CISS362: Introduction to Automata Theory, Languages, and Computation
Assignment 5

We'll take a break from proofs and do some DFA construction.

The questions are taken from our textbook. When I say "Textbook Q 1.1" I mean exercise/problem 1.1 in our textbook.

Q1 and Q2 are not graded since answers are provided in the textbook. Q3 is not graded because it's too easy – I'm giving you the solution.

For Q4, (a), (b), (d), and (g) will not be graded. I'm giving you the solution for 4(a) and (g). The textbook provides solutions for Q4(b) and Q4(d). Make sure you study my solutions and the author's solutions carefully.

Also, as I mentioned at the beginning of this course, I might not grade all questions. However you are strongly encouraged to solve every single problems.

Q1. Textbook Q 1.1. (Not graded.)

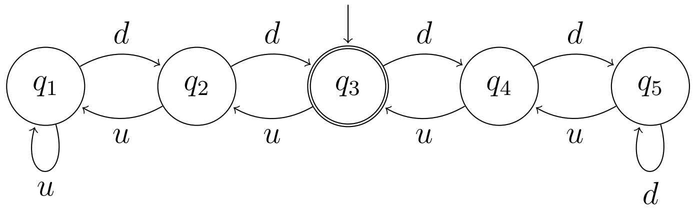
SOLUTION.

Q2. Textbook Q 1.2. (Not graded.)

SOLUTION.

Q3. Textbook Q 1.3

SOLUTION.



Q4. Textbook Q1.4. ((b), (d), and (g) not graded.)

Each language in this question can be written as an intersection of two languages that can accepted by DFAs. Given two DFAs, say M_1 and M_2, then it's always possible to design another DFA M such that

$$L(M) = L(M_1) \cap L(M_2).$$

See the textbook.

Try to layout the states in the "obvious" way. (Look at the solution for (a) and (g).)

SOLUTION.

4(a) Let

$$L = \{w \mid w \text{ has at least three } a\text{'s and at least two } b\text{'s}\}$$

Note that if

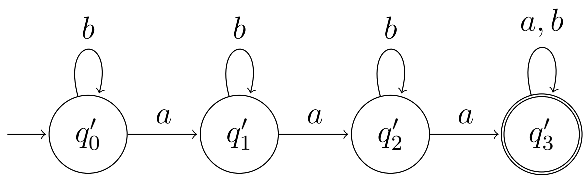
$$L' = \{w \mid w \text{ has at least three } a\text{'s}\}$$
$$L'' = \{w \mid w \text{ has at least two } b\text{'s}\}$$

then

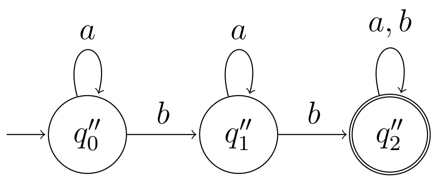
$$L = L' \cap L''$$

Therefore we construct DFAs for L' and L'' first.

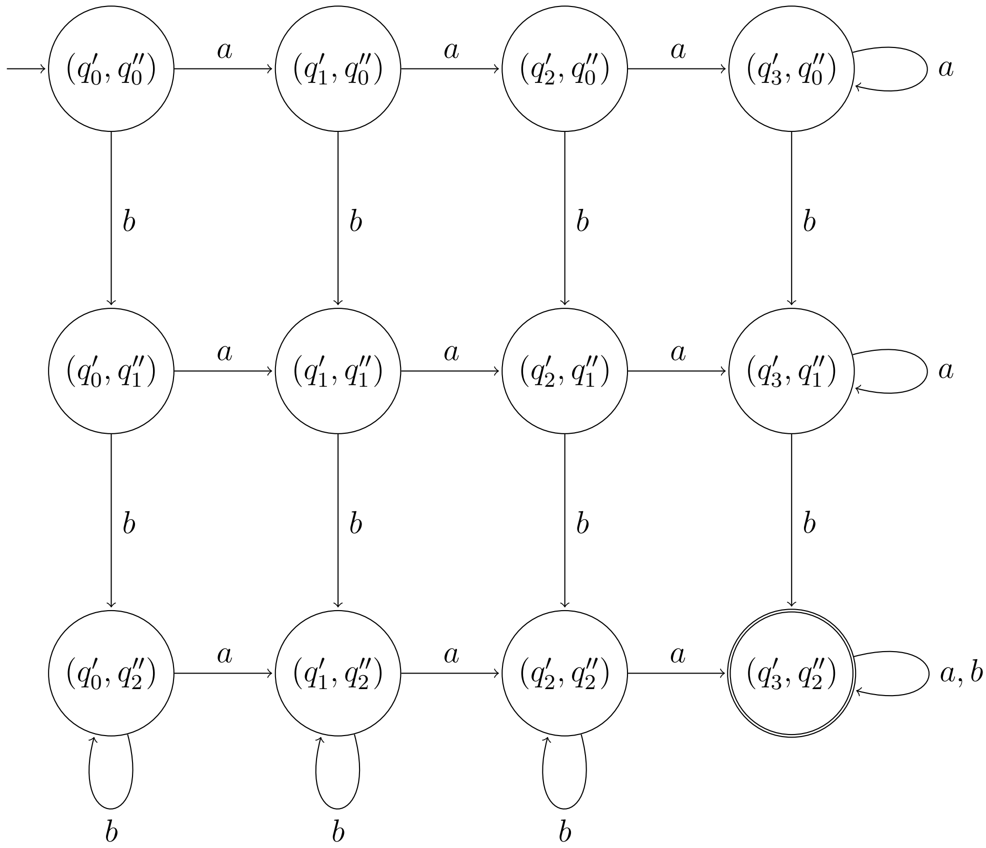
The following M' is a DFA such that $L(M') = L'$:



