AP Calculus 2016 – 2017 :: Summer Assignment – Minimum of 3 decimal places for numeric answers

Questions 1 – 6: Use the difference quotient $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ to compute the slope from the given

function, x , and Δx . Show the setup with x and Δx plugged in, then you may use your calculator to reduce.

1.
$$f(x) = 3x + 2$$
, $x = 2$, $\Delta x = 0.5$

1. _____

2.
$$f(x) = -4x^2 + 2$$
, $x = 4$, $\Delta x = 0.01$

2. _____

3.
$$f(x) = \sin\left(\frac{x}{2}\right)$$
, $x = \pi$, $\Delta x = 0.5$

3. _____

4.
$$f(x) = \frac{x+1}{x-3}$$
, $x = 2$, $\Delta x = -0.25$

4. _____

5.
$$f(x) = \frac{\sqrt{x+2}}{\sqrt{3-x}}$$
, $x = 1$, $\Delta x = 0.21$

5. _____

6.
$$f(x) = \ln\left(\frac{3}{2x}\right)$$
, $x = \frac{2}{5}$, $\Delta x = \frac{1}{10}$

6. _____

Questions 7 - 14: Given the following functions, find (a) the function value and (b) the requested composite function simplified.

7.
$$f(x) = \frac{4}{5}x + 6$$
, $f(10)$, $f(10+h) =$

7. (a) _____

(b) _____

8.
$$f(x) = -2x^2 + 4x - 5$$
, $f(2)$, $f(2+h) =$

8. (a) _____

(b) _____

9.
$$f(x) = \sqrt{7+4x}$$
, $f(-1)$, $f(h-1) =$

9. (a) _____

(b) _____

10.
$$f(x) = e^x + 4$$
, $f(3)$, $f(3+h) =$

10. (a) _____

(b) _____

11.
$$f(x) = \cos x$$
, $f\left(\frac{\pi}{3}\right)$, $f\left(\frac{\pi}{3} + h\right) =$

11. (a)_____

(b) _____

12.
$$f(x) = \cos x$$
, $f\left(-\frac{\pi}{4}\right)$, $f\left(-\frac{\pi}{4} + h\right) =$

12. (a) _____

(b) _____

13.
$$f(x) = \sin x$$
, $f\left(\frac{\pi}{3}\right)$, $f\left(\frac{\pi}{3} + h\right) =$

13. (a) _____

(b) _____

14.
$$f(x) = \sin x$$
, $f\left(-\frac{\pi}{4}\right)$, $f\left(-\frac{\pi}{4} + h\right) =$

14. (a) _____

(b) _____

Questions 15 - 22: Use your answers from questions 7 - 14 to complete/compute the difference quotient.

15. Question 7:
$$\frac{f\left(10+h\right)-f\left(10\right)}{h} =$$

15. _____

16. Question 8:
$$\frac{f(2+h)-f(2)}{h} =$$

16. _____

17. Question 9:
$$\frac{f(h-1)-f(-1)}{h} =$$
 (hint: conjugate the numerator)

17._____

18. Question 10:
$$\frac{f\left(3+h\right)-f\left(3\right)}{h}=$$
 (hint: factor out e^{3})

19. _____

20. Question 12:
$$\frac{f\left(-\frac{\pi}{4} + h\right) - f\left(-\frac{\pi}{4}\right)}{h} = \frac{\cos h - 1}{h} \text{ and } \frac{\sin h}{h}$$

20.

21. Question 13:
$$\frac{f\left(\frac{\pi}{3} + h\right) - f\left(\frac{\pi}{3}\right)}{h} =$$
 (hint: trig sum rule
$$get \frac{\cos h - 1}{h} \text{ and } \frac{\sin h}{h}$$

21.

22. Question 14:
$$\frac{f\left(-\frac{\pi}{4}+h\right)-f\left(-\frac{\pi}{4}\right)}{h} = \frac{\cos h - 1}{h} \text{ and } \frac{\sin h}{h}$$

22.

Questions 23 – 26: Use the difference quotient $\frac{f(x+h)-f(x)}{h}$ to compute a formula in terms of h given f(x) and x. Make sure the original single h in the denominator gets canceled out.

