

## AP Calculus – Spot Check 1

$t$ (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

1. The temperature in degrees Celsius, ( $^{\circ}\text{C}$ ), of the water in a pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.

- a. Use data from the table to find an approximation for  $W'(12)$ . Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) = \frac{W(15) - W(9)}{15 - 9} = \frac{21 - 24}{15 - 9} = -\frac{1}{2} \text{ } ^{\circ}\text{C/day}$$

- b. A student proposes the function  $P$ , given by  $P(t) = 20 + 10te^{-t/3}$ , as a model for the temperature of the water in the pond at time  $t$ , where  $t$  is measured in days and  $P(t)$  is measured in degrees Celsius. Find  $P'(12)$ . Using appropriate units, explain the meaning of your answer in terms of water temperature.

$$P'(12) = -0.54946$$

The temperature of the water is decreasing at 0.549  $^{\circ}\text{C/day}$  on day 12.

2. Consider the curve given by  $xy^2 - x^3y = 6$ .

- a. Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$$y^2 + 2xy \frac{dy}{dx} - 3x^2y - x^3 \frac{dy}{dx} = 0$$

$$(2xy - x^3) \frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

- b. Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.

$$xy^2 - x^3y = 6$$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3 \quad y = -2$$

$$\left. \frac{dy}{dx} \right|_{(1,3)} = \frac{3(1)^2(3) - 3^2}{2(1)(3) - 1^3} = 0$$

$$y - 3 = 0(x - 1)$$

$$\left. \frac{dy}{dx} \right|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3} = \frac{-10}{-5} = 2$$

$$y + 2 = 2(x - 1)$$