

Caching and pre-fetching: the role of hazard rates.

Andres Ferragut

joint work with Matias Carrasco and Fernando Paganini

Universidad ORT Uruguay

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The caching problem

Point processes and stochastic intensity

The optimal caching policy

Large scale asymptotics

Connection with timer-based policies

Conclusions

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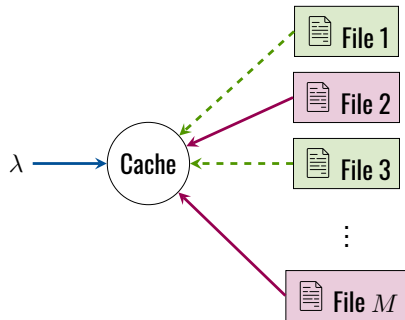
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Conclusions

The caching problem

- Consider a **cache system** with a catalog of M objects.
- Requests for objects arrive at random.
- The cache can locally store $C < M$ of them.
- If item is in cache, we have a **hit**. Otherwise, it is a **miss**.



Objective: for a given arrival stream, maximize the steady-state **hit rate**.

A sequential approach

- Consider a sequence of random variables Z_1, Z_2, \dots with values in $\{1, \dots, M\}$.
- Consider also the set:

$$\mathcal{C} = \{\{i_1, \dots, i_k\} \subset \{1, \dots, M\}, k \leq C\}$$

- A (causal) caching policy would be a sequence of maps π_n deciding which contents to store:

$$\pi_n(Z_1, \dots, Z_{n-1}) \rightarrow \mathcal{C}$$

- In probabilistic terms, let $\mathcal{F}_n = \sigma(Z_1, \dots, Z_n)$, then π_n is any \mathcal{C} -valued \mathcal{F}_n -predictable process (\mathcal{F}_{n-1} -measurable).

A simple case

The Independent Reference Model (IRM)

- Assume now that Z_n are *iid* with distribution $p_i = P(Z_n = i)$, where p_i is the **popularity** of content i . Wlog, we take $p_1 \geq p_2 \geq \dots$
- In this case, $Z_n \mid \mathcal{F}_{n-1} \sim p$, thus the hit probability at time n is:

$$P(Z_n \in \pi_n) = E[\mathbf{1}_{Z_n \in \pi_n}] = E[E[\mathbf{1}_{Z_n \in \pi_n} \mid \mathcal{F}_{n-1}]] = E\left[\sum_{i \in \pi_n} p_i\right] \leq \sum_{i=1}^C p_i$$

- Taking $\pi_n \equiv \{1, \dots, C\}$ achieves the bound.

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- Taking $\pi_n \equiv \{1, \dots, C\}$ achieves the bound.

Conclusion: under iid requests, the static “keep the most popular” policy is optimal.

In practice, popularities are not known. This leads to the **least-frequently-used (LFU)** eviction policy:

- Take π_n as the most requested objects so far (remove the least frequently used).
- In the long range, converges to the static policy.

Another popular eviction policy is **least-recently-used (LRU)**, which treats π_n as a list defined recursively:

- If $Z_n \in \pi_n$, serve the content, move Z_n to the front of the list.
- If $Z_n \notin \pi_n$, fetch the content, put Z_n in the front of the list, remove the last object in the list (which is the least recently requested).

- Typically, requests are correlated, and popularities evolve over time.
- For instance, requests for a file may arrive in bursts.
- **LRU** adapts to changes in popularity. Is good for bursts of requests. Tons of literature on this policy (also called move-to-front).
- However, performance metrics and optimality results are **hard** to establish.

The caching problem, take 2

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Point process approach [Fofack et al. 2014]:

- Assume requests for item i come from a **point process** of intensity $\lambda_i := \lambda p_i$.



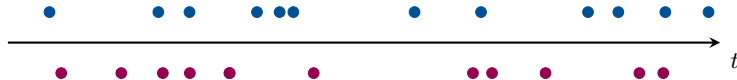
- At each point in time we must decide which items must be stored locally.

If inter-request times are **heavy tailed**, this can model burstiness.

Example: Pareto arrivals

Consider two items, with equal popularity...

■ Poisson arrivals:



Homogeneous

■ Heavy tailed arrivals (Pareto $\alpha = 2$):



Bursty!

