

## Report For Project2

### 1.Data Split:

Leter Data Set : This data set has 69623 rows and 46 columns

Synthetic Data Set : This data set has 20000 rows and 10 columns

The Leter Data Set is prepared by reading from Querylevelnorm.txt

The Synthetic data Set is read from the both input.csv file and output.csv file using the csv.reader

The Leter Data Set and Synthetic data Set are shuffled using `numpy.random.shuffle()` so as to minimize the Error values for prediction for the three datasets.

For Splitting the data a user defined class named `DataSplitClass` is used with parameters

1. `trainingrowsize` : Size of the training data Set
2. `validationrowsize` = Size of the validation data Set
3. `testingrowsize` = Size of the Testing data set
  
4. `colsize` = Number of Columns or the number of the input features
  
5. `trainingfeaturesmat` = Matrix to hold the training feature data values
6. `traininglabelsvec` = Matrix to hold the training label labels
  
7. `validationfeaturesmat` = Matrix to hold the validation feature data values
8. `validationlabelsvec` = Matrix to hold the validation label values
  
9. `testingfeaturesmat` = Matrix to hold the testing feature data values
10. `testinglabelsvec` = Matrix to hold the testing label values

The dataset is partitioned into 80% , 10% and 10 % to form training , validation and Testing data Sets correspondingly . Two objects of type `DataSplitClass`, are created one for the leter data set and another for the synthetic data set

The constructor of this class takes `trainingrowsize`, `validationrowsize`, `testingrowsize` and `colsize` as the input parameters , For Leter data set object

`Trainingrowsize` = 55698  
`validationrowsize` = 6962  
`testingrowsize` = 6963

For synthetic data set object

`Trainingrowsize` = 16000  
`validationrowsize` = 2000  
`testingrowsize` = 2000

The member function '`SplitData()`' is called which copies the values from original data using start index and data set size(`training` or `validation` or `testing`) parameter into either `training` ,`validation` or `testing` data sets

For Leter

Training Data Starting Index = 0  
Validation Data Starting Index = 55698  
Testing Data Starting Index = 62,660

For Synthetic

Training Data Starting Index = 0  
Validation Data Starting Index = 16000  
Testing Data Starting Index = 18,000

Using the above mentioned two class objects (one for leter and another for synthetic) the corresponding dataset is accessed

## 2. Hyper Parameter tuning :

### M and lambda:

As per the feature size, a range of M values are selected .

Considering the wiki link given for the grid search for the hyper-parameters in project specification range of lambda values are selected

For Letor data , The Set of M values considered is  $M = \{5, 7, 9, 11, 13, 15\}$  and  $\lambda = \{0.1, 0.3, 0.5, 0.9\}$

For Synthetic data , The Set of M values considered is  $M = \{3, 5, 8\}$  and  $\lambda = \{0.1, 0.3, 0.5, 0.9\}$

Number of iterations in Letor  $n = 6 * 4 = 24$

Number of iterations in Synthetic  $n = 3 * 4 = 12$

(2 nested for loops are run) for every M and Lambda combination i.e. n times mentioned above

For Every iteration weights and training rms error are obtained from the training data set .This weights are given as input to calculate Erms on the validation data set .

For all the 'n' iterations the lowest of all the validation RMS Error values is chosen , the corresponding M , lambda and weight vector are selected as the Final values .

The above mentioned selected final parameters are used to calculate the Erms error on testing data set which is the final Erms error and benchmark for the prediction accuracy for the trained model

Above Steps are similar for closed form and the Stochastic Gradient except that the method of calculating weights differs

### Closed form weight Determination :

The Basis function matrix is calculated as per the equation given in specification using  $\mu_j$  and  $\sigma_j$  . This is discussed more elaborately in upcoming section .

The closed form solution Equation (with regularization parameter) is used to find the weights

### Stochastic Gradient method of weight Determination :

As per the value of M , the random weight vector is prepared using `numpy.random.random()` . The size of this vector is M .This weight vector is used as input to the method that operates on training data set which calculates weights.

Learning Rate is selected as 1.

The gradient descent is run for the iterations equal to the size of the data set.

For first iteration Erms is calculated using the input random weight vector mentioned above. Thereafter Erms is calculated using the weights determined in previous iteration.

If the current rms is greater than Previous rms , the learning rate is decreased( dividing by2) and the weights are not updated .

If the current rms is less than previous rms by a factor 0.001 , weights are updated and the for loop is exited .

