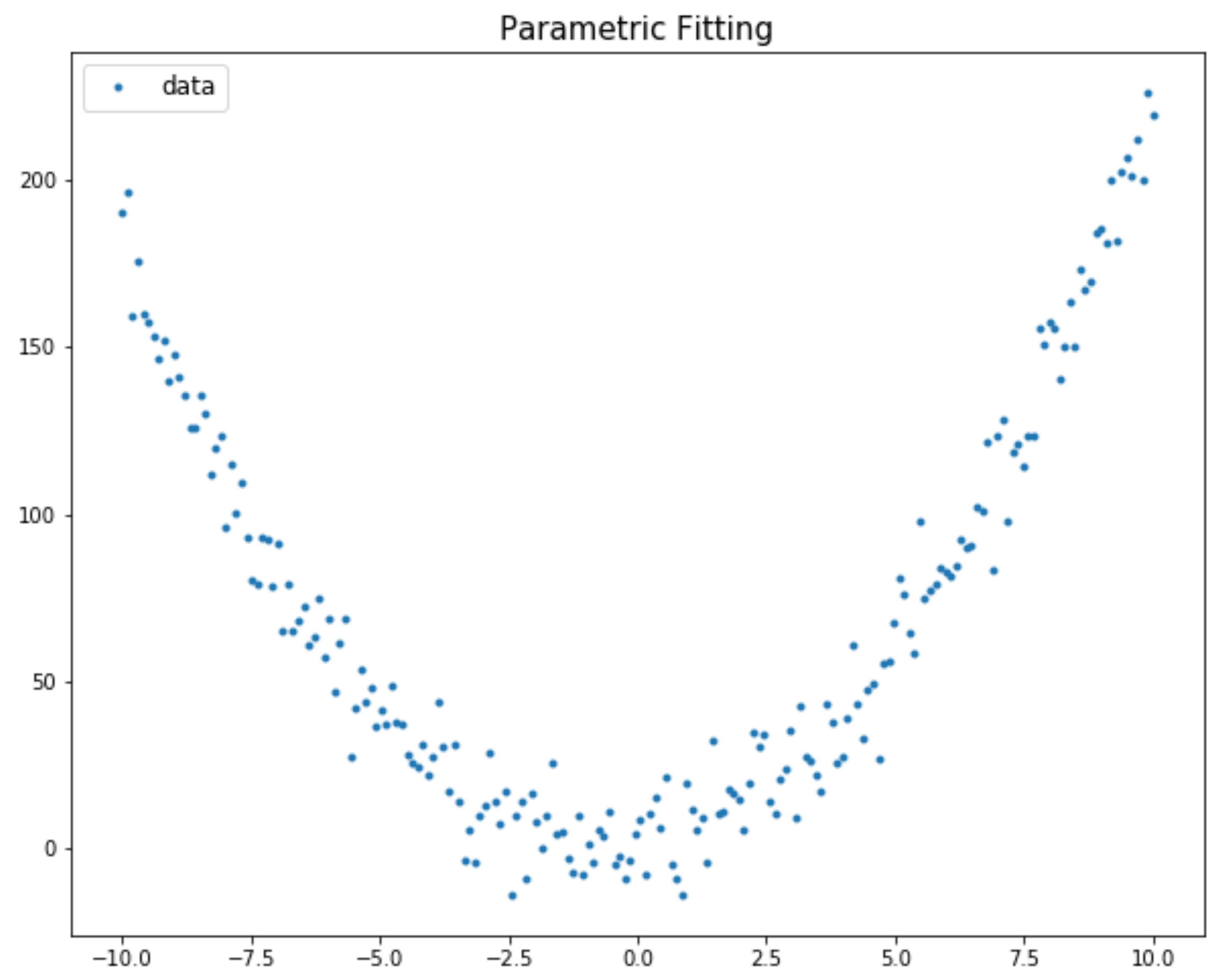
The What, Where, Why, and How of Gaussian Processes (GPs)

By: Ari Silburt

Why GPs?

Standard Fitting to Data: Parametric Modelling/Linear Regression



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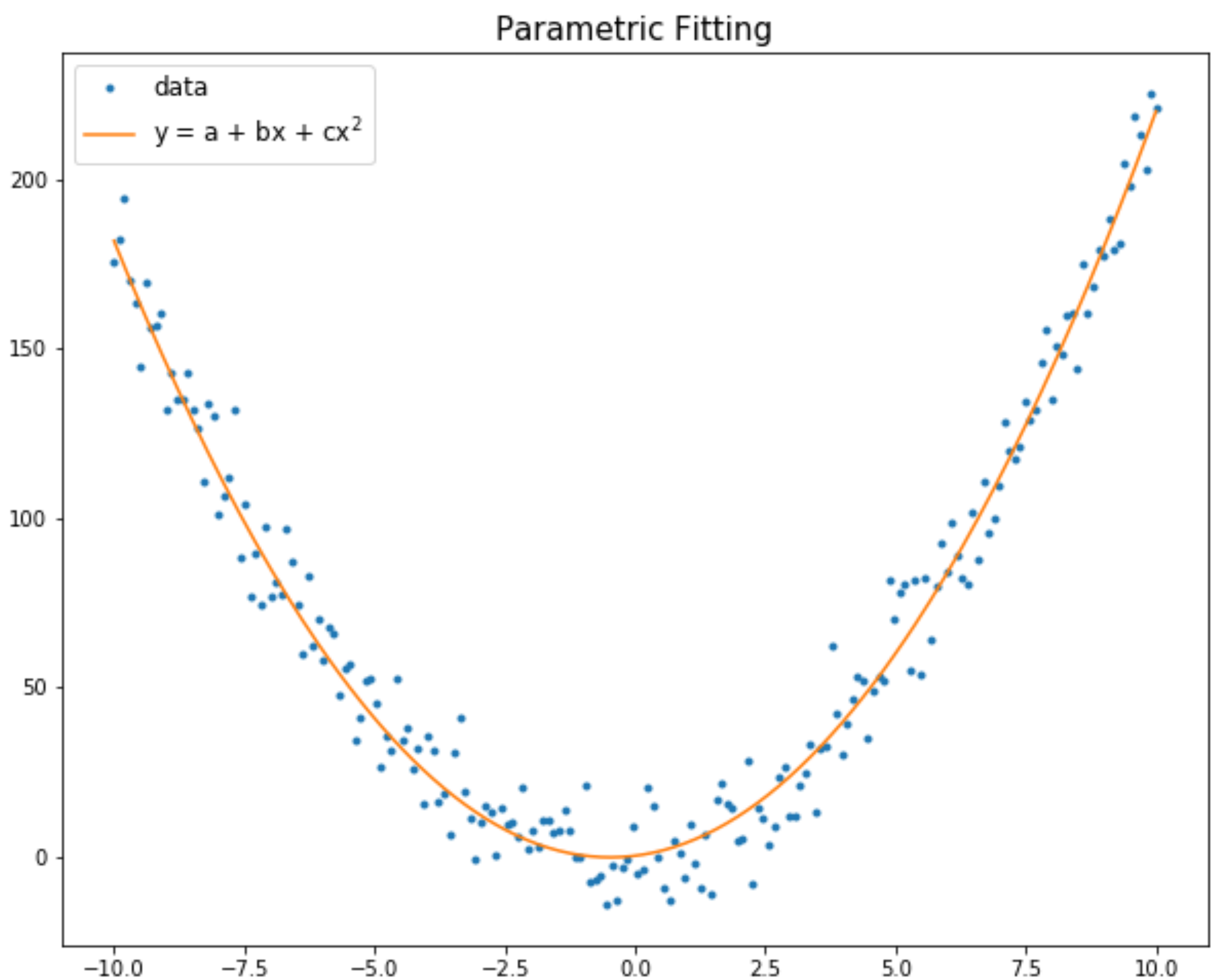

