

Implicit Differentiation (Examples)

Date _____ Period _____

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

1) $5x^3 + 1 = \cos 3y^3$

$$15x^2 = -\sin(3y^3) \left(9y^2 \frac{dy}{dx} \right)$$

$$-\frac{5x^2}{3y^2 \sin(3y^3)} = \frac{dy}{dx}$$

2) $x^2 = 2y^2 + 1$

$$2x = 4y \frac{dy}{dx}$$

$$\frac{x}{2y} = \frac{dy}{dx}$$

3) $5x^3 - 3y^2 = y^3$

$$15x^2 - 6y \frac{dy}{dx} = 3y^2 \frac{dy}{dx}$$

$$15x^2 = \frac{dy}{dx} (3y^2 + 6y)$$

$$\frac{5x^2}{y^2 + 2y} = \frac{dy}{dx}$$

4) $x - y^3 = 5y$

$$1 - 3y^2 \frac{dy}{dx} = 5 \frac{dy}{dx}$$

$$1 = \frac{dy}{dx} (5 + 3y^2)$$

$$\frac{dy}{dx} = \frac{1}{3y^2 + 5}$$

5) $5x = 3x^2y + 4$

$$5 = 3x^2 \frac{dy}{dx} + 6xy$$

$$\frac{5 - 6xy}{3x^2} = \frac{dy}{dx}$$

6) $3x^2 - 2x^2y^3 = 5$

$$6x - 4xy^3 - 6x^2y^2 \frac{dy}{dx} = 0$$

$$\frac{3x - 2xy^3}{3x^2y^2} = \frac{dy}{dx}$$

7) $-5y^3 + 3x^3y^3 = 3x^3$

$$-15y^2 \frac{dy}{dx} + 9x^3y^2 \frac{dy}{dx} + 9x^2y^3 = 9x^3$$

$$\frac{dy}{dx} = \frac{3x^3 - 3x^2y^3}{3x^3y^2 - 5y^2}$$

8) $3x^3 - x^3y = 3y$

$$9x^2 - 3x^2y - x^3 \frac{dy}{dx} = 3 \frac{dy}{dx}$$

$$\frac{9x^2 - 3x^2y}{3 + x^3} = \frac{dy}{dx}$$

9) $-x^2y + 3x^3y^3 = 3x$

$$-2xy - x^2 \frac{dy}{dx} + 9x^3y^2 \frac{dy}{dx} + 9x^2y^3 = 3$$

$$\frac{dy}{dx} = \frac{3 + 2xy - 9x^2y^3}{9x^3y^2 - x^2}$$

10) $-4xy + 4x^3y^2 = 2x$

$$-4y - 4x \frac{dy}{dx} + 8x^3y \frac{dy}{dx} + 12x^2y^2 = 2$$

$$\frac{dy}{dx} = \frac{1 + 2y - 6x^2y^2}{4x^3y - 2x}$$

$$11) (3x^3 + 5) \cdot 4y^2 = 2x^2$$

$$9x^2 \cdot 4y^2 + (3x^3 + 5) \cdot 8y \cdot \frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} = \frac{x - 9x^2 y^2}{2y(3x^3 + 5)}$$

$$12) x = (5x^2 + 4) \cdot 2y^3$$

$$1 = 10x \cdot 2y^3 + (5x^2 + 4) \cdot 6y^2 \frac{dy}{dx}$$

$$\frac{1 - 20xy^3}{6y^2(5x^2 + 4)} = \frac{dy}{dx}$$

$$13) \frac{3x + 5}{3y^2} = 5x^2$$

$$\frac{3y^2 \cdot 3 - (3x + 5) \cdot 6y \cdot \frac{dy}{dx}}{(3y^2)^2} = 10x$$

$$\frac{dy}{dx} = \frac{30xy^3 - 3y}{-2(3x + 5)}$$

$$14) 5x = \frac{5x^2 + 5}{y^2}$$

$$5 = \frac{y^2(10x) - (5x^2 + 5)(2y) \frac{dy}{dx}}{(y^2)^2}$$

$$\frac{5y^3 - 10xy}{-2(5x^2 + 5)} = \frac{dy}{dx}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y .

$$15) x^3 + 4y^2 = 5$$

$$6x + 8y \cdot y'' + \frac{9x^4}{8y^2} = 0$$

$$3x^2 + 8y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{3x^2}{8y}$$

$$6x + 8y \frac{d^2y}{dx^2} + 8 \cdot \frac{dy}{dx} \cdot \frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = \frac{-48xy^2 - 9x^4}{64y^3}$$

$$16) x = y^2 + 4$$

$$1 = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$0 = 2 \frac{dy}{dx} \cdot \frac{dy}{dx} + 2y \frac{d^2y}{dx^2}$$

$$\frac{-1}{4y^3} = \frac{d^2y}{dx^2}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ at the given point.

$$17) 2x^2 + y^2 = 3 \text{ at } (1, 1)$$

$$4x + 2y \frac{dy}{dx} = 0$$

$$4 + 2 \frac{dy}{dx} = 0 \quad \frac{dy}{dx} = -2$$

$$4 + 2 \cdot \frac{dy}{dx} \cdot \frac{dy}{dx} + 2y \frac{d^2y}{dx^2} = 0$$

$$4 + 2(-2)(-2) + 2 \frac{d^2y}{dx^2} = 0$$

$$\frac{d^2y}{dx^2} = -6$$

$$18) 4x = 2y^2 + 2 \text{ at } (1, 1)$$

$$4 = 4y \frac{dy}{dx}$$

$$4 = 4 \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1$$

$$0 = 4 \frac{dy}{dx} \cdot \frac{dy}{dx} + 4y \frac{d^2y}{dx^2}$$

$$0 = 4(1)(1) + 4 \frac{d^2y}{dx^2}$$

$$-1 = \frac{d^2y}{dx^2}$$