

Auto-Encoding Variational Bayes

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Introduction

We implement an Auto-Encoding Variational Bayes model and try and reproduce some of the results in (Kingma et. al., 2013). We start with an overview of the auto-encoding variational Bayes model and then describe experimental results and technical details of our implementation.

In our problem setting we are given some dataset $X = \{x^{(i)}\}_{i=1}^N$ of N i.i.d. samples of some random variable. We assume this data was generated from some latent random variable z where $z \sim p_{\theta^*}(z)$ some prior distribution, and $x \sim p_{\theta^*}(x|z)$ some conditional distribution.

In order to model this process in a tractable way we use variational inference. We introduce a new distribution $q_{\phi}(z|x)$, with variational parameters ϕ , to approximate the true posterior $p_{\theta}(z|x)$ and call this the encoder. We also refer to $p_{\theta}(x|z)$ as the decoder. We then minimize the evidence lower bound:

$$\log p_{\theta}(x^{(i)}) \geq \mathcal{L}(\theta, \phi; x^{(i)})$$

$$\mathcal{L}(\theta, \phi; x^{(i)}) = -D_{KL}(q_{\phi}(z|x^{(i)})||p_{\theta}(z)) + \mathbb{E}_{q_{\phi}(z|x^{(i)})}[\log p_{\theta}(x^{(i)}|z)]$$

Taking derivatives with respect to ϕ can be tricky so we reparamaterize z using a differentiable transform $g_{\phi}(\epsilon, x)$ where $\epsilon \sim p(\epsilon)$. $g_{\phi}(\epsilon, x)$ and $p(\epsilon)$ will change depending on what distribution $q_{\phi}(z|x)$ is. We describe the details of our implementation in the details section.

Details

For our model we used a Gaussian encoder and a Bernoulli decoder. Since our encoder is Gaussian we have:

$$\log q_{\phi}(z|x^{(i)}) = \log \mathcal{N}(z; \mu^{(i)}, \sigma^{2(i)} I)$$

$$z^{(i)} \sim q_{\phi}(z|x^{(i)}), \epsilon \sim \mathcal{N}(0, I)$$

$$z^{(i)} = g_{\phi}(\epsilon, x^{(i)}) = \mu^{(i)} + \sigma^{(i)} \odot \epsilon$$

Our encoding step is then:

$$\begin{aligned} h &= \tanh(W_3 x + b_3) \\ \mu &= W_4 h + b_4 \\ \log \sigma^2 &= W_5 h + b_5 \\ z &= \mu + \sigma \odot \epsilon \end{aligned} \quad \text{where } \epsilon \sim \mathcal{N}(0, I)$$

And our decoding step is:

$$\begin{aligned} y &= \sigma(W_2 \tanh(W_1 z + b_1) + b_2) \\ \log p_\theta(x|z) &= \sum_{i=1}^D x_i \log y_i + (1 - x_i) \log(1 - y_i) \end{aligned}$$

Where $\theta = \{W_1, W_2, b_1, b_2\}$ and $\phi = \{W_3, W_4, W_5, b_3, b_4, b_5\}$. We can use these to minimize an estimated evidence lower bound:

$$\mathcal{L}(\theta, \phi, x^{(i)}) \approx \frac{1}{2} \sum_{j=1}^J \left(1 + \log((\sigma_j^{(i)})^2) - (\mu_j^{(i)})^2 - (\sigma_j^{(i)})^2 \right) + \log p_\theta(x^{(i)}, z^{(i)})$$

Where J is the dimension of the latent variable z . The details of the KL divergence term can be found in the appendix of (Kingma et. al., 2013).

Experiments

For our experiments we used the MNIST data set. This gave us a dimension of 784 for our data, x . We used a dimension of 2 for our latent variable z to make it easier to visualize. We used a hidden dimension of 512 for h . We trained for 2000 epochs or roughly 110 million images with a batch size of 128.

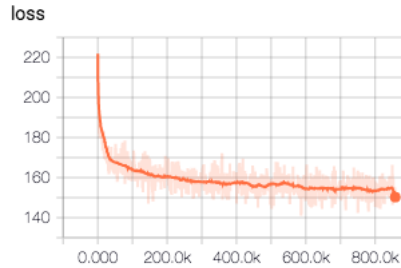


Figure 1: Training Loss

Figure 1 shows the negative log likelihood for the training data over iterations. We were able to achieve an average negative log likelihood of about

155 which is slightly worse than what (Kingma et. al., 2013) reported. We achieved a negative log likelihood of 156 on the test set. We tested 2 other models, one with a hidden dimension of 100 and a latent dimension of 3 and achieved a negative log likelihood of 150 on our test set, another model we used a hidden dimension of 512 and a latent dimension of 3 and achieved a negative log likelihood of 144 on our test set.

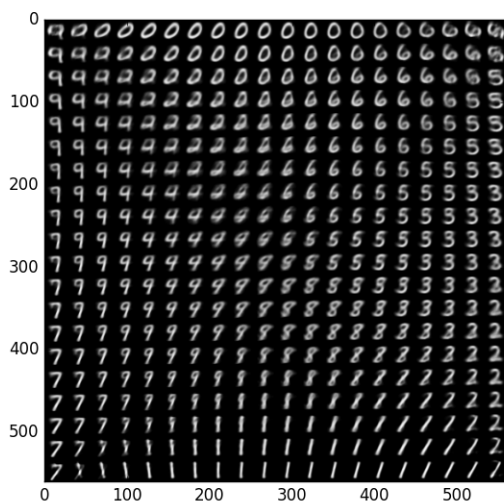


Figure 2: MNIST Manifold

In Figure 2 we sample a unit square from the latent variable and produce an image similar to Figure 4 in (Kingma et. al., 2013). What is interesting to note is that both images place 0 at the top and 1 at the bottom but the other digits are moved around slightly (except for possibly 7).

References

- [Kingma et. al. 2013] Diederik P. Kingma, Max Welling. 2013. Auto-Encoding Variational Bayes.