Name: _____ Date: _____ Period: _____ 2016

AP Calculus – Worksheet – Implicit Differentiation

Find dy/dx for each of the following.

1. $y^3 = x\sin x$

$3y^2 \cdot \frac{dy}{dx} = x\cos x + \sin x$

$\frac{dy}{dx} = \frac{x\cos x + \sin x}{3y^2}$

2. $\frac{\sqrt{2x}}{y} = \sin y$

$\frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2 = y\cos y \frac{dy}{dx} + \sin y \frac{dy}{dx}$

$\frac{1}{\sqrt{2x}(y\cos y + \sin y)} = \frac{dy}{dx}$

3. $\cos(xy) = x$

$-\sin(xy)\left[x\frac{dy}{dx} + y\right] = 1$

$\frac{dy}{dx} = \frac{1 + y\sin(xy)}{-x\sin(xy)}$

4. $\cos(x+y) = x$

$-\sin(x+y)\left[1 + \frac{dy}{dx}\right] = 1$

$\frac{dy}{dx} = \frac{1 + \sin(x+y)}{-\sin(x+y)}$

5. $x^4y^2 + 3x^2y + 2 = 0$

$4x^3y^2 + 2yx^4\frac{dy}{dx} + 3x^2\frac{dy}{dx} + 6xy = 0$

$\frac{dy}{dx} = \frac{-4x^3y^2 - 6xy}{2x^4y + 3x^2}$

$= \frac{-4x^2y^2 - 6y}{2x^3y + 3x}$

6. $7x(y+3) = \tan y - x$

$7(y+3) + 7x\frac{dy}{dx} = \sec^2 y \frac{dy}{dx} - 1$

$\frac{7y + 22}{\sec^2 y - 7x} = \frac{dy}{dx}$

7. $\frac{x^2+y^2}{x^2-y^2} = 4$

$\frac{(x^2-y^2)(2x+2y\frac{dy}{dx}) - (x^2+y^2)(2x-2y\frac{dy}{dx})}{(x^2-y^2)^2} = 0$

$0 = \frac{\left(2x^3 - 2xy^2 + 2x^2y\frac{dy}{dx} - 2y^3\frac{dy}{dx}\right) - \left(2x^3 + 2xy^2 - 2x^2y\frac{dy}{dx} - 2y^3\frac{dy}{dx}\right)}{(x^2-y^2)^2}$

$0 = -4xy^2 + 4x^2y\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{y}{x}$

8. $x^3 + x^2y^2 + y^3 = 30$

$3x^2 + 2xy^2 + 2x^2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy^2 - 3x^2}{2x^2y + 3y^2}$

9. $3x^2 + \tan y + (y+3)^2 = 5$

$6x + \sec^2 y \cdot \frac{dy}{dx} + 2(y+3)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-6}{\sec^2 y + 2y + 6}$

10. $\sin y \cos y = 2x^3 + x$

$\sin y(-\sin y)\frac{dy}{dx} + \cos y(\cos y)\frac{dy}{dx} = 6x^2 + 1$

$\frac{dy}{dx} = \frac{6x^2 + 1}{\cos^2 y - \sin^2 y}$

11. If $9x^2 + 4y^2 = 25$, find

$18x + 8y\frac{dy}{dx} = 0$

$18 + 8y\frac{d^2y}{dx^2} + 8\cdot\frac{dy}{dx}\cdot\frac{dy}{dx} = 0$

$18 + 8y\frac{d^2y}{dx^2} + \frac{81x^2}{2y^2} = 0$

a. $\frac{dy}{dx} = \frac{-9x}{4y}$

b. $\left.\frac{dy}{dx}\right|_{(1,2)} = -\frac{9}{8}$

c. $\frac{d^2y}{dx^2} = \frac{-81x^2 - 36y^2}{16y^3}$

d. $\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{-225}{128}$