Online learning in repeated matrix games

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Outline

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- Game repeated many times.

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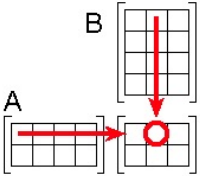
Mixed strategies in matrix notation



$$(A \times B)_{12} = \sum_{r=1}^{4} a_{1r} b_{r2} = a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} + a_{14}b_{42}$$

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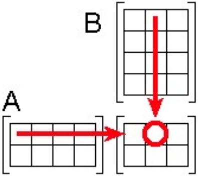

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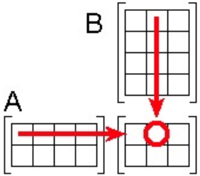

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$$\mathbf{M}(\mathbf{P}, \mathbf{Q}) = \mathbf{P}^T \mathbf{M} \mathbf{Q} = \sum_{i=1}^{n} \sum_{j=1}^{m} \mathbf{P}(i)\mathbf{M}(i,j)\mathbf{Q}(j)$$

The basic algorithm

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- $\eta > 0$ is the learning rate.

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- Any sequence of mixed strat. $\mathbf{Q}_1, \ldots, \mathbf{Q}_T$
- The sequence $\mathbf{P}_1, \ldots, \mathbf{P}_T$ produced by basic alg using $\eta > 0$ satisfies

$$\sum_{t=1}^{T} \mathbf{M}(\mathbf{P}_t, \mathbf{Q}_t) \leq \left(\frac{1}{1 - e^{-\eta}} \right) \min_{\mathbf{P}} \left[\eta \sum_{t=1}^{T} \mathbf{M}(\mathbf{P}, \mathbf{Q}_t) + \mathrm{RE}\left(\mathbf{P} \parallel \mathbf{P}_1 \right) \right]$$

Corollary

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- Where

$$\Delta_{T,n} = \sqrt{\frac{2\ln n}{T}} + \frac{\ln n}{T} = O\left(\sqrt{\frac{\ln n}{T}}\right).$$

Main Lemma

On any iteration t

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For any mixed strategy $\tilde{\mathbf{P}}$

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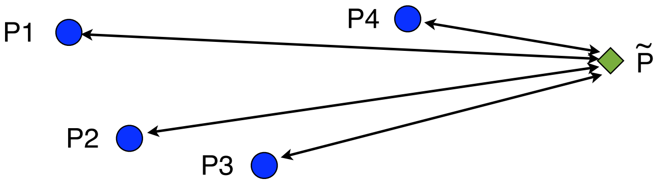
$$\mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t+1}\right) - \mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t}\right) \leq \eta \mathbf{M}(\tilde{\mathbf{P}}, \mathbf{Q}_t) - (1 - e^{-\eta})\mathbf{M}(\mathbf{P}_t, \mathbf{Q}_t)$$

Visual intuition

$$\mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t+1}\right) - \mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t}\right) \leq \eta \mathbf{M}(\tilde{\mathbf{P}}, \mathbf{Q}_t) - (1 - e^{-\eta})\mathbf{M}(\mathbf{P}_t, \mathbf{Q}_t)$$

Visual intuition

$$\mathrm{RE}\left(\tilde{\mathbf{P}} \parallel \mathbf{P}_{t+1}\right) - \mathrm{RE}\left(\tilde{\mathbf{P}} \parallel \mathbf{P}_t\right) \leq \eta \mathbf{M}(\tilde{\mathbf{P}}, \mathbf{Q}_t) - (1 - e^{-\eta}) \mathbf{M}(\mathbf{P}_t, \mathbf{Q}_t)$$



Proof of Lemma (1)

$$\mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t+1}\right) - \mathrm{RE}\left(\tilde{\mathbf{P}} \;\|\; \mathbf{P}_{t}\right)$$

Proof of Lemma (1)

$$\mathrm{RE}\left(\tilde{\mathbf{P}} \parallel \mathbf{P}_{t+1}\right) - \mathrm{RE}\left(\tilde{\mathbf{P}} \parallel \mathbf{P}_{t}\right)$$
$$= \sum_{i=1}^{n} \tilde{\mathbf{P}}(i) \ln \frac{\tilde{\mathbf{P}}(i)}{\mathbf{P}_{t+1}(i)} - \sum_{i=1}^{n} \tilde{\mathbf{P}}(i) \ln \frac{\tilde{\mathbf{P}}(i)}{\mathbf{P}_{t}(i)}$$

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&= \sum_{i=1}^{n} \tilde{\mathbf{P}}(i) \ln \frac{\mathbf{P}_{t}(i)}{\mathbf{P}_{t+1}(i)} \\
&= \sum_{i=1}^{n} \tilde{\mathbf{P}}(i) \ln \frac{Z_t}{e^{\eta \mathbf{M}(i,\mathbf{Q}_t)}}
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Proof of Lemma (2)

$$= \eta \sum_{i=1}^{n} \tilde{\mathbf{P}}(i)\mathbf{M}(i, \mathbf{Q}_t) + \ln Z_t$$

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&= \eta \mathbf{M}(\tilde{\mathbf{P}},\mathbf{Q}_t) + \ln\left(1-(1-e^{-\eta})\mathbf{M}(\mathbf{P}_t,\mathbf{Q}_t)\right) \\
&\leq \eta \mathbf{M}(\tilde{\mathbf{P}},\mathbf{Q}_t) + (1-e^{-\eta})\mathbf{M}(\mathbf{P}_t,\mathbf{Q}_t)
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The minmax Theorem

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In words: for mixed strategies, choosing second gives no advantage.

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Let $\overline{\mathbf{P}} \doteq \frac{1}{T}\sum_{t=1}^{T}\mathbf{P}_t$ and $\overline{\mathbf{Q}} \doteq \frac{1}{T}\sum_{t=1}^{T}\mathbf{Q}_t$

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\min_{\mathbf{P}} \max_{\mathbf{Q}} \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{Q} &\leq \max_{\mathbf{Q}} \overline{\mathbf{P}}^{\mathrm{T}}\mathbf{M}\mathbf{Q} \\
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&\le \frac{1}{T}\sum_{t=1}^{T} \max_{\mathbf{Q}} {\mathbf{P}_t}^{\mathrm{T}}\mathbf{M}\mathbf{Q}
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Proving minmax Theorem using online learning (2)

$$= \frac{1}{T}\sum_{t=1}^{T} \mathbf{P}_t^{\mathrm{T}} \mathbf{M} \mathbf{Q}_t \qquad \text{by definition of } \mathbf{Q}_t$$

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&\leq \min_{\mathbf{P}} \frac{1}{T}\sum_{t=1}^{T} \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{Q}_t + \Delta_{T,n} && \text{by the Corollary}
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but $\Delta_{T,n}$ can be set arbitrarily small.

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- If opponent is not optimally adversarial (limited by knowledge, computationa power...) then learning gives better performance than min-max.
- Is it realistic to assume that markets are at equilibrium?
- If game is not zero sum (allows incentives to collaborate) and all players use learning then game converges to correlated equilibrium.