Computation with Absolutely No Space Overhead

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Technical University of Berlin

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Outline

1. The Model of Overhead-Free Computation
   - The Standard Model of Linear Space
   - Our Model of Absolutely No Space Overhead
2. The Power of Overhead-Free Computation
   - Palindromes
   - Linear Languages
   - Context-Free Languages with a Forbidden Subword
   - Languages Complete for Polynomial Space
3. Limitations of Overhead-Free Computation
   - Linear Space is Strictly More Powerful

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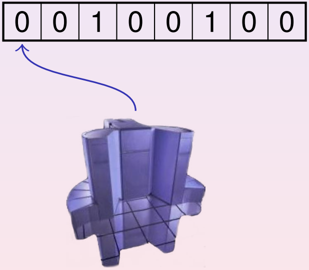
The Standard Model of Linear Space

tape



Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

The Standard Model of Linear Space

tape

$	0	1	0	0	1	0	0

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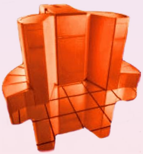

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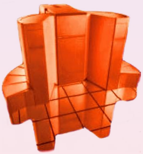

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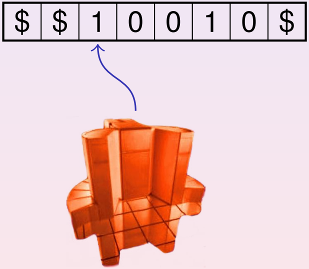

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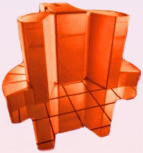

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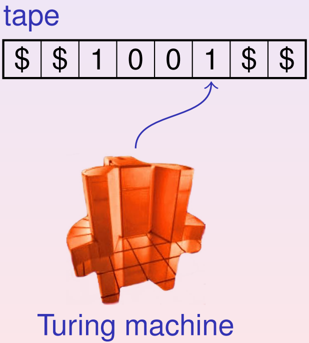

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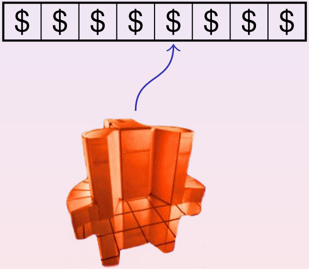

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The Standard Model of Linear Space

tape

$	$	$	$	$	$	$	$

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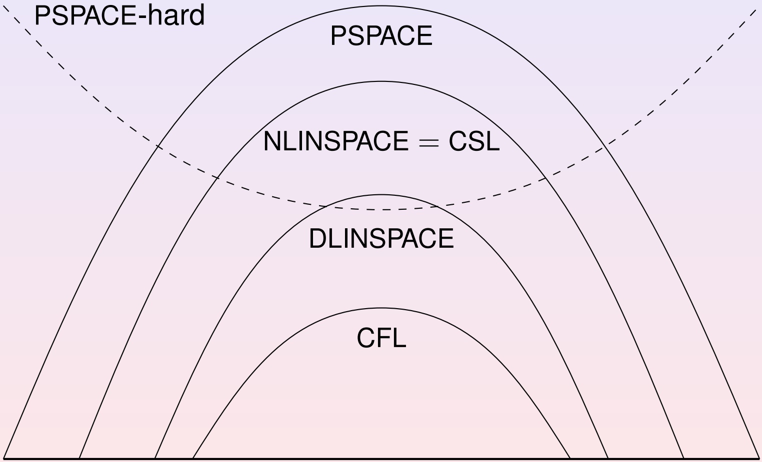
Linear Space is a Powerful Model



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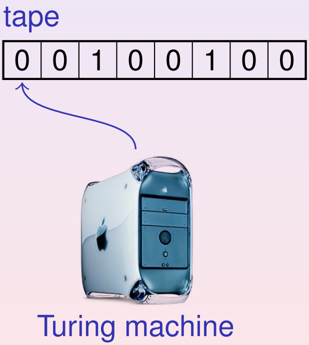
Our Model of “Absolutely No Space Overhead”



