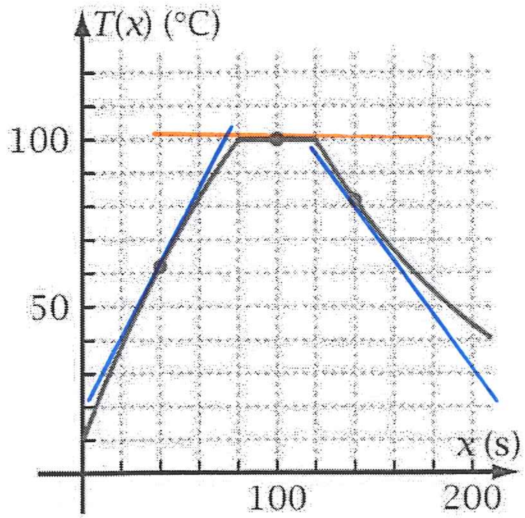
AP Calculus   Name ______________________________

Rate of Change by Graph and Table



Graph 1

1. Graph 1 above shows the temperature, $T(x)$, in degrees Celsius, of a kettle of water at time x, in seconds, since the burner was turned on.

(a) Use tangent lines at the points marked to estimate $T'(40)$, $T'(100)$ and $T'(140)$. Show how you arrived at your answers.

$T'(40) = \frac{80-40}{60-20} = 1$ $T'(140) = \frac{60-100}{160-120} = -1$

$T'(100) = \frac{100-100}{120-80} = 0$

(b) Using correct units, explain the meaning of the values estimated in part (a) in the context of the temperature of the water in the kettle. T' is the instantaneous rate of change of temperature, °C with respect to time, s.

(c) What do you suppose is happening to the temperature of the water for $0 < x < 80$? For $80 < x < 120$? For $x > 120$?

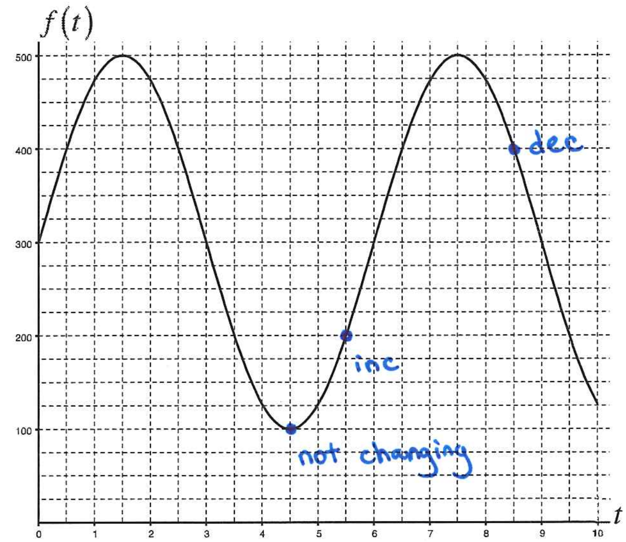
Increasing, Constant

decreasing

(d) On what intervals is $T'(x)$ increasing? Decreasing? Explain your reasoning.

T' is increasing on $x > 120$.

T' is decreasing on $0 < x < 80$.



Graph 2

2. The population of foxes in a particular region varies periodically due to fluctuating food supplies. Assume that the number of foxes $f(t)$ is shown in Graph 2 above where t is the time in years since a certain date.

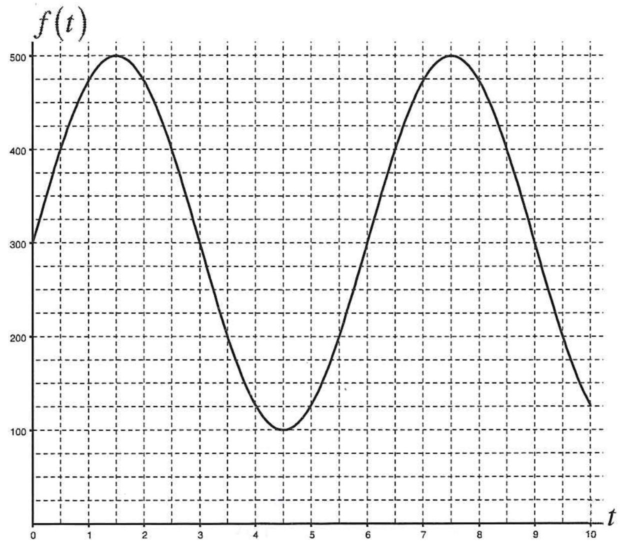
(a) On Graph 2, mark a point where $f(t)$ is increasing, a point where $f(t)$ is decreasing and a point where $f(t)$ is not changing much.

