Estimation, Tracking and control using **Hedge**(η)

Yoav Freund

January 12, 2010

Outline

Estimation

Outline

Outline

The static discrete estimation problem

- A system has an internal unobservable state: $s \in \{0, 1, \ldots, K\}$

The static discrete estimation problem

- A system has an internal unobservable state: $s \in \{0, 1, \ldots, K\}$
- observations are corrupted versions of the state: $o_1, o_2, \ldots \quad o_t \in \{0, 1, \ldots, K\}$

The static discrete estimation problem

- A system has an internal unobservable state: $s \in \{0, 1, \ldots, K\}$
- observations are corrupted versions of the state: $o_1, o_2, \ldots \quad o_t \in \{0, 1, \ldots, K\}$
- Goal: given $o_1, o_2, \ldots, o_t$ compute prediction p_{t+1} that is close to o_{t+1}

Generative modeling

- Prob. of observation o conditioned on state s:
  $P(O = o | S = s)$

Generative modeling

- Prob. of observation o conditioned on state s: $P(O = o|S = s)$
- Given $o_1, o_2, \ldots, o_T$ define likelihood of state s as $\prod_{t=1}^{T} P(O = o_t|S = s)$.

Generative modeling

- Prob. of observation o conditioned on state s:
  $P(O = o|S = s)$
- Given $o_1, o_2, \ldots, o_T$ define likelihood of state s as
  $\prod_{t=1}^{T} P(O = o_t|S = s)$.
- max-likelihood estimator $\hat{s}$: s that maximized likelihood or log-likelihood:
  $\sum_{t=1}^{T} \log P(O = o_t|S = s)$

Generative modeling

- Prob. of observation o conditioned on state s: $P(O = o | S = s)$
- Given $o_1, o_2, \ldots, o_T$ define likelihood of state s as $\prod_{t=1}^{T} P(O = o_t | S = s)$.
- max-likelihood estimator $\hat{s}$: s that maximized likelihood or log-likelihood: $\sum_{t=1}^{T} \log P(O = o_t | S = s)$
- Predict using the estimate: $p_{t+1} = \hat{s}$

Generative modeling

- Prob. of observation o conditioned on state s: $P(O = o|S = s)$
- Given $o_1, o_2, \ldots, o_T$ define likelihood of state s as $\prod_{t=1}^{T} P(O = o_t|S = s)$.
- max-likelihood estimator $\hat{s}$: s that maximized likelihood or log-likelihood: $\sum_{t=1}^{T} \log P(O = o_t|S = s)$
- Predict using the estimate: $p_{t+1} = \hat{s}$
- Real-world problem - often $P(O = o|S = s)$ - the correct conditional distribution - is not known!

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p, o) = |p - o|$, $\ell_2(p, o) = (p - o)^2$

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p, o) = |p - o|$, $\ell_2(p, o) = (p - o)^2$
- Given $o_1, o_2, \ldots, o_T$ use the minimal loss estimate.

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p,o) = |p-o|$, $\ell_2(p,o) = (p-o)^2$
- Given $o_1, o_2, \ldots, o_T$ use the minimal loss estimate.
- No such thing as a "correct" loss.

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p, o) = |p - o|$, $\ell_2(p, o) = (p - o)^2$
- Given $o_1, o_2, \ldots, o_T$ use the minimal loss estimate.
- No such thing as a "correct" loss.
- No direct relation to estimation of hidden state.

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p,o) = |p-o|$, $\ell_2(p,o) = (p-o)^2$
- Given $o_1, o_2, \ldots, o_T$ use the minimal loss estimate.
- No such thing as a "correct" loss.
- No direct relation to estimation of hidden state.
- Often useful to define an outlier insensitive loss: $\ell(p,o) = \min(c, |p-o|)$

Estimation by minimizing loss

- Define a loss function to measure the discrepency between the prediction p_{t+1} and observation o_{t+1}.
- Examples: $\ell_1(p,o) = |p-o|$, $\ell_2(p,o) = (p-o)^2$
- Given $o_1, o_2, \ldots, o_T$ use the minimal loss estimate.
- No such thing as a "correct" loss.
- No direct relation to estimation of hidden state.
- Often useful to define an outlier insensitive loss: $\ell(p,o) = \min(c, |p-o|)$
- Leads to non-convex optimization problems.

Estimation using **Hedge**(η)

- Consider each prediction $p = 0, 1, \ldots, K$ as an expert.

Estimation using **Hedge**(η)

- Consider each prediction $p = 0, 1, \ldots, K$ as an expert.
- **Hedge**(η)will define a distribution over $0, 1, \ldots, K$.