Characteristics

- Input fills fixed-size tape
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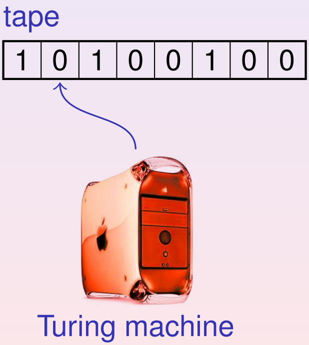
Our Model of "Absolutely No Space Overhead"



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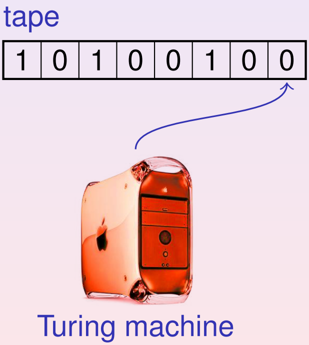

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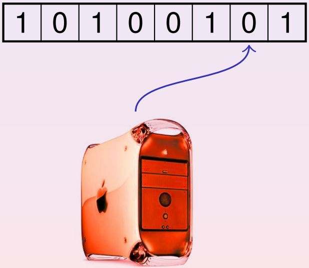

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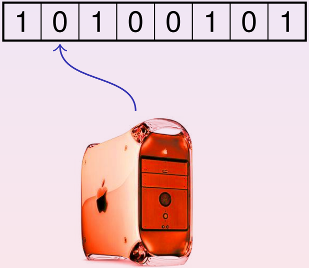

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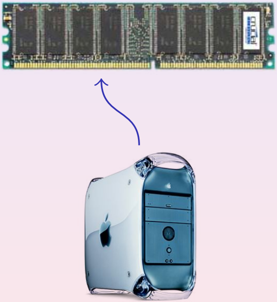

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

1. it has only a single tape,
2. writes only on input cells,
3. writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

- **DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- **$\mathrm{DOF}_{\mathrm{poly}}$** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- **NOF** is the nondeterministic version of DOF,
- **$\mathrm{NOF}_{\mathrm{poly}}$** is the nondeterministic version of $\mathrm{DOF}_{\mathrm{poly}}$.

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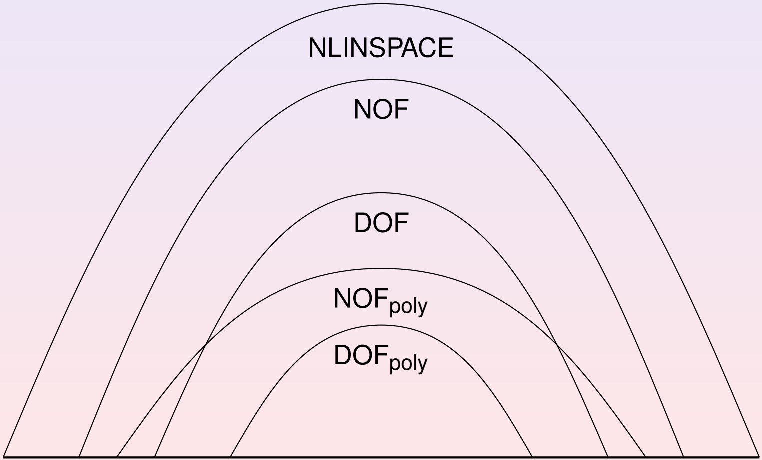
Simple Relationships among Overhead-Free Computation Classes



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Palindromes Can be Accepted in an Overhead-Free Way

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

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tape

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overhead-free machine

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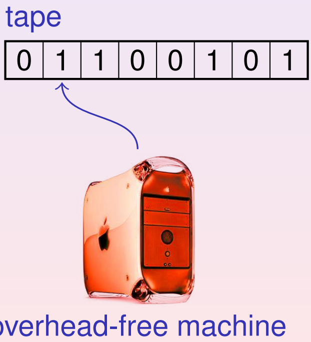