The following M'' is a DFA such that $L(M'') = L''$:



The following is a DFA M such that $L(M) = L(M') \cap L(M'')$:


(q'_0, q''_0)
(q'_1, q''_0)
(q'_2, q''_0)
(q'_3, q''_0)
(q'_0, q''_1)
(q'_1, q''_1)
(q'_2, q''_1)
(q'_3, q''_1)
(q'_0, q''_2)
(q'_1, q''_2)
(q'_2, q''_2)
(q'_3, q''_2)
a
b
a, b

4(b)

4(c)

4(d)

4(e)

4(f)

4(g) Let

$$L = \{w \mid w \text{ has even length and odd number of } a\text{'s}\}$$

Note that if

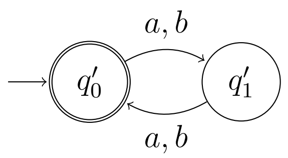
$$L' = \{w \mid w \text{ has even length}\}$$
$$L'' = \{w \mid w \text{ has odd number of } a\text{'s}\}$$

then

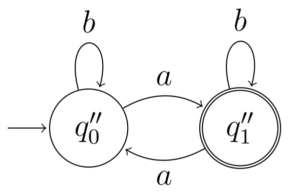
$$L = L' \cap L''$$

Therefore we construct DFAs for L' and L'' first.

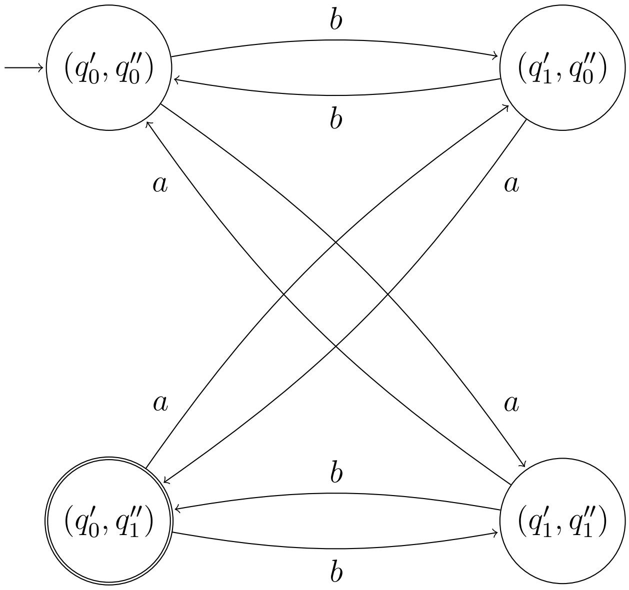
The following M' is a DFA such that $L(M') = L'$:



The following M'' is a DFA such that $L(M'') = L''$:



The following is a DFA M such that $L(M) = L(M') \cap L(M'')$:


b
b
a
a
a
a
b
b
(q'_0, q''_0)
(q'_1, q''_0)
(q'_0, q''_1)
(q'_1, q''_1)