Calculus

Name_____

Chain Rule

Date_____ Period_____

Differentiate each function with respect to x (CHAIN RULE = Deriv OUT * Deriv IN).

1) $f(x) = (x+1)^{\frac{1}{3}}$

$f' = \frac{1}{3}(x+1)^{\frac{1}{3}-1} \cdot \left(\frac{d}{dx}(x+1)\right)$

$= \frac{1}{3}(x+1)^{-\frac{2}{3}} \cdot 1$

2) $y = (5x^3 - 2)^{-5}$

$y' = -5(5x^3-2)^{-5-1}\left(\frac{d}{dx}(5x^3-2)\right)$

$y' = -75x^2(5x^3-2)^{-6}$

3) $f(x) = (-3x^4 - 2)^4$

$f' = 4(-3x^4-2)^3(-12x^3)$

$= -48x^3(-3x^4-2)^3$

4) $y = \sqrt[4]{-5x^5 + 1}$

$y' = \frac{1}{4}(-5x^5+1)^{-\frac{3}{4}}(-25x^4)$

$\frac{-25x^4}{4}(-5x^5+1)^{-\frac{3}{4}}$

5) $f(x) = (5x^3 - 1)^{\frac{1}{2}}$

$f' = \frac{15x^2}{2}(5x^3-1)^{-\frac{1}{2}}$

6) $y = (-2x^4 + 3)^{\frac{1}{5}}$

$y' = \frac{-8x^3}{5}(-2x^4+3)^{-\frac{4}{5}}$

7) $y = (-4x^3 - 5)^{-5}$

$y' = 60x^2(-4x^3-5)^{-6}$

8) $y = \sqrt{-x - 4}$

$y' = -\frac{1}{2}(-x-4)^{-\frac{1}{2}}$

9) $y = (4x^2 - 3)^3$

$24x\left(4x^2 - 3\right)^2$

10) $y = (-5x^2 - 4)^2$

$-20x\left(-5x^2 - 4\right)$

11) $y = (5x^4 + 3)^{-3}$

$-60x^3\left(5x^4 + 3\right)^{-4}$

12) $y = (-3x^5 - 2)^5$

$-75x^4\left(-3x^5 - 2\right)^4$

13) $y = (-5x^2 - 1)^4$

$-40x\left(-5x^2 - 1\right)^3$

14) $y = \dfrac{1}{(-5x - 4)^4}$

$20\left(-5x - 4\right)^{-5}$

15) $y = \dfrac{1}{(x^3 + 3)^{\frac{1}{4}}}$

$-\dfrac{3x^2}{4}\left(x^3 + 3\right)^{-\frac{3}{4}}$

16) $y = (-x^2 - 4)^{-2}$

$4x\left(-x^2 - 4\right)^{-3}$

17) $y = \cos 3x^3$

$$y' = -9x^2 \sin(3x^3)$$

18) $y = \tan 2x^2$

$$y' = 4x \sec^2(2x^2)$$

19) $f(x) = \cot 4x^4$

$$f' = -16x^3 \csc^2(4x^4)$$

20) $y = \sin x^4$

$$y' = 4x^3 (\cos(x^4))$$

21) $f(x) = \sec 5x^4$

$$f' = 20x^3 (\sec(5x^4))(\tan(5x^4))$$

22) $y = \csc 4x^3$

$$y' = -12x^2 (\csc(4x^3))(\cot(4x^3))$$

Use the PRODUCT rule and the CHAIN RULE to differentiate each function with respect to x.

23) $y = (2x^3 + 1)(-5x^5 - 3)^2$

$$y' = (2x^3+1)\left[-50x^4(-5x^5-3)\right] + (-5x^5-3)^2\left[6x^2\right]$$

$$y' = 2x^2(-5x^5-3)(-65x^5 - 25x^2 - 9)$$

24) $y = (-5x^2 - 4)^3(-5x^5 - 2)$

$$y' = (-5x^2-4)^3\left[-25x^4\right] + (-5x^5-2)\left[-30x(-5x^2-4)^2\right]$$

$$y' = 5x(-5x^2-4)^2(55x^5 + 20x^3 + 12)$$

25) $y = (-x^4 + 5)(-5x^3 - 1)^3$

$$y' = (-x^4 + 5)\left[-45x^2(-5x^3 - 1)^2\right] + (-5x^3 - 1)^3\left[-4x^3\right]$$

$$y' = x^2(-5x^3 - 1)^2\left(65x^4 + 24x - 225\right)$$

26) $y = (-3x^3 - 2)(-x^2 - 4)^3$

$$y' = (-3x^3 - 2)\left[-6x(-x^2 - 4)^2\right] + (-x^2 - 4)^3\left[-9x^2\right]$$

$$y' = 3x(-x^2 - 4)^2\left(9x^3 + 12x + 4\right)$$

Use the QUOTIENT rule and the CHAIN RULE to differentiate each function with respect to x.

27) $y = \left(\dfrac{-2x^3 + 5}{-x - 3}\right)^5$

$$y' = 5\left(\frac{-2x^3 + 5}{-x - 3}\right)^4\left[\frac{(-x - 3)(-6x^2) - (-2x^3 + 5)(-1)}{(-x - 3)^2}\right]$$

28) $y = \dfrac{(-4x + 1)^5}{-x^2 + 2}$

$$y' = \frac{(-x^2 + 2)\left[-20(-4x + 1)^4\right] - (-4x + 1)^5(-2x)}{(-x^2 + 2)^2}$$

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