Name: ______________________________ Date: ______________ Period: ______ 2016

AP Calculus – Integration by U-Substitution and Separable Differential Equations

1. $\int 3x^2\,dx$

$x^3 + C$

2. $\int (3x+4)^3\,dx$

$u = 3x+4 \quad du = 3dx$

$\frac{1}{3}\int u^3\,du$

$\frac{1}{12}(3x+4)^4 + C$

3. $\int 2x(x^2+5)\,dx$

$u = x^2+5 \quad du = 2x\,dx$

$\int u\,du$

$\frac{1}{2}(x^2+5)^2 + C$

4. $\int e^x e^{e^x}\,dx$

$u = e^x \quad du = e^x\,dx$

$\int e^u\,du$

$e^{e^x} + C$

5. $\int \frac{2x\,dx}{\sqrt{x^2-1}}$

$u = x^2-1 \quad du = 2x\,dx$

$\int u^{-1/2}\,du$

$2(x^2-1)^{1/2} + C$

6. $\int \frac{dx}{2x+5}$

$u = 2x+5 \quad du = 2\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|2x+5| + C$

7. $\int \frac{x^3\,dx}{\sqrt{1-x^2}}$

$u = 1-x^2$

$x^2 = 1-u$

$du = -2x\,dx$

$-\frac{1}{2}\int \frac{1-u}{\sqrt{u}}\,du$

$-\frac{1}{2}\int (u^{-1/2} - u^{1/2})\,du$

$-(1-x^2)^{1/2} + \frac{1}{3}(1-x^2)^{3/2} + C$

8. $\int \cos x \sin^3 x\,dx$

$u = \sin x \quad du = \cos x\,dx$

$\int u^3\,du$

$\frac{1}{4}\sin^4 x + C$

9. $\int \cos^3 x \sin^2 x\,dx$

$u = \sin x \quad du = \cos x$

$\int (1-u^2)u^2\,du$

$\int (u^2 - u^4)\,du$

$\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

Solve the separable differential equations below with their initial value.

10. $\frac{dy}{dx} = 3x^2 - 6,\ f(0) = 5$

$y = x^3 - 6x + C$

$y = x^3 - 6x + 5$

11. $\frac{dy}{dx} = y,\ f(3) = e$

$\int \frac{dy}{y} = \int dx$

$\ln|y| = x + C$

$\ln|y| = x - 2$

$y = e^{x-2}$

12. $\frac{dy}{dx} = \frac{2x}{y}$, $f(0) = -2$

$\int y \, dy = \int 2x \, dx$

$\frac{y^2}{2} = x^2 + C$

$y = \sqrt{2(x^2+2)}$

$y = -\sqrt{2(x^2+2)}$

13. $\frac{dy}{dx} = \frac{(x+3)}{2y}$, $f(4) = 12$

$\int 2y \, dy = \int (x+3) \, dx$

$y^2 = \frac{x^2}{2} + 3x + C$

$y = \sqrt{\frac{x^2}{2} + 3x + 124}$

14. $\frac{dy}{dx} = -2xy^2$, $f(1) = 0.25$

$\int -\frac{dy}{y^2} = \int 2x \, dx$

$\frac{1}{y} = x^2 + C$

$\frac{1}{y} = x^2 + 3$

$y = \frac{1}{x^2+3}$

15. $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$, $f(e) = 1$

$\int \frac{dy}{2\sqrt{y}} = \int \frac{2\ln x}{x} \, dx$

$\sqrt{y} = (\ln x)^2 + C$

$y = (\ln x)^4$

Solve the separable differential equations below without using an initial value.

16. $\frac{dy}{dx} = x\sqrt{y}\cos^2\sqrt{y}$

$\int \frac{dy}{\sqrt{y}\cos^2\sqrt{y}} = \int x \, dx$

$\int \frac{\sec^2\sqrt{y}}{\sqrt{y}} \, dy = \int x \, dx$

$2\tan\sqrt{y} = \frac{x^2}{2} + C$

$\tan\sqrt{y} = \frac{1}{2}\left(\frac{x^2}{2} + C\right)$

$y = \left[\tan^{-1}\left(\frac{1}{2}\left(\frac{x^2}{2} + c\right)\right)\right]^2$

17. $\frac{dy}{dx} = e^{x-y} = \frac{e^x}{e^y}$

$\int e^y \, dy = \int e^x \, dx$

$e^y = e^x + C$

$y = \ln(e^x + C)$