Name: _____ Date: _____ Period: _____ 2016

AP Calculus – QUIZ – U-Substitution, Partial Fraction Decomposition, and Improper Integrals

1. $\int \frac{x^2}{\sqrt{x^3+1}}\,dx$

$u = \sqrt{x^3+1}$

$du = \frac{3x^2}{2\sqrt{x^3+1}}\,dx$

$\int \frac{2}{3}\,du$

$\frac{2}{3}u + C$

$\frac{2}{3}\sqrt{x^3+1} + C$

2. $\int \frac{-6x-9}{2x^2-x-1}\,dx$

$\frac{-6x-9}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$

$-6x-9 = A(x-1) + B(2x+1)$

$x=1$: $-15 = 3B$, $-5 = B$

$x=-\frac{1}{2}$: $-6 = -\frac{3}{2}A$, $4 = A$

$\int \left(\frac{4}{2x+1} - \frac{5}{x-1}\right) dx$

$2\ln|2x+1| - 5\ln|x-1| + C$

3. $\int_0^3 \frac{1}{(x-1)^{2/3}}\,dx$

$\lim_{b\to 1^-} \int_0^b (x-1)^{-2/3}\,dx + \lim_{c\to 1^+} \int_c^3 (x-1)^{-2/3}\,dx$

$\lim_{b\to 1^-} 3(x-1)^{1/3}\Big|_0^b + \lim_{c\to 1^+} 3(x-1)^{1/3}\Big|_c^3$

$\lim_{b\to 1^-} 3(b-1)^{1/3} - 3(0-1)^{1/3} + \lim_{c\to 1^+} 3(3-1)^{1/3} - 3(c-1)^{1/3}$

0 $\qquad$ 0

$-3(-1)^{1/3} + 3(2)^{1/3}$

$3 + 3\sqrt[3]{2}$

4. $\int \left(e^{2x} + 5^x - \sin 2x\right)\,dx$

$\frac{e^{2x}}{2} + \frac{5^x}{\ln 5} + \frac{\cos(2x)}{2} + C$