AP Calculus Name _____

Project: Rate of Change

Directions
Complete the activities below. Think of these problems as your research. Then summarize the big ideas in a well-written paragraph or two. This report should demonstrate that you understand rate of change and include how your understanding of rate of change has increased.

US Population

Census figures for the US population (in millions) are listed in Table 2.12. Let f be the function such that $P = f(t)$ is the population (in millions) in year t.

Table 2.12 *US population (in millions), 1790–2000*

Year	Population	Year	Population	Year	Population	Year	Population
1790	3.9	1850	23.1	1910	92.0	1970	205.0
1800	5.3	1860	31.4	1920	105.7	1980	226.5
1810	7.2	1870	38.6	1930	122.8	1990	248.7
1820	9.6	1880	50.2	1940	131.7	2000	281.4
1830	12.9	1890	62.9	1950	150.7		
1840	17.1	1900	76.0	1960	179.0		

(a) (i) Estimate the rate of change of the population for the years 1900, 1945, and 2000.

(ii) When, approximately, was the rate of change of the population greatest?

(iii) Estimate the US population in 1956.

(iv) Based on the data from the table, what would you predict for the census in the year 2010?

(b) Assume that f is increasing (as the values in the table suggest). Then f is invertible.

(i) What is the meaning of $f^{-1}(100)$?

(ii) What does the derivative of $f^{-1}(P)$ at $P = 100$ represent? What are its units?

(iii) Estimate $f^{-1}(100)$.

(iv) Estimate the derivative of $f^{-1}(P)$ at $P = 100$.

(c) (i) Usually we think the US population $P = f(t)$ as a smooth function of time. To what extent is this justified? What happens if we zoom in at a point of the graph? What about events such as the Louisiana Purchase? Or the moment of your birth?

(ii) What do we in fact mean by the rate of change of the population at a particular time t?

(iii) Give another example of a real-world function which is not smooth but is usually treated as such.