The final weight vector would consist the values for which the Erms error is lowest

### Plots :

(note : The minimum values are already calculated in the code . Inference statement from plots shall reverfiy this )

### Closed Form Solution For Letor Data :

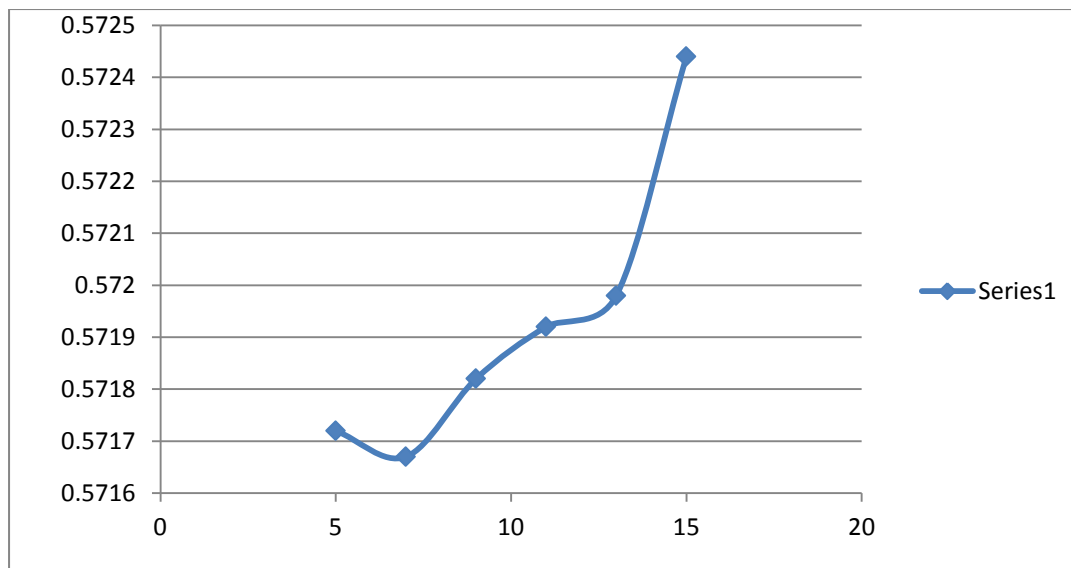
Table for M,lambda and Validation Rms error Values

		Validation
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M	Lambda	Erms Error
5	0.1	0.57207
5	0.3	0.57172
5	0.5	0.57172
5	0.9	0.57174
7	0.1	0.57174
7	0.3	0.57176
7	0.5	0.57167
7	0.9	0.57216
9	0.1	0.57179
9	0.3	0.57239
9	0.5	0.57182
9	0.9	0.57175
11	0.1	0.57211
11	0.3	0.57242
11	0.5	0.57192
11	0.9	0.57186
13	0.1	0.57183
13	0.3	0.57221
13	0.5	0.57198
13	0.9	0.57214
15	0.1	0.57167
15	0.3	0.57173
15	0.5	0.57244
15	0.9	0.57238

Plot between M and Erms

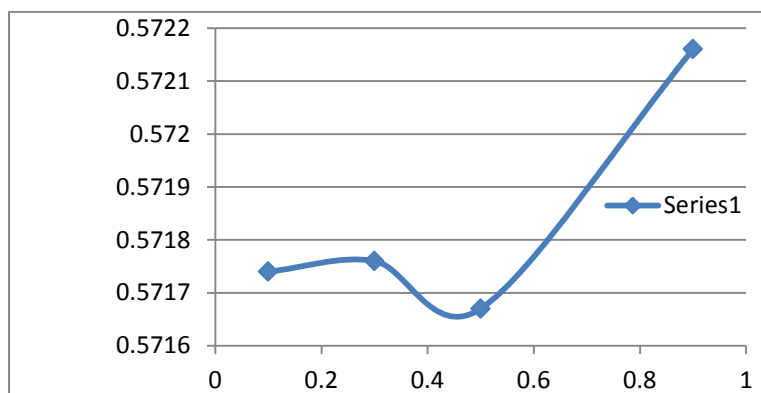
lambda = 0.5(minimum)						
M (X-axis)	5	7	9	11	13	15
Validation rms error(Y-axis)	0.57172	0.57167	0.57182	0.57192	0.57198	0.57244



Inference :  $M = 7$  (minimum value ) for minimum validation rms error = 0.57167

#### Plot Between lambda and Erms

M = 7 (minimum )				
lambda(X-axis)	0.1	0.3	0.5	0.9
Validation rms error(Y-axis)	0.57174	0.57176	0.57167	0.57216



lambda = 0.5 (minimum value ) for minimum validation rms error = 0.57167

#### 2.Plots for Closed Form Synthetic Data :

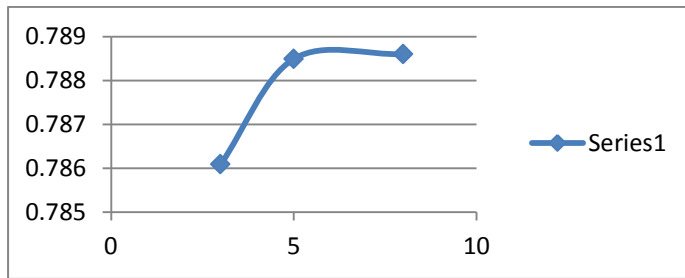
Table for M,lambda and Validation Rms error Values