(b) Estimate the instantaneous rate of change at each of the three points marked in part (a) using local linearity (tangent line). Show how you arrived at your answers. Using correct units, explain the meaning of the values estimated in the context of the fox population.

$f'(4.5) = \frac{105-105}{4.6-4.4} = 0$

$f'(5.5) = \frac{220-180}{5.6-5.4} = 200$

$f'(8.5) = \frac{380-420}{8.6-8.4} = -200$



Graph 2

2. (continued) The population of foxes in a particular region varies periodically due to fluctuating food supplies. Assume that the number of foxes $f(t)$ is shown in Graph 2 above where t is the time in years since a certain date.

(c) Write an equation of the tangent line $l(t)$ to $f(t)$ at the point $(0.5, 400)$.

$l(t) - 400 = 200(t - 0.5)$

(d) Use $l(t)$ to estimate $f(0.75)$ and $f(1)$.

$l(.75) = 200(0.75 - 0.5) + 400$
$= 450$
$l(1) = 200(1 - 0.5) + 400$
$= 500$

(e) Which estimate: $l(0.75)$ or $l(1)$ is closest to the exact value of the fox population? Explain your reasoning.

$l(0.75)$ since it is closer to the original $f(0.5)$ value of foxes.

t sec	d feet
3	90
3.25	64.8
3.5	42.2
3.75	24.5
4	13.4
4.25	10
4.5	14.8
4.75	27.2
5	46
5.25	69.3
5.5	94.6
5.75	119.5
6	141.4
6.25	158.1
6.5	167.8

t sec	d feet
6.75	169.6
7	163.4
7.25	149.7
7.5	129.9
7.75	106.1
8	80.6
8.25	56.1
8.5	35
8.75	19.5
9	11.2
9.25	10.8
9.5	18.5
9.75	33.4
10	54.1
10.25	78.5

Table 1

3. Lee Perr attaches himself to a strong bungee cord and jumps off a bridge. At time $t = 3$ seconds, the cord first becomes taut. From that time on, Lee's distance, d, in feet, from the river below the bridge is given in Table 1.

(a) Estimate $d'(4)$ using a symmetric difference quotient. Show how you arrived at your answer.

$d'(4) = \dfrac{10 - 24.5}{4.25 - 3.75} = -29$ feet/sec

(b) Using correct units, explain the meaning of the value estimated in part (a). Is Lee going up or going down at $t = 4$? Explain your reasoning.

Lee's height above the river is decreasing at a rate of 29 feet/sec.
Down because $d'(4) < 0$.

(c) Is Lee going up or going down at $t = 9.25$? How fast is he going? Explain how you arrived at your answer.

$d'(9.25) = \dfrac{18.5 - 11.2}{9.5 - 9} = 14.6 \dfrac{\text{feet}}{\text{sec}}$

Going up because $d'(9.25) > 0$

t minutes	$C(t)$ cars	t minutes	$C(t)$ cars
0	0	15	1235
1	83	20	1655
3	253	21	1734
4	339	24	1969
7	589	25	2050
9	747	27	2217
10	826	28	2303
13	1066	30	2474

Table 2

4. The number of cars that have passed through an intersection at t minutes after 8 AM is given in Table 2. Traffic flow is defined as the rate at which cars pass through an intersection measured in cars per minute.

(a) Estimate the traffic flow at $t = 15$. Show how you arrived at your answer. Explain the meaning of this value in the context of cars passing through an intersection.

$$C'(15) = \frac{C(20) - C(13)}{20 - 13}$$

$$= \frac{1655 - 1066}{7}$$

$$= 84.14285$$

(b) Using your answer from part (a), write the equation of the tangent line at the point $(15, 1235)$. Use the tangent line to find a linear approximation for $C(17)$.

$$\ell(t) - 1235 = 84.14285(t - 15)$$

(c) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.

Avg Rate of Traffic flow $= \dfrac{C'(15) - C'(10)}{15 - 10}$