Spatial estimation of EV energy demand based on aggregated measurements

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INFORMS APS 2023 – Nancy, France – June 2023

Introduction

- Electrical vehicle (EV) adoption is currently growing exponentially.
- Less carbon emissions, noise and other efficiency benefits.

Problems:

- We need to build the charging infrastructure to replace gas stations.
- Charging is power and energy intensive for the network, the grid must cope with the enlarged demand.

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We need good spatial estimates of energy demand!

Outline of the talk

Problem description

Radial basis functions approach

A Poisson parametric model

Final remarks

The Problem

- We need an spatial estimate of energy demand in order to upgrade the distribution network.
- Currently, we do not have measurements of this demand due to low EV penetration.

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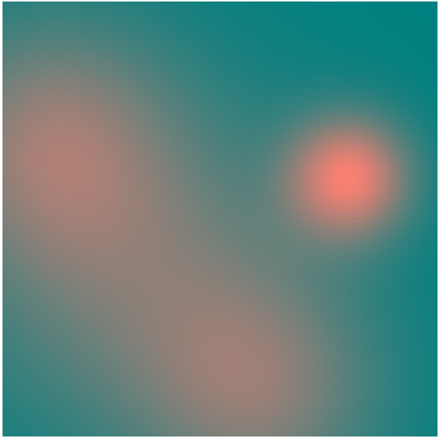
Challenge:

These measurements are concentrated at the gas stations. How to interpolate them?

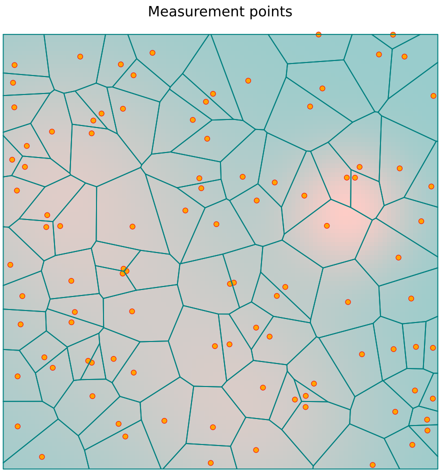
Energy density

- We have an unkonwn energy density $g(x)$ (in energy/km^2) over a region $\mathcal{X}$.
- Represents amount of energy demand coming from a small ball around x.



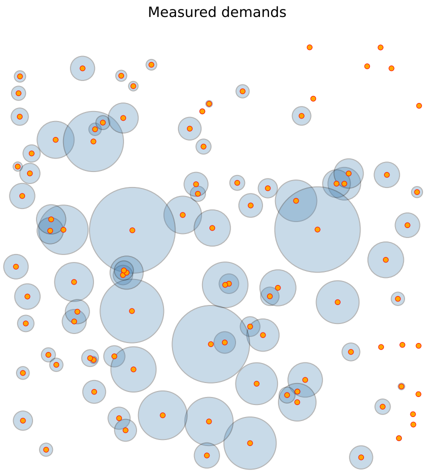
Measurement points

- We cannot sample from this density!
- All we have is some measurement points distributed over $\mathcal{X}$.
- What we can measure is the total demand coming from a cell around our measurement point.



Our dataset

- Each demand is measured at sites s_i.
- We have access to $y_i = \int_{V_i} g(x)dx$, where V_i is the Voronoi cell of site i.
- The size of the circle represents measured demand at the sites.



Mathematical formulation

- In a region of space $\mathcal{X} \subset \mathbb{R}^d$, we are given:
  - A list of fixed sites $\{s_1, \ldots, s_m\}$, $s_i \in \mathcal{X}$.
  - A list of measurements $\{y_1, \ldots, y_m\}$, $y_i \geqslant 0$.
- Goal: construct an estimate $\hat{g}(x;\theta)$ of the spatial density such that:

$$\int_{V_i} g(x;\theta)\, dx \approx y_i \quad \forall i$$

where V_i is a cell associated with site s_i (e.g. the Voronoi cell).

Non-parametric approach

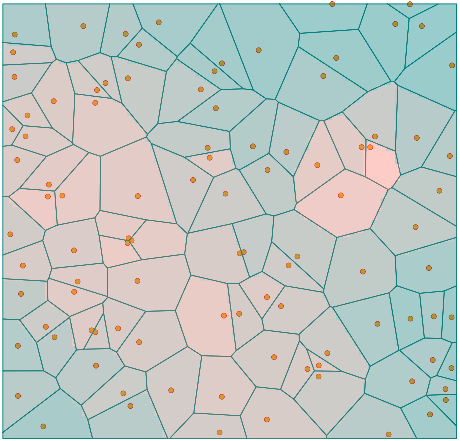
Not very fun...and maybe useless

- First approach: histogram counts.
- Estimate

$$g_H(x) = \sum_i \frac{y_i}{\mathrm{m}(V_i)} \mathbf{1}_{V_i}(x)$$

- Non-smooth. Low interpolation properties. Not suitable for low-dimensional representation.