Some open questions...

- What is the optimal causal policy in this framework?
- Can we compute the optimal hit rate/hit probability?
- What is its large scale behavior?
- How typical policies compare to the optimal one?

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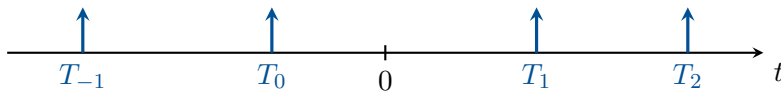
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A bit of point process theory...

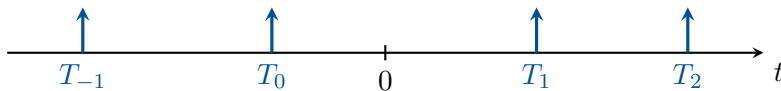
Let $N = \{T_k : k \in \mathbb{Z}\}$ be a **stationary point process** representing request times:



i.e. $N(B) = \sum_n \mathbf{1}_{\{T_n \in B\}}$ is a random counting measure.

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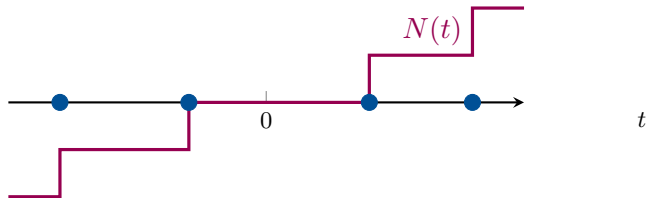
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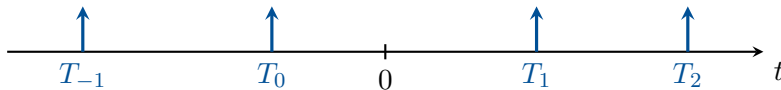
Counting process:

$$N(t) = \begin{cases} N([0, t]) & t \geq 0 \\ -N((t, 0]) & t < 0 \end{cases}$$



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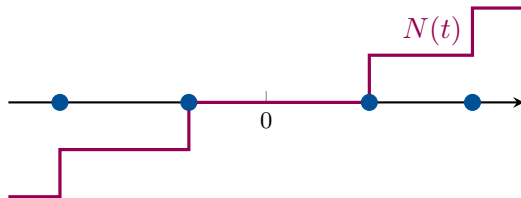
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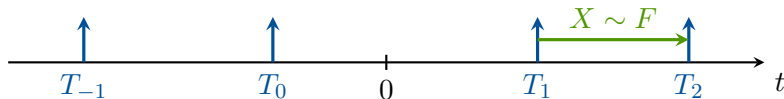
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Let $\mathcal{F}_t = \sigma(N(s), s \leq t)$ be its **internal history**.

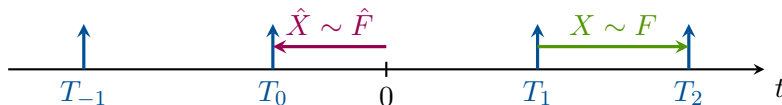
Two important distributions:



Inter-arrival distribution: $F(t) := P_N^0(T_1 - T_0 \leq t), \quad E_N^0[T_1] = 1/\lambda.$

Note: here P_N^0 is the **Palm probability** of the point process (conditioning on $T_0 = 0$).

Two important distributions:



Inter-arrival distribution: $F(t) := P_N^0(T_1 - T_0 \leq t), \quad E_N^0[T_1] = 1/\lambda.$

Age distribution: $\hat{F}(t) := P(-T_0 \leq t) = \lambda \int_0^t 1 - F(s) ds,$

Note: here P_N^0 is the **Palm probability** of the point process (conditioning on $T_0 = 0$).

Consider a simple stationary point process N with intensity λ , defined in some probability space (Ω, \mathcal{F}, P) . Let some filtration $\{\mathcal{F}_t\}_{t \in \mathbb{R}}$ be a **history** of the process.

Definition:

The random process $\lambda(t) \geq 0$ is a **stochastic intensity** for the history \mathcal{F}_t iff it is a.s. locally integrable, \mathcal{F}_t -adapted and:

$$E [N((a, b]) \mid \mathcal{F}_a] = E \left[\int_a^b \lambda(t) dt \mid \mathcal{F}_a \right]$$

for all $a, b \in \mathbb{R}$.