Estimation using **Hedge**(η)

- Consider each prediction $p = 0, 1, \ldots, K$ as an expert.
- **Hedge**(η)will define a distribution over $0, 1, \ldots, K$.
- The distribution defines our confidence interval.

Estimation using **Hedge**(η)

- Consider each prediction $p = 0, 1, \ldots, K$ as an expert.
- **Hedge**(η)will define a distribution over $0, 1, \ldots, K$.
- The distribution defines our confidence interval.
- Guaranteed to perform almost as well as the best estimate in hindsite.

Estimation using **Hedge**(η)

- Consider each prediction $p = 0, 1, \ldots, K$ as an expert.
- **Hedge**(η)will define a distribution over $0, 1, \ldots, K$.
- The distribution defines our confidence interval.
- Guaranteed to perform almost as well as the best estimate in hindsite.
- Similar to Bayesian posterior distribution, but does not assume known distribution of noise.

The discrete **tracking** problem

- The internal state of the system changes **slowly** over time.
  $s_1, s_2, \ldots \quad s_t \in \{0, 1, \ldots, K\}$

The discrete **tracking** problem

- The internal state of the system changes **slowly** over time.
  $s_1, s_2, \ldots \quad s_t \in \{0, 1, \ldots, K\}$
- observations are corrupted versions of the state:
  $o_1, o_2, \ldots \quad o_t \in \{0, 1, \ldots, K\}$

The discrete **tracking** problem

- The internal state of the system changes **slowly** over time.
  $s_1, s_2, \ldots \quad s_t \in \{0, 1, \ldots, K\}$
- observations are corrupted versions of the state:
  $o_1, o_2, \ldots \quad o_t \in \{0, 1, \ldots, K\}$
- Goal: for each t, given $o_1, o_2, \ldots, o_t$ compute p_{t+1} - a prediction of o_{t+1}.

Tracking using Generative models

- To define a time varying generative model we need to know the **correct** distribution of the state transitions

Tracking using Generative models

- To define a time varying generative model we need to know the **correct** distribution of the state transitions
- Hidden Markov Models: define $Pr(S_{t+1}|S_t)$

Tracking using Generative models

- To define a time varying generative model we need to know the **correct** distribution of the state transitions
- Hidden Markov Models: define $Pr(S_{t+1}|S_t)$
- likelihood of observed sequence equal to sum over all possible hidden sequences, but can be computed efficiently using dynamic programming.

Tracking using Generative models

- To define a time varying generative model we need to know the **correct** distribution of the state transitions
- Hidden Markov Models: define $Pr(S_{t+1}|S_t)$
- likelihood of observed sequence equal to sum over all possible hidden sequences, but can be computed efficiently using dynamic programming.
- Popular method in speech recognition.

Tracking by loss minimization

- Only assumption: state changes slowly.

Tracking by loss minimization

- Only assumption: state changes slowly.
- Instead of cumulative loss, use exponentially discounted loss $L_{t+1} = (1 - \alpha)L_t + \ell_t$

Tracking by loss minimization

- Only assumption: state changes slowly.
- Instead of cumulative loss, use exponentially discounted loss $L_{t+1} = (1 - \alpha)L_t + \ell_t$
- $\alpha = 0$ corresponds to standard cumulative loss.

Tracking by loss minimization

- Only assumption: state changes slowly.
- Instead of cumulative loss, use exponentially discounted loss $L_{t+1} = (1 - \alpha)L_t + \ell_t$
- $\alpha = 0$ corresponds to standard cumulative loss.
- $\alpha > 0$ corresponds approximately to averaging over the previous $1/\alpha$ iterations.

Using **Hedge**(η)for tracking

- Use exponentially discounted loss.

Using **Hedge**$(\eta)$for tracking

- Use exponentially discounted loss.
- **Hedge**(η)guaranteed to perform almost as well as best expert with respect to exp. discounted loss.

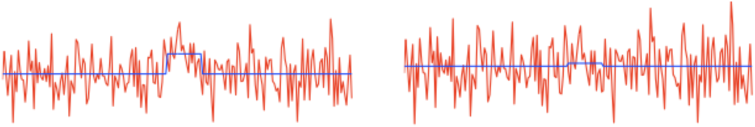
Using **Hedge**(η)for tracking

- Use exponentially discounted loss.
- **Hedge**(η)guaranteed to perform almost as well as best expert with respect to exp. discounted loss.
- Tracks state well when changes occur every $1/\alpha$ examples.

