The code length for arithmetic coding

- Given m bits of binary expansion we assume the rest are all zero.
- Distance between two m bit expansions is 2^{-m}
- If $l_T - u_T \geq 2^{-m}$ then there must be a point x described by m expansion bits such that $l_T \leq x < u_T$
- Required number of bits is $\lceil -\log_2(u_T - l_T) \rceil$.
- $u_T - l_T = \prod_{t=1}^{T} p\left(c_t \,|\, c_1, c_2, \ldots, c_{t-1}\right) \doteq p(c_1, \ldots c_T)$
- Number of bits required to code $c_1, c_2, \ldots, c_T$ is $\lceil -\sum_{t=1}^{T} \log_2 p_t(c_t) \rceil$.
- We call $-\sum_{t=1}^{T} \log_2 p_t(c_t) = -log_2 p(c_1, \ldots c_T)$ the Cumulative log loss
- Holds for all sequences.