Histogram counts



Radial basis functions

To obtain a lower dimensional representation we use radial basis functions to estimate $g(x)$. Namely, our estimator has the form:

$$g_{RBF}(x;\theta) = \sum_{j=1}^{n} w_j e^{-\frac{||x-\mu_j||^2}{2\sigma_j^2}}$$

where $\theta = (\{w_j\}, \{\mu_j\}, \{\sigma_j^2\})$.

- $w_j \in \mathbb{R}^+$ are the weights,
- $\mu_j \in \mathbb{R}^d$ are the nodes and
- $\sigma_j^2 \in \mathbb{R}^+$ the bandwidths.

Least squares approach

- Since we have access to the cell measurements, it makes sense to consider the loss function:

$$L(\theta) = \frac{1}{2} \sum_{i=1}^{m} \left(\int_{V_i} g_{RBF}(x;\theta)dx - y_i \right)^2$$

- Therefore, the least squares estimator becomes:

$$\hat{\theta}_{LS} = \arg\min_{\theta} L(\theta)$$

- We now show an algorithm to compute this estimator.

Computing the weights

Consider first given the nodes μ_j and the bandwidths σ_j^2, we have:

$$\int_{V_i} g_{RBF}(x,\theta)\,dx = \sum_{j=1}^{n} w_j \int_{V_i} e^{-\frac{||x-\mu_j||^2}{2\sigma_j^2}}\,dx =: \sum_{j=1}^{n} a_{ij} w_j.$$

The loss becomes:

$$L(\theta) = \frac{1}{2}\sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_{ij} w_j - y_i\right)^2 = \frac{1}{2}||Aw-y||^2$$

And thus we have the linear least squares problem:

$$\min \frac{1}{2}||Aw-y||^2, \quad s.t.\ w \geqslant 0.$$

It can be readily solved, typically the constraint is not active.

Estimating nodes and bandwidths

- To estimate μ_j and $\{\sigma_j^2\}$ we may use gradient descent. Note that:

$$\frac{\partial L}{\partial \theta_k} = \sum_{i=1}^{m} \left(\int_{V_i} g_{RBF}(x;\theta)\,dx - y_i \right) \left(\int_{V_i} \frac{\partial}{\partial \theta_k} g_{RBF}(x;\theta)\,dx \right)$$

- Moreover, due to the structure of the RBF functions:

$$\begin{aligned}
\frac{\partial}{\partial \mu_j} g_{RBF}(x;\theta) &= \left[\frac{x-\mu_j}{\sigma_j^2}\right] w_j e^{-\frac{||x-\mu_j||^2}{2\sigma_j^2}} \\
\frac{\partial}{\partial \sigma_j^2} g_{RBF}(x;\theta) &= \left[\frac{||x-\mu_j||^2}{2(\sigma_j^2)^2}\right] w_j e^{-\frac{||x-\mu_j||^2}{2\sigma_j^2}}
\end{aligned}$$

Computing the gradient

So in order to compute the gradient, we need to estimate the following moments of our current density estiamte:

$$\int_{V_i} g_{RBF}(x,\theta)\,dx, \quad \int_{V_i} \left[\frac{x-\mu_j}{\sigma_j^2}\right] g_{RBF}(x,\theta)\,dx, \quad \int_{V_i} \left[\frac{||x-\mu_j||^2}{2(\sigma_j^2)^2}\right] g_{RBF}(x,\theta)\,dx.$$

for each cell i.

Estimating the moment integrals

Monte Carlo approach

Sample N uniformly distributed points in the region $\mathcal{X}$ and estimate:

$$\int_{V_i} g_{RBF}(x;\theta)dx \approx \frac{\mathrm{m}(\mathcal{X})}{N} \sum_{k=1}^{N} g_{RBF}(u_k;\theta)\mathbf{1}_{V_i}(u_k)$$

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Two possible variants:

- Use a large N and fix the estimation points $\rightarrow$ slightly more bias, less variance, faster to compute.
- Resample a relatively small N on each step $\rightarrow$ less bias, high variance, amounts to Stochastic Gradient Descent.