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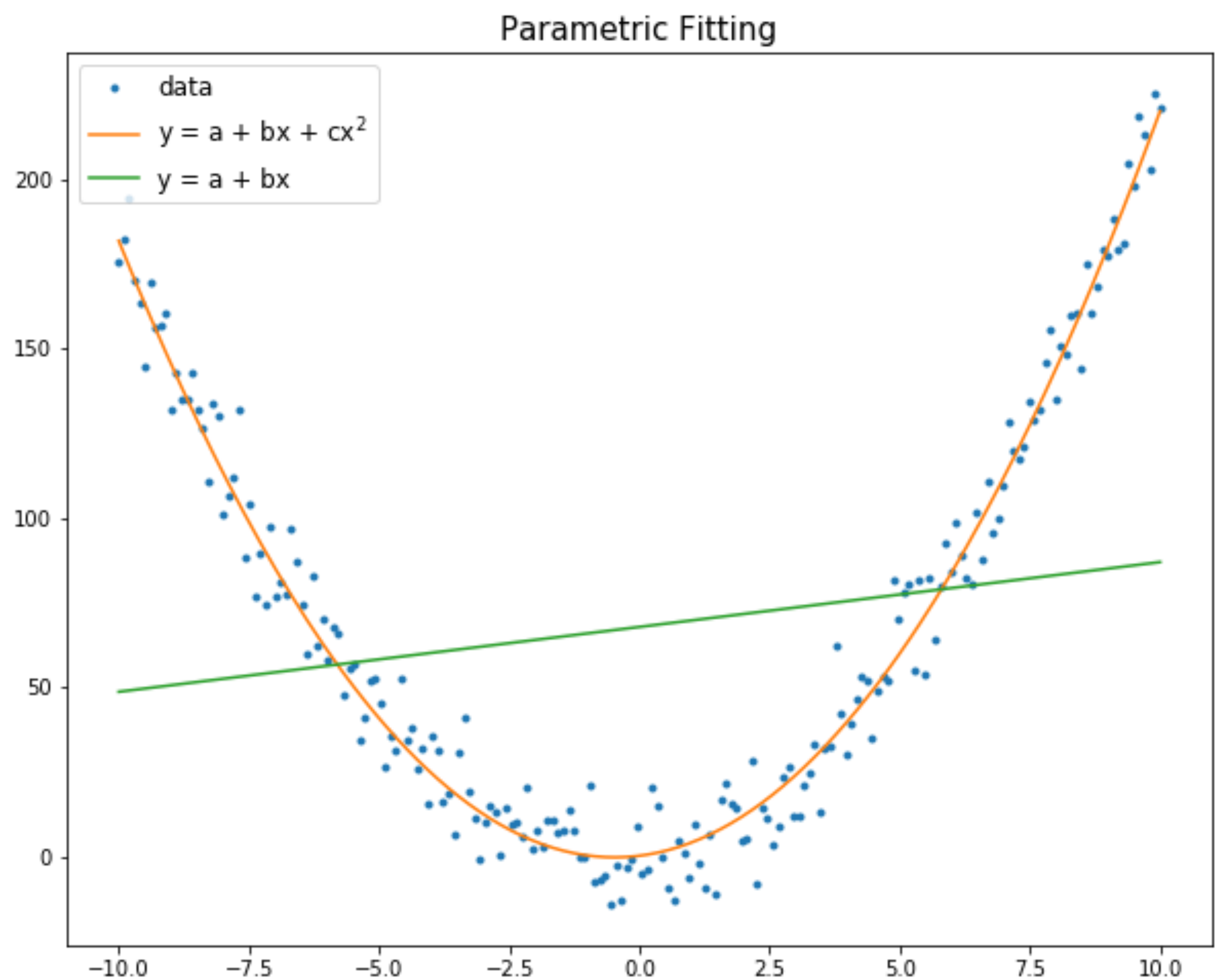
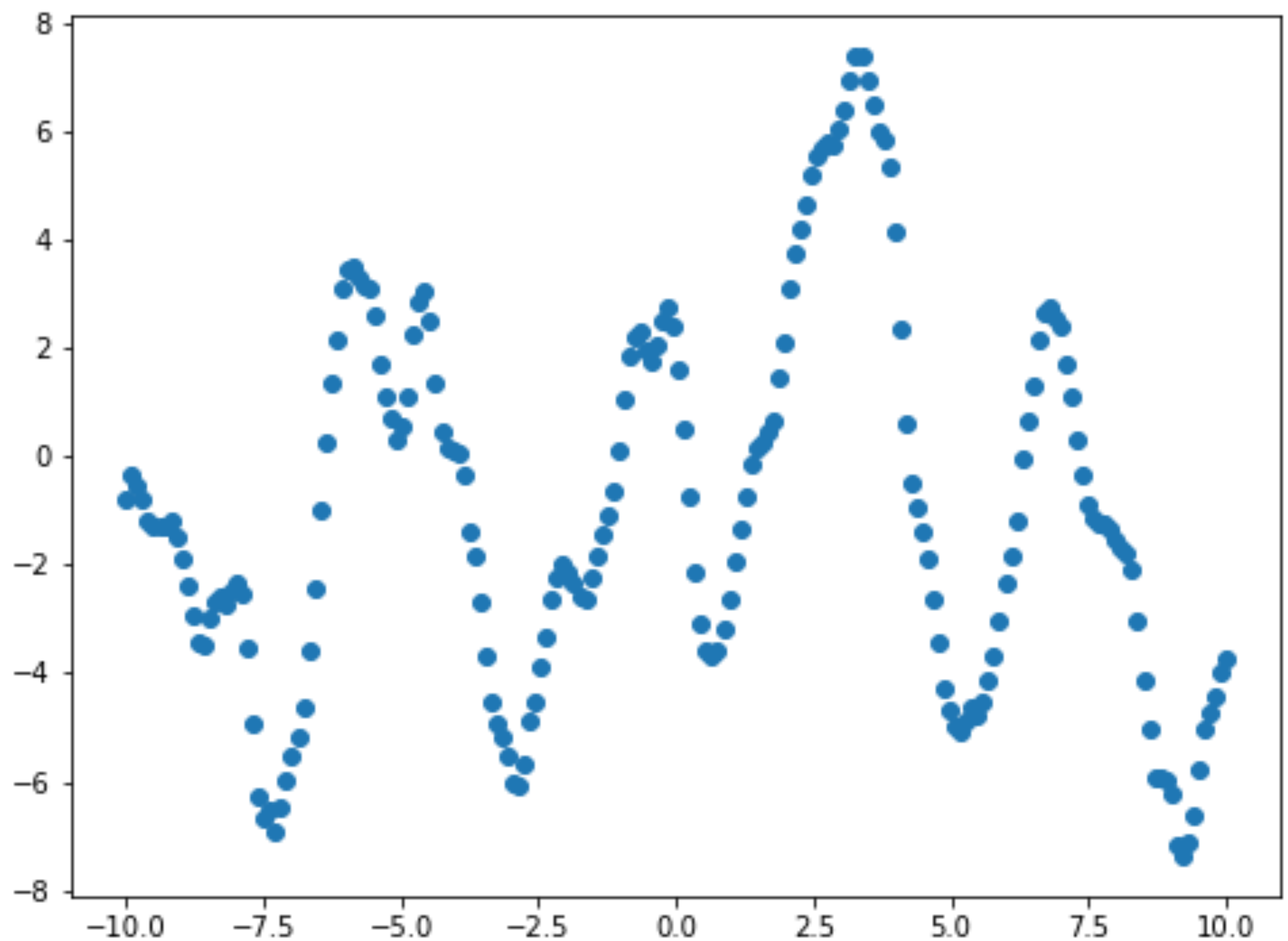


Why GPs?

So… what function do we want to fit this?



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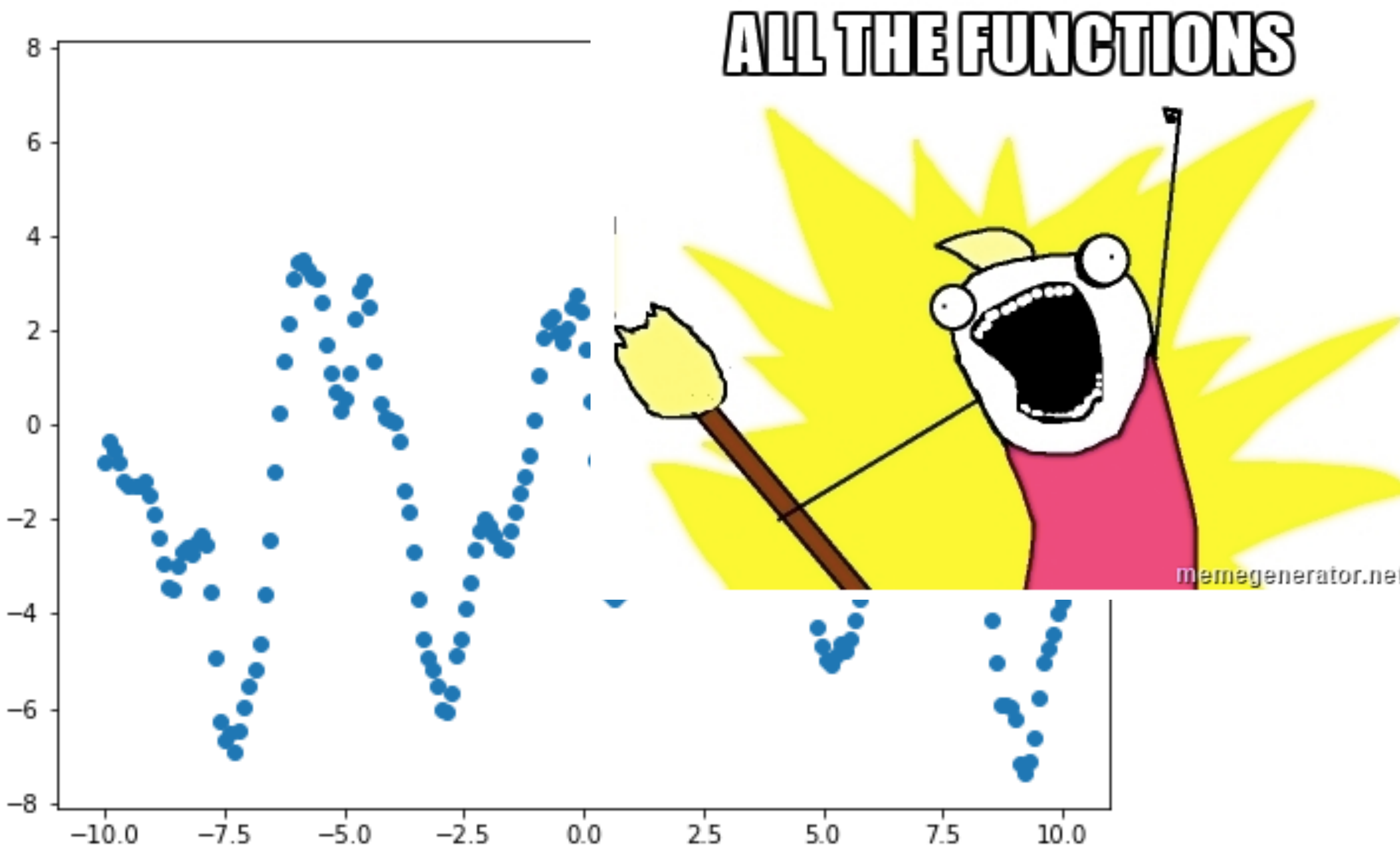
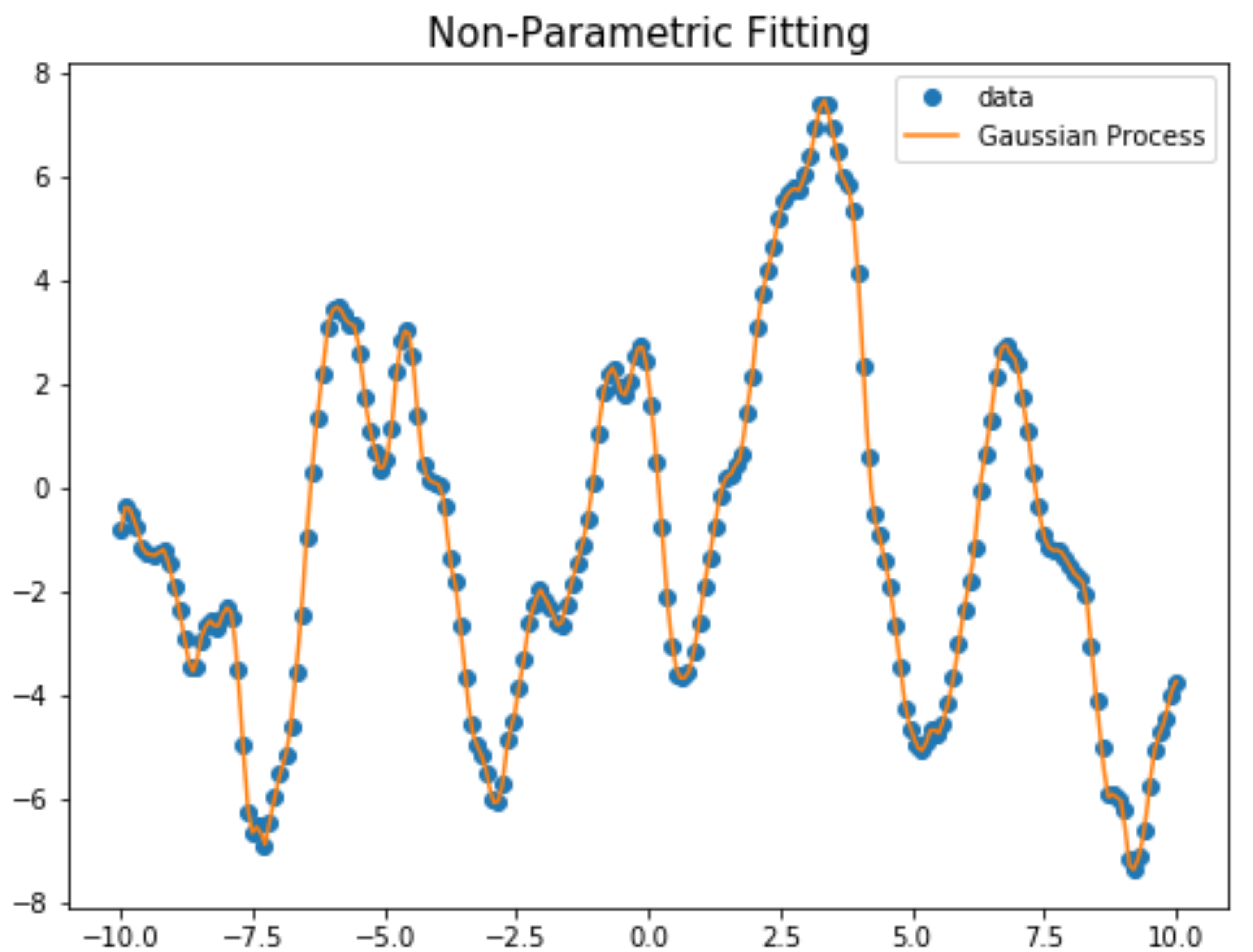


