Mixable losses
and
Tracking the best Expert

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Outline

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  - c^t is revealed.
- **Goal:** minimize regret:

$$-\sum_{t=1}^{T} \log p_A^t(c^t) + \min_{i=1,\ldots,N} \left(-\sum_{t=1}^{T} \log p_i^t(c^t) \right)$$

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$$\mathbf{p}_A^t = \frac{\sum_{i=1}^{N} w_i^t \mathbf{p}_i^t}{\sum_{i=1}^{N} w_i^t}$$

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EQUALITY not bound!

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4. Each expert incurs loss $\ell_i^t = \lambda(\omega^t, \gamma_i^t)$
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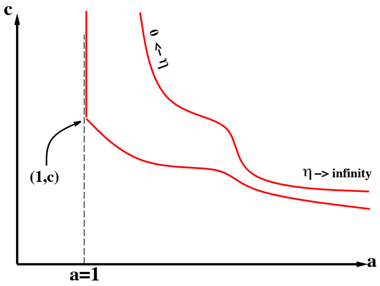

Some useful loss functions

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- No triangle inequality
  $\exists p_1, p_2, p_3 \;\; \lambda(p_1, p_3) > \lambda(p_1, p_2) + \lambda(p_2, p_3)$

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- Corresponds to regression.

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- Loss is bounded.
- Defines a metric.
- $\lambda_{\text{hel}}(p, q) \approx \lambda_{\text{ent}}(p, q)$ when $p \approx q$ and $p, q \in (0, 1)$

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- If $P[\omega^t = 1] = q, \quad P[\omega^t = 0] = 1 - q$, then the optimal prediction is
$$\gamma^t = \begin{cases} 1 & \text{if } q > 1/2, \\ 0 & \text{otherwise} \end{cases}$$

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- **Which losses behave like entropy loss and which behave like hedge loss?**

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- There is a universally reasonable prediction $\exists \gamma \in \Gamma, \forall \omega \in \Omega, \lambda(\omega, \gamma) < \infty$
- There is no universally optimal prediction $\neg \exists \gamma \in \Gamma, \forall \omega \in \Omega, \lambda(\omega, \gamma) = 0$

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Choose γ_t so that, for all $\omega^t \in \Omega$:

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- 2. If choice of γ^t always exists, then the total loss satisfies:

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- Vovk's result: *yes!* a good choice for γ_t always exists!

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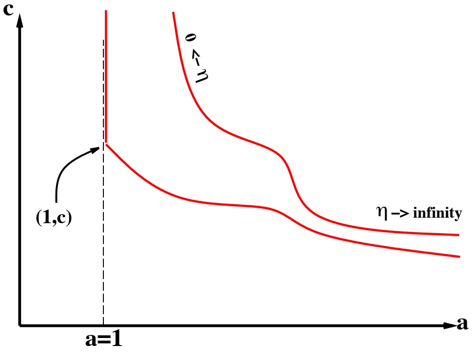

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- Equivalently - the image of the set Γ under the mapping $F(\gamma) = \left\langle e^{-\eta \lambda(\omega, \gamma)} \right\rangle_{\omega \in \Omega}$ is concave.

convexity condition: Pictorially

- **Example:** Suppose $\Omega = \{0, 1\}$, $\Gamma = [0, 1]$. then

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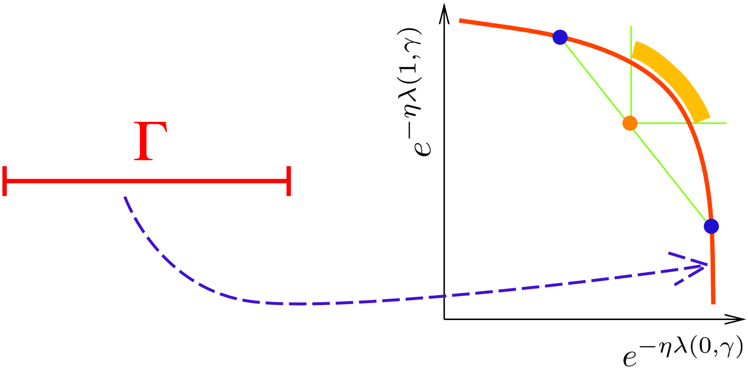

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- We are back to the online Bayes algorithm.