M	lambda	Validation Rms Error
3	0.1	0.78623
3	0.3	0.78609
3	0.5	0.78667
3	0.9	0.78692

5	0.1	0.78685
5	0.3	0.78849
5	0.5	0.78748
5	0.9	0.78707
8	0.1	0.79003
8	0.3	0.78756
8	0.5	0.7886
8	0.9	0.78727

Plot between M and Validation Rms Error

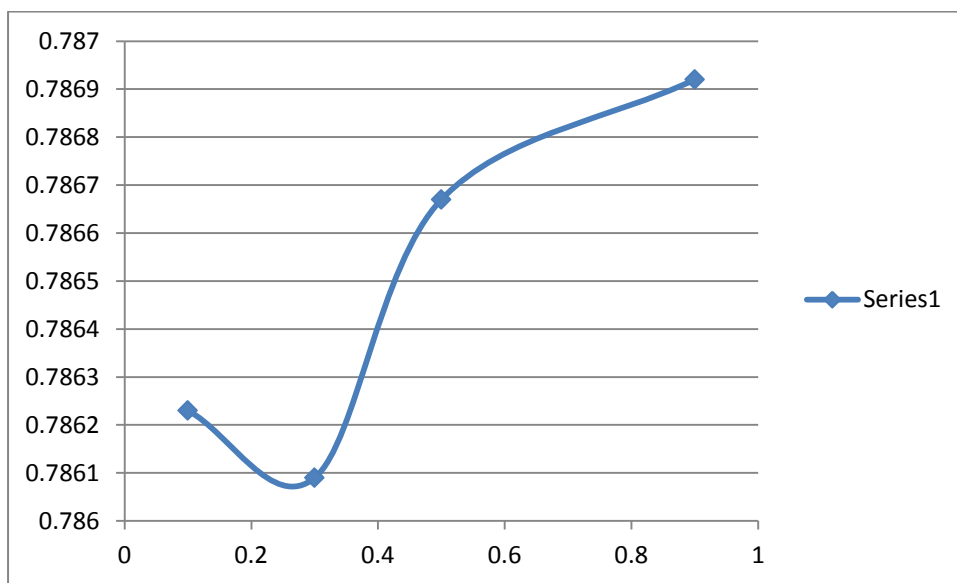
lambda(min) =0.3			
M(X-axis)	3	5	8
Validation rms error(Y-axis)	0.78609	0.78849	0.7886



Inference : M =3 is the minimum value for minimum validation error = 0.78609

Plot between lambda and Validation Rms Error

M= 3(minimum)				
lambda (X-axis)	0.1	0.3	0.5	0.9
Validation rms error(Y-axis)	0.78623	0.78609	0.78667	0.78692



Inference :lambda(minimum) =0.3 for minimum Validation Rms error = 0.78609

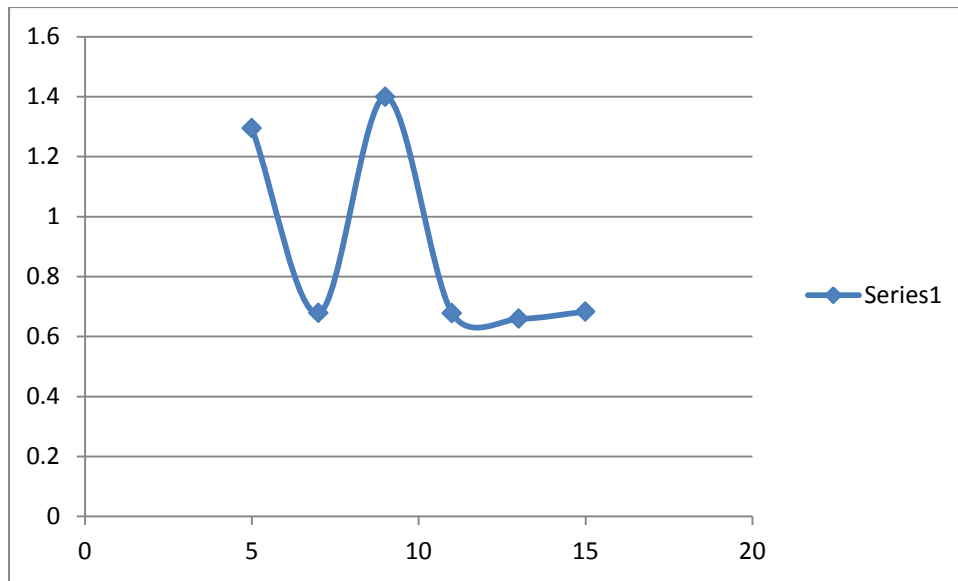
**Plots for stochastic gradient descent Letor Data :**