Why GPs?

Nonparametric = infinitely many parameters (not zero parameters)



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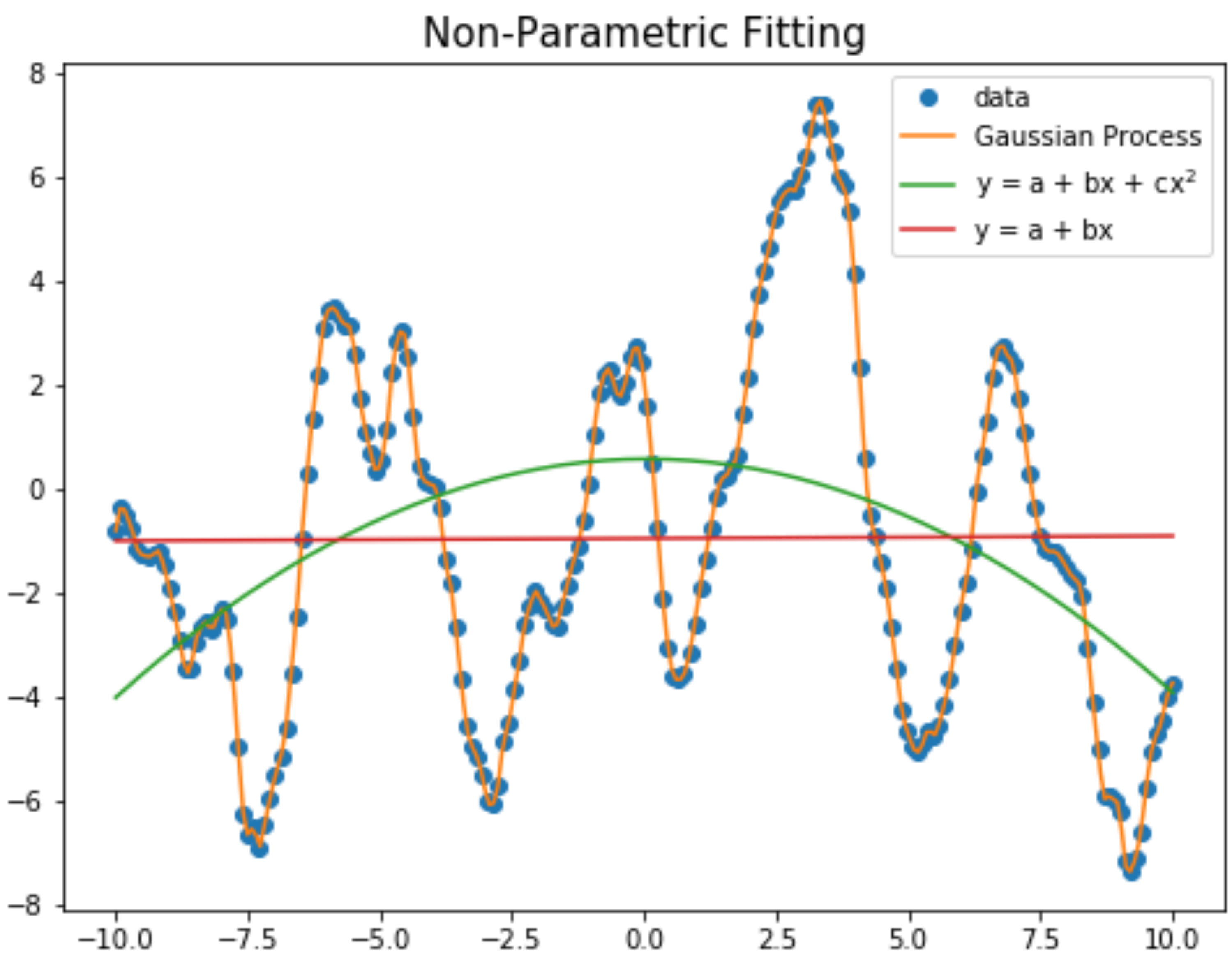

What (are) GPs?

Carl Rasmussen (i.e. GP God):

A Gaussian process is fully specified by its mean function m(x) and covariance function k(x, x'). This is a natural generalization of the Gaussian distribution whose mean and covariance is a vector and matrix, respectively. The Gaussian distribution is over vectors, whereas the Gaussian process is over functions.

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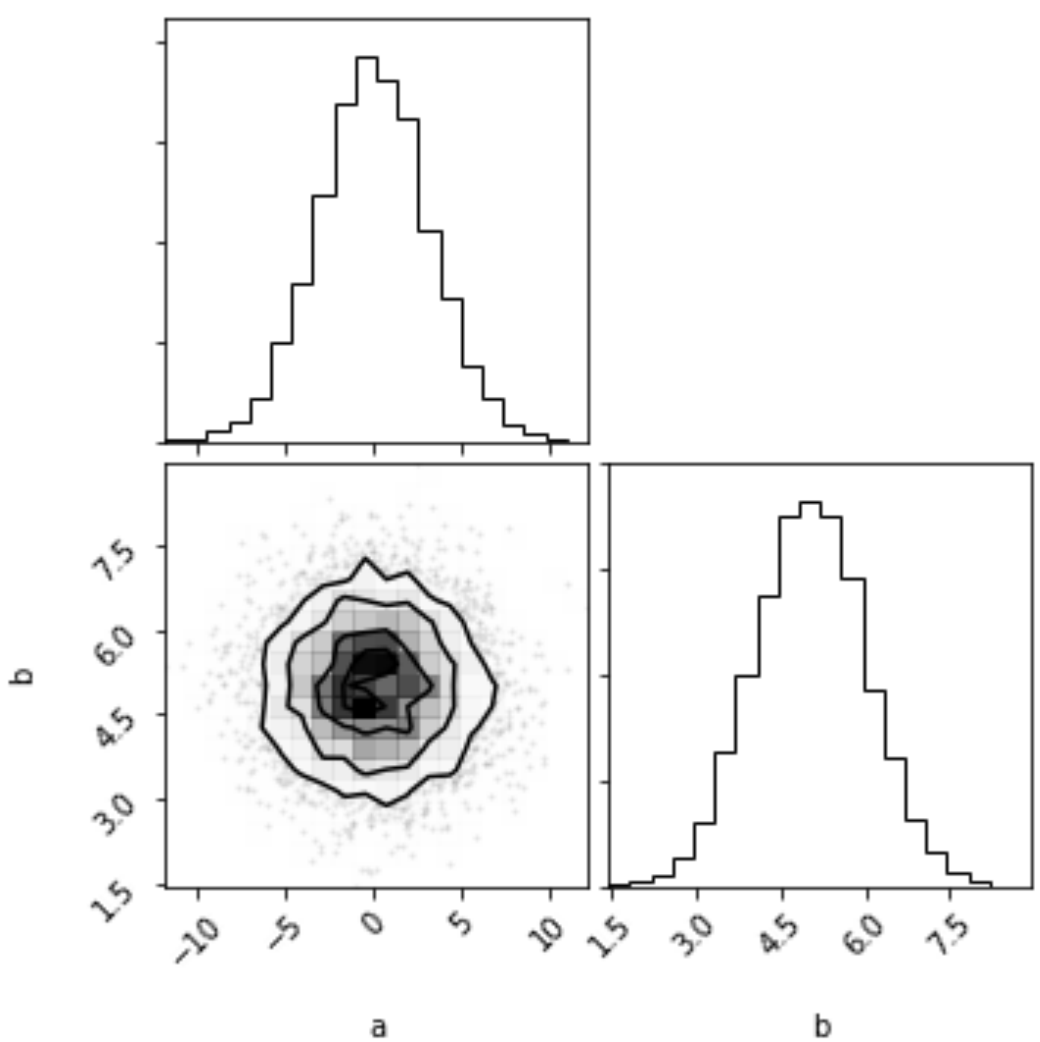

