Name: _____ Date: _____ Period: _____ 2016

AP Calculus – WS: Partial Fraction Decomposition and Integration

1. $\int \frac{3(x+1)-4(x-2)}{(x+1)(x-2)}dx$

$\int \left(\frac{3}{x-2} + \frac{-4}{x+1}\right) dx$

$3\ln|x-2| - 4\ln|x+1| + C$

$\ln\left|\frac{(x-2)^3}{(x+1)^4}\right| + C$

2. $\int \frac{5(2x+3)+6(x+1)}{2x^2+5x+3}dx$

$(2x+3)(x+1)$

$\int \left(\frac{5}{x+1} + \frac{6}{2x+3}\right) dx$

u-sub required

$5\ln|x+1| + 3\ln|2x+3| + C$

$\ln\left|(x+1)^5(2x+3)^3\right| + C$

3. $\int \frac{10}{(x+2)(x-3)}dx$

$\frac{10}{(x+2)(x-3)} = \frac{A}{x-3} + \frac{B}{x+2}$

$10 = A(x+2) + B(x-3)$

$x=-2$: $10 = -5B$, $-2 = B$

$x=3$: $10 = 5A$, $2 = A$

$\int \left(\frac{2}{x-3} + \frac{-2}{x+2}\right) dx$

$2\ln|x-3| - 2\ln|x+2| + C$

$\ln\left|\frac{(x-3)^2}{(x+2)^2}\right| + C$

4. $\int \frac{8}{(7x-1)(2x-3)}dx$

$\frac{8}{(7x-1)(2x-3)} = \frac{A}{7x-1} + \frac{B}{2x-3}$

$8 = A(2x-3) + B(7x-1)$

$x=\frac{3}{2}$: $8 = \frac{19}{2}B$, $\frac{16}{19} = B$

$x=\frac{1}{7}$: $8 = -\frac{19}{7}A$, $-\frac{56}{19} = A$

$\int \left(\frac{16}{19(2x-3)} + \frac{-56}{19(7x-1)}\right) dx$

$\frac{8}{19}\ln|2x-3| - \frac{8}{19}\ln|7x-1| + C$

$\frac{8}{19}\ln\left|\frac{2x-3}{7x-1}\right| + C$

5. $\int \frac{1}{x^2+x}dx$

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$1 = A(x+1) + Bx$

$x=-1 \quad 1=-B \quad B=-1$

$x=0 \quad 1=A$

$\int \left(\frac{1}{x} + \frac{-1}{x+1}\right)dx$

$\ln|x| - \ln|x+1| + C$

$\ln\left|\frac{x}{x+1}\right| + C$

6. $\int \frac{24}{x^2-6x+8}dx$

$\frac{24}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$

$24 = A(x-4) + B(x-2)$

$x=4 \quad 24=2B \quad B=12$

$x=2 \quad 24=-2A \quad A=-12$

$\int \left(\frac{12}{x-4} + \frac{-12}{x-2}\right)dx$

$12\ln|x-4| - 12\ln|x-2| + C$

$12\ln\left|\frac{x-4}{x-2}\right| + C$

7. $\int \frac{3x}{x^2-3x-4}dx$

$\frac{3x}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$

$3x = A(x+1) + B(x-4)$

$x=-1 \quad -3=-5B \quad \frac{3}{5}=B$

$x=4 \quad 12=5A \quad \frac{12}{5}=A$

$\int \left(\frac{3}{5(x+1)} + \frac{12}{5(x-4)}\right)dx$

$\frac{3}{5}\ln|x+1| + \frac{12}{5}\ln|x-4| + C$

$\frac{3}{5}\ln\left|(x+1)(x-4)^4\right| + C$

8. $\int \frac{3x+5}{5x^2-4x-1}dx$

