

Prove that the set of prime numbers is infinite

Proof by Contradiction

The set of primes can be written as  $\{p_1 < p_2 < \dots < p_n\}$  for some  $n < \infty$

Define  $q = \prod_{i=1}^n p_i + 1$

The remainder from dividing  $q$  by  $p_i$  (for any  $1 \leq i \leq n$ ) is 1

$q$  has no prime dividers, therefor  $q$  is prime

There is a prime outside the set  $\{p_1, p_2, \dots, p_n\}$

QED