$$\text{mean} = [0, 5]$$

$$\text{cov} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$$

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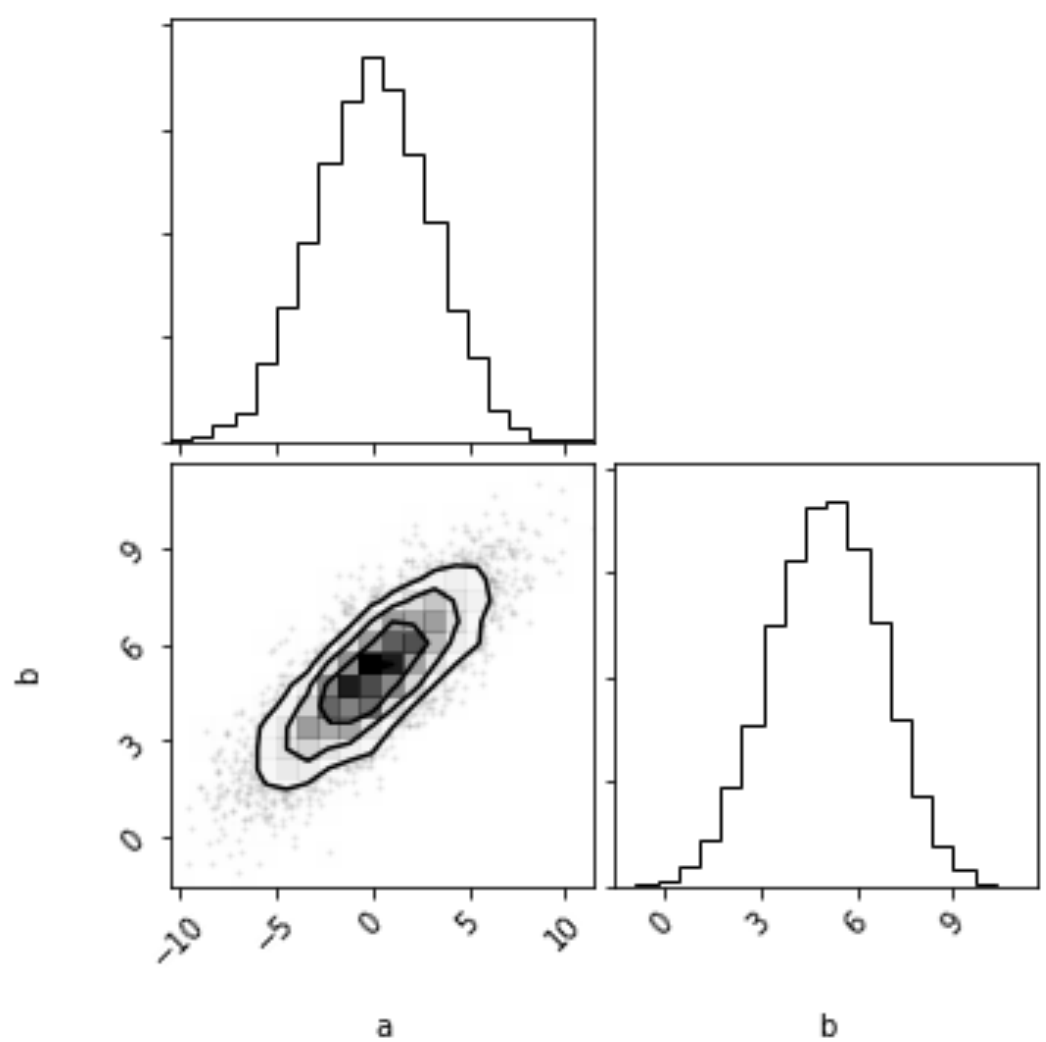

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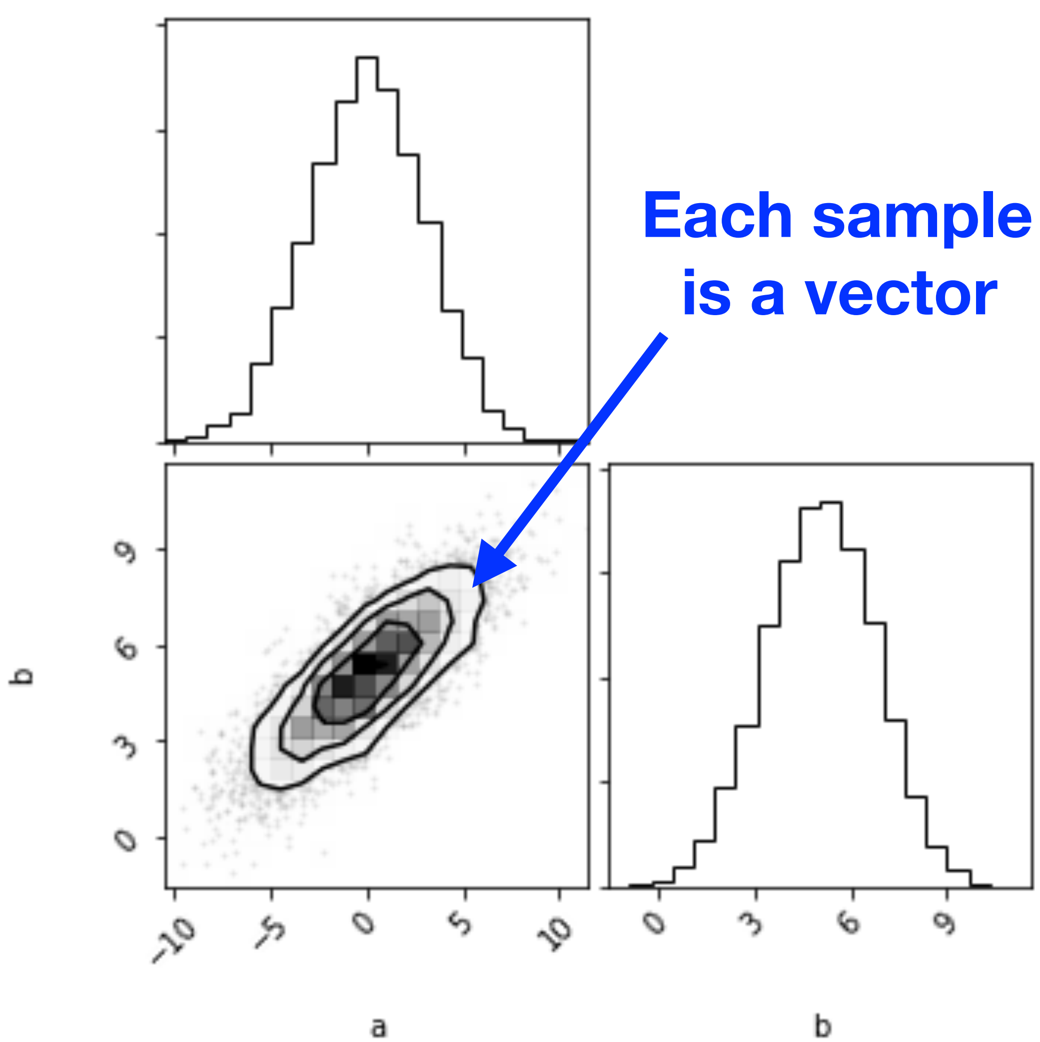

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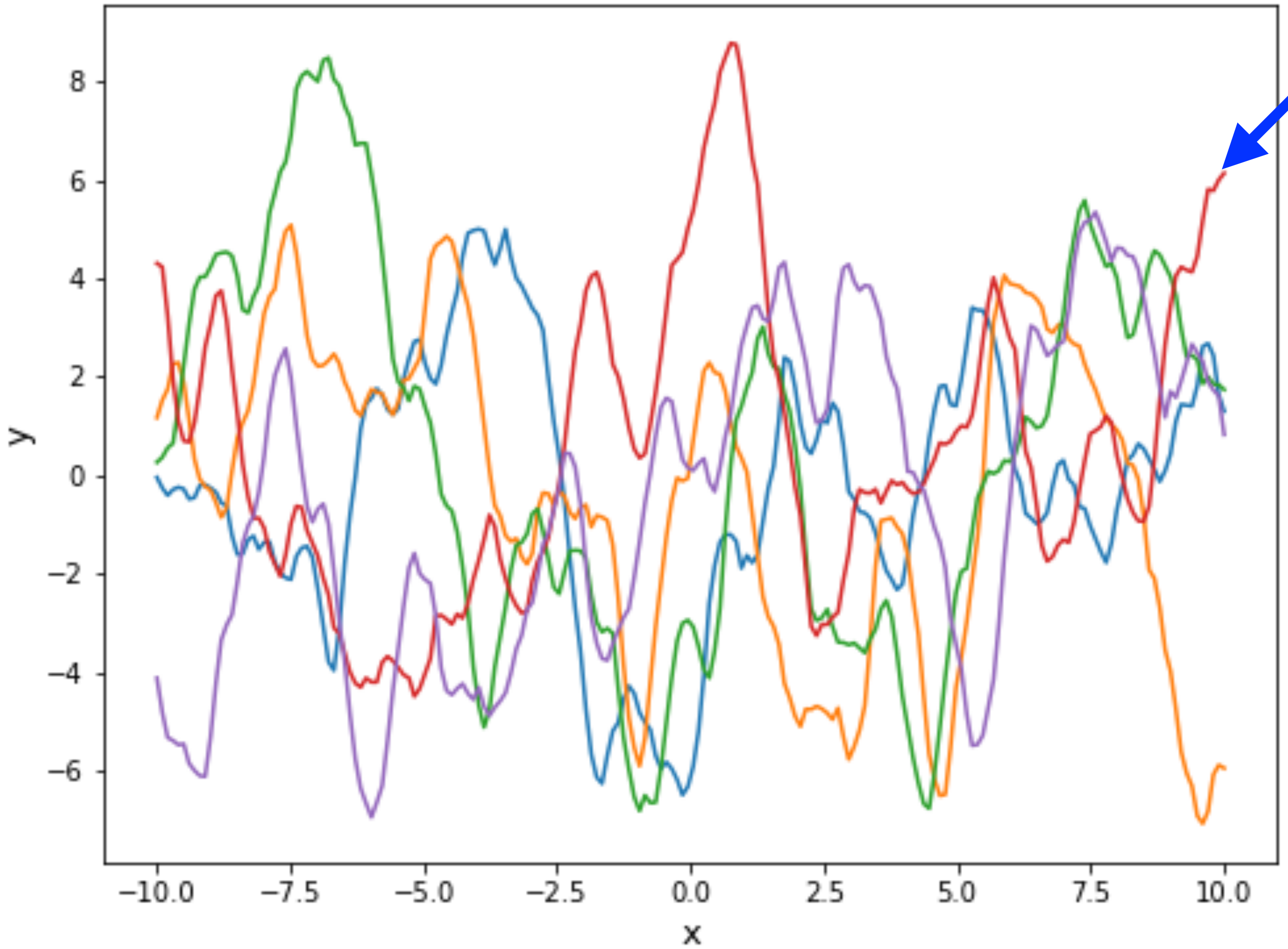

