki*Ka],[m 0 -Kx]);

%%% Root Locus Plot
rlocus(plant);
title('Root Locus for G(s)');
plant_cl = plant/(1+plant);

%%% Bode Plot
bode(plant);
title('Frequency Response of G(s)');

%%% Designing the controller
z = 20;
p = 300;
Gc = tf([1 z],[1 p]);
K = .2375*(p/z);
H = K*Gc*plant;
our_dc_gain = dcgain(H)
[Gm,Pm,a,b] = margin(H)
```