Local interpretation:

$$E[N((t, t + h]) \mid \mathcal{F}_t] = \lambda(t)h + o(h) \quad P - a.s.,$$

So $\lambda(t)$ acts as a **local** notion of intensity based on previous history.

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Martingale interpretation:

$$M_a(t) = N(t) - N(a) - \int_a^t \lambda(s)ds$$

is a local (P, \mathcal{F}_t) martingale for any $a \in \mathbb{R}$.

Namely, $A(t) = N(a) + \int_a^t \lambda(s)ds$ is the **compensator** of the counting process.

- If $N(t)$ is a Poisson process, then we know that

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is a martingale, so the stochastic intensity of a Poisson process is just $\lambda(t) \equiv \lambda$.

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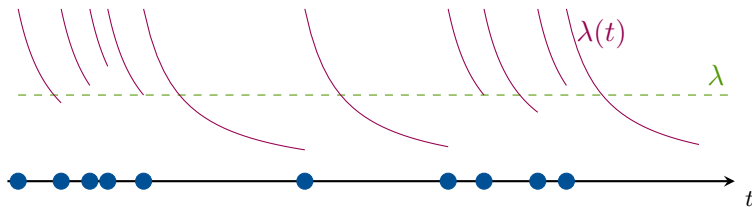
is a martingale, so the stochastic intensity of a Poisson process is just $\lambda(t) \equiv \lambda$.

- In fact, this **characterizes** the Poisson process. The stochastic intensity $\lambda(t)$ is **deterministic** if and only if N is a Poisson process of (possible time-varying) intensity $\lambda(t)$.

Stochastic intensity

A local notion of intensity...

However, if traffic is **bursty**, the stochastic intensity **rises** after arrivals:



Note: for stationary processes, $E[\lambda(t)] = E[\lambda(0)] = \lambda$, the average intensity.

Renewal processes

- Let now N be a **stationary renewal process**, i.e. inter request times $T_{n+1} - T_n$ are $iid \sim F$.
- Assume that F has a density, and define the **hazard rate** of F as:

$$\eta(t) = \frac{f(t)}{1 - F(t)}$$

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Theorem (Daley-Vere Jones, Chapter 7)

For a renewal process and its natural history, the stochastic intensity is:

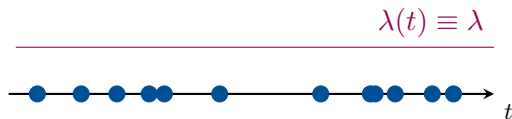
$$\lambda(t) = \eta(t - T^*(t)),$$

where

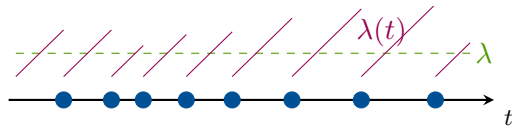
$$T^*(t) = \sup\{T_n : T_n < t\}$$

is the last point before t .

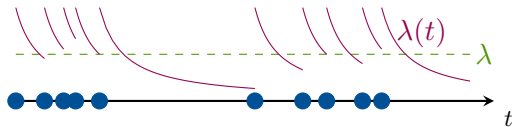
Some examples...



Constant hazard rate \rightarrow Poisson process.



Increasing hazard rate \rightarrow more periodic!



Decreasing hazard rate \rightarrow more bursty!

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Causal caching policies

- Consider again a cache system fed by M **independent** request processes $N_i(t)$ with stochastic intensities $\lambda_i(t)$.
- Let $\mathcal{F}_t = \sigma(\{\mathcal{F}_t^{(i)} : i = 1, \dots, M\})$ their aggregate history.

Definition

A **causal** caching policy is an \mathcal{F}_t **predictable** stochastic process

$$\pi(t) : \Omega \times \mathbb{R} \rightarrow \mathcal{C}$$

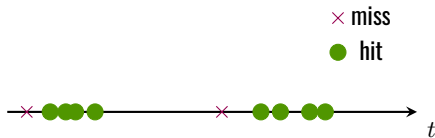
i.e. $\pi(t) = \{i_1, \dots, i_k\}$ (with $k \leq C$) is the subset kept at time t , and only depends on the past history of item requests.