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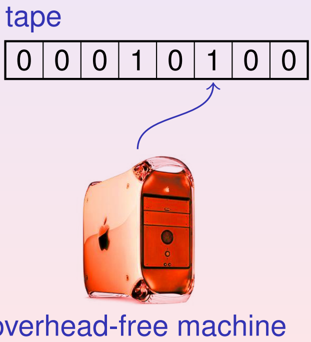

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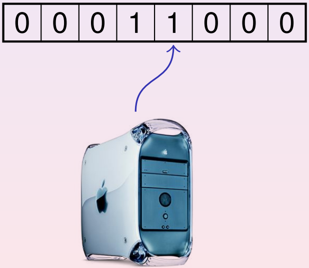

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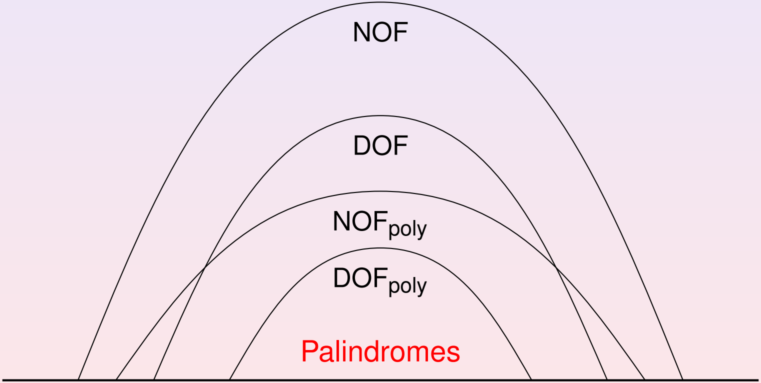
Relationships among Overhead-Free Computation Classes



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A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

Example

$G_1: S \to 00S0 \mid 1$ and $G_2: S \to 0S10 \mid 0$.

Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

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$G_1: S \to 00S0 \mid 1$ is deterministic.
$G_2: S \to 0S10 \mid 0$ is not deterministic.

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Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathrm{DOF}_{\mathrm{poly}}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly}.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

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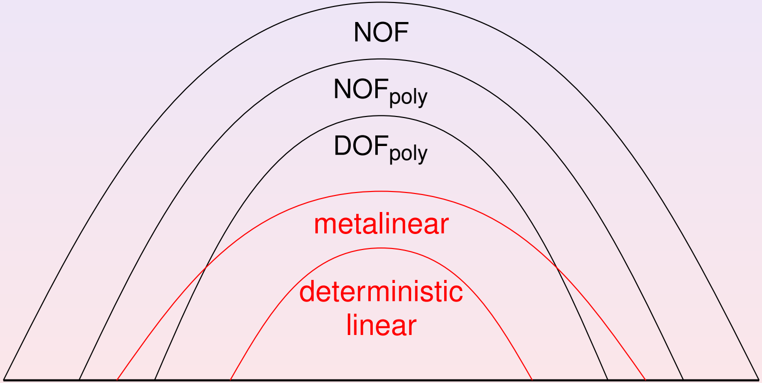

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Definition of Almost-Overhead-Free Computations

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A Turing machine is almost-overhead-free if

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Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word. Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

Skip proof

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

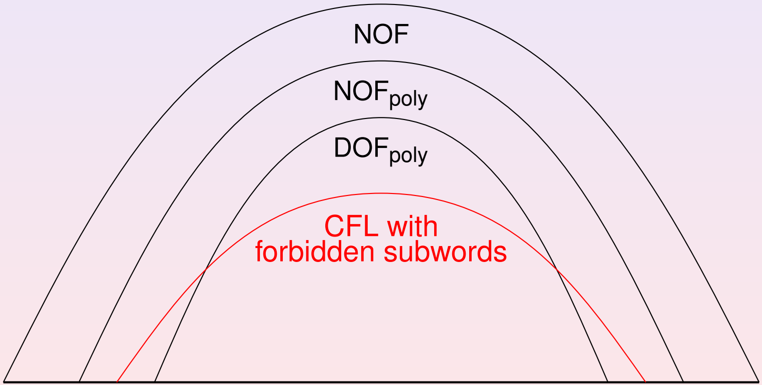
Theorem

Let L be a context-free language with a forbidden word. Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time. □

