Name: ______________________________ Date: ______________ Period: _______ 2016

AP Calculus – Worksheet – Integration by Parts

1. $\int x^2 \ln x\,dx$

$u = \ln x \quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^3}{3}$

$\frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx$

$\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx$

$\frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C$

2. $\int x\tan^{-1}x\,dx$

$u = \tan^{-1}x \quad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx \quad v = \frac{x^2}{2}$

$\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

$\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$

$\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \tan^{-1}x + C$

3. $\int x\sin x\,dx$

$u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$-x\cos x - \int -\cos x\,dx$

$-x\cos x + \sin x + C$

4. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\int e^u\,du$

$2e^{\sqrt{x}} + C$

5. $\int 5^x x\,dx$

$u = x \quad dv = 5^x\,dx$

$du = dx \quad v = \frac{5^x}{\ln 5}$

$\frac{x5^x}{\ln 5} - \int \frac{5^x}{\ln 5}\,dx$

$\frac{x5^x}{\ln 5} - \frac{5^x}{(\ln 5)^2} + C$

6. $\int x\ln x^2\,dx$

$u = \ln(x^2) \quad dv = x\,dx$

$du = \frac{2}{x}\,dx \quad v = \frac{x^2}{2}$

$\frac{x^2}{2}\ln(x^2) - \int x\,dx$

$\frac{x^2}{2}\ln(x^2) - \frac{x^2}{2} + C$

7. $\int \frac{\ln x}{x^2}\,dx$

$u = \ln x$ $\quad dv = x^{-2}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = -x^{-1}$

$-x^{-1}\ln x - \int -\frac{1}{x}(-x^{-1})\,dx$

$-\frac{\ln x}{x} + \int x^{-2}\,dx$

$-\frac{\ln x}{x} - \frac{1}{x} + C$

9. $\int e^x \sin x\,dx$ $= -e^x\cos x - \int -e^x \cos x\,dx$

$u = e^x$ $\quad dv = \sin x\,dx$

$du = e^x\,dx$ $\quad v = -\cos x$

$\int e^x \sin x\,dx = -e^x\cos x + \int e^x\cos x\,dx$

$u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x\,dx$ $\quad v = \sin x$

$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x$

$\int e^x \sin x\,dx = \frac{-e^x\cos x + e^x \sin x}{2} + C$

8. $\int \sin^{-1} x\,dx$

$u = \sin^{-1} x$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{1}{2}du = -x\,dx$

$x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$

$x\sin^{-1}x + \sqrt{1-x^2} + C$

10. $\int_1^2 xe^x\,dx$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$ $\quad v = e^x$

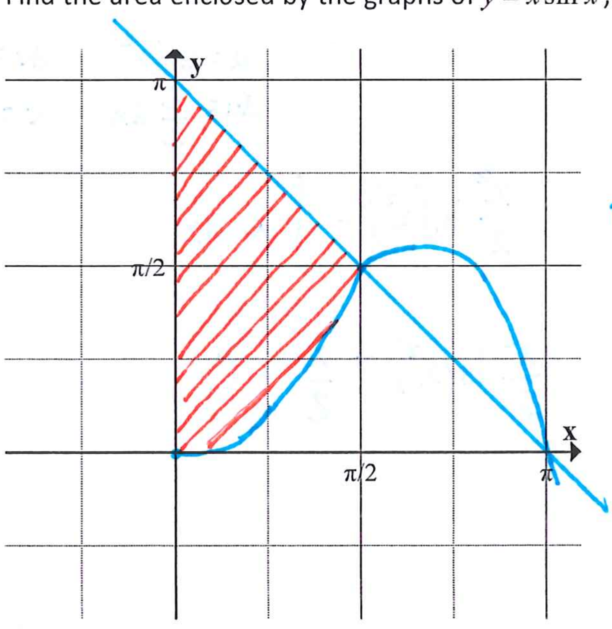
$xe^x\Big|_1^2 - \int_1^2 e^x\,dx$

$(xe^x - e^x)\Big|_1^2$

$(2e^2 - e^2) - (1e^1 - e^1)$

$e^2$

11. Find the area enclosed by the graphs of $y = x\sin x$, $y = \pi - x$, and the y-axis.




$\int_0^{\pi/2} (\pi - x - x\sin x)\,dx$

$u = x$ $\quad dv = \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$\pi x - \frac{x^2}{2}\Big|_0^{\pi/2} - \left(-x\cos x\Big|_0^{\pi/2} - \int_0^{\pi/2} -\cos x\,dx\right)$

$\left(\pi x - \frac{x^2}{2} + x\cos x - \sin x\right)\Big|_0^{\pi/2}$

$\frac{\pi^2}{2} - \frac{\pi^2}{8} + \frac{\pi}{2}(0) - 1 - (0 - 0 + 0(1) - 0)$

$\frac{3\pi^2}{8} - 1$