Calculus Name________________________________

Related Rates

Date________________ Period____

Solve each related rate problem.

1) A perfect cube shaped ice cube melts so that the length of its sides are decreasing at a rate of 2 mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 6 mm each?

$V = s^3$

$\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt}$

$\frac{dV}{dt} = 3(6)^2(-2)$

$\frac{dV}{dt} = -216 \frac{mm^3}{sec}$

2) A hypothetical square grows so that the length of its sides are increasing at a rate of 7 m/min. How fast is the area of the square increasing when the sides are 2 m each?

$A = s^2$

$\frac{dA}{dt} = 2s\left(\frac{ds}{dt}\right)$

$\frac{dA}{dt} = 2(2)(7)$

$\frac{dA}{dt} = 28 \frac{m^2}{min}$

3) A spherical snowball melts so that its radius decreases at a rate of 2 in/sec. At what rate is the volume of the snowball changing when the radius is 8 in?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(8)^2(-2)$

$\frac{dV}{dt} = -512\pi \frac{in^3}{sec}$

4) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 7 m/min. How fast is the area of the spill increasing when the radius is 11 m?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(11)(7)$

$\frac{dA}{dt} = 154\pi \frac{m^2}{min}$

5) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 3 cm/sec. At what rate is the volume of water in the cup changing when the water level is 2 cm?

$V = \frac{1}{3}\pi r^2 h$

$\frac{10}{10} = \frac{h}{r}$

$r = h$

$V = \frac{1}{3}\pi (h)^2 h$

$V = \frac{\pi}{3} h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi (2)^2 (-3)$

$\frac{dV}{dt} = -12\pi \ \frac{cm^3}{sec}$

6) A hypothetical cube grows at a rate of 27 m³/min. How fast are the sides of the cube increasing when the sides are 4 m each?

$V = s^3$

$\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt}$

$27 = 3(4)^2 \frac{ds}{dt}$

$\frac{ds}{dt} = \frac{9}{16} \ \frac{m}{min}$

7) A 13 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 8 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 5 ft from the wall?

$x^2 + y^2 = 13^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2(5) \frac{dx}{dt} + 2(12)(-8) = 0$

$\frac{dx}{dt} = \frac{96}{5} \ \frac{ft}{sec}$

8) An observer stands 400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 300 ft from the ground?

$\tan\theta = \frac{y}{400}$

$(\sec^2\theta)\frac{d\theta}{dt} = \frac{1}{400}\left(\frac{dy}{dt}\right)$

$\frac{d\theta}{dt} = \frac{1}{400}(200)\cos^2\theta$

$\frac{d\theta}{dt} = \frac{200}{400}\left(\frac{400}{500}\right)^2$

$\frac{d\theta}{dt} = \frac{8}{25} \ \frac{radians}{sec}$

9) A spherical balloon is deflated at a rate of $\frac{32\pi}{3}$ cm³/sec. At what rate is the radius of the balloon changing when the radius is 4 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-\frac{32\pi}{3} = 4\pi(4)^2 \frac{dr}{dt}$

$\frac{-32\pi}{64 \cdot 3\pi} = \frac{dr}{dt}$

$-\frac{1}{6} \frac{cm}{sec} = \frac{dr}{dt}$

10) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 4π m²/min. How fast is the radius of the spill increasing when the radius is 3 m?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$4\pi = 2\pi(3)\frac{dr}{dt}$

$\frac{2}{3} \frac{m}{min} = \frac{dr}{dt}$

11) An observer stands 1200 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 500 ft from the ground?

$(1200)^2 + y^2 = z^2$

$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(500)(200) = 2(1300)\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{2(500)(200)}{2(1300)}$

$\frac{dz}{dt} = \frac{1000}{13} \frac{ft}{sec}$

12) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{4}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 9 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi\left(\frac{4}{9}\right)(9)^2$

$\frac{dV}{dt} = 144\pi \frac{cm^3}{sec}$

13) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius (r) of the spill increases at a rate of $\frac{9}{r}$ m/min. How fast is the area of the spill increasing when the radius is 9 m?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r\left(\frac{9}{r}\right) \qquad \frac{dA}{dt} = 18\pi \ \frac{m^2}{min}$$

14) A conical paper cup is 10 cm tall with a radius of 20 cm. The cup is being filled with water so that the water level (h) rises at a rate of $\frac{4}{h}$ cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?

$$V = \frac{1}{3}\pi r^2 h \qquad \frac{h}{r} = \frac{10}{20}$$

$$V = \frac{1}{3}\pi (2h)^2 \cdot h \qquad r = 2h$$

$$V = \frac{4}{3}\pi h^3$$

$$\frac{dV}{dt} = 4\pi h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = 4\pi(8)^2\left(\frac{4}{8}\right)$$

$$\frac{dV}{dt} = 128\pi \ \frac{cm^3}{sec}$$

15) A perfect cube shaped ice cube melts so that the length of its sides (s) are decreasing at a rate of $\frac{3}{s}$ mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 6 mm each?

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$\frac{dV}{dt} = 3(6)^2\left(-\frac{3}{6}\right) \qquad \frac{dV}{dt} = -54 \ \frac{mm^3}{sec}$$

16) A hypothetical square grows so that the length of its sides (s) are increasing at a rate of $\frac{5}{s}$ m/min. How fast is the area of the square increasing when the sides are 6 m each?

$$A = s^2$$

$$\frac{dA}{dt} = 2s \frac{ds}{dt}$$

$$\frac{dA}{dt} = 2(6)(5/6) \qquad \frac{dA}{dt} = 10 \ \frac{m^2}{min}$$