Table for M ,lambda and Validation Rms Error Values

		validation rms error
M	lambda	
5	0.1	1.29495
5	0.3	0.7637
5	0.5	0.71221
5	0.9	1.19777
7	0.1	0.67821
7	0.3	0.77685
7	0.5	0.68723
7	0.9	0.94671
9	0.1	1.39909
9	0.3	0.69484
9	0.5	0.66973
9	0.9	0.85657
11	0.1	0.67792
11	0.3	0.70981
11	0.5	0.72145
11	0.9	0.79617
13	0.1	0.65897
13	0.3	0.78726
13	0.5	0.90892
13	0.9	0.7014
15	0.1	0.68309
15	0.3	0.70674
15	0.5	0.86993
15	0.9	0.75042

Plot between M and Minimum Validation Rms Error

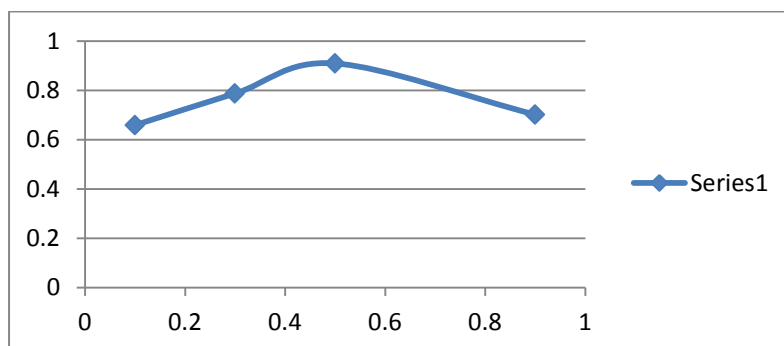
lambda(min) =0.1						
M(X-axis)	5	7	9	11	13	15
Validation rms error(Y-axis)	1.29495	0.67821	1.39909	0.67792	0.65897	0.68309



Inference :  $M(\text{minimum}) = 13$  for the minimum validation rms error = 0.65897

Plot between lambda and Minimum Validation Rms Error

M = 13(minimum)				
lambda (X-axis)	0.1	0.3	0.5	0.9
Validation rms error(Y-axis)	0.65897	0.78726	0.90892	0.7014



Inference :  $\lambda(\text{minimum}) = 0.1$  for minimum validation error = 0.65897

**Plots for stochastic gradient descent Synthetic Data :**

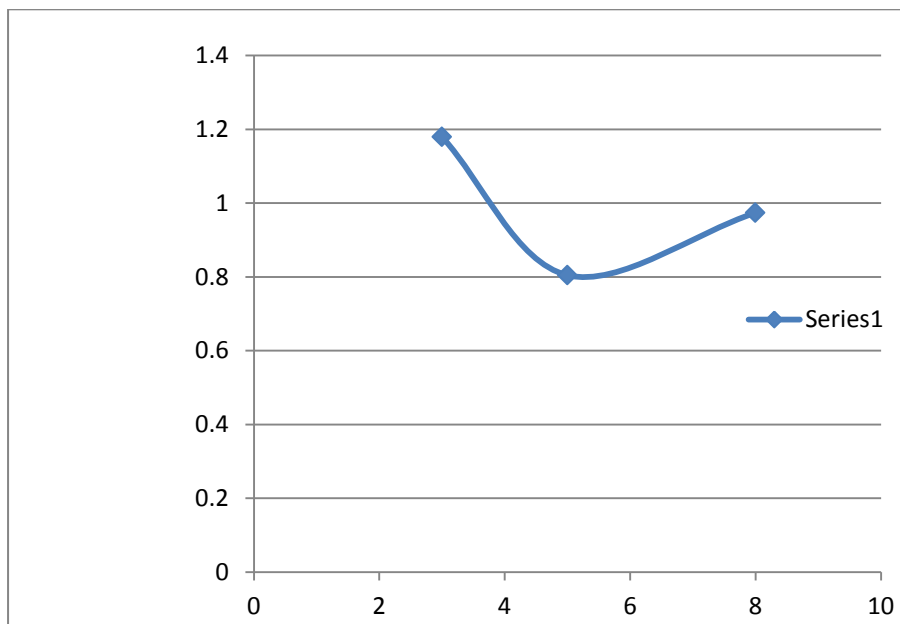
Table for M , lambda and Validation Rms Error Values

		validation rms error
M	lambda	
3	0.1	1.25442
3	0.3	1.26854
3	0.5	1.00844

3	0.9	1.17993
5	0.1	1.27484
5	0.3	1.12645
5	0.5	1.27911
5	0.9	0.80472
8	0.1	1.24579
8	0.3	1.16682
8	0.5	0.9768
8	0.9	0.97407

Plot between M and validation error

lambda(min) =0.9				
M(X-axis)		3	5	8
Validation rms error(Y-axis)		1.17993	0.80472	0.97407

