

Related Rates

Date _____ Period _____

Solve each related rate problem.

- 1) A perfect cube shaped ice cube melts so that the length of its sides are decreasing at a rate of 2 mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 6 mm each?

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt}$$

$$\frac{dV}{dt} = 3(6)^2(-2)$$

$$\frac{dV}{dt} = -216 \frac{\text{mm}^3}{\text{sec}}$$

- 2) A hypothetical square grows so that the length of its sides are increasing at a rate of 7 m/min. How fast is the area of the square increasing when the sides are 2 m each?

- 3) A spherical snowball melts so that its radius decreases at a rate of 2 in/sec. At what rate is the volume of the snowball changing when the radius is 8 in?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(8)^2(-2)$$

$$\frac{dV}{dt} = -512\pi \frac{\text{in}^3}{\text{sec}}$$

- 4) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 7 m/min. How fast is the area of the spill increasing when the radius is 11 m?

- 5) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 3 cm/sec. At what rate is the volume of water in the cup changing when the water level is 2 cm?

$$V = \frac{1}{3} \pi r^2 h \quad \frac{10}{10} = \frac{h}{r} \quad \frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$$

$$r = h$$

$$\cancel{V} = \frac{1}{3} \pi (h)^2 h$$

$$V = \frac{\pi}{3} h^3$$

$$\frac{dV}{dt} = \pi (2)^2 (-3)$$

$$\frac{dV}{dt} = -12\pi \frac{\text{cm}^3}{\text{sec}}$$

- 6) A hypothetical cube grows at a rate of 27 m³/min. How fast are the sides of the cube increasing when the sides are 4 m each?

- 7) A 13 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 8 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 5 ft from the wall?

$$x^2 + y^2 = 13^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(5) \frac{dx}{dt} + 2(12)(-8) = 0$$

$$\frac{dx}{dt} = \frac{96}{5} \frac{\text{ft}}{\text{sec}}$$

- 8) An observer stands 400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 300 ft from the ground?

- 9) A spherical balloon is deflated at a rate of $\frac{32\pi}{3}$ cm³/sec. At what rate is the radius of the balloon changing when the radius is 4 cm?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-\frac{32\pi}{3} = 4\pi(4)^2 \frac{dr}{dt}$$

$$\frac{-32\pi}{64 \cdot 3\pi} = \frac{dr}{dt}$$

$$-\frac{1}{6} \frac{\text{cm}}{\text{sec}} = \frac{dr}{dt}$$

- 10) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 4π m²/min. How fast is the radius of the spill increasing when the radius is 3 m?

- 11) An observer stands 1200 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 500 ft from the ground?

$$(1200)^2 + y^2 = z^2$$

$$2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(500)(200) = 2(1300) \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{2(500)(200)}{2(1300)}$$

$$\frac{dz}{dt} = \frac{1000}{13} \frac{\text{ft}}{\text{sec}}$$

- 12) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{4}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 9 cm?

- 13) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius (r) of the spill increases at a rate of $\frac{9}{r}$ m/min. How fast is the area of the spill increasing when the radius is 9 m?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \left(\frac{9}{r} \right)$$

$$\frac{dA}{dt} = 18\pi \frac{\text{m}^2}{\text{min}}$$

- 14) A conical paper cup is 10 cm tall with a radius of 20 cm. The cup is being filled with water so that the water level (h) rises at a rate of $\frac{4}{h}$ cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?

- 15) A perfect cube shaped ice cube melts so that the length of its sides (s) are decreasing at a rate of $\frac{3}{s}$ mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 6 mm each?

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$\frac{dV}{dt} = 3(6)^2 \left(-\frac{3}{6} \right)$$

$$\frac{dV}{dt} = -54 \frac{\text{mm}^3}{\text{sec}}$$

- 16) A hypothetical square grows so that the length of its sides (s) are increasing at a rate of $\frac{5}{s}$ m/min. How fast is the area of the square increasing when the sides are 6 m each?