Name: ______________________ Date: ______________ Period: _______ 2016

AP Calculus – Worksheet – Chain Rule

Find the derivative of each of the following functions and parametric equations.

1. $f(x)=\left(3x^2+5x\right)^3$

$f' = 3(3x^2+5x)^2(6x+5)$

2. $g(x)=\sqrt[3]{4x^2+5x}\,(3x+5)$

$g' = \frac{1}{3}(4x^2+5x)^{-2/3}(8x+5)(3x+5) + \sqrt[3]{4x^2+5x}\,(3)$

3. $s(d)=\dfrac{\sqrt{d-4d^2}}{(d+1)^2}$

$$\frac{(d+1)^2\frac{1}{2}(d-4d^2)^{-1/2}(1-8d) - \sqrt{d-4d^2}\,(2(d+1))}{(d+1)^4}$$

4. $g(x)=\dfrac{6-x-x^2}{x+3} = \dfrac{(3+x)(2-x)}{x+3}$

$$g' = \frac{(x+3)(-1-2x)-(6-x-x^2)(1)}{(x+3)^2}$$

$g' = -1$

5. $x(t)=t$
$y(t)=3t^2+6$

$\frac{dy}{dx} = \frac{6t}{1} = 6t$

6. $x(t)=6t+1$
$y(t)=4t^2-6t+5$

$\frac{dy}{dx} = \frac{8t-6}{6} = \frac{4t-3}{3}$

7. $x(t)=\cos(2t)$
$y(t)=\sin(4t)$

$\frac{dy}{dx} = \frac{4\cos(4t)}{-2\sin(2t)} = \frac{-2\cos(4t)}{\sin(2t)}$

8. $x(\theta)=\theta$
$y(\theta)=2-2\cos\theta$

$\frac{dy}{dx} = \frac{2\sin\theta}{1} = 2\sin\theta$

9. $f(x)=2\sin\left(\tan(3x)\right)$

$f' = 2\cos(\tan(3x))\sec^2(3x)\cdot 3$

$= 6\cos(\tan(3x))\sec^2(3x)$

10. $g(x)=\left(\sqrt{x+2}\right)^{1/2}$

$g' = \frac{1}{2}\left(\sqrt{x+2}\right)^{-1/2}\left(\frac{1}{2}(x+2)^{-1/2}\right)\cdot 1$

$= \frac{1}{4}(x+2)^{-3/4}$

11. $y=\cos^2(4x)$

$$\frac{dy}{dx}=2\cos(4x)(-\sin(4x))(4)$$
$$=-8\cos(4x)\sin(4x)$$

13. $f(x)=\sqrt[4]{1+2x+x^3}$

$$f'=\frac{1}{4}(1+2x+x^3)^{-3/4}(2+3x^2)$$

12. $f(x)=\dfrac{\sin(2x)}{(4x+1)^2}$

$$f'=\frac{(4x+1)^2(2\cos(2x))-\sin(2x)(2(4x+1)(4))}{(4x+1)^4}$$

$$\frac{2\cos(2x)(4x+1)-8\sin(2x)}{(4x+1)^3}$$

14. $y=\dfrac{x^2-x^{-2}}{x^2+x^{-2}}$ $\nearrow$ $y=\dfrac{x^{-2}(x^4-1)}{x^{-2}(x^4+1)}$

$$\frac{(x^2+x^{-2})(2x+2x^{-3})-(x^2-x^{-2})(2x-2x^{-3})}{(x^2+x^{-2})^2}$$

$$\frac{8x^{-1}}{(x^2+x^{-2})^2}$$

Convert Problem 5 and 6 from parametric equations to functions and then take the derivative.

15. $x(t)=t \quad y(t)=3t^2+6$

$y(x)=3x^2+6$

$y'(x)=6x$

16. $x(t)=6t+1 \quad y(t)=4t^2-6t+5$

$t=\frac{x-1}{6}$

$y(x)=4\left(\frac{x-1}{6}\right)^2-6\left(\frac{x-1}{6}\right)+5$

$y'(x)=8\left(\frac{x-1}{6}\right)\left(\frac{1}{6}\right)-6\left(\frac{1}{6}\right)$

$=\frac{4}{3}\left(\frac{x-1}{6}\right)-1$

17. If $F(x)=f(g(x))$, where $f(-2)=8$, $f'(-2)=4$, $f'(5)=3$, $g(5)=-2$, and $g'(5)=6$, find $F'(5)$.

$F'(x)=f'(g(x))\cdot g'(x)\cdot 1$

$F'(5)=f'(g(5))\cdot g'(5)$

$=f'(-2)\cdot 6 \Rightarrow 4\cdot 6 = 24$

18. The following table of values contains f, g, f', and g', use them to find:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. If $h(x)=f(g(x))$, find $h'(1)$.

$h'(1)=f'(g(1))\cdot g'(1) \Rightarrow 30$

b. If $H(x)=g(f(x))$, find $H'(1)$.

$H'(1)=g'(f(1))\cdot f'(1) \Rightarrow 36$