23.
$$f(x) = 2x^2 + 3$$
, $x = 1$

24.
$$f(x) = 4x^3 - 6$$
, $x = 2$

25.
$$f(x) = \sqrt{x}$$
, $x = 4$ (hint: conjugate)

26.
$$f(x) = \frac{1}{\sqrt{6-x}}$$
, $x = -3$ (hint: common denom., then conjugate) 26.

Questions 27 – 30: Use the difference quotient $\frac{f(x+h)-f(x)}{h}$ to **(a)** compute a formula in terms of h and x given f(x) and **(b)** then substitute 0 into h to compute a formula only in terms of x. Make sure the original single h in the denominator gets canceled out. **Show all work!**

27.
$$f(x) = (2x+3)(3x-4)$$

(h)

28.
$$f(x) = \frac{1}{x}$$
 (hint: common denominator)

(b)

29.
$$f(x) = \frac{x-1}{x+1}$$
 (hint: common denominator)

29. (a) _____

(b) _____

30.
$$f(x) = \sqrt{x^2 + 1}$$
 (hint: conjugate numerator)

30. (a) _____

(b) _____

Questions 31 – 34: Write the equation of the line in **point-slope** form, from the given slope and point.

31.
$$m = \frac{3}{5}$$
 and $p = (2, -4)$

32.
$$m = -\frac{7}{3}$$
 and $p = (5,8)$

33.
$$m = \frac{3}{e}$$
 and $p = (\ln 4, e^3)$

34.
$$m = \frac{9}{\ln 2}$$
 and $p = (-2,1)$

Questions 35 - 40: A graphing calculator is required. Use the steps below to compute the equations of the tangent line and **normal** (perpendicular slope) line in **point-slope** form, from the slope and point you find.

Step 1: Type your function into Y1.

Step 2: Press Trace to evaluate the function at the given x value and record it below.

Step 3: Press 2nd then Trace (CALC) and choose option 6 (dy/dx or slope).

Step 4: Type in the given x value to obtain your slope for your line and record it below.

35.
$$f(x) = 2x^3 - 3x + 5$$
, $x = 2$

36. $f(x) = \sqrt{3x^2 - 4x}$, x = 2

tangent:

normal:

37.
$$f(x) = \frac{3-x}{x^2+1}$$
, $x = 4$

normal:

38.
$$f(x) = \tan x$$
, $x = \frac{\pi}{6}$

normal:

39.
$$f(x) = \ln(5x^3 - 4x^2 + 6)$$
, $x = 1$

normal:

40.
$$f(x) = -4.9x^2 + \cos x - e^{3x}$$
, $x = 0$

normal:

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FORMULAS FROM GEOMETRY

Triangle

$$h = a \sin \theta$$

Area =
$$\frac{1}{2}bh$$

(Law of Cosines)

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Sector of Circular Ring

(p = average radius,

$$w =$$
width of ring,

 θ in radians)

Area =
$$\theta pw$$



Right Triangle

(Pythagorean Theorem)

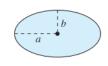
$$c^2 = a^2 + b^2$$



Ellipse

Area = πab

Circumference
$$\approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$$



Equilateral Triangle

$$h = \frac{\sqrt{3}s}{2}$$

Area =
$$\frac{\sqrt{3}s^2}{4}$$



Cone

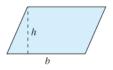
(A = area of base)

Volume =
$$\frac{Ah}{3}$$



Parallelogram

Area = bh



Right Circular Cone

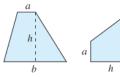
Volume =
$$\frac{\pi r^2 h}{3}$$

Lateral Surface Area = $\pi r \sqrt{r^2 + h^2}$



Trapezoid

$$Area = \frac{h}{2}(a+b)$$



Frustum of Right Circular Cone

$$Volume = \frac{\pi(r^2 + rR + R^2)h}{3}$$

Lateral Surface Area = $\pi s(R + r)$



Circle

Area =
$$\pi r^2$$

Circumference = $2\pi r$



Right Circular Cylinder

Volume =
$$\pi r^2 h$$

Lateral Surface Area = $2\pi rh$



Sector of Circle

 $(\theta \text{ in radians})$

Area =
$$\frac{\theta r^2}{2}$$

 $s = r\theta$



Sphere

Volume =
$$\frac{4}{3}\pi r^3$$

Surface Area = $4\pi r^2$



Circular Ring

(p = average radius,

$$w = \text{width of ring})$$

Area =
$$\pi (R^2 - r^2)$$

= $2\pi pw$



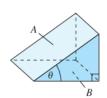


Wedge

(A = area of upper face,

$$B = area of base)$$

$$A = B \sec \theta$$



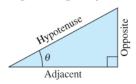
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TRIGONOMETRY

