Part I

1. Solve any number of problems having credits that sum to 60.
2. Your score will be scaled to 60 if solve questions having credits more than 60. (If scored 63 out of 70, then it will be 54 out of 60).
3. You should use completely different notation than that one used in the class or in Prof Narahari's book. (e.g. You may use (M,w) for (N,v) or wherever we used coalition C you can use D or A or S or any other your favorite letter).
4. Submit all the answers in pdf. (If you use latex, good. If you use word/Libreoffice/any other editor, ensure all the equations are written using equation editor or equivalent. Don't paste images of equations copied from somewhere else.)
5. (10C)

Explain Nash bargaining (NB) problem and prove Nash's Theorem related to this. (Theorem 26.1 in Y Narahari's book)

1. (10C)

Consider a game with two players, two actions (0 and 1). u(0,0) = (5,5) , u(0,1) = (1,1), u (1,0) =(1,1) and u(1,1) = (3,3) (10C)

* 1. Find out space of all utilities that could be achieved by correlated strategies.
  2. Find out space of all utilities that could be achieved by correlated strategies that are individually rational.
  3. Correlated equilibrium that maximizes the sum of utilities of both the agents

1. (30C)

John and Mary are discussing whether to go to the ballet, to the boxing match, or to stay home tonight. They have a random-number generator, and they can agree to let their decision depend on its output in any way, so as to create a lottery with any probability distribution over the three possible outcomes (go to the ballet, go to boxing, or stay home). If they cannot agree, they will stay home. John wants to go to the boxing match, and he is indifferent between going to the ballet and staying home. For Mary, going to the ballet would be best, staying home would be worst, and she would be indifferent between going to the boxing match or taking a lottery with probability 1/4 of going to the ballet and probability 3 /4 of staying home. They both satisfy the axioms of von Neumann-Morgenstern utility theory.

* 1. Describe this situation by a two-person bargaining problem (F,v) and compute its Nash bargaining solution. How should they implement the Nash bargaining solution?

1. (10C)

In NB, F= convex hull of points A=(1,8), B=(6,7), C=(8,6), D=(9,5), E=(10,3), F=(11,-1), G=(-1,-1). v= (2,1). Draw a picture representing F and find out NB solution and explain how did you get the solution. (Note don’t draw F by hand and attach photo. Use any good picture editor or graph plotters).

1. (20C)

Construct TU games

* 1. that is both monotonic and super additive
  2. monotonic but not super additive
  3. super additive but not monotonic
  4. neither monotonic nor super additive

Explain clearly why the game you constructed is of one of the types above.

1. (20C)

Consider the following characteristic function for coalitions of four agents A, B, C and D:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A | 1 | AB | 3 | BD | 4 | ACD | 6 |
| B | 1 | AC | 4 | CD | 4 | BCD | 6 |
| C | 2 | AD | 4 | ABC | 6 | ABCD | 8 |
| D | 1 | BC | 3 | ABD | 7 |  |  |

                                                Game in characteristic function form

* 1. Is this game super additive? If not, what set of coalitions would be a counterexample? Is the grand coalition (ABCD) the best coalition structure? If not, what is the best coalition structure? What are Shapley values of all the agents?
  2. In the above game, suppose that the payoff of the grand coalition is changed from 8 to 9. Is the game super additive? Is the game convex? If not, what set of coalitions would be a counterexample? What is the Shapley value of each agent in the coalition (ABD)? Is this payoff distribution in the core?

1. (20C)

Show that Shapley value satisfies all the three axioms of TU game (symmetry, linearity and carrier set). Show that any other function satisfying the three axioms, must be Shapley value.

Part II (40)

Form a team of two.

Write a java program to compute a Shapley value of a characteristic form game.

File format:

N=n

v(1),…..,v(1,2,….,n)

(

Number of players in first row and valuations for all the coalitions in the second row except empty set. Valuations are ordered like v(1),…,v(n),v(12),..,v(1n),v(23)…,v(2n),….,v(n-1n),v(123),v(124),…,v(12n),v(234)…,…,v(12…n)

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For example: Divide the Dollar Version 2 would be

N = 3

0,0,0,30,0,0,30