Name: ______________________________ Date: ______________ Period: ______ 2016

AP Calculus – QUIZ – Related Rates – **NO CALCULATOR ALLOWED**

1. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V ?

$$\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$$

(A) 10π (B) 12π (C) 22.5π (D) 25π $\boxed{\text{(E) } 30\pi}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 100\pi(0.3)$$

2. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π $\boxed{\text{(C) } 24\pi}$ (D) 54π (E) 108π

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$$

$$\frac{\pi}{6}(36 + 108) \qquad \pi(6 + 18)$$

3. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

$\boxed{\text{(A) } 6}$ (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

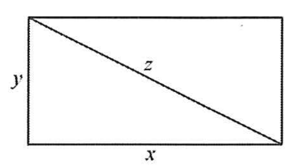
$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad \frac{96\cancel{\pi}}{2\cancel{\pi}(8)} = \frac{dr}{dt}$$

4. The sides of the rectangle below increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

$$\frac{1}{3}\frac{dx}{dt} = \frac{dy}{dt}$$

$$x^2 + y^2 = z^2$$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\cancel{2}x\frac{dx}{dt} + \frac{y}{3}\frac{dx}{dt}$$

(A) $\frac{1}{3}$ $\boxed{\text{(B) } 1}$ (C) 2 (D) $\sqrt{5}$ (E) 5

$$\frac{1}{3}\frac{dx}{dx}(3x + y) = z\frac{dz}{dt}$$

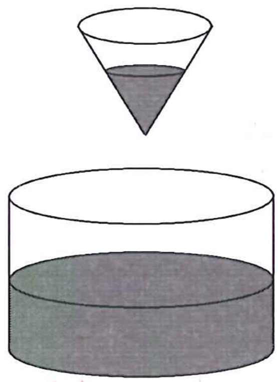
$$\frac{dx}{dt} = \frac{3z\frac{dz}{dt}}{3x + y}$$

5. As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at a rate of $(h - 12)$ feet per minute.

$$\left(V = \pi r^2 h \text{ and } V = \frac{1}{3}\pi r^2 h \right)$$

$3r = h$

$r = \frac{1}{3}h$



(part a) Write an equation for the volume of the water in the conical tank as a function of h.

2: V with h's only

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{\pi}{27}h^3$

(part b) At what rate is the volume of the water in the conical tank changing when $h = 3$? Indicate units of measure.

1 dv/dt
1 unit
1 Ans
1 dh/dt = h−12

$\frac{dV}{dt} = \frac{\pi}{9}h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9}h^2(h-12)$

$\frac{dV}{dt} = -9\pi \; \frac{ft^3}{min}$

(part c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

1 unit
1 ans
1 dv/dt equ
1 $\frac{dr}{dt} = 0$

$V = \pi r^2 y$

$\frac{dV}{dt} = \pi r^2 \frac{dy}{dt}$

$9\pi = 400\pi \frac{dy}{dt}$

$\frac{9}{400}$ ft/min $= \frac{dy}{dt}$