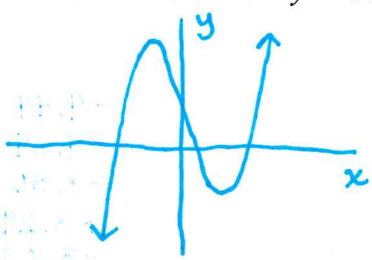
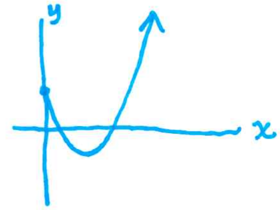
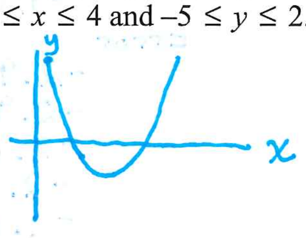
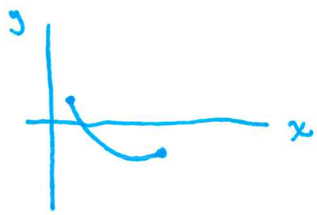
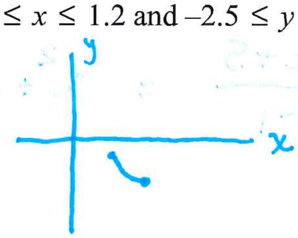
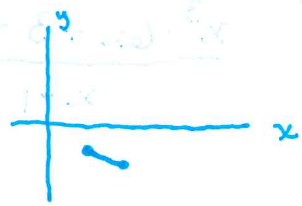
AP Calculus Name ______________________________

Discovering The Derivative
As you work on this project, use technology to perform as much work as possible. This project should be well-polished work. Your explanations should be written so that an ignorant, but intelligent peer could learn the ideas from your essay without consulting any other material.

Let (x_0, y_0) and (x_1, y_1) be points on a non-vertical line. The slope of the line determined by these points is $\dfrac{\Delta y}{\Delta x} = \dfrac{y_1 - y_0}{x_1 - x_0}$. Most of the functions we study in calculus have the property of **local linearity**, that is, locally they appear to be like a line. If we pick a point on the graph and zoom in, we will see a line.

1. Sketch the function $f(x) = x^3 - 6x + 3$ for each of the following windows. Mark the point (1, –2) on each graph.

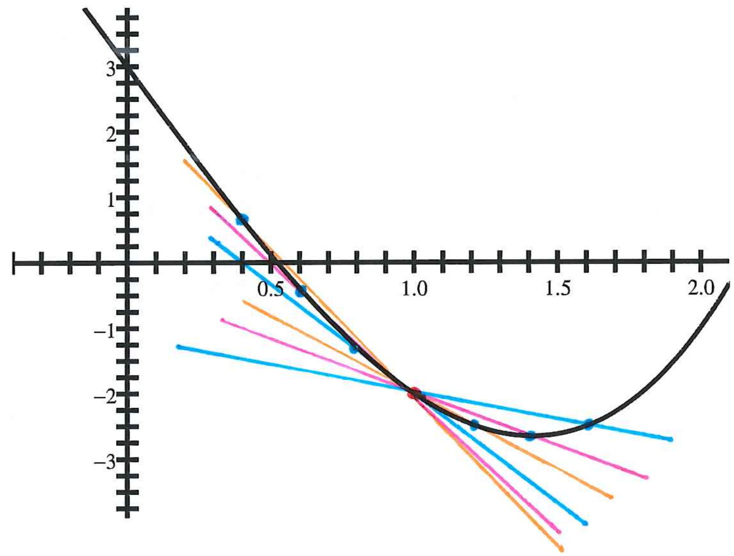
$-10 \le x \le 10$ and $-10 \le y \le 10$.  	$0 \le x \le 10$ and $-10 \le y \le 10$. 
$0 \le x \le 4$ and $-5 \le y \le 2.5$  	$.5 \le x \le 1.5$ and $-3 \le y \le .5$. 
$.8 \le x \le 1.2$ and $-2.5 \le y \le -1.5$.  	$.9 \le x \le 1.1$ and $-2.3 \le y \le -1.6$. 

Notice that the graph becomes more and more like a line as we zoom in. This line is called **a tangent line at the point (1, –2).** Pick a point on this tangent line close to (1, –2) and use technology to estimate the coordinates of this new point. Calculate the slope of the line through these two points.

Points: (1, –2) and (1.02, -2.058792) Slope: -2.9396

The slope of the tangent line is the "slope" of the function $f(x) = x^3 - 6x + 3$ at the point (1, –2). This slope is called **the instantaneous rate of change or derivative of f at $x = 1$** and is denoted by $f'(1)$.

2. a. We will now consider the graph of the function $f(x) = x^3 - 6x + 3$ for $0 \le x \le 2$. On the graph to the right, draw a line from $(1, f(1))$ to $(1.6, f(1.6))$ <u>using a straight edge</u>. This line is called a **secant line.** Be sure your "line" goes beyond each of these points given. Now draw five more secant lines from $(1, f(1))$ to $(x, f(x))$ for $x = 1.4, 1.2, 0.4, 0.6$, and 0.8.



b. We want to calculate the slope of each of the secant lines you have drawn and other similar lines. Complete the following table. Use technology! Hint: If you create a table with $y_1 = x^3 - 6x + 3$ and $y_2 = \dfrac{y_1(x) - y_1(1)}{x-1}$, the computations will be rapidly completed. In your calculator you need to set your table to INPUT.

x	slope $= \dfrac{f(x)-f(1)}{x-1}$
0.4	-4.44
0.6	-4.04
0.8	-3.56
0.9	-3.29
0.99	-3.0299
0.999	-3.00299
1	Error
1.001	-2.99699
1.01	-2.9699
1.1	-2.69
1.2	-2.36
1.4	-1.64
1.6	-0.84

c. As the value of x gets closer to 1, what happens to the secant lines you have drawn? Try to guess a limiting value for the slopes of the secant lines you have drawn.

d. Write an expression, in terms of x, for the slope of a secant line to f from $(1, -2)$ and $(x, f(x))$ referred to in part b of this problem.

$$\frac{f(x)-f(1)}{x-1} = \frac{x^3 - 6x + 3 - (-2)}{x-1} = \frac{x^3 - 6x + 5}{x-1} = x^2 + x - 5$$

e. Evaluate the slope of the secant line for the function when $x = 1$. What happens? Simplify the expression and then evaluate when $x = 1$? What happens now? How is this answer related to the previous values of the slope of the tangent line?

secant $= x^2 + x - 5$

secant$(1) = 1^2 + 1 - 5$

$= -3$