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Summary of bounds for mixable losses

TRACKING THE BEST EXPERT

Loss Functions:	c values: $(\eta = 1/c)$	
	$\mathbf{pred}_{\text{wmean}}(v, x)$	$\mathbf{pred}_{\text{Vovk}}(v, x)$
$L_{\text{sq}}(p, q)$	2	1/2
$L_{\text{ent}}(p, q)$	1	1
$L_{\text{hel}}(p, q)$	1	$1/\sqrt{2}$

Figure 2. $(c, 1/c)$-realizability: c values for loss and prediction function pairin

Switching experts setup

- Usually: compare algorithm's total loss to total loss of the best expert.

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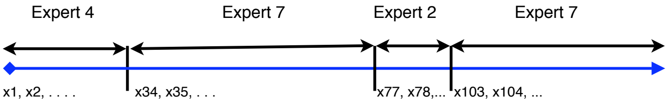
- 

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- Then using the partition-expert algorithm for the switching-experts case we get a bound on the regret $\frac{1}{\eta}\left((k+1)\log n + k \log \frac{l}{k} + k\right)$

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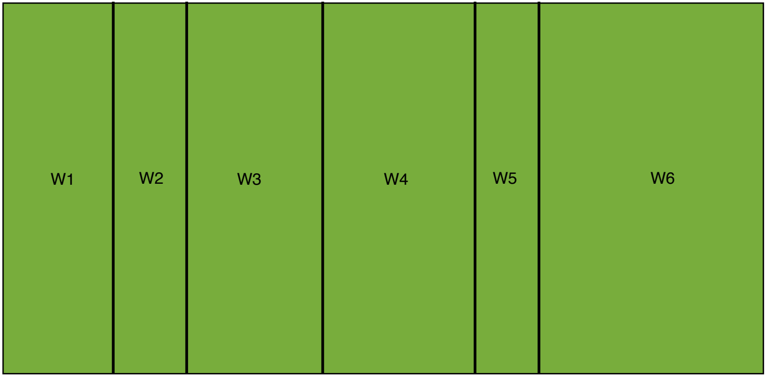
- Fixed-share:

$$\begin{aligned} pool &= \alpha \sum_{i=1}^{n} w^m_{t,i} \\ w^s_{t+1,i} &= (1-\alpha) w^m_{t,i} + \frac{1}{n-1} \left(pool - \alpha w^m_{t,i} \right) \end{aligned}$$

The fixed-share algorithm



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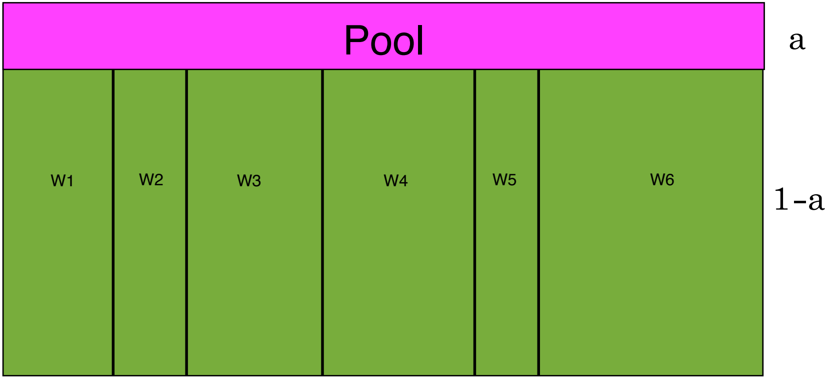

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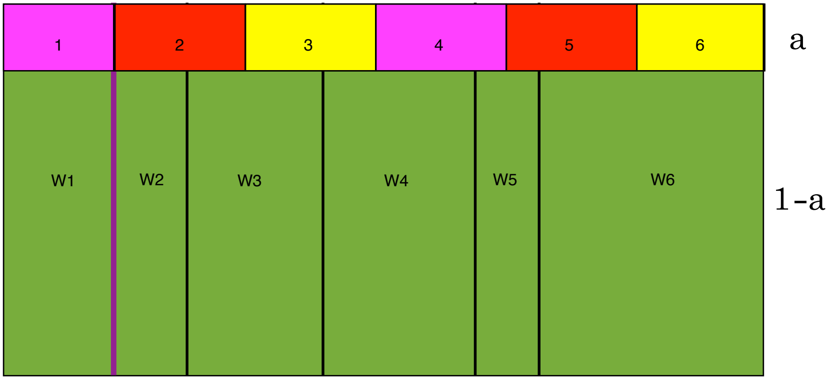