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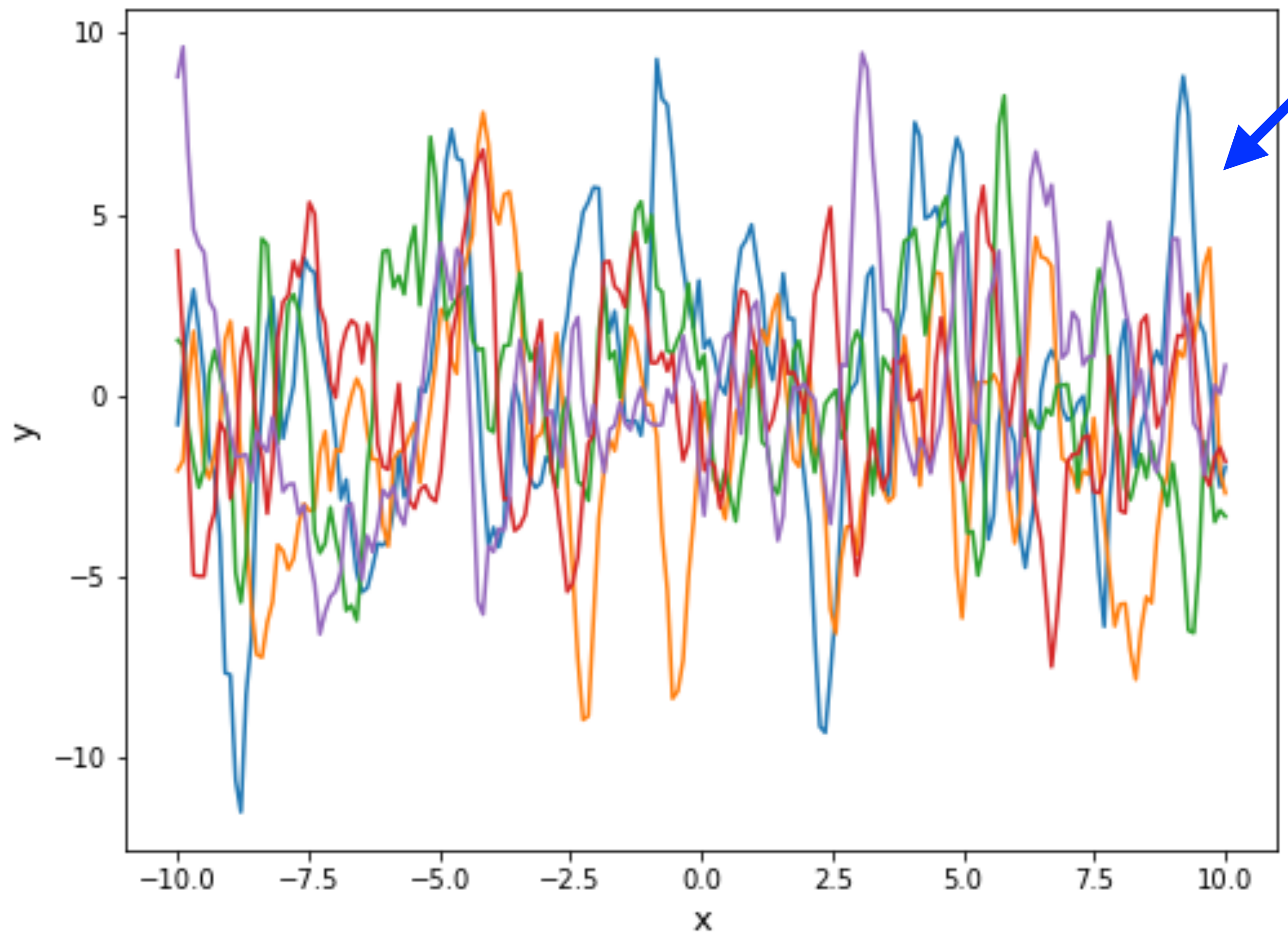

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How (to use) GPs

GPs in a Bayesian Framework:

- Prior information about model.

- See some data.

- Update model by conditioning on data, arriving at a posterior distribution.

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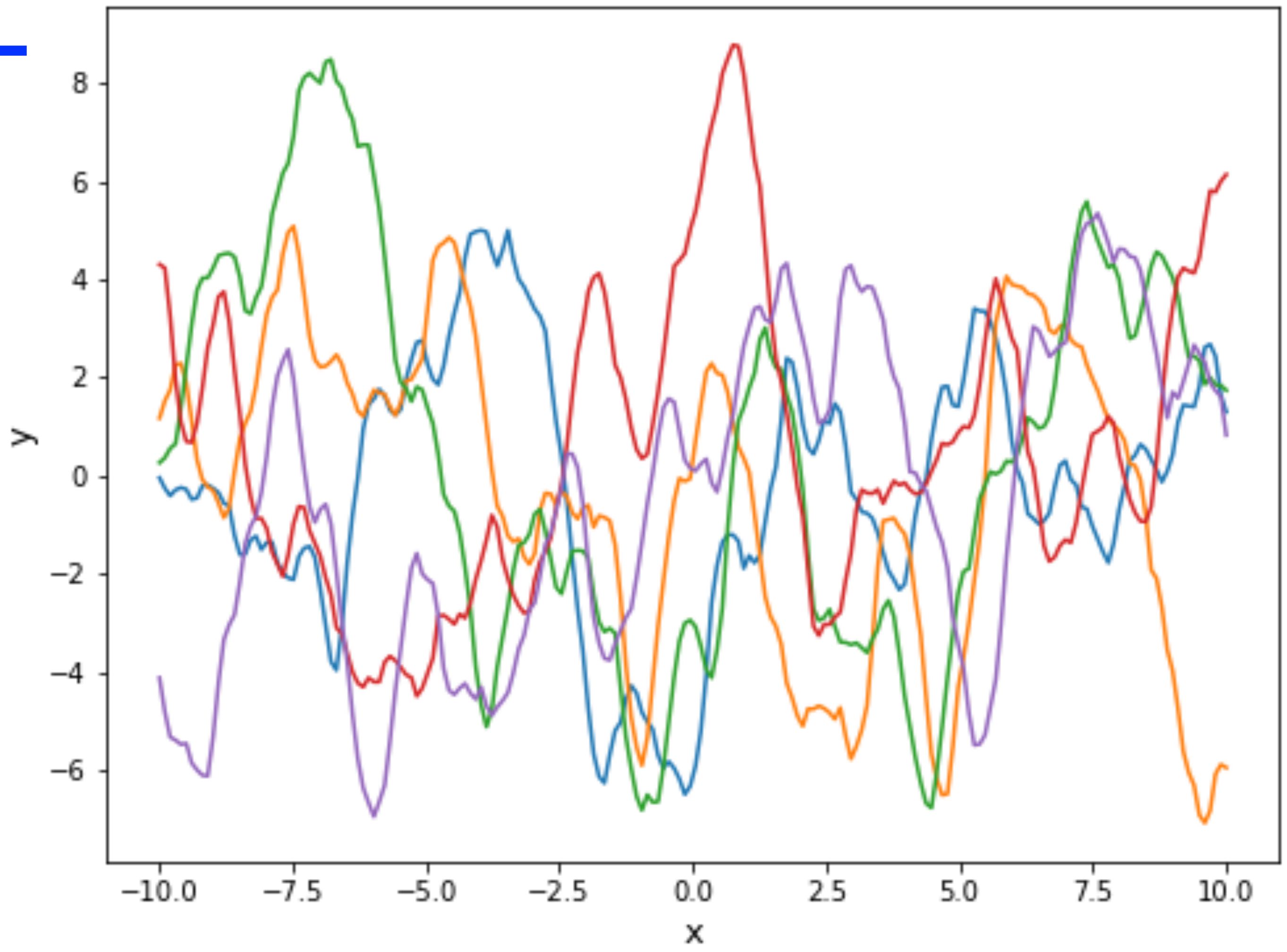

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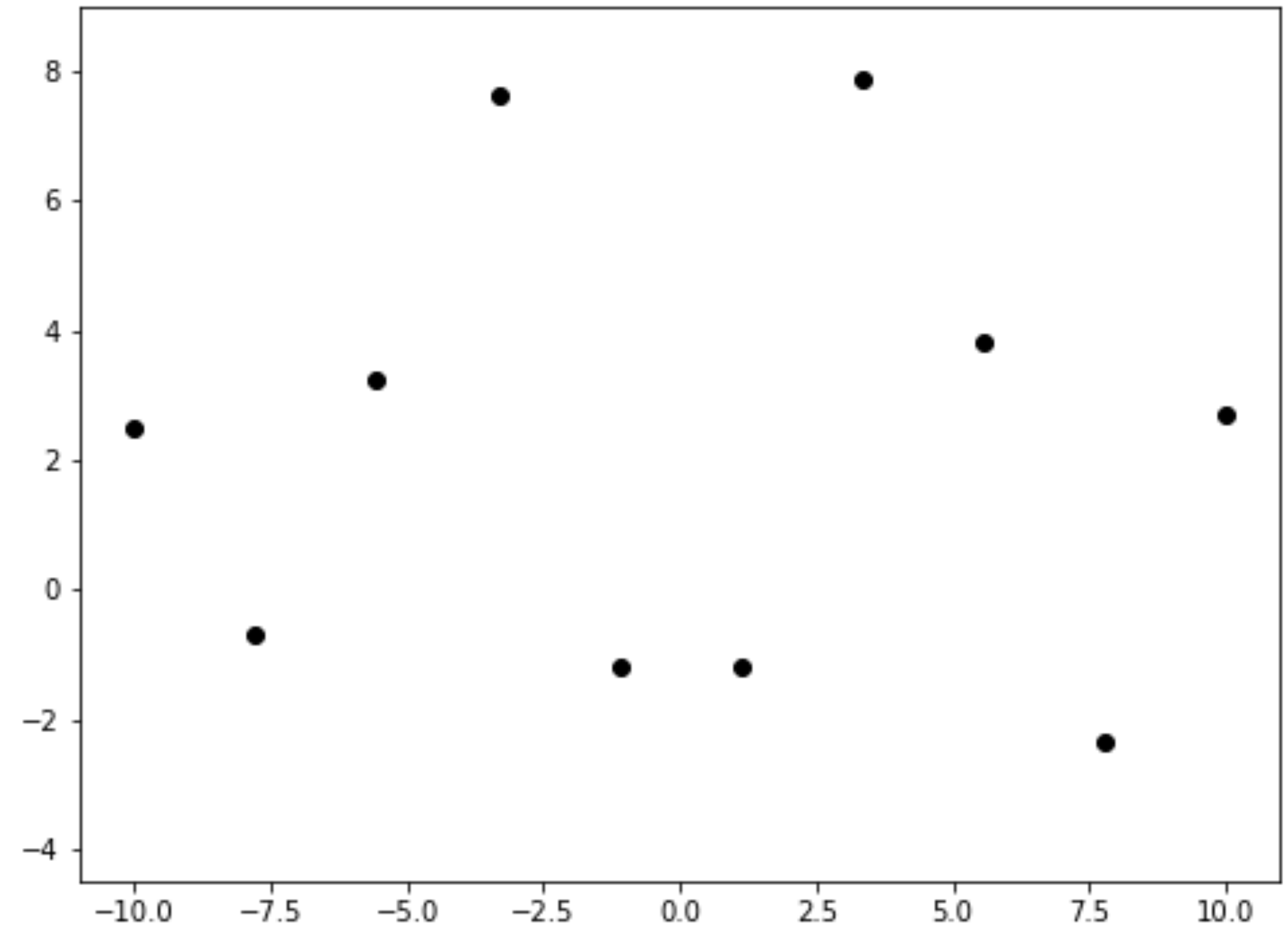

