Optimal Play

- minmax theorem:

$$\min_{\mathbf{P}} \max_{\mathbf{Q}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) = \max_{\mathbf{Q}} \min_{\mathbf{P}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) = \text{value } v \text{ of game}$$

- optimal strategies:
  - $\mathbf{P}^* = \arg\min_{\mathbf{P}} \max_{\mathbf{Q}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) =$ minmax strategy
  - $\mathbf{Q}^* = \arg\max_{\mathbf{Q}} \min_{\mathbf{P}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) =$ maxmin strategy
- in words:
  - Mindy's minmax strategy $\mathbf{P}^*$ guarantees loss $\leq v$ (regardless of Max's play)
  - optimal because Max has maxmin strategy $\mathbf{Q}^*$ that can force loss $\geq v$ (regardless of Mindy's play)
- e.g.: in RPS, $\mathbf{P}^* = \mathbf{Q}^* =$ uniform
- solving game $=$ finding minmax/maxmin strategies