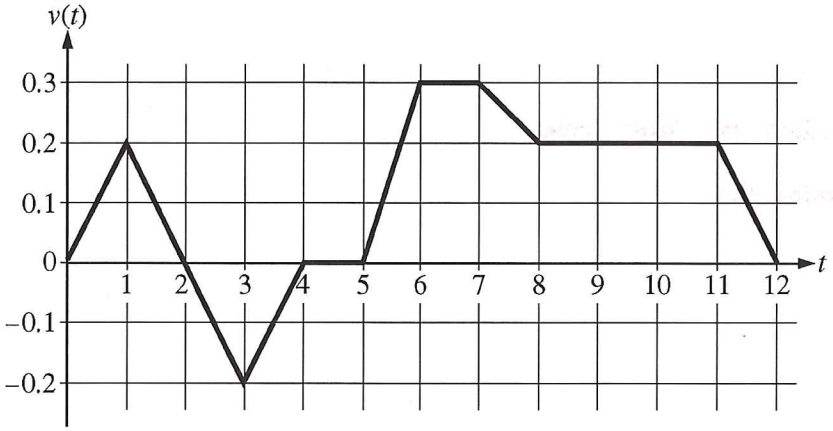
AP Calculus Name_____

Properties of Definite Integrals



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

1. Write a story about Caren's 12-minute bicycle ride to school based on the graph of Caren's velocity given above.

2. Shortly after leaving home, Caren realizes she left her calculus book at home, and she returns home to get it. At what time does she turn around to go back home? Give a reason for your answer.

t = 2

V(t) goes from positive to negative

3. What time did she leave home the second time? Give a reason for your answer.

t = 5

t from 0 to 2 same distance as t from 2 to 4. V(t) = 0 from 4 to 5.

4. Using correct units, explain the meaning of $\int_0^2 v(t)\,dt$ in terms of Caren's bicycle ride to school.

distance in miles of bike ride for 2 minutes

5. Using correct units, explain the meaning of $\int_2^4 v(t)\,dt$ in terms of Caren's bicycle ride to school.

distance going home in miles of bike ride of 2 minutes

6. If $h(t)$ is Caren's distance from home in miles t minutes after leaving home. Let $t=0$ be the time Caren left home the first time. What is the meaning of $h(4)$ in terms of Caren's bicycle ride to school?

Where is she at 4 minutes?

She is 0 miles from home.

According to the Fundamental Theorem of Calculus applied to this situation

$$h(t)=h(0)+\int_0^t v(m)\,dm$$

(Note: The variable inside the integrand should not be the same as the variable used as the upper limit.)

7. Izzy Wright uses the above equation and calculates

$$h(4)=h(0)+\int_0^4 v(m)\,dm=\frac{1}{2}(2)(0.2)+\frac{1}{2}(2)(0.2)=0.4$$

Izzy's twin sister Mae B. Wright uses the above equation and calculates

$$h(4)=h(0)+\int_0^4 v(m)\,dm=\int_0^2 v(m)\,dm-\int_2^4 v(m)\,dm=\frac{1}{2}(2)(0.2)-\frac{1}{2}(2)(0.2)=0$$

Which Wright do you think is right? Explain your reasoning in terms of Caren's bicycle ride to school.

Mae is correct. Direction matters.

FACT: The definite integral of a rate, $\int_a^b f'(x)\,dx = f(b) - f(a)$, measures the amount of change in $f(x)$ where $a \le b$.

8. Suppose that $f'(x) < 0$ on the interval $a \le x \le b$. Is $f(x)$ increasing, decreasing or both on $a \le x \le b$? How do you know?

f(x) is decreasing since f'(x) is the rate of f(x).

9. If $f(x)$ is decreasing for $a \le x \le b$, which is larger, $f(a)$ or $f(b)$? Is the value of $f(b) - f(a)$ greater than or less than zero? How do you know?

f(a) is larger $f(a) > f(b)$

$f(b) - f(a) < 0$ or negative

Fact: $f'(x) < 0$ on the interval $a \le x \le b$ then $\int_a^b f'(x)\,dx < 0$.

10. Returning to the context of Caren's ride to school, why does $\int_2^4 v(t)\,dt$ have to be negative? What direction is Caren riding her bicycle during the time interval $2 < t < 4$. Explain the meaning of $\int_2^4 v(t)\,dt$.

11. How far does Caren live from school? Explain how you arrived at your answer. Write a definite integral that represents how far Caren lives from school.

Area from 5 to 12

1.4 miles

$\int_5^{12} v(t)\,dt$

12. How far did Caren ride her bike during the 12 minutes, $0 \le t \le 12$? Is this the same answer as problem 11? Explain your reasoning.

1.8 miles

No different, since 0.2 miles in both directions before leaving home for good.

13. Using correct units, explain the meaning of $\int_0^{12} |v(t)|\,dt$ in terms of Caren's bicycle ride to school?

Finds total distance ridden.