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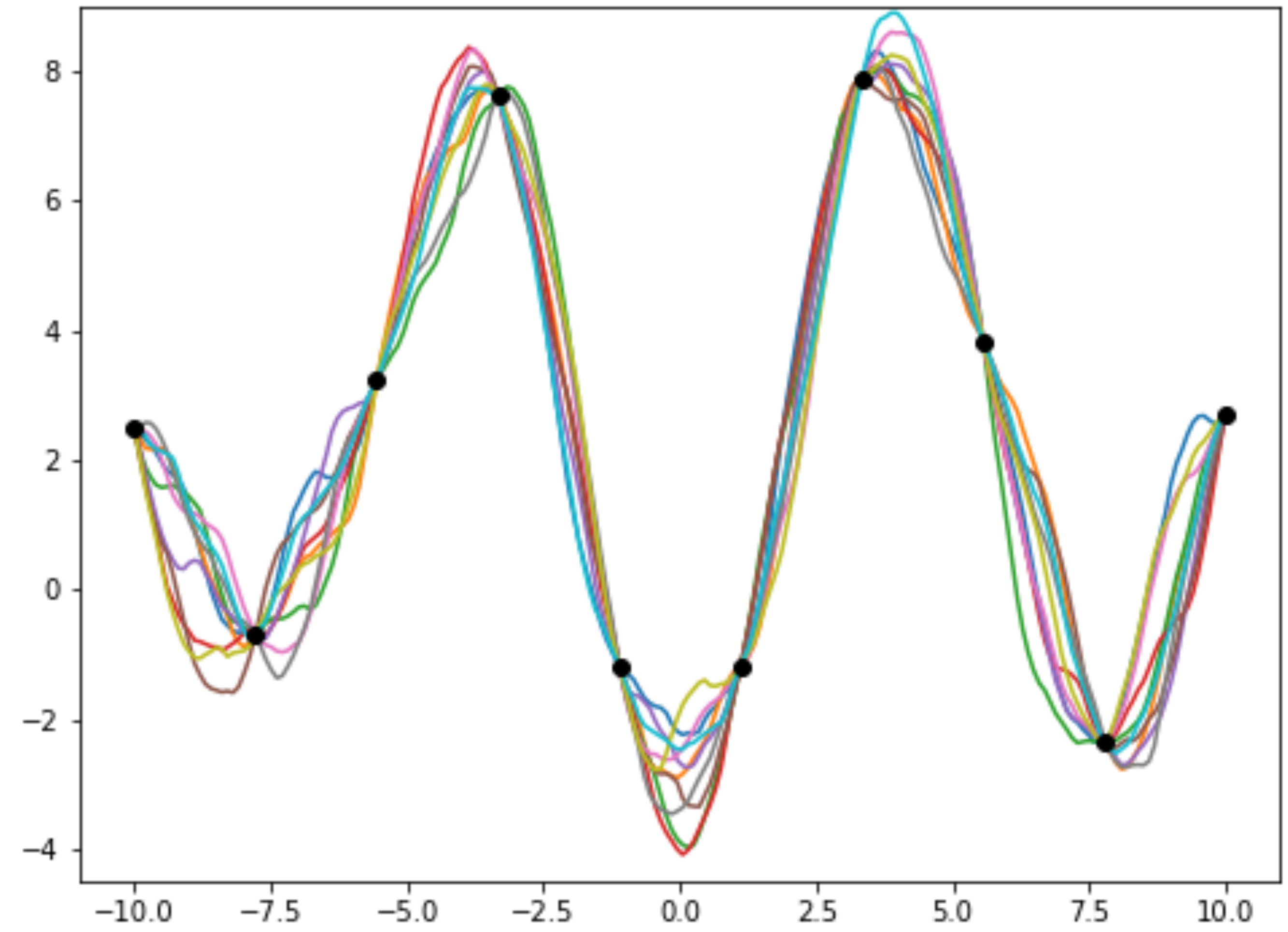

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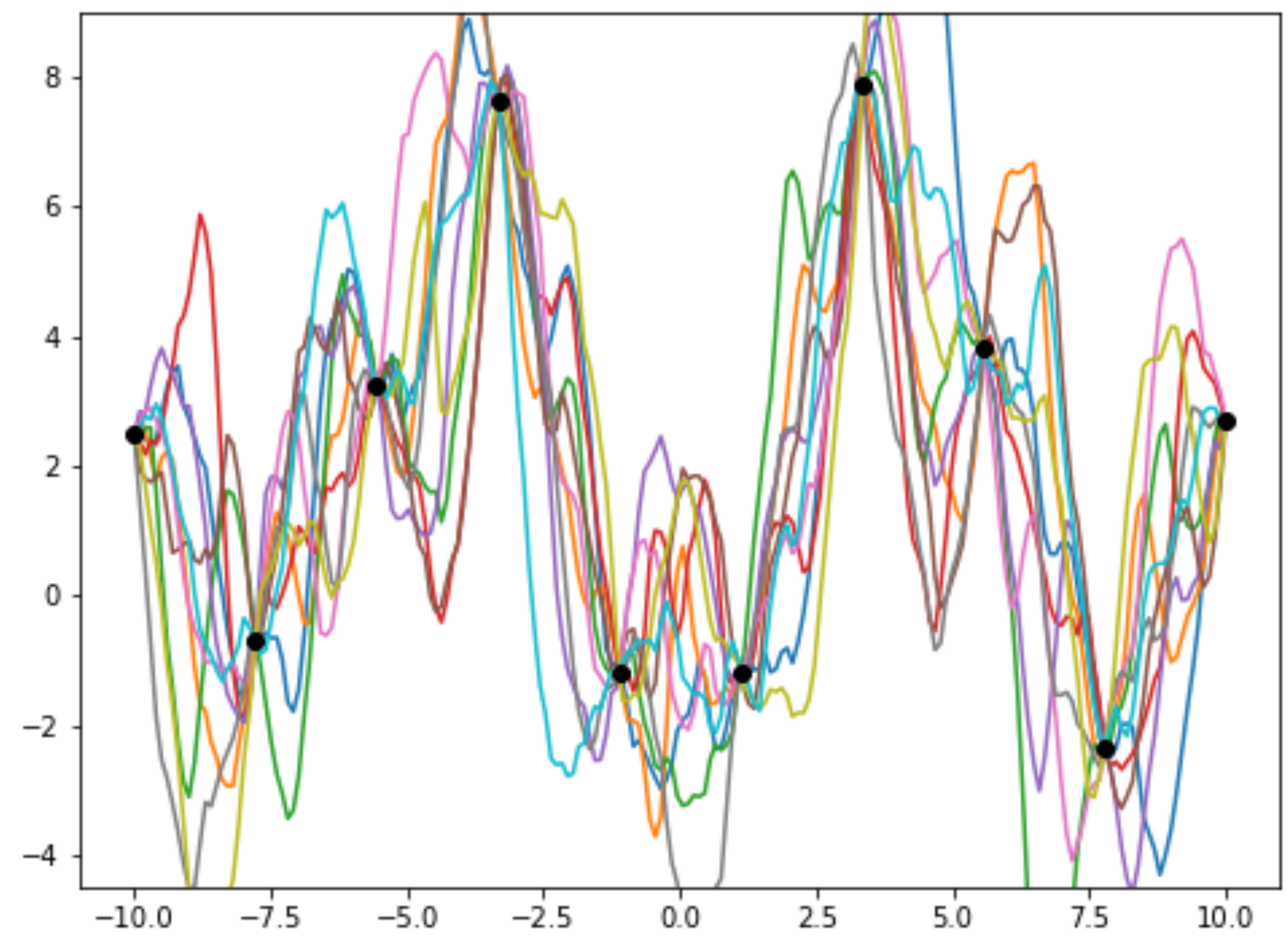

What are the optimal parameters?

How (to use) GPs

Maximize the Marginal Likelihood:

$$L = \log p(y|x, \theta) = -\frac{1}{2}\log|\Sigma| - \frac{1}{2}(y-\mu)^\top \Sigma^{-1}(y-\mu) - \frac{n}{2}\log(2\pi)$$

