Calculus

Name_____

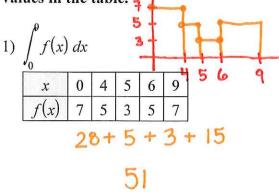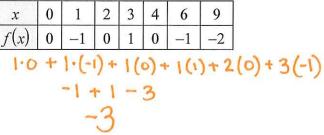# Riemann Sums

Date_____ Period_____

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table.

1) $\int_0^9 f(x)\,dx$

x	0	4	5	6	9
$f(x)$	7	5	3	5	7

$28 + 5 + 3 + 15$

51

2) $\int_0^{10} f(x)\,dx$

x	0	1	4	6	9	10
$f(x)$	6	4	5	3	2	4

$1\cdot6 + 3\cdot4 + 2\cdot5 + 3\cdot3 + 1\cdot2$
$6 + 12 + 10 + 9 + 2$

39

3) $\int_0^8 f(x)\,dx$

x	0	1	2	3	5	6	8
$f(x)$	2	3	5	3	5	6	8

$1\cdot2 + 1\cdot3 + 1\cdot5 + 2\cdot3 + 1\cdot5 + 2\cdot6$
$2 + 3 + 5 + 6 + 5 + 12$

33

4) $\int_0^9 f(x)\,dx$

x	0	1	2	3	4	6	9
$f(x)$	0	−1	0	1	0	−1	−2

$1\cdot0 + 1\cdot(-1) + 1(0) + 1(1) + 2(0) + 3(-1)$
$-1 + 1 - 3$

-3

For each problem, use a right-hand Riemann sum to approximate the integral based off of the values in the table.

5) $\int_0^9 f(x)\,dx$

x	0	6	7	8	9
$f(x)$	6	4	5	7	6

$6\cdot4 + 1\cdot5 + 1\cdot7 + 1\cdot6$
$24 + 5 + 7 + 6$

42

6) $\int_0^9 f(x)\,dx$

x	0	4	5	6	8	9
$f(x)$	0	1	0	−1	0	−1

$4\cdot1 + 1\cdot0 + 1(-1) + 2\cdot0 + 1(-1)$
$4 - 1 - 1$

2

7) $\int_0^8 f(x)\,dx$

x	0	2	3	4	6	7	8
$f(x)$	6	7	5	3	2	3	2

$2\cdot7 + 1\cdot5 + 1\cdot3 + 2\cdot2 + 1\cdot3 + 1\cdot2$
$14 + 5 + 3 + 4 + 3 + 2$

31

8) $\int_0^9 f(x)\,dx$

x	0	1	2	5	6	7	9
$f(x)$	−3	−2	−1	−2	−3	−2	−1

$1(-2) + 1(-1) + 3(-2) + 1(-3) + 1(-2) + 2(-1)$
$-2 - 1 - 6 - 3 - 2 - 2$

-16

For each problem, approximate the area under the curve over the given interval using L-*RAM*$_4$, M-*RAM*$_4$, R-*RAM*$_4$, *TRAP*$_4$, and Simpson's-4.

9) $y = -\dfrac{x^2}{2} + 6;\ [-3, 1]$

x	Y
-3	3/2 L
-2	4 L R
-1	11/2 L R
0	6 L R
1	11/2 R

X	Y
-5/2	23/8 m
-3/2	39/8 m
-1/2	47/8 m
1/2	47/8 m

$LRAM_4 = 1\left(\frac{3}{2} + 4 + \frac{11}{2} + 6\right) = 17$

$MRAM_4 = 1\left(\frac{23}{8} + \frac{39}{8} + \frac{47}{8} + \frac{47}{8}\right) = \frac{39}{2}$

$RRAM_4 = 1\left(4 + \frac{11}{2} + 6 + \frac{11}{2}\right) = 21$

$TRAP_4 = \frac{1}{2}\left(\frac{3}{2} + 2(4) + 2\left(\frac{11}{2}\right) + 2(6) + \frac{11}{2}\right) = 19$

$Simp_4 = \frac{1}{3}\left(\frac{3}{2} + 4(4) + 2\left(\frac{11}{2}\right) + 4(6) + \frac{11}{2}\right) = 19\frac{1}{3} = 19.333$

10) $y = x^2 - 2x + 3;\ [-1, 3]$

x	Y
-1	6
0	3
1	2
2	3
3	6

x	Y
-1/2	17/4
1/2	9/4
3/2	9/4
5/2	17/4

$LRAM_4 = 14$

$MRAM_4 = 13$

$RRAM_4 = 14$

$TRAP_4 = 14$

$Simp_4 = 13.333$

11) $y = \dfrac{2}{x};\ [2, 6]$

X	Y
2	1
3	2/3
4	1/2
5	2/5
6	1/3

X	Y
5/2	4/5
7/2	4/7
9/2	4/9
11/2	4/11

$LRAM_4 = \dfrac{77}{30} = 2.56666$

$MRAM_4 = \dfrac{7552}{3465} = 2.17950$

$RRAM_4 = \dfrac{19}{10} = 1.900$

$TRAP_4 = \dfrac{67}{30} = 2.23333$

$Simp_4 = \dfrac{11}{5} = 2.200$

12) $y = \dfrac{5}{x^2 + 1};\ [-6, -2]$

X	Y
-6	5/37
-5	5/26
-4	5/17
-3	5/10
-2	5/5

X	Y
-11/2	4/25
-9/2	4/17
-7/2	20/53
-5/2	20/29

$LRAM_4 = \dfrac{9171}{8177} = 1.12156$

$MRAM_4 = \dfrac{955216}{653225} = 1.46230$

$RRAM_4 = \dfrac{439}{221} = 1.98642$

$TRAP_4 = \dfrac{12707}{8177} = 1.55399$

$Simp_4 = \dfrac{36736}{24531} = 1.49753$