Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

1. A metal wire of length 8 centimeters is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius $(°C)$, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   a. Estimate $T'(7)$. Show the work that lead to your answer. Indicate units of measure.

   $$T'(7) = \frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{8 - 6} = -\frac{7}{2} \; °C/cm$$

   b. Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

   $T'(.5) = -\frac{7}{2}$ $T'(3) = -\frac{23}{4}$

   T'' not greater than 0 $T'(5.5) = -8$ $T'(7) = -\frac{7}{2}$

   because slopes steeper in negative direction.

2. Consider the function given by $y = \dfrac{1}{x^2 - 6x + 13}$ and the coordinate of $\left(3, \dfrac{1}{4}\right)$ located on that function.

   a. Show that $\dfrac{dy}{dx} = y^2(6 - 2x)$

   $$\frac{dy}{dx} = -1(x^2 - 6x + 13)^{-2}(2x - 6)$$

   $$\frac{dy}{dx} = \frac{-2x + 6}{(x^2 - 6x + 13)^2} \longrightarrow$$

   $$\frac{dy}{dx} = y^2(6 - 2x)$$
   $$\uparrow$$
   $$\frac{dy}{dx} = \left(\frac{1}{x^2 - 6x + 13}\right)^2 (6 - 2x)$$

   b. Find $\dfrac{d^2y}{dx^2}$ and evaluate it at the coordinate $\left(3, \dfrac{1}{4}\right)$.

   $$\frac{d^2y}{dx^2} = -2y^2 + 2y(6 - 2x)\frac{dy}{dx}$$

   $$= -2\left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right)(6 - 2(3))\left(\left(\frac{1}{4}\right)^2(6 - 2(3))\right)$$

   $$\frac{d^2y}{dx^2} = -\frac{1}{8}$$

   c. If $\dfrac{dy}{dx}$ is velocity and $\dfrac{d^2y}{dx^2}$ is acceleration, then at the coordinate $\left(3, \dfrac{1}{4}\right)$ is the object speeding up, slowing down, or neither?

   $$\left.\frac{dy}{dx}\right|_{(3, \frac{1}{4})} = 0 \qquad \left.\frac{d^2y}{dx^2}\right|_{(3, \frac{1}{4})} = -\frac{1}{8}$$