Tracking the best Expert

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February 9, 2006

Outline

Review

mixable loss functions

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4. Each expert incurs loss $\ell_i^t = \lambda(\omega^t, \gamma_i^t)$
   The learner incurs loss $\ell_A^t = \lambda(\omega^t, \gamma^t)$

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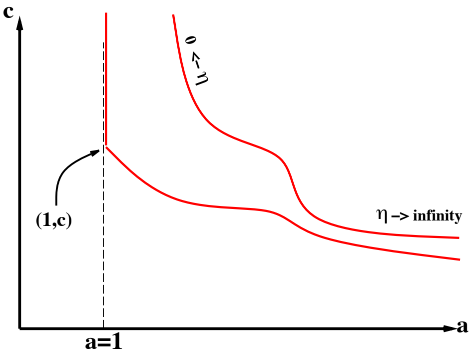

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- No triangle inequality
  $\exists p_1, p_2, p_3 \;\; \lambda(p_1,p_3) > \lambda(p_1,p_2) + \lambda(p_2,p_3)$

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- Corresponds to regression.

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- Probability of making a mistake if predicting 0 or 1 using a biased coin
- If $P[\omega^t = 1] = q, \;\; P[\omega^t = 0] = 1 - q$, then the optimal prediction is
$$\gamma^t = \begin{cases} 1 & \text{if } q > 1/2, \\ 0 & \text{otherwise} \end{cases}$$

Mixable Loss Functions

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Summary of bounds for mixable losses

TRACKING THE BEST EXPERT

Loss Functions:	c values: $(\eta = 1/c)$	
	$\mathbf{pred}_{\mathrm{wmean}}(v, x)$	$\mathbf{pred}_{\mathrm{Vovk}}(v, x)$
$L_{\mathrm{sq}}(p, q)$	2	$1/2$
$L_{\mathrm{ent}}(p, q)$	1	1
$L_{\mathrm{hel}}(p, q)$	1	$1/\sqrt{2}$

Figure 2. $(c, 1/c)$-realizability: c values for loss and prediction function pairin

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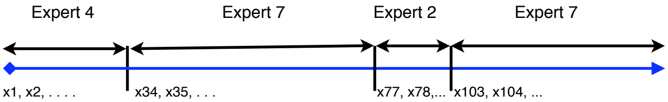

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- The log-loss regret is at most $(k+1)\log n + k \log \frac{l}{k} + k$
- Requires maintaining $O\left(n^{k+1}\left(\frac{el}{k}\right)^k\right)$ weights.

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- There is an exponential weights algorithm with learning rate η that achieves (in the non-switching case) a bound

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- Then using the partition-expert algorithm for the switching-experts case we get a bound on the regret $\frac{1}{\eta}\left((k+1)\log n + k \log \frac{l}{k} + k\right)$

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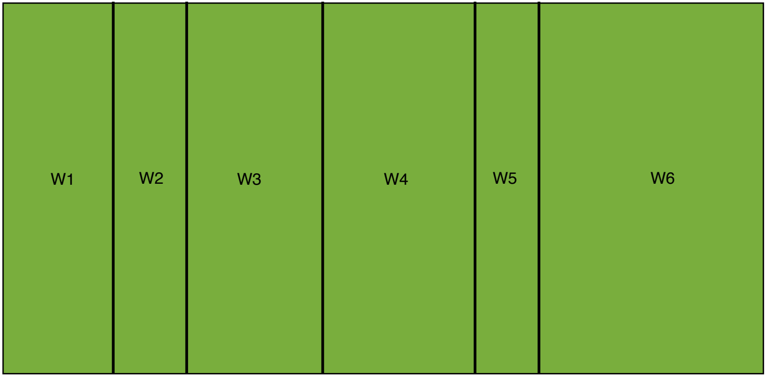
- Fixed-share:

$$\begin{aligned} pool &= \alpha \sum_{i=1}^{n} w^m_{t,i} \\ w^s_{t+1,i} &= (1-\alpha) w^m_{t,i} + \frac{1}{n-1}\left(pool - \alpha w^m_{t,i}\right) \end{aligned}$$

The fixed-share algorithm



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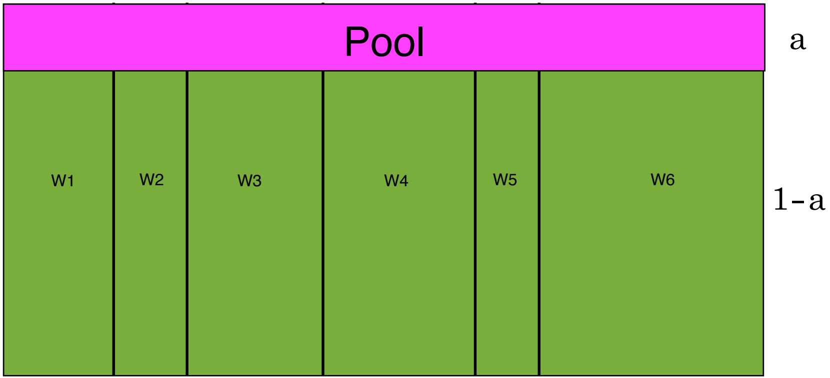

