Thrust-Vectored Hovering Rocket

Dynamic Modeling and Optimal Control Algorithm

In this project, we aim to design the dynamic of hovering rocket with its trust control. The hovering rocket has a jet propulsion which can exert the force on its bottom. The system can also be considered as an inverted pendulum which can be controlled by force acting parallel and perpendicular to the rocket: $u_1(t), u_2(t)$. Let (x, y, θ) as a position and orientation of the rocket respectively (see figure in page 2), we can define state and control of the system as $X(t) = [x(t)\ y(t)\ \theta(t)\ \dot{x}(t)\ \dot{y}(t)\ \dot{\theta}(t)]^T$ and $U(t) = [u_1(t)\ u_2(t)]^T$. In this simulation, the rocket hovers over x-axis while minimize their error over y-position and its direction θ.

The dynamic of the hovering rocket: $\dot{X}(t) = f(X(t), U(t))$ can be derived using force Lagrange Equation (or Newton's law of motion). The simple model of the system can be modeled as shown in [2] and shown as follows. More complicated model can be developed by representing the drag coefficient d as a function of $\theta(t)$.

$$m\ddot{x} = -mg\sin\theta - d\dot{x} + u_1\cos\theta - u_2\sin\theta \tag{1}$$
$$m\ddot{y} = mg(\cos\theta - 1) - d\dot{y} + u_1\sin\theta + u_2\cos\theta \tag{2}$$
$$J_\theta\ddot{\theta} = ru_1 \tag{3}$$

the parameter of the system are defined as given in [1] and [2]: $m = 11.2$ kg, $J_\theta = 0.0462$ kg m^2, $r = 0.156$ m, $d = 0.1$ Ns/m where d represented the drag parameter due to the rocket speed. Simulation time is set to $T_0 = 0$ sec, $T_f = 30$ sec with initial condition $X(0) = [0, 0, -\pi/10, 0, 0, 0]$ and cost of the system contains LQR term defined below where $Q = \text{diag}[100, 30, 30, 1, 1, 1]$, $R = \text{diag}[0.1, 0.1]$ and $P_1 = \text{diag}[100, 30, 30, 1, 1, 1]$. We weight the cost high over $x(t)$ since we want to achieve the trajectory over x-axis. We also assume low cost of control i.e. we can exert much control force in this simulation. Cost function of the system from time T_0 to T_f can fully determined as:

$$J = \int_{T_0}^{T_f} l(X(t), U(t))dt + m(X(T_f)) \tag{4}$$
$$l(X(t), U(t)) = \frac{1}{2}(X(t) - X_d(t))^T Q(X(t) - X_d(t)) + \frac{1}{2}(U(t) - U_d(t))^T R(U(t) - U_d(t)) \tag{5}$$
$$m(X(T_f)) = \frac{1}{2}(X(T_f) - X_d(T_f))^T P_1(X(T_f) - X_d(T_f)) \tag{6}$$

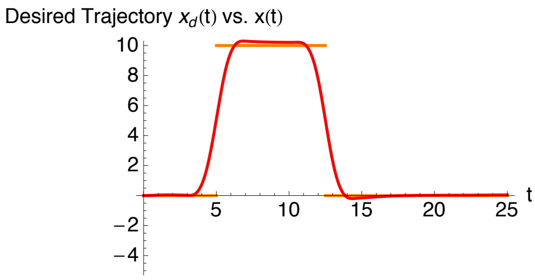
where J is the total cost function, $l(X(t), U(t))$ is a running cost, $m(X(T_f))$ is a terminal cost and $X_d(t)$ is the desired trajectory over the given state. The descent direction can be found by minimizing quadratic term over the linearize dynamics using gradient descent method:

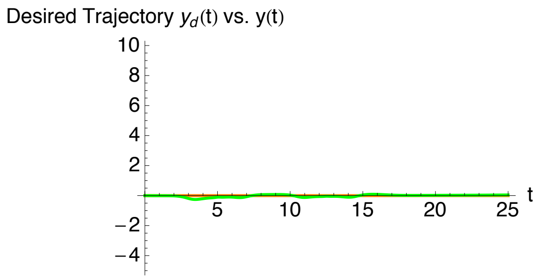
$$g(\zeta) = DJ(\xi)\cdot\zeta + \frac{1}{2}\int_{T_0}^{T_f} ||\zeta||^2 dt \text{ subject to } \ \dot{z} = A(t)z(t) + B(t)v(t) \tag{7}$$