The hit process

Stochastic intensity

Focus now on a particular content i , its **hit process** is the point process given by:

$$H_i(B) = \sum_{n \in \mathbb{Z}} \mathbf{1}_{\{T_n^i \in B\}} \mathbf{1}_{\{i \in \pi(T_n^i)\}}$$



Now $\mathbf{1}_{\{i \in \pi(t)\}}$ is \mathcal{F}_t -predictable, so the stochastic intensity of H_i is:

$$h_i(t) = \lambda_i(t) \mathbf{1}_{\{i \in \pi(t)\}}$$

i.e., $h_i(t) = \lambda_i(t)$ while i is cached and otherwise 0.

The hit process

The hit rate

If we now consider the aggregate of requests, the **total hit process** is given by:

$$H = \sum_{i=1}^M H_i$$

And its stochastic intensity is just:

$$h(t) = \sum_{i=1}^M h_i(t) = \sum_{i=1}^M \lambda_i(t) \mathbf{1}_{\{i \in \pi(t)\}}$$

The steady state **hit rate** of the policy is:

$$\text{hit rate} = \lambda_{\text{hit}} := E[h(t)]$$

Maximizing the hit rate

In order to maximize λ_{hit} , consider the causal policy:

$$\pi^*(t) = \{i_1, \dots, i_C\} \quad \text{such that} \quad \sum_{i \in \{i_1, \dots, i_C\}} \lambda_i(t) \text{ is maximized.}$$

Then, for any causal policy π and for each realization:

$$h(t) = \sum_{i \in \pi(t)} \lambda_i(t) \leq \sum_{i \in \pi^*(t)} \lambda_i(t) = h^*(t).$$

Theorem

The **optimal causal policy** is to keep in the cache the C objects with the **highest stochastic intensity** at any time.

Back to the Poisson case

- Assume the N_i are Poisson processes of intensities λ_i .
- We take $\lambda_1 > \lambda_2 > \dots$ as the popularities.
- The total request process is also Poisson of intensity $\sum_i \lambda_i$.
- In that case, the optimal policy is:

$$\pi^*(t) \equiv \{1, \dots, C\}$$

since $\lambda_i(t) \equiv \lambda_i$ and these are is decreasing.

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Conclusion: under Poisson arrivals, statically keeping the most popular objects is optimal (compare to the IRM before).

The renewal case

- If now the N_i are renewal processes of (decreasing) intensities λ_i .
- The total request process is no longer renewal, but its intensity is again $\sum_i \lambda_i$.
- Since $\lambda_i(t) = \eta_i(t - T_i^*(t))$, the optimal policy is:
 - Keep track of the **current hazard rate** of each content i .
 - Choose to keep in $\pi^*(t)$ the C highest.

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Conclusion: under renewal arrivals, the optimal policy only depends on the current hazard rates since the last request.

An interesting observation

Decreasing hazard rates

- If hazard rates are **decreasing**, caching makes sense! After an arrival it becomes more likely to get another request.
- After some time, we will evict the content to make room for more recent ones (as in LRU).

An interesting observation

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Increasing hazard rates

- If instead hazard rates are **increasing**, then when a request arrives, the item becomes less likely to be requested again!
- It may be better to remove it and make room for other ones (i.e. LRU makes no sense!).
- If we haven't seen it for a while, then we may have to fetch it **anticipating** the upcoming request.

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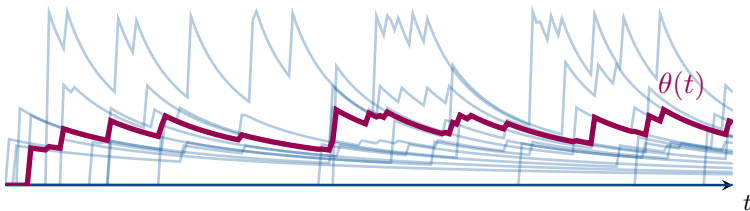
Understanding the optimal policy

The threshold process

We can rewrite this optimal policy as a **threshold** policy:

$$i \in \pi^*(t) \Leftrightarrow \lambda_i(t) \geq \theta(t) := \text{the } C \text{ largest stochastic intensity}$$

Example: Pareto requests, Zipf popularities, $N = 20$, $C = 4$.



