**CISS362: Introduction to Automata Theory, Languages, and Computation**
**Assignment 2**

Q1. Textbook Q 0.3.

**SOLUTION.**

Q2. Textbook Q 0.4.

**SOLUTION.**

Q3. Textbook Q 0.5.

**SOLUTION.**

Q4. Textbook Q 0.6.

**SOLUTION.**

Q5. Textbook Q 0.7.

**SOLUTION.**

Q6. Textbook Q 0.10

**SOLUTION.** .

Q7. Textbook Q 0.11.

**SOLUTION.**

Q8. Prove or disprove the following: Let $X$ and $Y$ be sets.

(a) $P(X \times Y) = P(X) \times P(Y)$

(b) $P(X \cup Y) = P(X) \cup P(Y)$

Remember that to disprove a statement, all you need is a counterexample. When constructing examples, always construct the simplest.

**SOLUTION.**

Q9. Textbook Q 0.12.

**SOLUTION.**

Q10. Is there a finite set $X$ such that $P(X) = X$? Prove your claim.

**SOLUTION.**

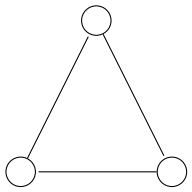
Q11. Using mathematical induction, prove that for $n \geq 4$, $2^n \geq n^2$. [Can you also prove this using Calculus? Any other methods you can think of?]

**SOLUTION.**

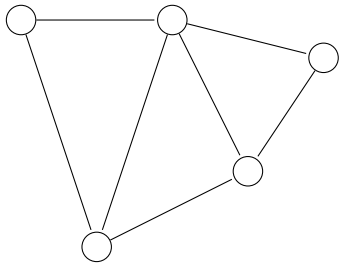
Q12. Using mathematical induction, prove that $3 \mid n^3 - n + 3$ for $n \geq 0$. [Can you prove it without using induction?]

**SOLUTION.**

Q13. Q5. A triangulation graph is a graph which looks like a patchwork of triangles, i.e., the graph is constructed by "glueing" triangles on a tabletop.



along edges. Note that line (edge) crossings are not allowed. For instance here's one 5 nodes (i.e. circles) and 7 edges (i.e. lines):



(a) Is it true that using the above construction (i.e., glueing triangle graphs along edges), all the "pieces" (i.e. finite regions) that you see are always triangles (i.e. a shape that is bounded by exactly 3 edges)? For instance in the above graph, there are 3 triangles.

(b) The degree of a node is just the number of edges (lines) joined to it. For the above type of graphs is it possible to draw one with exactly one node of degree 10? Show that it is possible by drawing such a graph or prove it's not possible.

(c) Is it possible to have a graph with 5 nodes of degree 2? Show that it is possible by drawing such a graph or prove it's not possible.

(d) Is it possible to have a graph with no nodes of degree 2? Show that it is possible by drawing such a graph or prove it's not possible.

(e) The degrees of the nodes in the above example is 2, 2, 3, 3, 4. Is it possible to have a graph with degrees 2, 2, 2, 2, 4? Show that it is possible by drawing such a graph or prove it's not possible.

(f) The degrees of the nodes in the above example is 2, 2, 3, 3, 4. Is it possible to have a graph with degrees 2, 2, 3, 3, 3? Show that it is possible by drawing such a graph or prove it's not possible.

(g) Is it possible to have a triangulation with no nodes of degree 2 or 3?

(h) Suppose you have a triangular graph $G$ with a node $v$ of degree 2. If there are at least 4 nodes in $G$, after removing $v$ from $G$ and removing all the edges attached to $v$, is the resulting graph a triangulation graph? Prove that this is true or provide a counterexample.

**SOLUTION.**

Q14. A graph is a bunch of dots (i.e. nodes) and lines (i.e. edges). There are no arrows on the lines. We are talking about an undirected graph. A graph is a tree if the graph has not loops. Make sure you draw a few trees for yourself. For each tree, count the number of nodes and the number of edges. You will see that there's an interesting relationship.

(a) What is the relationship between $n$ and $e$ where $n$ is the number of nodes of a tree $T$ and $e$ is the number of edges of $T$?

(b) Given a tree with $n$ nodes and $e$ edges, prove your claim in (a) using by mathematical induction. (Well, it's either an induction on $n$ or $e$. Which one are you going to use?)

**SOLUTION.**