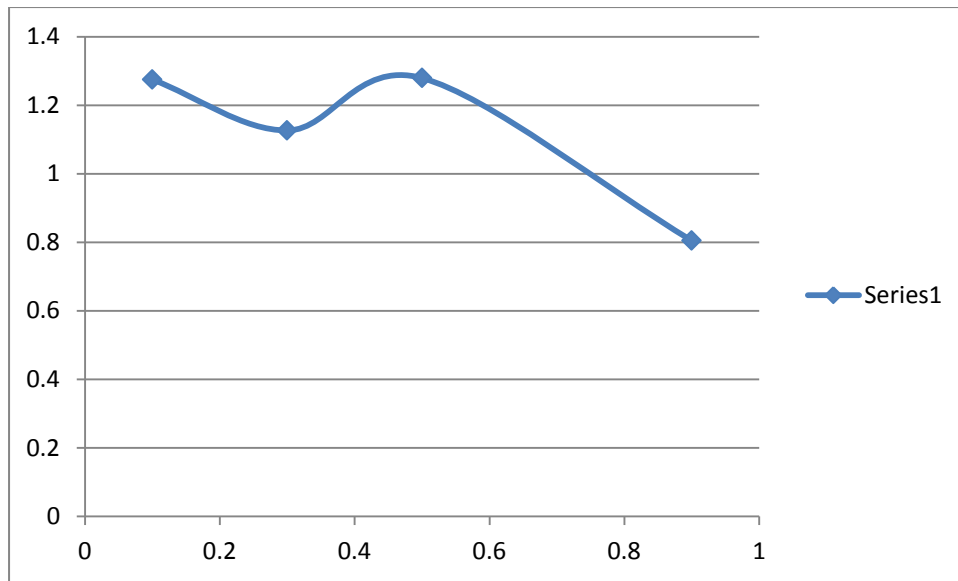


Inference : M(minimum) = 5 for minimum validation error = 0.80472

Plot between lambda and Validation Rms Error

M = 5(minimum)				
lambda (X-axis)	0.1	0.3	0.5	0.9
Validation rms error(Y-axis)	1.27484	1.12645	1.27911	0.80472





Inference :  $\lambda_{\text{minimum}} = 0.9$  for minimum validation error = 0.80472

#### **Muj and Sigmaj :**

##### Muj:

The Muj Matrix contains 'M' rows which are drawn randomly from the corresponding partition are found using the following function `mujmat = featuresmat[np.random.choice(featuresmat.shape[0],M, replace=False),:]`. It contains M number of rows and columns of this matrix is equal to the number of features. The Basis Function is calculated using the MujMat. The Basis function Matrix contains M columns and number of rows is equal to size of the data set. The first column in the basis function is '1' for every row of the Basis Function Matrix.

##### Sigmaj :

The variance of every column in given input data set is calculated. Variance is divided by 10 as mentioned in main.pdf project specification document (to maintain proportionality). If the variance value is equal to zero, it is substituted by a very small value i.e. 0.001 so as to prevent Sigmaj from becoming a singular matrix. This variance value of every column is taken as diagonal element in the diagonal matrix. The dimension of this matrix is 'Number of columns by Number of columns'.

Using the `numpy.linalg.inv()` function the inverse of the sigma matrix is calculated which is used in the calculation of the basis function matrix.

#### **Learning Rate (eta):**

In stochastic Gradient Descent, for both cases i.e. letor data and synthetic data, the initial learning rate value is chosen as '1' as suggested in the recitation class. Though the code is present to handle the division of eta by 2 when the present rms error is greater than previous rms error, it has been observed that convergence was achieved in few iterations and there was never a situation where eta has to be divided. Hence eta (learning rate) was kept constant throughout for Stochastic Gradient descent in Letor or Synthetic data cases.