Using **Hedge**$(\eta)$for tracking

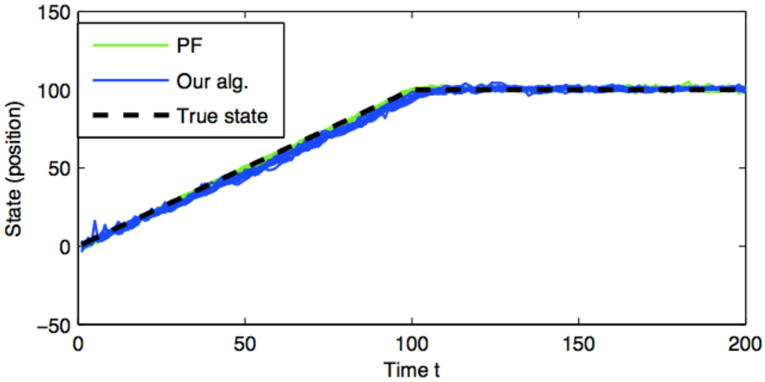
- Use exponentially discounted loss.
- **Hedge**(η)guaranteed to perform almost as well as best expert with respect to exp. discounted loss.
- Tracks state well when changes occur every $1/\alpha$ examples.
- Choosing the learning rate η is a significant problem.

Tracking using a noisy echo

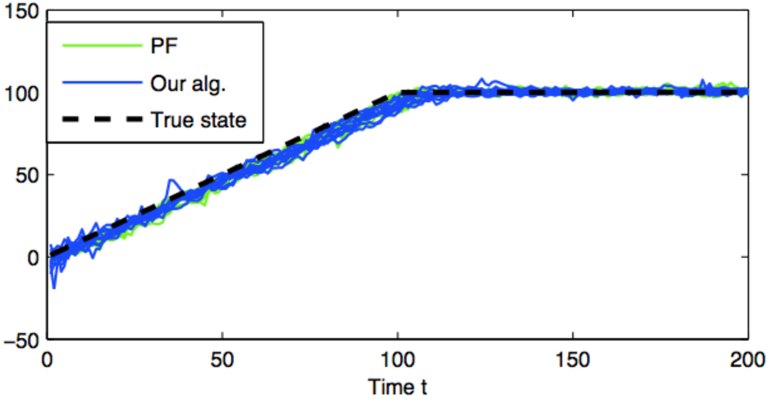


Low Noise High Noise

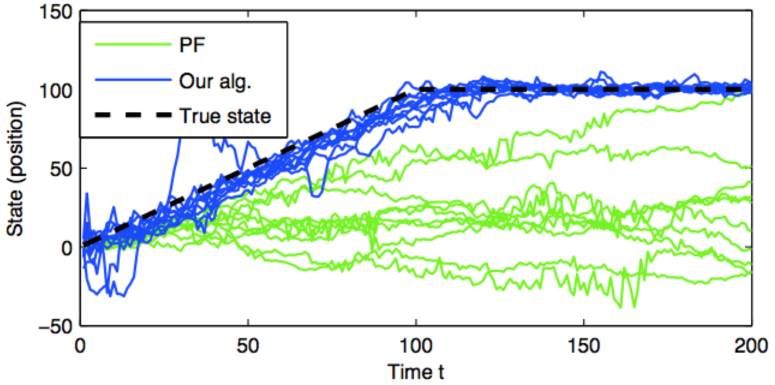
Tracking for $\sigma = 1$



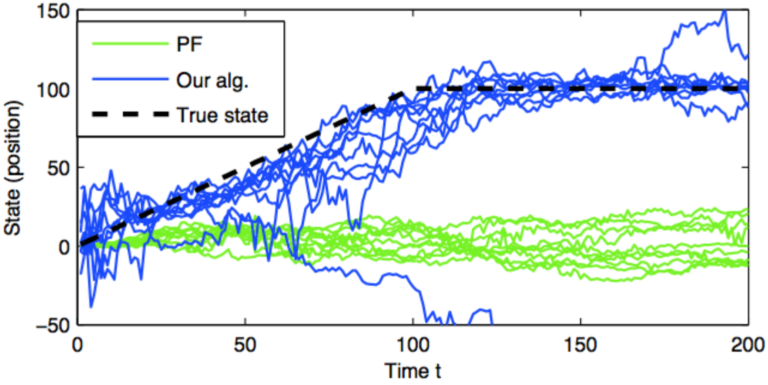
Tracking for $\sigma = 2$



Tracking for $\sigma = 4$



Tracking for $\sigma = 8$



Tracking dynamical systems

- Each expert can corresponds to a trajectory in state space.