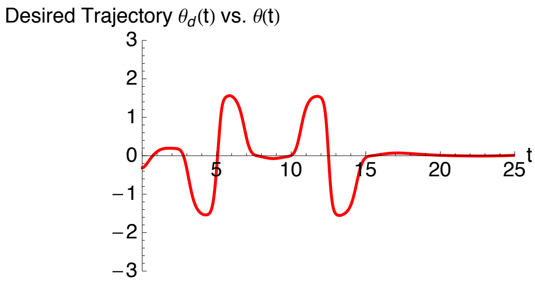
where $A(t), B(t)$ is linearized matrix and $\zeta(t) = (z(t), v(t))$ is the descent direction on direction $\eta = (X(t), U(t))$ respectively. The projection operation involve the feedback/feedforward term of the control over the dynamics as $U(t) = \mu(t) + K(t)[\alpha(t) - X(t)]$ where $\xi(t) = (\mu(t), \alpha(t))$ is an infeasible trajectory resulted from the summation of two curve i.e. we can write projection operator from infeasible to feasible curve as $\mathcal{P} : (\alpha(t), \mu(t)) \to \eta = (X(t), U(t))$. In the optimization algorithm, we use constant step size to optimize the trajectory in early iteration and end with Armijo line search in order to speed up the optimization loop. The integration symbolic processing is also turned off.

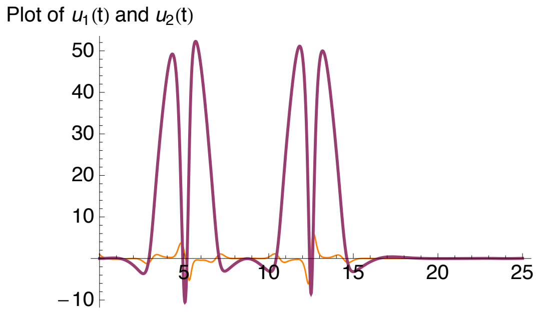
Result

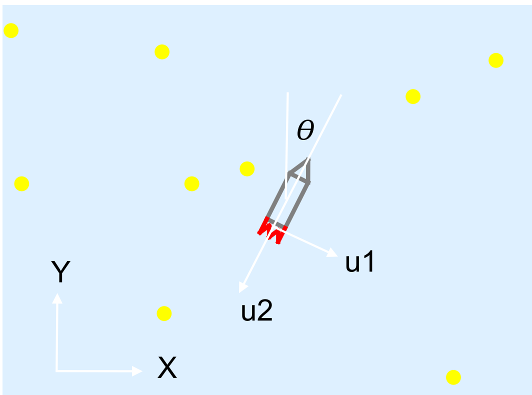
The desired trajectory and simulated trajectory in following desired trajectory in x-axis are shown in figures below. Direction $\theta(t)$ does **not** weight high but is important in controlling the vector thrust u_1, u_2 and thus control its position x, y. We can observe that the rocket can track given step trajectory in x direction while maintaining altitude y. The animation of the hovering rocket are also provided in the given code.

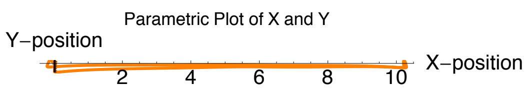












Conclusion

In this project, we can be able to use projection operator approach applying to optimal control problem in order to control the nonlinear system of hovering rocket using Mathematica. The simulation result of the hovering rocket give the promising result as shown in animation.

References

[1] M. B. Milan, R. Franz, J. E. Hauser, R. M. Murray, *Receding Horizon Control of a Vectored Thrust Flight Experiment*, IEE Proceedings on Control Theory and Applications, 2003.

[2] Ali Jadbabaie, John Hauser, *Control of a thrust-vectored flying wing: a receding horizon LPV approach*, Int. J. Robust Nonlinear Control, 2002.