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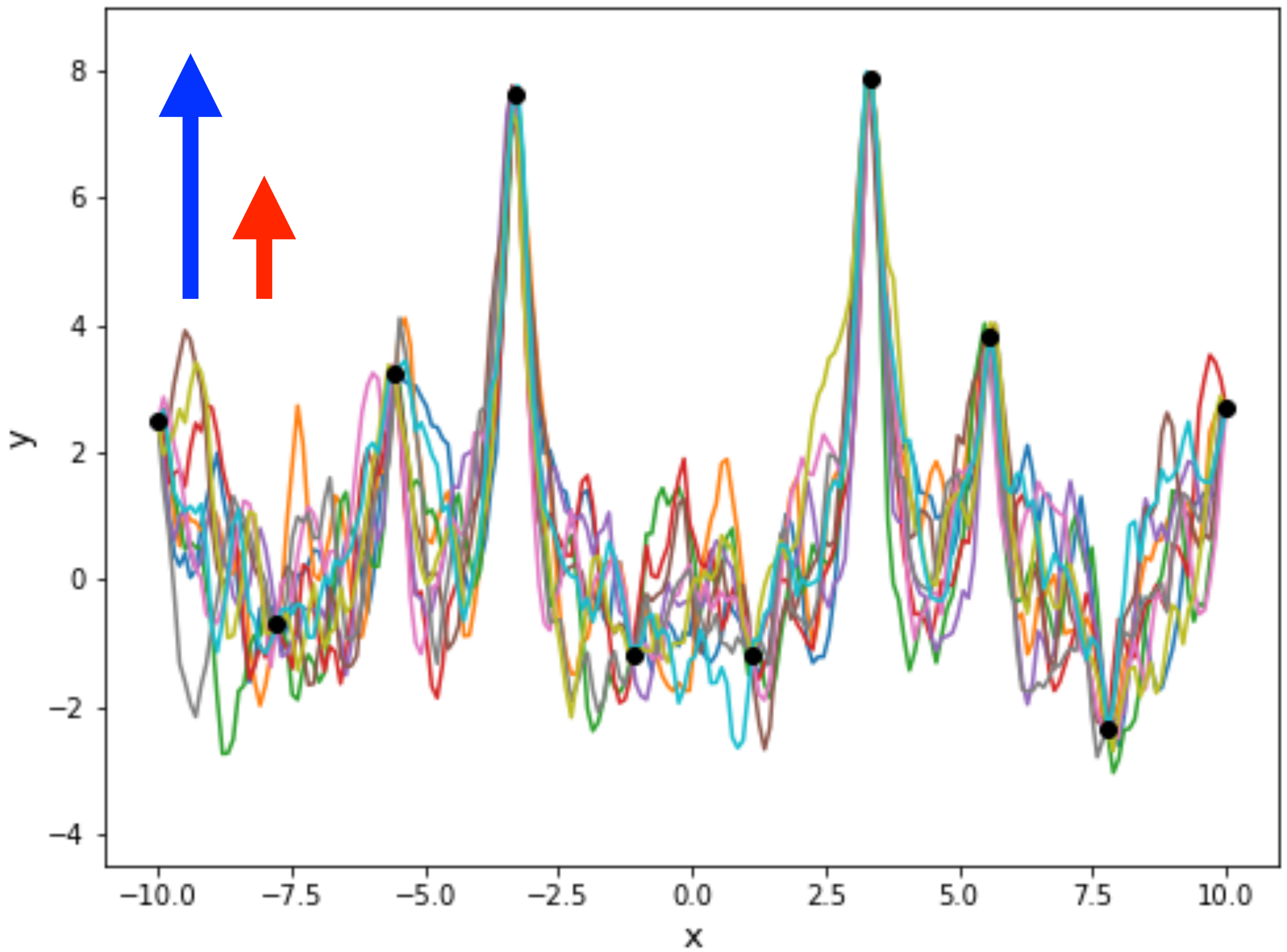
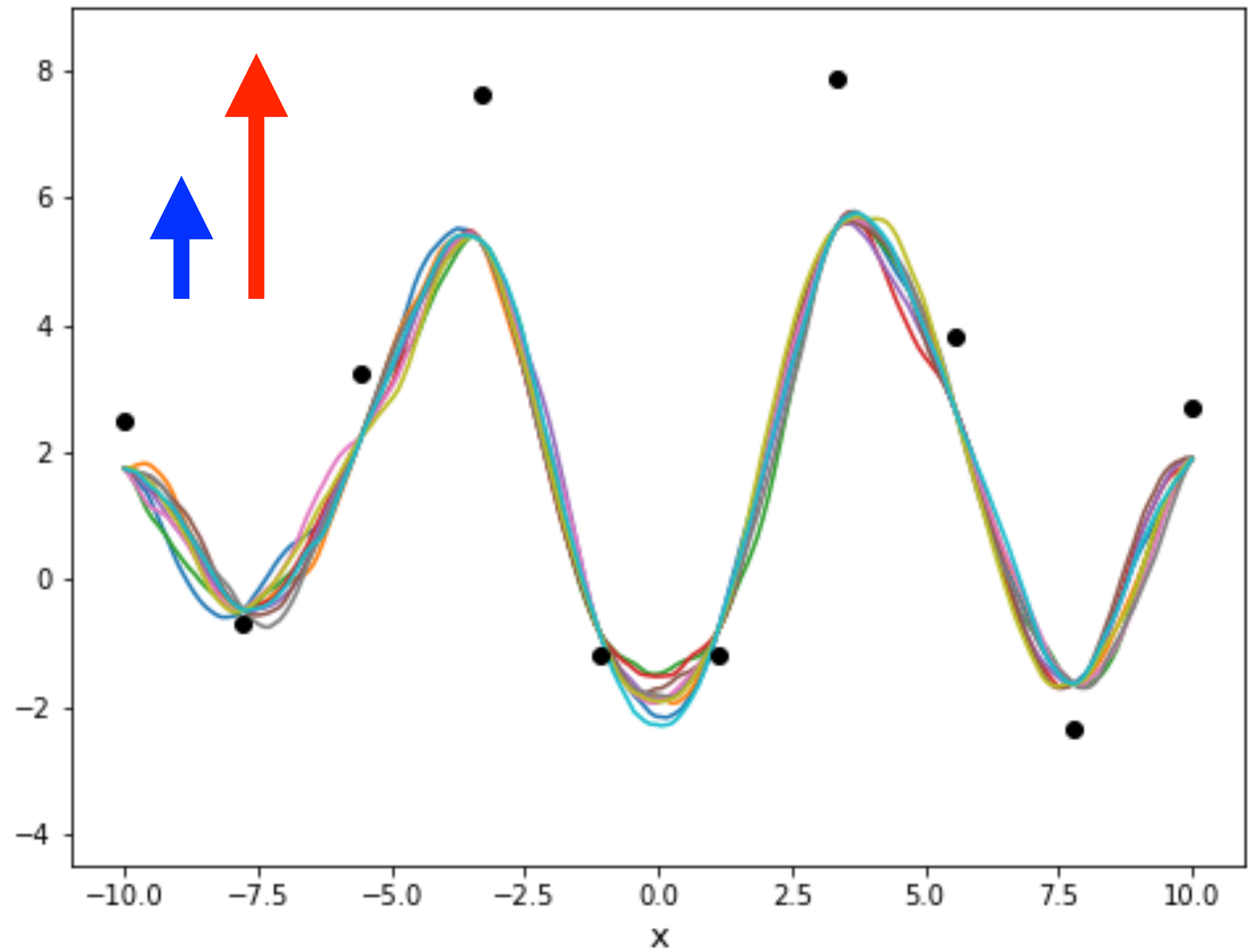