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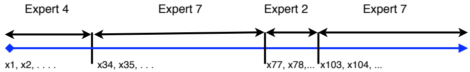
- The harder question is how to lower bound $\sum_{i=1}^{n} w^s_{l+1,i}$

Lower bounding the final total weight

- Fix some switching experts sequence:



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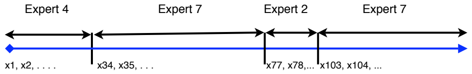

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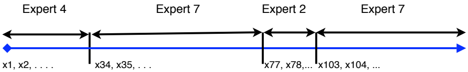

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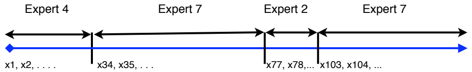

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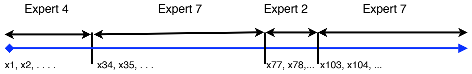

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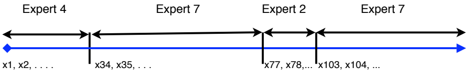

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Bound for arbitrary α

- Combining we lower bound the final weight of the last expert in the sequence

$$w^{s}_{l+1,e_k} \geq \frac{1}{n} e^{-\eta L_*} (1-\alpha)^{l-k-1} \left(\frac{\alpha}{n-1} \right)^{k}$$

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- Combining the upper and lower bounds we get that for any sequence

$$L_A \leq L_* + \frac{1}{\eta}\left(\ln n + (l-k-1)\ln\frac{1}{1-\alpha} + k\left(\ln\frac{1}{\alpha} + \ln(n-1)\right)\right)$$

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- Not so for square loss!

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- Works for square loss, but not for log loss!

Variable-share

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Shares the weight quickly if $\ell_{t,i} > 0$

Bound for variable share

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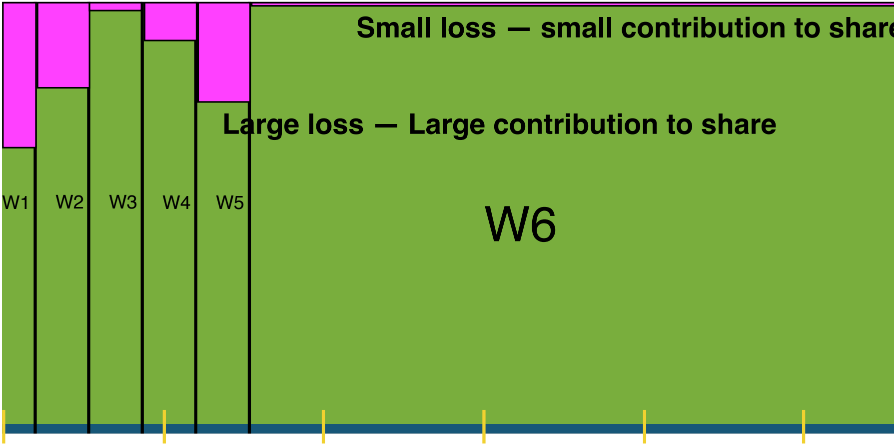
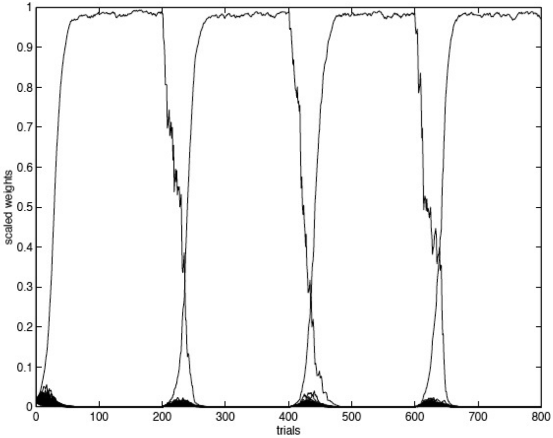
- α should be tuned so that it is (close to) $\frac{k}{2k+L_*}$

Variable share figure



An experiment using variable share



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- Can track switches much more closely.
- Easy to describe an inefficient algorithm (consider all $\binom{n}{n'}$ subsets.)

Next Class

- Suppose the best switching sequence is repeatedly switching among a small subset of the experts $n' \ll n$
- In the context of speech recognition - the speaker repeatedly uses a small number of phonemes.
- If we know the subset, we can pay $\ln n'$ per switch rather than $\ln n$
- Can track switches much more closely.
- Easy to describe an inefficient algorithm (consider all $\binom{n}{n'}$ subsets.)
- Next class - how to do as well with just one weight per expert.