¿What is the large scale behavior of $\theta(t)$ in steady state?

The threshold value in steady state

- Now we have M independent renewal processes with intensities $\lambda_i(t)$.
- At time $t = 0$, we have a sample $\{X_1, \dots, X_M\}$ of independent, but **not identically distributed** random variables, with distribution:

$$X_i \sim \eta_i(-T_0^i), \quad -T_0 \sim \hat{F}_i(t)$$

- The threshold $\theta(0)$ is the C -th **order statistic** (in decreasing order) of the sample.

Problem: for non *iid* random variables, no closed form \rightarrow Can we say something about the large scale limit?

A little more structure

Assume now that the request processes come from a common scale family, i.e. their inter-arrival distributions satisfy:

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In this case:

- The distribution of $-T_0^i$ is $\hat{F}_i(t) = \hat{F}_0(\lambda_i t)$.
- The hazard-rate of F_i is $\eta_i(t) = \lambda_i \eta_0(t/\lambda_i)$.
- The random variable $X_i \sim G_i(x) := G_0(x/\lambda_i)$

where $G_0(x) = P(\eta_0(-T_0) \leq x)$ is the observed hazard rate distribution for the base process.

Consider now the popularities $\lambda_1 > \dots > \lambda_M$ and define:

$$\phi_M(\lambda) = \frac{1}{M} \sum_{i=1}^M \mathbf{1}_{\{\lambda_i \leq \lambda\}}$$

their empirical (deterministic) distribution.

The distribution of popularities

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Assumption:

$$\phi_M(\lambda) \rightarrow \phi(\lambda) \quad \text{as } M \rightarrow \infty$$

where $\phi(\lambda)$ is a probability distribution.

Example: Zipf popularities

- A common model for popularities is the **Zipf** distribution, where $\lambda_i \propto \frac{1}{i^\beta}$.

- In our framework, take:

$$\lambda_i = \left(\frac{M}{i} \right)^\beta$$

- Then we can show that:

$$\phi_M(\lambda) \rightarrow \phi(\lambda) = \left[1 - \lambda^{-1/\beta} \right] \mathbf{1}_{\{\lambda \geq 1\}}$$

Remark: note that $\sum_i \lambda_i$ diverges, so the system is scaling up...

Theorem (Carrasco,F',Paganini)

Consider a caching system fed by M independent and stationary renewal processes, with intensities $\{\lambda_i\}$, and inter-arrival distributions $F_i(t) = F_0(\lambda_i t)$. Let X_1, \dots, X_M denote the observed hazard-rates at time 0. Then, under the preceding assumption, the empirical distribution:

$$\hat{G}_M(x) = \frac{1}{M} \sum_{i=1}^M \mathbf{1}_{\{X_i \leq x\}} \rightarrow_M G_\infty(x) = \int_0^\infty G_0\left(\frac{x}{\lambda}\right) \phi(d\lambda)$$

A law of large numbers for the threshold

Assume further that the cache has capacity $C = cM$ with $0 < c < 1$ is the fraction of the catalog that can be stored.

A law of large numbers for the threshold

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Then, the optimal policy threshold $\theta_M^*(0)$ is the random variable:

$$\theta_M^* : \sum_{i=1}^M \mathbf{1}_{\{X_i \leq \theta_M^*\}} = (1 - c)M$$

or equivalently θ_M^* is such that $\hat{G}_M(\theta_M^*) = 1 - c$.

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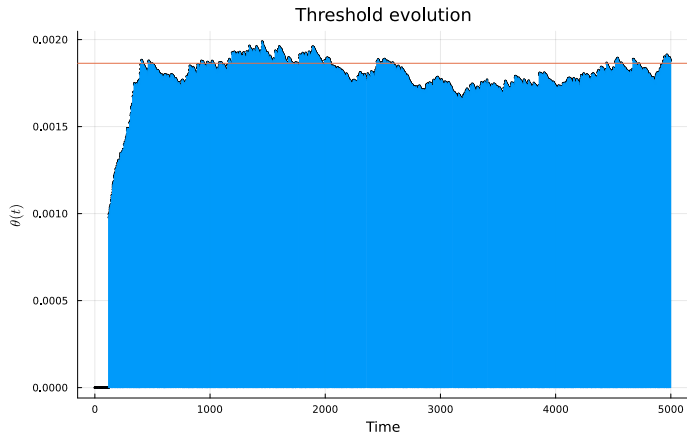
Corollary

If the cache size scales linearly with the catalog as $C_M = cM$, then:

$$\theta_M^* \rightarrow \theta^* : G_\infty(\theta^*) = 1 - c$$

So the optimal policy becomes a **fixed** threshold policy.

