Internal Regret and Calibration

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Outline

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- For log loss we have $\max_i R_i^n = O(\ln N)$

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- We want an algorithm such that $\max_{(i,j)} R_{(i,j),n} = o(n)$

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- No deterministic algorithm can be calibrated for all sequences.

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- If the prediction is not ϵ calibrated, then the internal regret has to be large.

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- We solve the fixed point equation

$$\mathbf{p}_t = \sum_{i \neq j} \Delta_{(i,j),t} \mathbf{p}_t^{i \to j}$$