Deep Reinforcement Learning

Special Focus: Continuous Control

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Markov Decision Processes and Reinforcement Learning

Problem setting

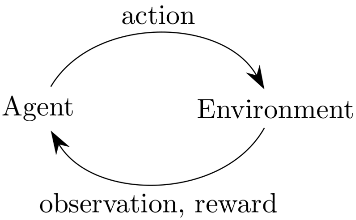


Figure 1: RL General Idea

- An agent is within an environment
- The agent is to complete some task and receive reward
- It solves this task over some amount of time steps

Markov Decision Processes

The environment in reinforcement learning can be described as a Markov Decision Process

This relies on the Markov Property (here described as a Markov State):

$$\mathbb{P}[S_{t+1}|S_t] = \mathbb{P}[S_{t+1}|S_1 ... S_t]$$

This means:

- The future is independent of the past, given the present
- The state is a sufficient statistic of the future
- All previous states can be thrown away and the same result will still be calculated

Note: For the Markov property to hold, the environment must be fully observable.

MDP: Observability

- In a *fully observable environment*, the agent's internal state is the same as the environment's internal state
  - i.e., the agent knows how the environment works exactly, and can therefore predict what each of its action will do with 100% accuracy
  - Put formally, the observation at time t is the same as both the agent's and environment's internal representations $O_t = S_t^a = S_t^e$
  - Can be represented with an MDP
- In a *partially observale environment*, the agent only indirectly observes the environment's state
  - The agent must construct it's own internal state based on its belief/construction of the environment state
  - Can be represented with a *Partially Observable Markov Decision Process*, POMDP

MDP: From Chains to Reward Processes

Chain: A *Markov Process* or *Chain* is a random sequence of states with the Markov property, defined by tuple:

$$\langle S, P \rangle$$

where S is the (finite) state space and P is the state transition matrix (matrix of state transition probabilities)

MDP: From Chains to Reward Processes

Markov Reward Process: Add in reward values to a Markov chain. Our tuple becomes:

$$\langle S, P, R, \gamma \rangle$$

where R is a reward function and γ is a discount factor, $\gamma \in [0, 1]$

Now that we have reward, we can calculate the total reward of a sequence/chain:

$$G_t = R_{t+1} + \gamma R_{t+2} + ... = \sum_{k=0} \gamma^k R_{t+k+1}$$

MDP: Value Functions

The *State-Value Function* gives the long term value of state s, i.e. the expected reward if the agent starts in this state

$$V(s) = \mathbb{E}(G_t | S_t = s)$$

We have to take the expectation because G_t is random; we need to know the expected value based on all random permutations of traversals through the Markov process

MDP: Now to the Markov Decision Process

Up until now, actions have been completely random–now we add a policy to choose actions

It can be described as an "MRP with decisions." We add in the agent to our tuple representation:

$$\langle S, A, P, R, \gamma \rangle$$

where A is a finite set of actions

P and R now depend on actions taken; formally:

$$P_{ss'}^{a} = \mathbb{P}(R_{t+1} | S_t = s, A = a)$$
$$R_{s}^{a} = \mathbb{E}(R_{t+1} | S_t = s, A = a)$$

Reminder: A *policy* is a function mapping states to actions; it tells the agent what to do given the state

MDP: Value Functions Revisited

The *State-Value Function* remains mostly the same in MDP as in MRP, except it depends on the policy:

$$V^{\pi}(s) = \mathbb{E}_{\pi}(G_t | S_t)$$

"The expectation when we sample all actions according to this policy π"; the value of a state

The *Action-Value Function* is defined as how good it is to take a particular action when the agent is in a particular state:

$$Q^{\pi}(s, a) = \mathbb{E}_{\pi}(G_t | S_t = s, A_t = a)$$

"The expected return starting from state s, taking action a, and then following policy π"; the value of an action

Note: we can also define these as recursive *Bellman Equations* where they refer to themselves instead of with G_t

MDP: Solving Reinforcement Learning

We want to maximize the value of our actions based on future reward, therefore (* denotes max/optimal function):

$$V^*(s) = \max_a Q^*(s, a)$$

$$Q^*(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V^*(s')$$

We can nest these to get:

$$Q^*(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a \max_a Q^*(s, a)$$

This is the *Bellman Optimality Equation* (note: it can be nested in the other direction too to solve for $V_*(s)$)

Solve this, and the reinforcement learning problem is solved.

Method to solve Q^*

- Iteratively act through *episodes*
- Backpropagate reward in order to calculate Q values which tell the values of actions
- Take the maximum Q value at each time step
- Stores Q values into a table

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t \cdot (R_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$$

DQN

- Uses a neural network to learn Q^*
- Can be thought of as looking at a state row in the Q-table, then taking the argmax

Continuous Actions

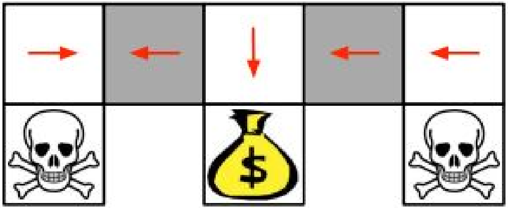
- If we can't (or don't want to) discretize the action space, the Q-table becomes incalculable
- Would equate to an infinite length table
- Therefore, Q-learning (and by extension DQN) will not work when dealing with continuous actions

Action	State				
	Stay	Left	Right	Forward	Backward
0	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
1	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
2	0	0	-1	-1	10
3	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
4	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

Figure 2: Q-Table

Q-Learning – Deterministic Policies

- A deterministic policy can lead an agent into an infinite loop
- Imagine this rule from some policy:
  - "Whenever there is a wall to the north and south, go left"
- If we applied this policy to the problem pictured, the agent would get stuck
- This can be solved by using a stochastic policy and leaving γ active during test time



Some Helpful Definitions

To follow everything to come, it is necessary to have a general grasp of the following concepts:

- **Agent**: what performs actions in the environment; wants to maximize future reward
- **Environment**: where the agent resides and what gives observations and reward; interacted with by agent
- **Reward**: R_t; reward at time step t, scalar
- **Observation**: O_t; what the environment shows to the agent at step t, after an action
- **Action**: A_t; the action taken at step t, performed by agent
- **History**: sequence of observations, actions, and rewards up to current time step; i.e. $H_t = A_1, O_1, R_1 ... A_t, O_t, R_t$
- **State**: a function of history; $S_t = f(H_t)$; the information used to determine what happens next

Some Helpful Definitions

- ***Fully observable***: the environment state equals the agent state; $S_t^a = S_t^e$; the agent knows the complete dynamics of the environment; MDP
- ***Partially observable***: the agent must make an assumption about the environment because it doesn't know it's dynamics
- **Model**: the agent's internal representation of the environment
- **Policy**: π; what the agent uses to map states to actions; tells the agent what to do
- ***Deterministic policy***: a state will always lead to a certain action; $a = \pi(s)$
- ***Stochastic policy***: a state will yield a probability of actions to choose from; $\pi(a|s) = \mathbb{P}(A = a|S = s)$
- ***State-Value Function***: tells the value of a state based on the expected future reward
- ***Action-Value Function***: tells the value of an action based on expected future reward

Types of Reinforcement Algorithms

- Value Based (e.g. Q-Learning)
  - No policy (implicit)
  - Value function
- Policy Based
  - Policy
  - No value function
- **Actor-Critic**
  - Policy
  - Value function
- Model Based/Model Free
  - Policy and/or Value function
  - Based: has model; Free; no model