CISS362: Introduction to Automata Theory, Languages, and Computation
Assignment 18

The questions are taken from the textbook.

Q1. Textbook Q 1.36.

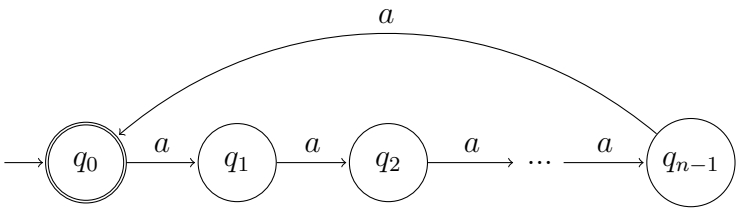
Solution is provided. Study the solution very carefully. I expect the same level of rigor in your solution for the other questions.

SOLUTION.

SOLUTION 1: Let $n \geq 1$. We want to show

$$B_n = \{a^k \mid k \text{ is a multiple of } n\}$$

is regular. Define the following DFA M (over $\Sigma = \{a\}$) as follows:



Formally the DFA M is defined as $M = (\Sigma, Q, q_0, \delta, F)$ where:

1. $\Sigma = \{a\}$
2. $Q = \{q_0, q_1, \ldots, q_{n-1}\}$
3. $\delta : Q \times \Sigma \to Q$ is defined by

$$\delta(q_i, a) = \begin{cases} q_{i+1} & \text{if } 0 \leq i < n-1 \\ q_0 & \text{if } i = n-1 \end{cases}$$

4. $F = \{q_0\}$

Clearly the strings accepted by M are ϵ, a^n, a^{2n}, ..., i.e. $L(M) = \{a^k \mid k \text{ is a multiple of } n\} = B_n$. Hence B_n must be regular.

SOLUTION 2: Let $n \geq 1$. We want to show

$$B_n = \{a^k \mid k \text{ is a multiple of } n\}$$

is regular. Note that

$$\begin{aligned} B_n &= \{a^k \mid k \text{ is a multiple of } n\} \\ &= \{a^{mn} \mid m \geq 0\} \\ &= \{(a^n)^m \mid m \geq 0\} \\ &= \{a^n\}^* \\ &= L(a^n)^* \\ &= L((a^n)^*) \end{aligned}$$

i.e. B_n is the language accepted by the regular expression $r = (a^n)^*$. We already know that the language generated by a regular expression is also accepted by a DFA. Hence B_n must be regular.

Note. Note that the second solution is a lot clearer since $L(r) = \{a^k \mid k \text{ is a multiple of } n\}$ is shown completely. However in the first solution the statement

$$L(M) = \{a^m \mid m \text{ is a multiple of } n\}$$

is not so immediate and properly speaking requires some proof. You can formally show that

$$L(M) = \{a^m \mid m \text{ is a multiple of } n\}$$

using mathematical induction. This is the reason why in CS and Math, we frequently have several different ways of looking at the same concept. (In the case of regular languages, a regular language is one that is accepted by or DFA, *or* is accepted by an NFA, *or* is generated by a regular expression.) Sometimes one way of looking at a problem will yield a more natural solution/proof or one that is shorter.

Q2. Textbook Q 1.40.

You only need to do (a). Note that the answer to (a) is given in the book. You need to explain very clearly why the construction works.

SOLUTION.

Q3. Textbook Q 1.41.

Provide an informal description of the DFA and explain why the construction works.

Provide a formal description of the construction.

SOLUTION.