Relationships among Overhead-Free Computation Classes



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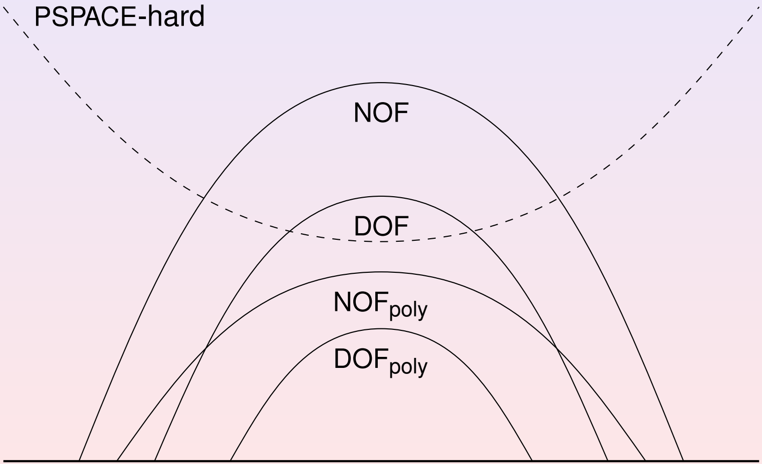
Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF *contains languages that are complete for* PSPACE.

Proof details

Relationships among Overhead-Free Computation Classes



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Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

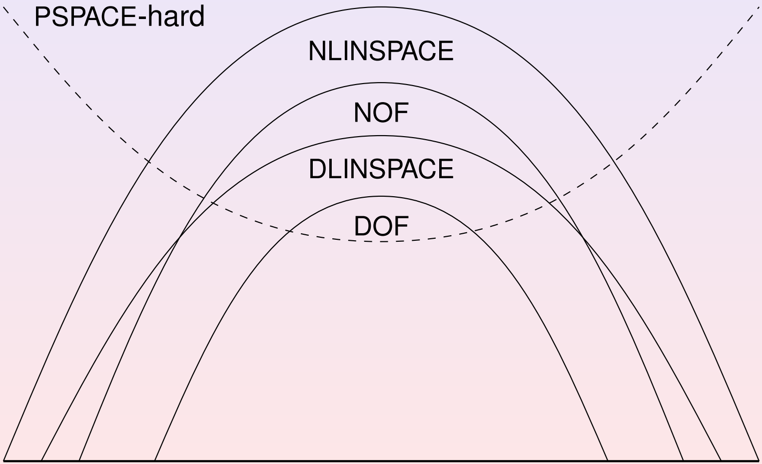
DOF $\subsetneq$ DLINSPACE.

Theorem

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Theorem

DOUBLE-PALINDROMES $\in$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

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Appendix Outline

4 Appendix
- Complete Languages
- Improvements for Context-Free Languages
- Abbreviations

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF *contains languages that are complete for* PSPACE.

Proof.

1. Let $A \in$ DLINSPACE be PSPACE-complete. Such languages are known to exist.
2. Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ.
3. Let $h \colon \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
4. Then $h(L)$ is in DOF and it is PSPACE-complete. $\square$

Return

Improvements

Theorem

1. $\mathrm{DCFL} \subseteq \mathrm{DOF}_{\mathrm{poly}}$.
2. $\mathrm{CFL} \subseteq \mathrm{NOF}_{\mathrm{poly}}$.

Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
$\mathrm{DOF}_{\mathrm{poly}}$	Deterministic Overhead-Free, polynomial time.
$\mathrm{DOF}_{\mathrm{poly}}$	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.