AP Calculus – Worksheet – 4.4 – Optimization

- 1. The sum of two nonnegative numbers is 20. Find the numbers if
  - a. the sum of their squares is as large as possible; as small as possible.

b. one number plus the square root of the other is as large as possible; as small as possible.

2. What is the largest possible area for a right triangle whose hypotenuse is 5 cm long, and what are its dimensions?

3. A rectangle has its base on the x-axis and its upper two vertices on the parabola  $y = 12 - x^2$ . What is the largest area the rectangle can have, and what are its dimensions?

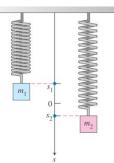
4. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

5. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

6. Suppose  $r(x) = \frac{x^2}{x^2 + 1}$  represents revenue and  $c(x) = \frac{(x-1)^3 - 1}{3}$  represents cost, with x measured in thousands of units. Is there a production level that maximizes profit? If so, what is it?

7. How close does the curve  $y = \sqrt{x}$  come to the point  $\left(\frac{3}{2}, 0\right)$ ? [Hint: If you minimize the square of the distance, you can avoid square roots.]

8. Two masses hanging side by side from springs have positions  $s_1 = 2\sin t$  and  $s_2 = \sin 2t$ , respectively, with  $s_1$  and  $s_2$  in meters and t in seconds.



a. At what times in the interval t > 0 do the masses pass each other? [Hint:  $\sin 2t = 2\sin t \cos t$ .]

b. When in the interval  $0 \le t \le 2\pi$  is the vertical distance between the masses the greatest? What is this distance? [Hint:  $\cos 2t = 2\cos^2 t - 1$ .]