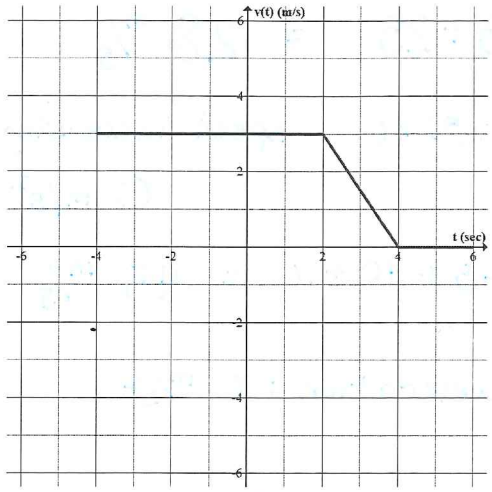
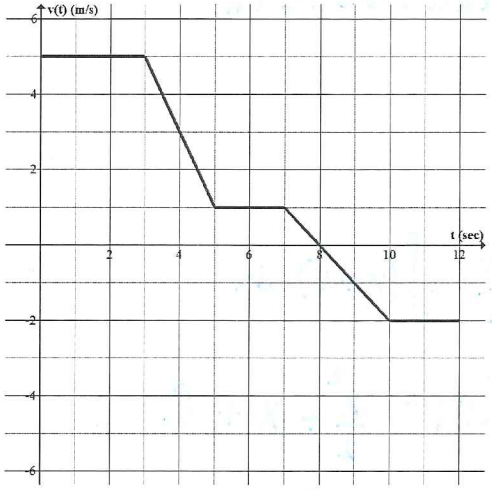
Name: _____ Date: _____ Period: _____ 2016

AP Calculus – Displacement, Distance, Position from Velocity: Velocity, Speed from Acceleration

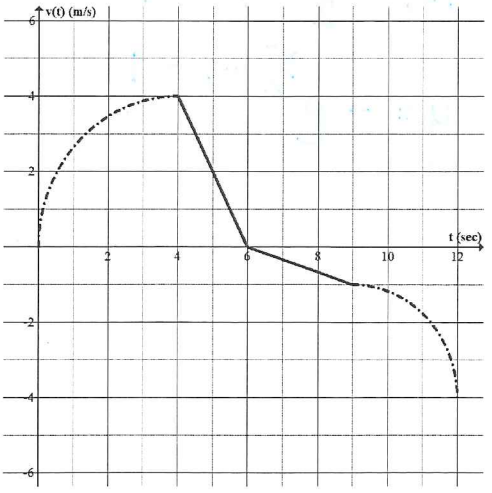
Given the velocity by time graphs below, find the following.



1. How far did the object travel from 0 to 2 seconds? 6 m
2. How far did the object travel from 0 to 6 seconds? 9 m
3. How far did the object travel from -4 to 0 seconds? 12 m
4. If the object had an initial position of 2m at -2 seconds, where is the object at 5 seconds? $2m + 15m = 17m$
5. If the object had an initial position of 3m at -1 second, what is the average velocity of the object at over the next 5 seconds? $\frac{12}{5}\ \frac{m}{s}$



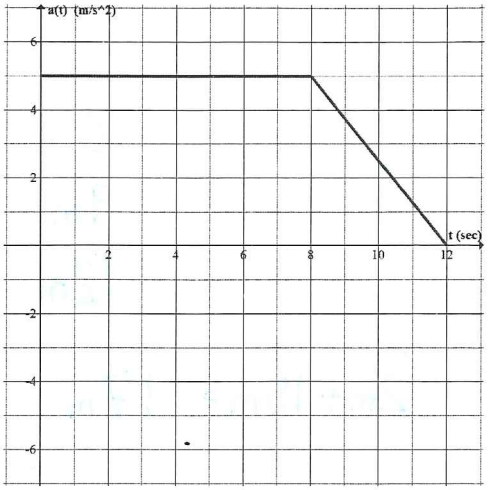
6. How far did the object travel from 0 to 2 seconds? 10 m
7. How far did the object travel from 0 to 6 seconds? 22 m
8. How far did the object travel from 4 to 0 seconds? 19 m
   distance
9. If the object had an initial position of 2m at 3 seconds, where is the object at 6 seconds? $2m + 7m = 9m$
10. If the object had an initial position of 5m to the left at 0 seconds, where is the object at 12 seconds?
    $-5m + 23.5m + (-2m - 4m) = 12.5m$ to the right
11. What is the average **speed** of the object for 0 to 12 seconds?
    $\frac{23.5m + (6m)}{12s} = \frac{59}{24}\ \frac{m}{s} = 2.45833\ m/s$



The two dashed curves are quarter-circles.