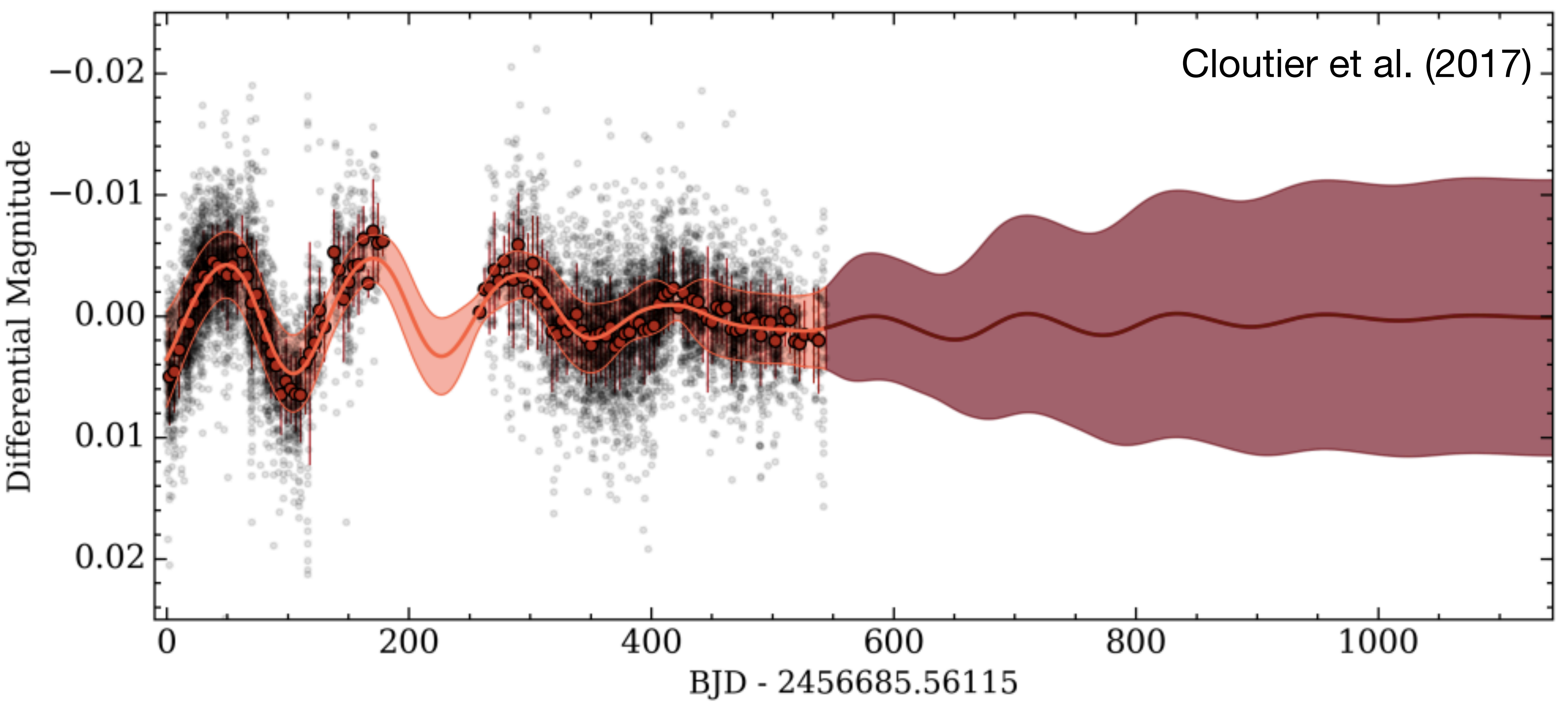
How (to use) GPs

Daniel Foreman-Mackey's live demo

http://dfm.io/gp.js/

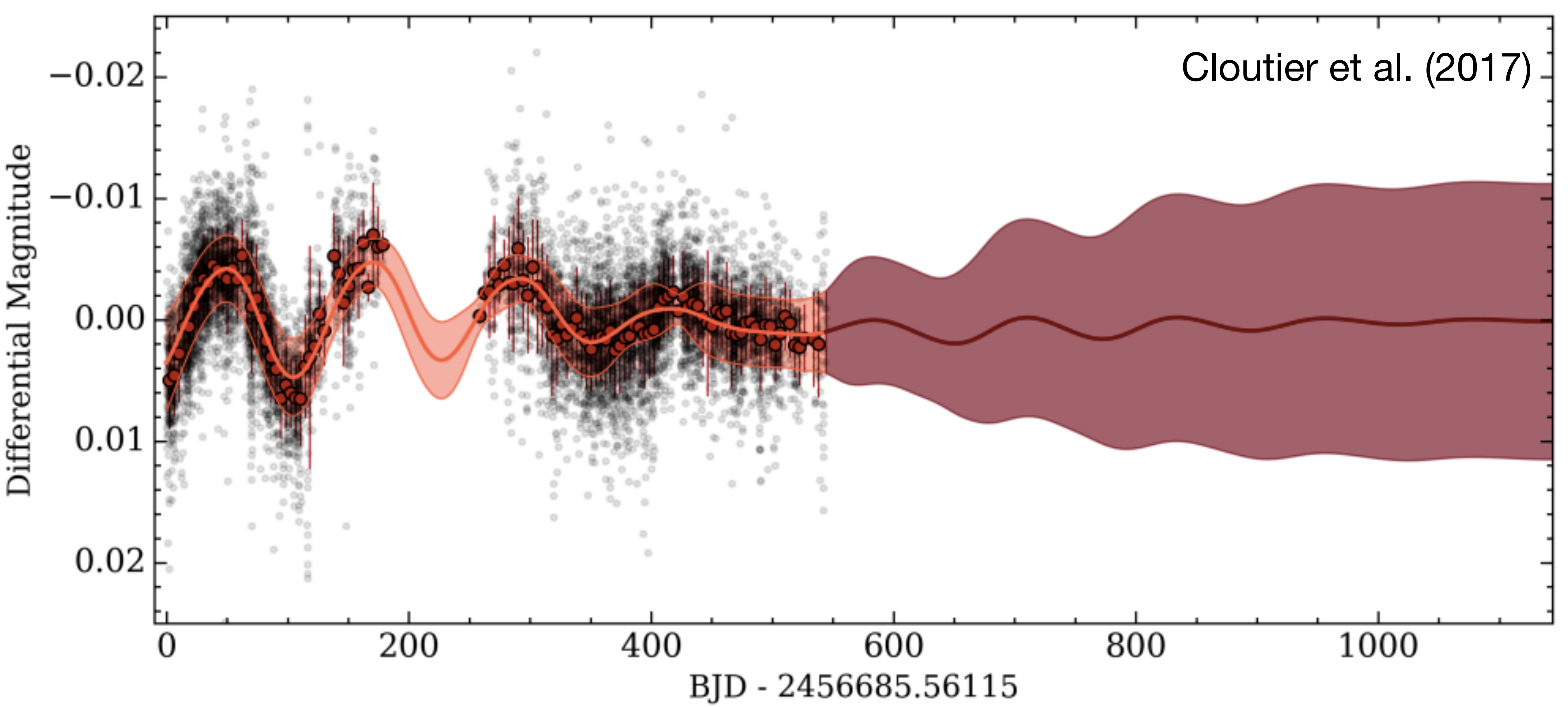
Where (are) GPs (used)

"Parametric models of stellar variability due to active regions feature degenerate model parameters including the sizes and spatial distribution of active regions thus making it difficult to accurately constrain model parameters of active regions."



Where (are) GPs (used)

$$\text{cov} = a^2 \exp\left[-\frac{(x-x')^2}{2\lambda^2} - \Gamma^2 \sin^2\left(\frac{\pi|x-x'|}{P_{rot}}\right)\right]$$



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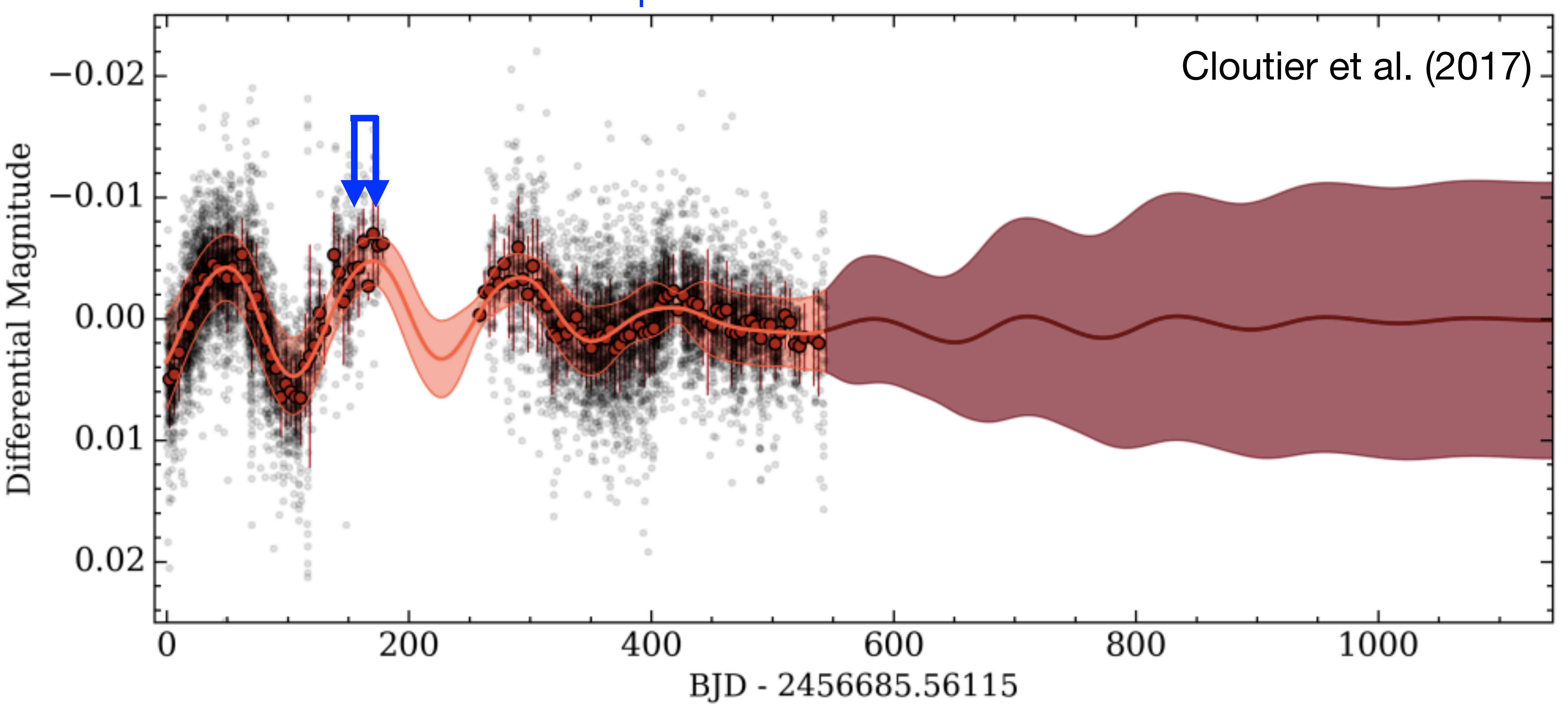
Nearby points more correlated than far points



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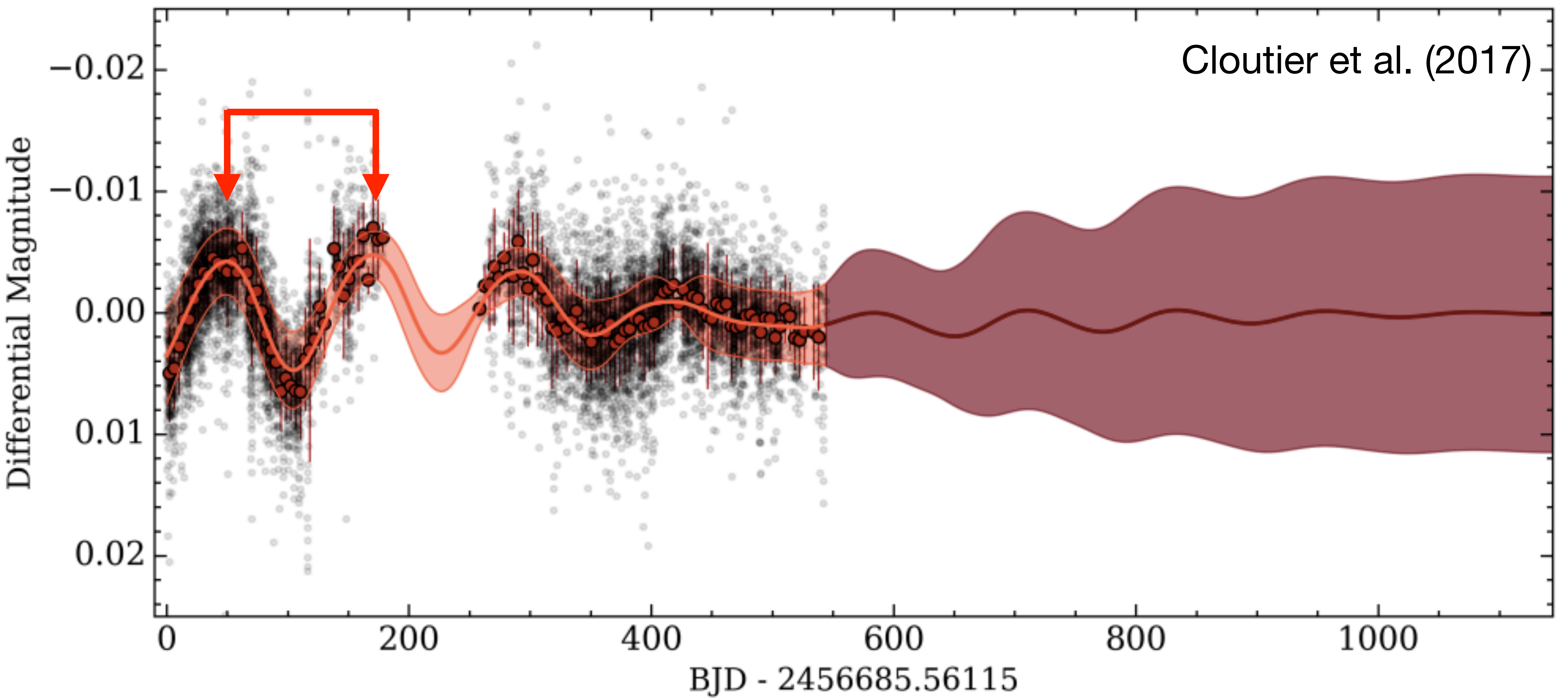
Far terms correlated according to some characteristic period



Where (can I learn more about) GPs

Learning about GPs:

- Rasmussen & Williams - http://www.gaussianprocess.org/gpml/chapters/
- Gaussian Processes for Timeseries Modelling - http://www.robots.ox.ac.uk/~sjrob/Pubs/philTransA_2012.pdf
- Daniel Foreman-Mackey's Python code George - http://dfm.io/george/current/
- Daniel Foreman-Mackey's live demo - http://dfm.io/gp.js/
- Blog post - http://katbailey.github.io/post/gaussian-processes-for-dummies/

GPs in Astro:

- https://arxiv.org/pdf/1610.09667.pdf
- https://arxiv.org/pdf/1501.00369.pdf
- https://arxiv.org/pdf/1609.07617.pdf
- https://arxiv.org/pdf/1506.07304.pdf