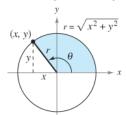
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{r} \quad \cot \theta = \frac{x}{y}$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

1 + $\tan^2 x = \sec^2 x$ 1 + $\cot^2 x = \csc^2 x$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

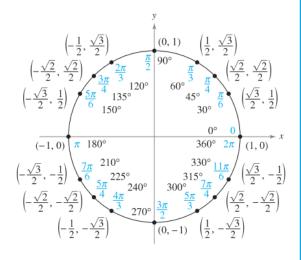
$$\sin(-x) = -\sin x$$
 $\cos(-x) = \cos x$
 $\csc(-x) = -\csc x$ $\tan(-x) = -\tan x$
 $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$



Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

Pre-Calc.	Pre-Calc.	Pre-Calc.
Area of a triangle	Area of a triangle (SAS Formula)	Area of a trapezoid
1	2	3
Pre-Calc.	Pre-Calc.	Pre-Calc.
Area of a circle	Circumference of a Circle	Lateral surface area of a cylinder
4	5	6
Pre-Calc.	Pre-Calc.	Pre-Calc.
Total surface area of a cylinder	Surface area of a sphere	Volume of a prism
7 Pre-Calc.	8 Pre-Calc.	9 Pre-Calc.
Volume of a pyramid	Volume of a cylinder	Volume of a cone
10 Pre-Calc.	11 Pre-Calc.	12 Pre-Calc.
Volume of a sphere	Double angle identity for $\cos^2 heta$	Double angle identity for $\sin^2 heta$
13 Pre-Calc.	14 Pre-Calc.	15 Pre-Calc.
$\sin\left(\frac{\pi}{2}-\theta\right)$	$\tan\left(\frac{\pi}{2}-\theta\right)$	$\sec\left(\frac{\pi}{2} - \theta\right)$
10	17	10

$$A = \frac{h}{2} \left(b_1 + b_2 \right)$$

$$A = \frac{1}{2}ab\sin C$$

$$A = \frac{1}{2}bh$$

$$S = 2\pi rh$$

$$C = 2\pi r$$
$$C = \pi d$$

$$A = \pi r^2$$

$$V = Bh$$

 $B =$ Area of Base

$$S = 4\pi r^2$$

$$S = 2\pi rh + 2\pi r^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \pi r^2 h$$

$$V = \frac{1}{3}Bh$$

$$B = \text{Area of Base}$$

$$\frac{1}{2}(1-\cos 2\theta)$$

$$\frac{1}{2}(1+\cos 2\theta)$$

$$V = \frac{4}{3}\pi r^3$$

 $\csc\theta$

 $\cot \theta$

 $\cos\theta$

Pre-Calc.	Pre-Calc.	Pre-Calc.
Fre-Calc.	rie-caic.	rie-caic.
$\cos^2\theta + \sin^2\theta$	$1+\cot^2\theta$	$1 + \tan^2 \theta$
19	20	21
19 Pre-Calc.	Pre-Calc.	Pre-Calc.
1	_1_	
$\sin heta$	$\cos heta$	an heta
22 Pre-Calc.	Pre-Calc.	24 Pre-Calc.
rie-caic.	Fie-Caic.	Fie-Calc.
$\sin heta$	$\cos heta$	
$\frac{\sin \theta}{\cos \theta}$	$\frac{\cos \theta}{\sin \theta}$	$\sin 2\theta$
coso	SIII O	
25	26	27
Pre-Calc.	Pre-Calc.	Pre-Calc.
$\cos 2\theta$	$\cos 2\theta$	$\cos 2\theta$
(in terms of $\cos \theta$ and $\sin \theta$)	(in terms of $\cos \theta$)	(in terms of $\sin heta$)
28	29	30
Pre-Calc.	Pre-Calc.	Pre-Calc.
$\sin(-\theta)$	$\cos(- heta)$	anig(- hetaig)
31	32	33
Pre-Calc.	Pre-Calc.	Pre-Calc.
Law of sinos	Law of society	$\log h = a$
Law of sines	Law of cosines	$\log_a b = c$
34	35	36