12. How far did the object travel from 0 to 4 seconds? 4π m
13. How far did the object travel from 0 to 6 seconds? $(4\pi + 4)$ m
14. How far did the object travel from 0 to 9 seconds?
    $4\pi + 4 + \frac{3}{2}$ $\left(4\pi + \frac{11}{2}\right)$ m
15. How far did the object travel from 0 to 12 seconds?
    $4\pi + 4 + \frac{3}{2} + \left(12 - \frac{9\pi}{4}\right)$ $\left(\frac{7\pi}{4} + \frac{35}{2}\right)$ m
16. If the object had an initial position of 2m at 4 seconds, where is the object at 12 seconds?
    $2m + 4m - \frac{3}{2}m - \left(12 - \frac{9\pi}{4}\right)$ $\left(\frac{9\pi}{4} - \frac{15}{2}\right)$ m
17. If the object had an initial position of 0m at 0 seconds, where is the object at 12 seconds?
    $4\pi + 4 - \frac{3}{2} - \left(12 - \frac{9\pi}{4}\right)$ $\left(\frac{25\pi}{4} - \frac{19}{2}\right)$ m
18. If a new graph $h(t)$ is defined by $h(t) = v(t) + 3$, where is the object at 12 seconds, if it had an initial position of 0m at 0 seconds?
    $\frac{25\pi}{4} - \frac{19}{2} + 3 \cdot 12 = \left(\frac{25\pi}{4} + \frac{53}{2}\right)$ m

Given the acceleration by time graphs below, find the following.



19. What is the objects' acceleration at 5 seconds? $5\ m/s^2$

20. What is the objects' velocity at 5 seconds if it had an initial velocity of 3 m/s? $3 + 25 = 28\ m/s$

21. Is the objects' speed increasing or decreasing at 5 seoncds?
$a(5) > 0$, $v(5) > 0$ speed increasing

22. What is the objects' acceleration at 12 seconds? $0\ m/s^2$

23. What is the objects' velocity at 12 seconds if it had an initial velocity of -5 m/s? $-5 + 40 + 10 = 45\ m/s$

24. Is the objects' speed increasing or decreasing at 12 seconds?
neither, acceleration = $0\ m/s^2$

I would find equations of v(t) to solve →

25. What is the position of the object at 12 seconds if its initial position and velocity were 5m and -10m/s respectively?

$$x(12) - x(0) = \frac{695}{3} - 5 = \frac{680}{3} m = 226.66667 m$$

Solve the following indefinite integrals for the general solution and the particular solution from the given initial value.

26. $\frac{dy}{dx} = 3x^3 - \sin x$, $y(0) = 10$

$$\int dy = \int (3x^3 - \sin x)\, dx$$

$$y = \frac{3}{4}x^4 + \cos x + C$$

$$y = \frac{3}{4}x^4 + \cos x + 9$$

27. $\frac{dy}{dx} = xy + y$, $y(4) = -e^3$

$$\int \frac{dy}{y} = \int (x+1)\, dx$$

$$\ln|y| = \frac{x^2}{2} + x + C$$

$$\ln|y| = \frac{x^2}{2} + x - 9$$

$$y = -e^{\frac{x^2}{2} + x - 9}$$

28. $v(t) = 5t - 4t^{-1} + 6t^2$, $x(1) = 6$

$$\int dx = \int (5t - 4t^{-1} + 6t^2)\, dt$$

$$x(t) = \frac{5}{2}t^2 - 4\ln|t| + 2t^3 + C$$

$$x(t) = \frac{5}{2}t^2 - 4\ln|t| + 2t^3 + \frac{3}{2}$$

29. $a(t) = \sqrt{t} + 3\sin t - e^t$, $v(0) = 4$, $x(0) = 8$

$$v(t) = \frac{2}{3}t^{3/2} - 3\cos t - e^t + C$$

$$v(t) = \frac{2}{3}t^{3/2} - 3\cos t - e^t + 8$$

$$x(t) = \frac{4}{15}t^{5/2} - 3\sin t - e^t + 8t + C$$

$$x(t) = \frac{4}{15}t^{5/2} - 3\sin t - e^t + 8t + 9$$

$$a(t) = \begin{cases} 5 & , \ 0 \le t \le 8 \\ -\frac{5}{4}t + 15 & , \ 8 \le t \le 12 \end{cases}$$

$$v(t) = \begin{cases} 5t + C \\ -\frac{5}{8}t^2 + 15t + C_1 \end{cases}$$

$v(0) = -10 \text{ m/s} \Rightarrow -10 = 5(0) + C$

$C = -10$

$v(8) = 5(8) - 10$

$v(8) = 30$

$30 = -\frac{5}{8}(8)^2 + 15(8) + C_1$

$-50 = C_1$

$$v(t) = \begin{cases} 5t - 10 & , \ 0 \le t \le 8 \\ -\frac{5}{8}t^2 + 15t - 50 & , \ 8 \le t \le 12 \end{cases}$$

$$x(t) = \begin{cases} \frac{5}{2}t^2 - 10t + C \\ -\frac{5}{24}t^3 + \frac{15}{2}t^2 - 50t + C_1 \end{cases}$$

$x(0) = 5\text{m} \Rightarrow 5 = \frac{5}{2}(0)^2 - 10(0) + C$

$5 = C$

$x(8) = \frac{5}{2}(8)^2 - 10(8) + 5$

$x(8) = 85$

$85 = -\frac{5}{24}(8^3) + \frac{15}{2}(8)^2 - 50(8) + C_1$

$\frac{335}{3} = C_1$

$$x(t) = \begin{cases} \frac{5}{2}t^2 - 10t + 5 & , \ 0 \le t \le 8 \\ -\frac{5}{24}t^3 + \frac{15}{2}t^2 - 50t + \frac{335}{3} & , \ 8 \le t \le 12 \end{cases}$$

$x(12) = \frac{695}{3}$