#### **Evaluation Results :**

##### Closed Form Solution For Letor Data

M=5

$\lambda_{\text{reg}}=0.1$

```
letor_trainingrmseerror=[[ 0.5718]]
letor_validationrmseerror=[[ 0.57207]]
minweight=
[[ 0.31438085]
 [ 0.62327833]
 [ 0.62329014]
 [ 0.62329149]
 [-0.28580159]]
M=5
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57182]]
letor_validationrmseerror=[[ 0.57172]]
minweight=
[[ 0.31442785]
 [-0.25100952]
 [-0.24186757]
 [-0.26611259]
 [-0.24250344]]
M=5
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57179]]
letor_validationrmseerror=[[ 0.57172]]
M=5
lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57181]]
letor_validationrmseerror=[[ 0.57174]]
M=7
lambda_reg=0.1
letor_trainingrmseerror=[[ 0.57181]]
letor_validationrmseerror=[[ 0.57174]]
M=7
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57178]]
letor_validationrmseerror=[[ 0.57176]]
M=7
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57181]]
letor_validationrmseerror=[[ 0.57167]]
minweight=
[[ 0.3144177 ]
 [-0.21118822]
 [-0.2096118 ]
 [-0.20961558]
 [-0.2096118 ]
 [-0.20959667]
 [ 0.45704477]]
M=7
lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57177]]
letor_validationrmseerror=[[ 0.57216]]
M=9
lambda_reg=0.1
letor_trainingrmseerror=[[ 0.57179]]
letor_validationrmseerror=[[ 0.57179]]
M=9
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57172]]
letor_validationrmseerror=[[ 0.57239]]
M=9
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57181]]
letor_validationrmseerror=[[ 0.57182]]
M=9
```

```

lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57181]]
letor_validationrmseerror=[[ 0.57175]]
M=11
lambda_reg=0.1
letor_trainingrmseerror=[[ 0.57178]]
letor_validationrmseerror=[[ 0.57211]]
M=11
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57176]]
letor_validationrmseerror=[[ 0.57242]]
M=11
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57178]]
letor_validationrmseerror=[[ 0.57192]]
M=11
lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57115]]
letor_validationrmseerror=[[ 0.57186]]
M=13
lambda_reg=0.1
letor_trainingrmseerror=[[ 0.57179]]
letor_validationrmseerror=[[ 0.57183]]
M=13
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57178]]
letor_validationrmseerror=[[ 0.57221]]
M=13
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57115]]
letor_validationrmseerror=[[ 0.57198]]
M=13
lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57122]]
letor_validationrmseerror=[[ 0.57214]]
M=15
lambda_reg=0.1
letor_trainingrmseerror=[[ 0.57179]]
letor_validationrmseerror=[[ 0.57167]]
M=15
lambda_reg=0.3
letor_trainingrmseerror=[[ 0.57179]]
letor_validationrmseerror=[[ 0.57173]]
M=15
lambda_reg=0.5
letor_trainingrmseerror=[[ 0.57156]]
letor_validationrmseerror=[[ 0.57244]]
M=15
lambda_reg=0.9
letor_trainingrmseerror=[[ 0.57171]]
letor_validationrmseerror=[[ 0.57238]]

```

```

min_rmseerror=[ 0.57167]
min_lambda=0.5
min_M= 7

```

```

Final M=7
Final lambda=0.5
Final minimum Validation rmseerror=[[ 0.57167]]
Final Weight =
[[ 0.3144177 ]
[-0.21118822]
[-0.2096118 ]

```

```
[-0.20961558]
[-0.2096118 ]
[-0.20959667]
[ 0.45704477]]
```

**FinalTesting rms Error** = 0.56787

#### **Explanantion :**

The above highlighted results shows that the testing rms error is lower than minimum validation rms error for Letor Data using Closed Form Solution which is desirable as the Testing Rms Error should be lower . When above M ,lambda and Weights are used ,57% of the predictions are incorrect while 43% of predictions are correct which shows this model is better comparing to the below mentioned models , but this is not the excellent prediction

#### Closed Form Solution For Synthetic Data :

```
M=3
lambda_reg=0.1
synth_trainingrmerror=[[ 0.79047]]
synth_validationrmerror=[[ 0.78623]]
minweight=
[[ 0.96074456]
 [ 0.03568897]
 [ 0.95000236]]
M=3
lambda_reg=0.3
synth_trainingrmerror=[[ 0.79046]]
synth_validationrmerror=[[ 0.78609]]
minweight=
[[ 0.9607907 ]
 [ 0.79943254]
 [-0.73913073]]
M=3
lambda_reg=0.5
synth_trainingrmerror=[[ 0.79053]]
synth_validationrmerror=[[ 0.78667]]
M=3
lambda_reg=0.9
synth_trainingrmerror=[[ 0.79052]]
synth_validationrmerror=[[ 0.78692]]
M=5
lambda_reg=0.1
synth_trainingrmerror=[[ 0.79048]]
synth_validationrmerror=[[ 0.78685]]
M=5
lambda_reg=0.3
synth_trainingrmerror=[[ 0.7904]]
synth_validationrmerror=[[ 0.78849]]
M=5
lambda_reg=0.5
synth_trainingrmerror=[[ 0.79042]]
synth_validationrmerror=[[ 0.78748]]
M=5
lambda_reg=0.9
synth_trainingrmerror=[[ 0.7905]]
synth_validationrmerror=[[ 0.78707]]
M=8
lambda_reg=0.1
synth_trainingrmerror=[[ 0.79029]]
synth_validationrmerror=[[ 0.79003]]
M=8
lambda_reg=0.3
```

```

synth_trainingrmseerror=[[ 0.79033]]
synth_validationrmseerror=[[ 0.78756]]
M=8
lambda_reg=0.5
synth_trainingrmseerror=[[ 0.79039]]
synth_validationrmseerror=[[ 0.7886]]
M=8
lambda_reg=0.9
synth_trainingrmseerror=[[ 0.79044]]
synth_validationrmseerror=[[ 0.78727]]

min_rmseerror=[ 0.78609]
min_lambda=0.3
min_M= 3

