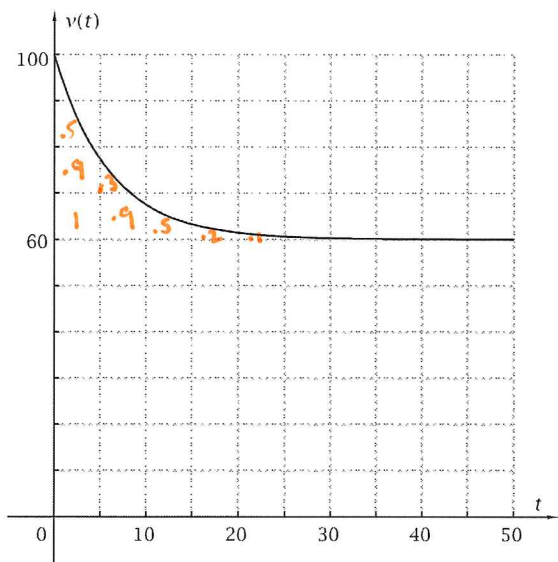


Exploration 1-3a: Introduction to Definite Integrals

Date: _____

Objective: Find out what a definite integral is by working a real-world problem that involves the speed of a car.

As you drive on the highway you accelerate to 100 ft/s to pass a truck. After you have passed, you slow down to a more moderate 60 ft/s. The diagram shows the graph of your velocity, $v(t)$, as a function of the number of seconds, t , since you started slowing.



1. What does your velocity seem to be between $t = 30$ and $t = 50$ s? How far do you travel in the time interval $[30, 50]$?

i) velocity seems constant $60 \frac{\text{ft}}{\text{s}}$

ii) $(60 \frac{\text{ft}}{\text{s}})(20 \text{ s}) = 1200 \text{ ft}$

2. Explain why the answer to Problem 1 can be represented as the area of a rectangular region of the graph. Shade this region.

(velocity)(time) = position
or
displacement

3. The distance you travel between $t = 0$ and $t = 20$ can also be represented as the area of a region bounded by the (curved) graph. Count the number of squares in this region. Estimate the area of parts of squares to the nearest 0.1 square space. For instance, how would you count this partial square?



$1200 + 4.3(50) = 28.3 \text{ Boxes}$

1415 ft

4. How many feet does each small square on the graph represent? How far, therefore, did you go in the time interval $[0, 20]$?

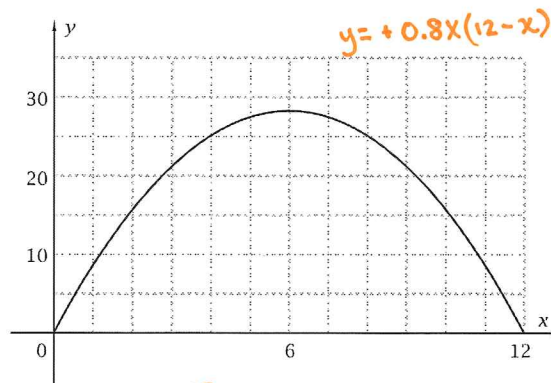
i) 50 ft

ii) 1415 ft

5. Problems 3 and 4 involve finding the product of the x -value and the y -value for a function where y may vary with x . Such a product is called the **definite integral** of y with respect to x . Based on the units of t and $v(t)$, explain why the definite integral of $v(t)$ with respect to t in Problem 4 has feet for its units.

$\int_0^{20} v(t) dt$, unit conversion

6. The graph shows the cross-sectional area, y , in in.^2 , of a football as a function of the distance, x , in in. , from one of its ends. Estimate the definite integral of y with respect to x .



$\int_0^{12} 0.8x(12-x) dx = 230.4 \text{ in}^3$

7. What are the units of the definite integral in Problem 6? What, therefore, do you suppose the definite integral represents?

in^3 or volume

8. What did you learn as a result of doing this Exploration that you did not know before?