AP Calculus – Area and Definite Integrals

Use the given integrals and their values to answer the questions 1 – 10.

$$\int_0^4 f(x)\,dx = 5, \; \int_3^4 f(x)\,dx = -1, \; \int_3^5 f(x)\,dx = -4$$

$$\int_0^5 g(x)\,dx = 12, \; \int_1^4 g(x)\,dx = -6, \; 2\int_0^1 g(x)\,dx = \int_4^5 g(x)\,dx, \text{ and } \int_4^3 g(x)\,dx = -9$$

1. $\int_0^3 f(x)\,dx$

$\int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx = \int_0^4 f(x)\,dx$

$\int_0^3 f(x)\,dx - 1 = 5$

$\int_0^3 f(x)\,dx = 6$

2. $\int_4^5 f(x)\,dx$

$\int_3^4 f(x)\,dx + \int_4^5 f(x)\,dx = \int_3^5 f(x)\,dx$

$-1 + \int_4^5 f(x)\,dx = -4$

$\int_4^5 f(x)\,dx = -3$

3. $\int_0^5 f(x)\,dx$

$\int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx = \int_0^5 f(x)\,dx$

$6 + (-4) = \int_0^5 f(x)\,dx$

$\int_0^5 f(x)\,dx = 2$

4. $\int_1^3 g(x)\,dx$

$\int_1^3 g(x)\,dx + \int_3^4 g(x)\,dx = \int_1^4 g(x)\,dx$

$\int_1^3 g(x)\,dx + 9 = -6$

$\int_1^3 g(x)\,dx = -15$

5. $\int_4^5 g(x)\,dx$

$\int_0^1 g(x)\,dx + \int_1^4 g(x)\,dx + \int_4^5 g(x)\,dx = \int_0^5 g(x)\,dx$

$\int_0^1 g(x)\,dx + (-6) + \left(2\int_0^1 g(x)\,dx\right) = 12$

$3\int_0^1 g(x)\,dx = 18$

$\int_0^1 g(x)\,dx = 6 \quad \Rightarrow \quad \int_4^5 g(x)\,dx = 12$

6. $\int_1^5 g(x)\,dx$

$\int_1^4 g(x)\,dx + \int_4^5 g(x)\,dx = \int_1^5 g(x)\,dx$

$-6 + 12 = \int_1^5 g(x)\,dx$

$\int_1^5 g(x)\,dx = 6$

7. $\int_0^3 (f(x) + g(x))\,dx$

$\int_0^3 g(x)\,dx + \int_3^4 g(x)\,dx + \int_4^5 g(x)\,dx = \int_0^5 g(x)\,dx$

$\int_0^3 g(x)\,dx + (9) + (12) = 12 \qquad \Rightarrow \qquad \int_0^3 g(x)\,dx = -9$

$\int_0^3 (f(x) + g(x))\,dx = 6 + (-9) = -3$

8. $\int_0^4 (2f(x) - 3g(x))\,dx$

$\int_0^4 (2f(x) - 3g(x))\,dx = 2\int_0^4 f(x)\,dx - 3\int_0^4 g(x)\,dx$

$\int_0^3 g(x)\,dx + \int_3^4 g(x)\,dx = \int_0^4 g(x)\,dx \qquad \Rightarrow (-9) + 9 = \int_0^4 g(x)\,dx$

$\int_0^4 (2f(x) - 3g(x))\,dx = 2(5) - 3(0) = 10$

9. $\int_0^5 (4f(x) + g(x) + 8)\,dx$

$\int_0^5 (4f(x) + g(x) + 8)\,dx = 4\int_0^5 f(x)\,dx + \int_0^5 g(x)\,dx + \int_0^5 8\,dx$

$\int_0^5 (4f(x) + g(x) + 8)\,dx = 4(2) + (12) + 5(8) = 60$

10. $\int_3^4 f(x+1)\,dx$

$\int_3^4 f(x+1)\,dx$ Let $j = x+1$, $j(3) = 4$, $j(4) = 5$

$\dfrac{dj}{dx} = 1 \Rightarrow \qquad dj = dx$ now substitute

$\int_4^5 f(j)\,dj = \int_4^5 f(x)\,dx = -3$