```

```

Final minimum validation rmseerror =[[ 0.78609]]
Final_lambda=0.3
Final_M=3
Final_weight=[[ 0.96074456]
               [ 0.03568897]
               [ 0.95000236]]

```

```

Final Testing rmseerror = [ 0.80026]

```

#### **Explanantion :**

The above highlighted results shows that the testing rms error is lower than minimum validation rms error for Synthetic Data using Closed Form Solution which is not desirable as the Testing Rms Error should be lower .The prediction accuracy would have been better . When above M ,lambda and Weights are used ,80% of the predictions are incorrect while 20% of predictions are correct which shows this model needs more data samples for sufficient training

### **Stochastic Gradient Solution For Letor Data :**

```

Convergence is achieved
M=5
lambda_reg=0.1
letor_trainingrmseerror=[ 1.29827]
letor_validationrmseerror=[ 1.29495]
minweight=
[-0.85098548 0.6410765 0.31364313 0.18663343 0.50722349]
Convergence is achieved
M=5
lambda_reg=0.3
letor_trainingrmseerror=[ 0.76751]
letor_validationrmseerror=[ 0.7637]
minweight=
[-0.19696979 0.6817619 0.52301359 0.26289888 0.18633128]
Convergence is achieved
M=5
lambda_reg=0.5
letor_trainingrmseerror=[ 0.71615]
letor_validationrmseerror=[ 0.71221]
minweight=
[-0.11602345 0.31574379 0.34443905 0.2230774 0.31839392]
Convergence is achieved
M=5

```

lambda\_reg=0.9  
letor\_trainingrmseerror=[ 1.2009]  
letor\_validationrmseerror=[ 1.19777]  
Convergence is achieved  
M=7  
lambda\_reg=0.1  
letor\_trainingrmseerror=[ 0.68233]  
letor\_validationrmseerror=[ 0.67821]  
minweight=  
[-0.05706061 0.31274085 0.63169951 0.41235758 0.11172135 0.79176589  
0.55040891]  
Convergence is achieved  
M=7  
lambda\_reg=0.3  
letor\_trainingrmseerror=[ 0.78063]  
letor\_validationrmseerror=[ 0.77685]  
Convergence is achieved  
M=7  
lambda\_reg=0.5  
letor\_trainingrmseerror=[ 0.69135]  
letor\_validationrmseerror=[ 0.68723]  
Convergence is achieved  
M=7  
lambda\_reg=0.9  
letor\_trainingrmseerror=[ 0.95021]  
letor\_validationrmseerror=[ 0.94671]  
Convergence is achieved  
M=9  
lambda\_reg=0.1  
letor\_trainingrmseerror=[ 1.4022]  
letor\_validationrmseerror=[ 1.39909]  
Convergence is achieved  
M=9  
lambda\_reg=0.3  
letor\_trainingrmseerror=[ 0.69881]  
letor\_validationrmseerror=[ 0.69484]  
Convergence is achieved  
M=9  
lambda\_reg=0.5  
letor\_trainingrmseerror=[ 0.67384]  
letor\_validationrmseerror=[ 0.66973]  
minweight=  
[-0.04115007 0.39376918 0.23272234 0.13661678 0.43999043 0.05327082  
0.15255113 0.22347034 0.10971165]  
Convergence is achieved  
M=9  
lambda\_reg=0.9  
letor\_trainingrmseerror=[ 0.86027]  
letor\_validationrmseerror=[ 0.85657]  
Convergence is achieved  
M=11  
lambda\_reg=0.1  
letor\_trainingrmseerror=[ 0.68192]  
letor\_validationrmseerror=[ 0.67792]  
Convergence is achieved  
M=11  
lambda\_reg=0.3  
letor\_trainingrmseerror=[ 0.71381]  
letor\_validationrmseerror=[ 0.70981]  
Convergence is achieved  
M=11  
lambda\_reg=0.5  
letor\_trainingrmseerror=[ 0.72537]

```

letor_validationrmerror=[ 0.72145]
Convergence is achieved
M=11
lambda_reg=0.9
letor_trainingrmerror=[ 0.8]
letor_validationrmerror=[ 0.79617]
Convergence is achieved
M=13
lambda_reg=0.1
letor_trainingrmerror=[ 0.66293]
letor_validationrmerror=[ 0.65897]
minweight=
[-0.02001276 0.0771293 0.10899979 0.65586121 0.39351467 0.7837358
 0.40337427 0.05893762 0.50965642 0.49628657 0.11680944 0.08320048
 0.77428499]
Convergence is achieved
M=13
lambda_reg=0.3
letor_trainingrmerror=[ 0.79115]
letor_validationrmerror=[ 0.78726]
Convergence is achieved
M=13
lambda_reg=0.5
letor_trainingrmerror=[ 0.91276]
letor_validationrmerror=[ 0.90892]
Convergence is achieved
M=13
lambda_reg=0.9
letor_trainingrmerror=[ 0.70546]
letor_validationrmerror=[ 0.7014]
Convergence is achieved
M=15
lambda_reg=0.1
letor_trainingrmerror=[ 0.687]
letor_validationrmerror=[ 0.68309]
Convergence is achieved
M=15
lambda_reg=0.3
letor_trainingrmerror=[ 0.71084]
letor_validationrmerror=[ 0.70674]
Convergence is achieved
M=15
lambda_reg=0.5
letor_trainingrmerror=[ 0.87381]
letor_validationrmerror=[ 0.86993]
Convergence is achieved
M=15
lambda_reg=0.9
letor_trainingrmerror=[ 0.75435]
letor_validationrmerror=[ 0.75042]

min_rmerror=[ 0.65897]
min_lambda=0.1
min_M= 13