$\frac{3x+5}{(5x+1)(x-1)} = \frac{A}{5x+1} + \frac{B}{x-1}$

$3x+5 = A(x-1) + B(5x+1)$

$x=1 \quad 8=6B \quad B=4/3$

$x=-1/5 \quad \frac{22}{5}=-\frac{6}{5}A \quad -\frac{11}{3}=A$

$\int \left(\frac{4}{3(x-1)} + \frac{-11}{3(5x+1)}\right)dx$

$\frac{4}{3}\ln|x-1| - \frac{11}{15}\ln|5x+1| + C$

$\frac{1}{3}\ln\left|\frac{(x-1)^4}{\sqrt[5]{(5x+1)^{11}}}\right| + C$

9. $\int \frac{6x-4}{(3x-7)(x+1)}dx$

$\frac{6x-4}{(3x-7)(x+1)} = \frac{A}{3x-7} + \frac{B}{x+1}$

$6x-4 = A(x+1) + B(3x-7)$

$x=-1$: $-10 = -10B$, $B=1$

$x=\frac{7}{3}$: $10 = \frac{10}{3}A$, $3=A$

$\int\left(\frac{3}{3x-7} + \frac{1}{x+1}\right)dx$

$\ln|3x-7| + \ln|x+1| + C$

$\ln|(3x-7)(x+1)| + C$

11. $\int_2^4 \frac{4x^2+3x+5}{15x^2-34x+15}dx$

$15x^2-34x+15 \overline{)\,4x^2+3x+5}$ quotient $\frac{4}{15}$, remainder $\frac{181}{15}x+1$

$\int_2^4\left(\frac{4}{15} + \frac{181x+15}{15(3x-5)(5x-3)}\right)dx$

$\frac{181x+15}{(3x-5)(5x-3)} = \frac{A}{3x-5} + \frac{B}{5x-3}$

$181x+15 = A(5x-3) + B(3x-5)$

$x=\frac{3}{5}$: $\frac{618}{5} = -\frac{16}{5}B$, $-\frac{309}{8} = B$

$x=\frac{5}{3}$: $\frac{950}{3} = \frac{16}{3}A$, $\frac{475}{8} = A$

$\frac{1}{15}\int_2^4\left(\frac{475}{8(3x-5)} + \frac{-309}{8(5x-3)} + 4\right)dx$

$\left(\frac{95}{72}\ln|3x-5| - \frac{103}{200}\ln|5x-3| + 4x\right)\Big|_2^4$

2.64389

10. $\int \frac{x^2+1}{x^2-5x-6}dx$

Same power, long divide first

$x^2-5x-6 \overline{)\,x^2+0x+1}$ quotient 1, remainder $5x+7$

$\int\left(1 + \frac{5x+7}{(x-6)(x+1)}\right)dx$

$\frac{5x+7}{(x-6)(x+1)} = \frac{A}{x-6} + \frac{B}{x+1}$

$5x+7 = A(x+1) + B(x-6)$

$x=-1$: $2 = -7B$, $B = -\frac{2}{7}$

$x=6$: $37 = 7A$, $A = \frac{37}{7}$

$\int\left(1 + \frac{37}{7(x-6)} + \frac{-2}{7(x+1)}\right)dx$

$x + \frac{37}{7}\ln|x-6| - \frac{2}{7}\ln|x+1| + C$

12. $\int_0^{\ln 2} \frac{-11e^x}{e^{2x}-5e^x-24}dx$

change limits also; u-sub, to convert to a simpler form

$u = e^x$

$du = e^x dx$

$\int_1^2 \frac{-11}{u^2-5u-24}du$

$\frac{-11}{(u-8)(u+3)} = \frac{A}{u-8} + \frac{B}{u+3}$

$-11 = A(u+3) + B(u-8)$

$u=-3$: $-11 = -11B$, $1 = B$

$u=8$: $-11 = 11A$, $-1 = A$

$\int_1^2\left(\frac{1}{u+3} + \frac{-1}{u-8}\right)du$

$\left(\ln|u+3| - \ln|u-8|\right)\Big|_1^2$

0.37729