$$\sec^2 \theta$$

$$\csc^2 \theta$$

1

$$\cot\theta$$

$$\sec \theta$$

 $\csc\theta$

$$2\sin\theta\cos\theta$$

$$\cot \theta$$

 $\tan \theta$

$$1-2\sin^2\theta$$

$$2\cos^2\theta-1$$

$$\cos^2\theta - \sin^2\theta$$

$$-\tan\theta$$

$$\cos \theta$$

$$-\sin\theta$$

$$a^c = b$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Pre-Calc.	Pre-Calc.	Pre-Calc.
$\log \left(AB ight)$	$\log\!\left(rac{A}{B} ight)$	$\log A^x$
37	38	39
Pre-Calc.	Pre-Calc.	Pre-Calc.
ln e	ln 1	$e^{\ln x}$
40	41	42
Pre-Calc.	Pre-Calc.	Pre-Calc.
Pythagorean Theorem	Distance between points $\left(x_1,y_1\right)$ and $\left(x_2,y_2\right)$	Slope of the line between points $\left(x_1,y_1\right)$ and $\left(x_2,y_2\right)$
43 Pre-Calc.	44 Pre-Calc.	45 Pre-Calc.
Point-Slope equation of a line	Slope-Intercept equation of a line	Standard Form of the equation of a line
46	47	48
Pre-Calc.	Pre-Calc.	Pre-Calc.
Circle with center at point $ig(h,kig)$ and radius r	Ellipse	Hyperbola
49 Pre-Calc.	50 Pre-Calc.	51 Pre-Calc.
Hyperbola with axes as asymptotes	Parabola (vertical axis of symmetry)	Parabola (horizontal axis of symmetry)
52	53	54

x

0

1

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$$

$$a^2 + b^2 = c^2$$

$$Ax + By = C$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$Ax^2 - By^2 = C$$

$$Ax^2 + By^2 = C$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x = ay^2 + by + c$$

$$y = ax^2 + bx + c$$

$$xy = k$$

Pre-Calc.	Pre-Calc.	Pre-Calc.
Domain of a function	Range of a function	Function symmetric across the <i>y</i> -axis
55	56	57
Pre-Calc.	Pre-Calc.	Pre-Calc.
Even function	Function symmetric through the origin	Odd function
58	59	60
Pre-Calc.	Pre-Calc.	Pre-Calc.
Quadratic formula (roots of $y = ax^2 + bx + c$)	Inverse of a function	To graph the inverse of a function
61	62	63
Pre-Calc.	Pre-Calc.	Pre-Calc.
To find the equation of the inverse of a function	If point $\left(a,b\right)$ lies on the inverse function f^{-1} then	Inverse of $y = e^x$
64	65	66
Pre-Calc.	Pre-Calc.	Pre-Calc.
Inverse of $y = \ln x$	Line parallel to $y = mx + b$ through point (x_1, y_1) .	Line perpendicular to $y = mx + b$ through point (x_1, y_1) .
67 Pre-Calc.	68 Pre-Calc.	69 Pre-Calc.
70	71	72

$$f(-x) = f(x)$$

The set of all possible values of y for a function

The set of all possible values of x for a function

$$f(-x) = -f(x)$$

$$f(-x) = -f(x)$$

$$f(-x) = f(x)$$

reflect the graph of the function across the line y = x

$$(f\circ f^{-1})(x)=x$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \ln x$$

point (b,a) lies on function f

interchange x and y, then solve the equation for y

$$y-y_1=-\frac{1}{m}\big(x-x_1\big)$$
 or
$$y=-\frac{1}{m}x+b_1\text{, where }b_1=y_1+\frac{x_1}{m}$$

$$y-y_1=m\big(x-x_1\big)$$
 or
$$y=mx+b_1\text{, where }b_1=\big(y_1-mx_1\big)$$

$$y = e^x$$