```

```

Final M=13
Final lambda=0.1
Final minimum Validation rmerror=[ 0.65897]
Final Weight =
[-0.02001276 0.0771293 0.10899979 0.65586121 0.39351467 0.7837358
 0.40337427 0.05893762 0.50965642 0.49628657 0.11680944 0.08320048
 0.77428499]

```

**FinalTesting rms Error** = [ 0.66016]

**Explanation:**

The above highlighted results shows that the testing rms error is higher than minimum validation rms error for Letor Data using Stochastic Gradient Solution which is not desirable as the Testing Rms Error should be lower .This happened even after doing a random shuffle . However the difference between them is minimum .The prediction accuracy is of average standard . When above M ,lambda and Weights are used ,66% of the predictions are incorrect while 34% of predictions are correct which shows this model needs more data samples for sufficient training

**Stochastic Gradient Solution For Synthetic Data :**

Convergence is achieved

M=3

lambda\_reg=0.1

synth\_trainingrmerror=[ 1.24981]

synth\_validationrmerror=[ 1.25442]

minweight=

[ 1.93089273 0.73667915 0.30937971]

Convergence is achieved

M=3

lambda\_reg=0.3

synth\_trainingrmerror=[ 1.26415]

synth\_validationrmerror=[ 1.26854]

Convergence is achieved

M=3

lambda\_reg=0.5

synth\_trainingrmerror=[ 1.00499]

synth\_validationrmerror=[ 1.00844]

minweight=

[ 1.5837407 0.39630155 0.2337651 ]

Convergence is achieved

M=3

lambda\_reg=0.9

synth\_trainingrmerror=[ 1.17548]

synth\_validationrmerror=[ 1.17993]

Convergence is achieved

M=5

lambda\_reg=0.1

synth\_trainingrmerror=[ 1.27035]

synth\_validationrmerror=[ 1.27484]

Convergence is achieved

M=5

lambda\_reg=0.3

synth\_trainingrmerror=[ 1.12233]

synth\_validationrmerror=[ 1.12645]

Convergence is achieved

M=5

lambda\_reg=0.5

synth\_trainingrmerror=[ 1.2738]

synth\_validationrmerror=[ 1.27911]

Convergence is achieved

M=5

lambda\_reg=0.9

synth\_trainingrmerror=[ 0.80403]

synth\_validationrmerror=[ 0.80472]

minweight=

[ 1.11208520e+00 1.00175337e-03 7.56676761e-02 9.09355132e-03  
9.18295757e-02]

Convergence is achieved



M=8  
lambda\_reg=0.1  
synth\_trainingrmseerror=[ 1.23979]  
synth\_validationrmseerror=[ 1.24579]  
Convergence is achieved  
M=8  
lambda\_reg=0.3  
synth\_trainingrmseerror=[ 1.16185]  
synth\_validationrmseerror=[ 1.16682]  
Convergence is achieved  
M=8  
lambda\_reg=0.5  
synth\_trainingrmseerror=[ 0.97364]  
synth\_validationrmseerror=[ 0.9768]  
Convergence is achieved  
M=8  
lambda\_reg=0.9  
synth\_trainingrmseerror=[ 0.97065]  
synth\_validationrmseerror=[ 0.97407]  
  
min\_rmseerror=[ 0.80472]  
min\_lambda=0.9  
min\_M= 5

**Final lambda**=0.9

**Final M**=5

**Final minimum Validation rmseerror**=[ 0.80472]

**Final weight** = [ 1.11208520e+00 1.00175337e-03 7.56676761e-02 9.09355132e-03  
9.18295757e-02]

**FinalTesting Rms Error** = 0.80399

**Explanation:**

The above highlighted results shows that the testing rms error is lower than minimum validation rms error for Synthetic Data using Stochastic Gradient Solution which is desirable as the Testing Rms Error should be lower .The prediction accuracy would have been better . When above M ,lambda and Weights are used ,80% of the predictions are incorrect while 20% of predictions are correct which shows this model needs more data samples for sufficient training