Simulation example



$M = 1000, C = 100$. Pareto $\alpha = 2$ requests, Zipf $\beta = 0.5$ popularities.

Moreover, we can calculate the asymptotic performance:

Theorem

Under all the above assumptions, the asymptotic **miss rate** verifies:

$$\lambda_{\text{miss},M} \rightarrow_M \int_0^\infty \lambda \tilde{G}_0 \left(\frac{\theta^*}{\lambda} \right) \phi(d\lambda) = E \left[\Lambda \tilde{G}_0 \left(\frac{\theta^*}{\Lambda} \right) \right]$$

where $\Lambda \sim \phi$, and \tilde{G}_0 is the distribution of the hazard-rate prior to an arrival:

$$\tilde{G}_0(x) = \int_0^\infty \mathbf{1}_{\{\eta_0(t) \leq x\}} F_0(dt).$$

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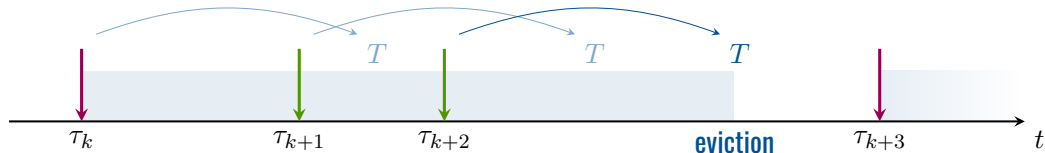
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Conclusions

Populating a cache: timer based policies

Timer based (TTL) policies:

- Upon request arrival for item i , check for presence.
- If new, store item and start a **timer** T_i to evict.
- If present, reset timer to T_i .
- Keep timers T_i such that **average** cache occupation is C .



Choosing the optimal timers

Requests come from independent sources with intensities λ_i and inter-arrival distribution F_i :

Problem (Optimal TTL policy)

Choose timers $T_i \geq 0$ such that:

$$\max_{T_i \geq 0} \sum_i \lambda_i F_i(T_i)$$

subject to:

$$\sum_i \hat{F}_i(T_i) \leq C$$

Remark: non-convex non-linear program. But it can be solved by a change of variables!!! [Ferragut et al. 2018].

Theorem

For the following cases, the optimal timers are:

- Constant hazard rate (Poisson) or increasing hazard rate: keep the most popular objects ($T_i = \infty$ or 0).
- Decreasing hazard rate:

$$\eta_i(T_i^*) \geq \theta^*$$

for every stored content.

Theorem

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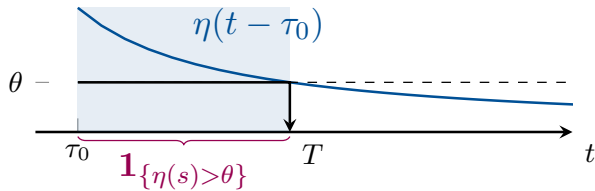
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So the optimal timer policy is a threshold policy?

Why this happens?



Theorem (F', Carrasco, Paganini)

In the scaling regime considered earlier, for renewal processes with DHR, the TTL policy is asymptotically optimal.

Idea: prove that the thresholds are the same in the limit.

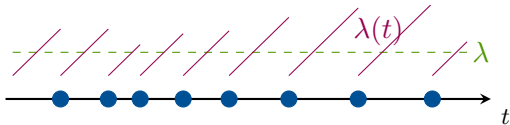
Theorem (F', Carrasco, Paganini)

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But what about **increasing hazard rates**?

Back to increasing hazard rates...