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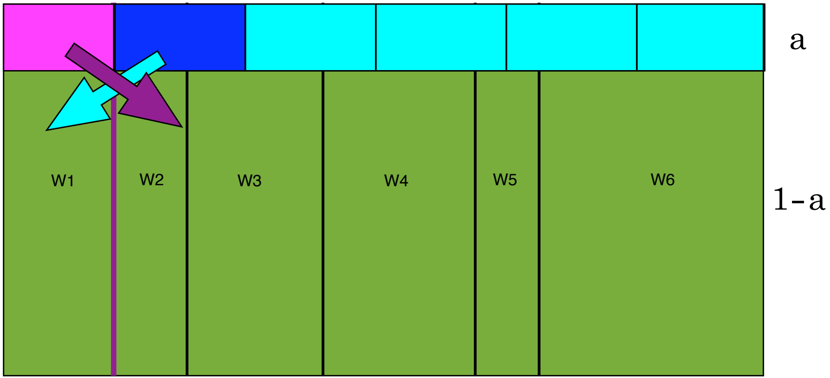

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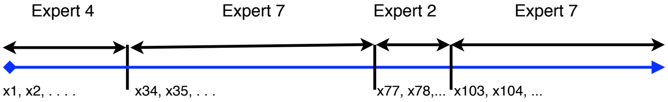
$$L_A \leq \frac{1}{\eta} \sum_{i=1}^{n} w^s_{l+1,i}$$

- The harder question is how to lower bound $\sum_{i=1}^{n} w^s_{l+1,i}$

Lower bounding the final total weight

- Fix some switching experts sequence:



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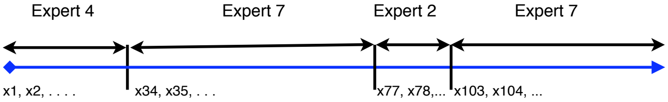

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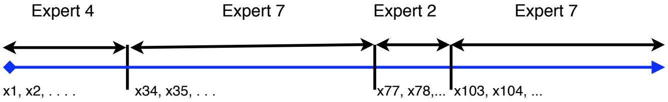

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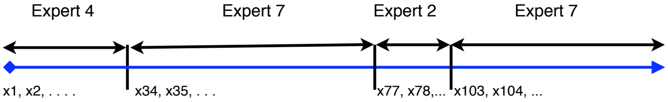

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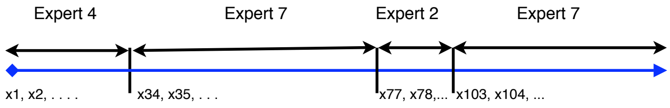

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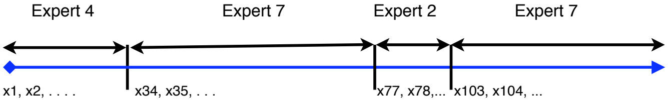

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  - $1 - \alpha$ on iterations with no switch.
  - $\frac{\alpha}{n-1}$ on iterations where a switch occurs.

Bound for arbitrary α

- Combining we lower bound the final weight of the last expert in the sequence

$$w^s_{l+1,e_k} \geq \frac{1}{n} e^{-\eta L_*} (1-\alpha)^{l-k-1} \left(\frac{\alpha}{n-1} \right)^k$$

Where L_* is the cumulative loss of the switching sequence of experts.

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- Combining the upper and lower bounds we get that for any sequence

$$L_A \leq L_* + \frac{1}{\eta} \left(\ln n + (l-k-1) \ln \frac{1}{1-\alpha} + k \left(\ln \frac{1}{\alpha} + \ln(n-1) \right) \right)$$

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Where

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- Not so for square loss!

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- In the fixed-share algorithm, the weight of a suboptimal expert never decreases below α/n.
- The algorithm does not concentrate only on the best expert, even if the last switch is in the distant past.
- The regret depends on the length of the sequence.

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- we can get a regret bound that depends only on the number of switches, not on the lenght of the sequence.
- Requires that the loss be bounded.
- Works for square loss, but not for log loss!

Variable-share

$$
\begin{aligned}
pool &= \sum_{i=1}^{n} \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^{m}_{t,i} \\
w^{s}_{t+1,i} &= (1-\alpha)^{\ell_{t,i}} w^{m}_{t,i} + \frac{1}{n-1}\left(pool - \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^{m}_{t,i}\right)
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If $\ell_{t,i} = 0$, then expert i does not contribute to the pool.
Expert can get fraction of the total weight arbitrarily close to 1.
Shares the weight quickly if $\ell_{t,i} > 0$

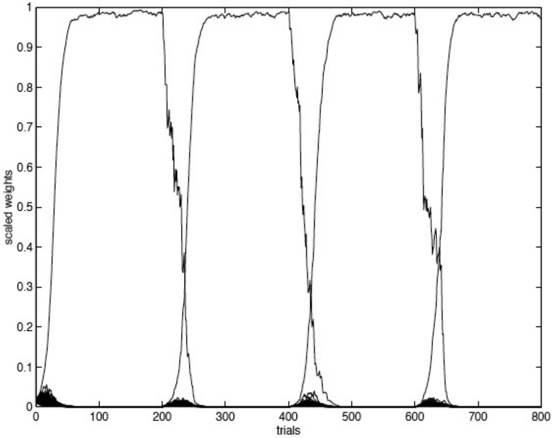
Bound for variable share

- $$\frac{1}{\eta}\ln n + \left(1 + \frac{1}{(1-\alpha)\eta}\right)L_* + k\left(1 + \frac{1}{\eta}\left(\ln n - 1 + \ln\frac{1}{\alpha} + \ln\frac{1}{1-\alpha}\right)\right)$$

Bound for variable share

- $$\frac{1}{\eta}\ln n + \left(1 + \frac{1}{(1-\alpha)\eta}\right)L_* + k\left(1 + \frac{1}{\eta}\left(\ln n - 1 + \ln\frac{1}{\alpha} + \ln\frac{1}{1-\alpha}\right)\right)$$
- α should be tuned so that it is (close to) $\frac{k}{2k+L_*}$

An experiment using variable share



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- Next class - how to do as well with just one weight per expert.