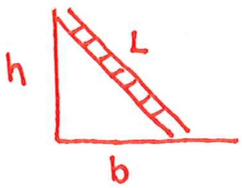
Name: ______________________________ Date: _______________ Period: _______ 2016

AP Calculus – Notes / Examples – 2.6 Related Rates

1. Draw a picture.
2. Write down known information.
3. Write down what you are looking for.
4. Write an equation to relate the variables.
5. Differentiate both sides with respect to t.
6. Evaluate.

1. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder starts to slide away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



$b = 6$, $\frac{db}{dt} = 1$ ft/s

$h = 8$, $\frac{dh}{dt} = ?$

$L = 10$, $\frac{dL}{dt} = 0$

comes from formula and knowns

$b^2 + h^2 = L^2$

$b^2 + h^2 = 10^2$

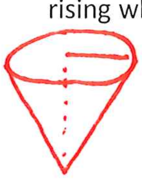
implicit deriv with t

$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 0$

$2(6)(1) + 2(8)\frac{dh}{dt} = 0$

$\frac{dh}{dt} = \frac{-3}{4}$ ft/s

2. A water tank has the shape of an inverted right circular cone with a base radius of 2m and height of 4m. If water is being pumped into the tank at a rate of $2\ \text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.



Top $R = 2$

Height $= 4$

proportional h and r

$\frac{4}{2} = \frac{h}{r}$

$r = h/2$

$h = 3$

$\frac{dV}{dt} = 2\ \frac{m^3}{min}$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

use proportional r to h

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$2 = \frac{\pi}{4}(3)^2\frac{dh}{dt}$

$\frac{8}{9\pi}$ m/min $= \frac{dh}{dt}$

3. If a snowball melts so that its surface area decreases at a rate of $1\text{cm}^2/\text{min}$, find the rate at which the diameter decreases when the diameter is 10cm.

$d = 2r$

$\frac{d}{2} = r$

$d = 10$

$\frac{dd}{dt} = ?$

$\frac{dSA}{dt} = -1\ \frac{cm^2}{min}$

$SA = 4\pi r^2$

$SA = 4\pi\left(\frac{d}{2}\right)^2$

$SA = \pi d^2$

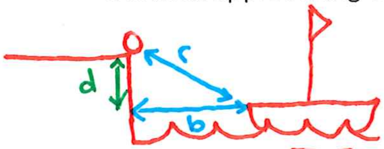
$\frac{dSA}{dt} = 2\pi d \cdot \frac{dd}{dt}$

$-1 = 2\pi(10)\frac{dd}{dt}$

$\frac{-1}{20\pi}\ \frac{cm}{min} = \frac{dd}{dt}$

4. A street light is mounted at the top of a 15ft tall pole. A man 6ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40ft from the pole?

5. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8m from the dock?

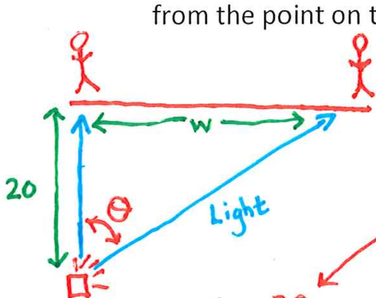


$b = 8$ $\quad r = \sqrt{65}$ $\quad d = 1$

$\frac{db}{dt} = ?$ $\quad \frac{dr}{dt} = 1 \text{ m/s}$ $\quad$ no change

$$d^2 + b^2 = r^2$$
$$1^2 + b^2 = r^2$$
$$2b\frac{db}{dt} = 2r\frac{dr}{dt}$$
$$2(8)\frac{db}{dt} = 2\sqrt{65}\,(1)$$
$$\frac{db}{dt} = \frac{\sqrt{65}}{8}\ \frac{m}{s}$$

6. A man walks along a straight path at a speed of 4ft/s. A searchlight is located on the ground 20ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15ft from the point on the path closest to the searchlight?



$w = 15$

$\frac{dw}{dt} = 4 \text{ ft/s}$

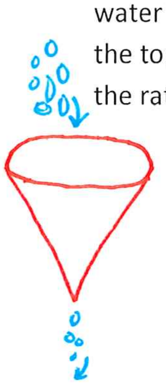
$\theta = \tan^{-1}\left(\frac{15}{20}\right)$

$\frac{d\theta}{dt} = ?$

$\cos\theta = \frac{20}{25}$

$$\tan\theta = \frac{w}{20}$$
$$(\sec^2\theta)\frac{d\theta}{dt} = \frac{1}{20}\cdot\frac{dw}{dt}$$
$$\frac{d\theta}{dt} = \frac{1}{20}\cdot 4 \cdot \cos^2\theta$$
$$\frac{d\theta}{dt} = \frac{1}{20}\cdot 4\cdot\left(\frac{4}{5}\right)^2$$
$$\frac{d\theta}{dt} = \frac{16}{125}\ \frac{\text{radians}}{s}$$

7. Water is leaking out of an inverted conical tank at a rate of $10{,}000\ \mathrm{cm}^3/\mathrm{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has a height of 6m and the diameter of the top is 4m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2m, find the rate at which water is being pumped into the tank. $h = 200$ cm



$\frac{dV_{out}}{dt} = -10000\ \frac{cm^3}{min}$ $\quad h_{top} = 600\,cm$ $\quad \frac{dh}{dt} = 20\ \frac{cm}{min}$

$r_{top} = 200\,cm$

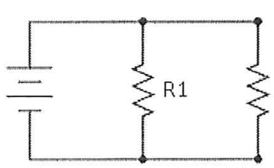
$\frac{dV_{in}}{dt} = ?$

$\frac{600}{200} = \frac{h}{r}$

$r = \frac{1}{3}h$

$$V_{in} + V_{out} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$
$$V_{in} + V_{out} = \frac{\pi}{27}h^3$$
$$\frac{dV_{in}}{dt} + \frac{dV_{out}}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$$
$$\frac{dV_{in}}{dt} = 10000 + \frac{\pi}{9}(200)^2(20)$$
$$\frac{dV}{dt} = 10000 + \frac{800000\pi}{9}$$
$$\frac{dV}{dt} = 289252.6803\ \frac{cm^3}{min}$$

8. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance is R, measured in ohms Ω, is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 and R_2 are increasing at rates of $0.3\ \Omega/\mathrm{s}$ and $0.2\ \Omega/\mathrm{s}$ respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



$\frac{dR_1}{dt} = 0.3$

$\frac{dR_2}{dt} = 0.2$

$R = \frac{400}{9}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R^{-1} = R_1^{-1} + R_2^{-1}$$
$$-R^{-2}\frac{dR}{dt} = -R_1^{-2}\frac{dR_1}{dt} - R_2^{-2}\frac{dR_2}{dt}$$
$$\frac{dR}{dt} = R^2\left(\frac{dR_1/dt}{R_1^2} + \frac{dR_2/dt}{R_2^2}\right)$$