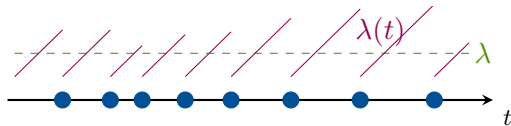
- Recall the increasing hazard rate behavior:



- Once you have seen a request, it's less likely to see another one for a while.

Back to increasing hazard rates...

- Recall the increasing hazard rate behavior:



- Once you have seen a request, it's less likely to see another one for a while.

What is the timer based equivalent of this case?

Timer based pre-fetching policies

Key insight

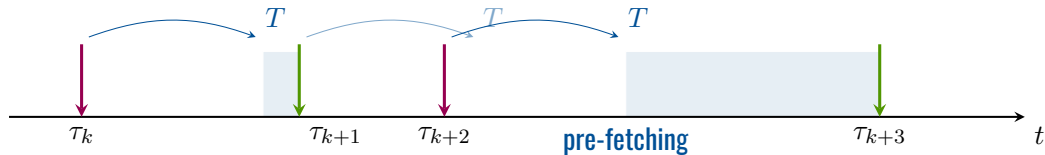
The question now is not **how long we should remember something**, but instead **how long we should forget about it!**

Timer based pre-fetching policies

Key insight

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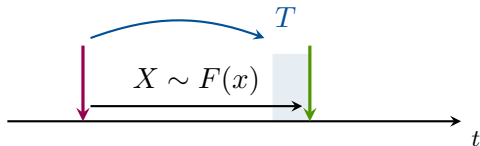
Timer based pre-fetching policy:



Timer based pre-fetching

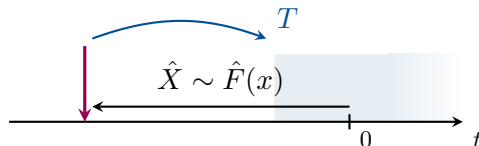
Consider a single item with a timer T and its request process:

Hit probability: **next** arrival occurs **after** timer expires.



$$\text{Hit probability} = 1 - F(T)$$

Occupation probability: probability that timer **has** expired by 0 since last arrival.



$$\text{Avg. occupation} = 1 - \hat{F}(T)$$

Choosing the optimal timers

Requests come from independent sources with intensities λ_i and inter-arrival distribution F_i :

Problem (Optimal pre-fetching policy)

Choose timers $T_i \geq 0$ such that:

$$\max_{T_i \geq 0} \sum_i \lambda_i (1 - F_i(T_i))$$

subject to:

$$\sum_i (1 - \hat{F}_i(T_i)) \leq C$$

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Choose timers $T_i \geq 0$ such that:

$$\min_{T_i \geq 0} \sum_i \lambda_i F_i(T_i)$$

subject to:

$$\sum_i \hat{F}_i(T_i) \geq N - C$$

Remark: we can use the same change of variables again!

Pre-fetching for increasing hazard rates

Optimal pre-fetching policy, IHR, [F',Carrasco, Paganini].

The optimal timer based pre-fetching policy for IHR is such that:

$$\eta_i(T_i^*) \geq \theta^*$$

for every stored content.

Remark: Again we have to equalize hazard-rates. The policy is a threshold policy.

Theorem (F', Carrasco, Paganini)

In the scaling regime considered earlier, for renewal processes with IHR, the **timer based pre-fetching policy** is asymptotically optimal.

Idea: as before, prove that the thresholds are the same in the limit.

The caching problem

Point processes and stochastic intensity

The optimal caching policy

Large scale asymptotics

Connection with timer-based policies

Conclusions

- The main result characterizes the optimal policy completely in the large-scale scenario.
- For particular distributions of interest (e.g. Pareto requests, Zipf popularities) the threshold can be computed explicitly.
- Once the threshold is computed, we can compute the asymptotic hit probability.
- Therefore, we have a computable absolute performance bound in the limit.

- The main result characterizes the optimal policy completely in the large-scale scenario.
- For particular distributions of interest (e.g. Pareto requests, Zipf popularities) the threshold can be computed explicitly.
- Once the threshold is computed, we can compute the asymptotic hit probability.
- Therefore, we have a computable absolute performance bound in the limit.
- There is much more to do! **Students Welcome!**

Thanks!

Andres Ferragut

`ferragut@ort.edu.uy`

`aferragu.github.io`