Tracking dynamical systems

- Each expert can corresponds to a trajectory in state space.
- full state: a description of the state that is sufficient to predict the future.

Tracking dynamical systems

- Each expert can corresponds to a trajectory in state space.
- full state: a description of the state that is sufficient to predict the future.
- State of physical rigid object: location, speed, rotation, rotations speeds. In physics: "phase space"

Tracking dynamical systems

- Each expert can corresponds to a trajectory in state space.
- full state: a description of the state that is sufficient to predict the future.
- State of physical rigid object: location, speed, rotation, rotations speeds. In physics: “phase space”
- Without drift (dynamic noise) the trajectory is deterministically determined from the state.

Dealing with drift

- s_{t+1} is not fully determined by s_t.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.
- Split each expert s_t into a distribution over experts: $Pr(S_{t+1}|S_t)$.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.
- Split each expert s_t into a distribution over experts: $Pr(S_{t+1}|S_t)$.
- Use exponentially discounted loss to emphasize recent history.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.
- Split each expert s_t into a distribution over experts: $Pr(S_{t+1}|S_t)$.
- Use exponentially discounted loss to emphasize recent history.
- Number of experts grows exponentially.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.
- Split each expert s_t into a distribution over experts: $Pr(S_{t+1}|S_t)$.
- Use exponentially discounted loss to emphasize recent history.
- Number of experts grows exponentially.
- We can use dynamic programming - stil very expensive.

Dealing with drift

- s_{t+1} is not fully determined by s_t.
- We borrow from generative modeling.
- Split each expert s_t into a distribution over experts: $Pr(S_{t+1}|S_t)$.
- Use exponentially discounted loss to emphasize recent history.
- Number of experts grows exponentially.
- We can use dynamic programming - stil very expensive.
- We use monte-carlo sampling.

The discrete **control** problem

- The internal state of the system (plant) changes due to:

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.
  - Extermal control signal.

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.
  - Extermal control signal.
  - Drift.

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.
  - Extermal control signal.
  - Drift.
- Goal of the controller given the observation history, generate a control signal to bring the plant close to desired state.

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.
  - Extermal control signal.
  - Drift.
- Goal of the controller given the observation history, generate a control signal to bring the plant close to desired state.
  - In short amount of time.

The discrete **control** problem

- The internal state of the system (plant) changes due to:
  - Internal dynamics.
  - Extermal control signal.
  - Drift.
- Goal of the controller given the observation history, generate a control signal to bring the plant close to desired state.
  - In short amount of time.
  - Using small amount of power.

control based on generative models

- Requires generative model of plant

control based on generative models

- Requires generative model of plant
  - Dynamics of plant.

control based on generative models

- Requires generative model of plant
  - Dynamics of plant.
  - Distribution model of drift.

control based on generative models

- Requires generative model of plant
  - Dynamics of plant.
  - Distribution model of drift.
  - Distribution of observation noise.

control based on generative models

- Requires generative model of plant
  - Dynamics of plant.
  - Distribution model of drift.
  - Distribution of observation noise.
- Analysis: combine controller and plant into a single dynamic system and analyze its properties under the generative model.

control based on experts

- Expert is a mapping from past observations to a control signal.

control based on experts

- Expert is a mapping from past observations to a control signal.
- Loss is the difference between observation of desired observation (corresponding to desired state).

control based on experts

- Expert is a mapping from past observations to a control signal.
- Loss is the difference between observation of desired observation (corresponding to desired state).
- Similar to driving a car without understanding mechanics.

control based on experts

- Expert is a mapping from past observations to a control signal.
- Loss is the difference between observation of desired observation (corresponding to desired state).
- Similar to driving a car without understanding mechanics.
- Experts can be based on generative models. Can estimate the state of the plant, etc.

control based on experts

- Expert is a mapping from past observations to a control signal.
- Loss is the difference between observation of desired observation (corresponding to desired state).
- Similar to driving a car without understanding mechanics.
- Experts can be based on generative models. Can estimate the state of the plant, etc.
- Experts can be also be dumb!

control based on experts

- Expert is a mapping from past observations to a control signal.
- Loss is the difference between observation of desired observation (corresponding to desired state).
- Similar to driving a car without understanding mechanics.
- Experts can be based on generative models. Can estimate the state of the plant, etc.
- Experts can be also be dumb!
- When system is complex, dumb expert is likely to perform better than complex expert.