Algorithm

Stochastic gradient descent version

Given a suitable initial condition $\theta^{(0)} = (\{w_j^{(0)}\}, \{\mu_j^{(0)}\}, \{\sigma_j^{2\,(0)}\})$, at each step k:

1. Sample N uniformly distributed random points in $\mathcal{X}$.
2. Estimate the moment integrals and compute the gradient $\nabla L(\theta^{(k)})$.
3. Perform a gradient step:

$$\mu_j \leftarrow \mu_j - \alpha_k \nabla L(\theta^{(k)})_{\mu_j}, \quad \sigma_j^2 \leftarrow \sigma_j^2 - \alpha_k \nabla L(\theta^{(k)})_{\sigma_j^2}.$$

   with step size $\alpha_k \sim O(1/k)$.
4. For the new nodes and bandwidths, recompute w_j using linear least squares.
5. Update $\theta^{(k+1)}$ and iterate until convergence.

Choosing the initial condition

We need a good first estimate $\theta^{(0)}$. We propose the following method:

Bootstrapping:

Fix the number of kernels n as an hyperparameter and do:

1. Given the sites $\{s_1, \ldots, s_m\}$ and the measurements $\{y_1, \ldots, y_m\}$, run weighted $k-$means with n clusters to optimize:

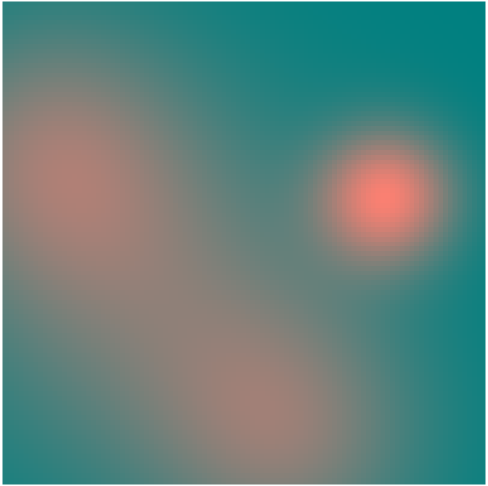
$$\min_{\mu_j} \sum_{j=1}^{n} \sum_{i \text{ closest to } \mu_j} y_i ||s_i - \mu_j||^2$$

2. Estimate the bandwidths σ_j^2 as the mean square distance of the allocated sites to node j.
3. Compute a first estimate of w_j by solving the linear least squares problem with the above initial estimates.

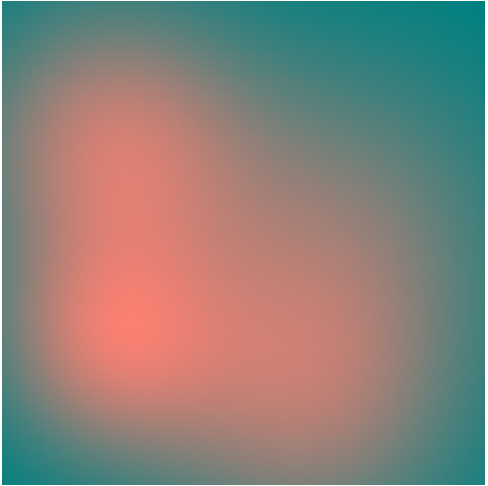
Example: reconstructing the original density

Initial condition

Baseline density

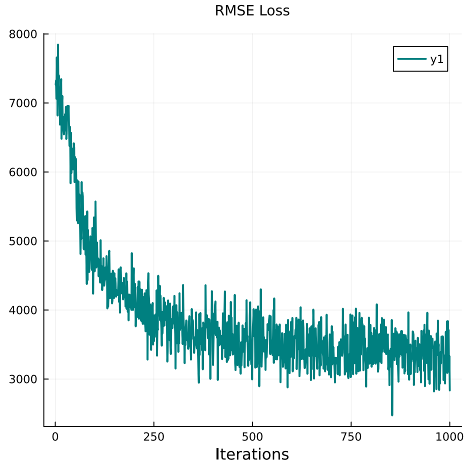


Initial density estimation



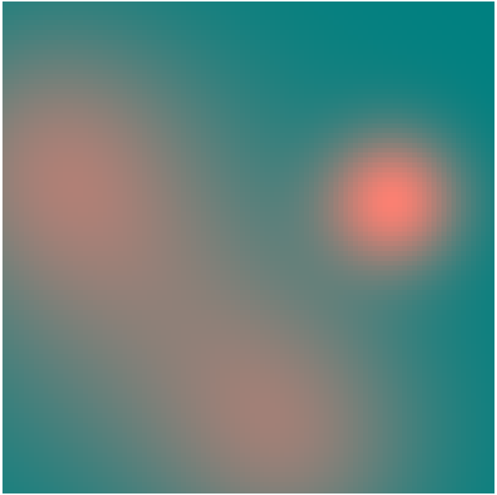
Example: reconstructing the original density

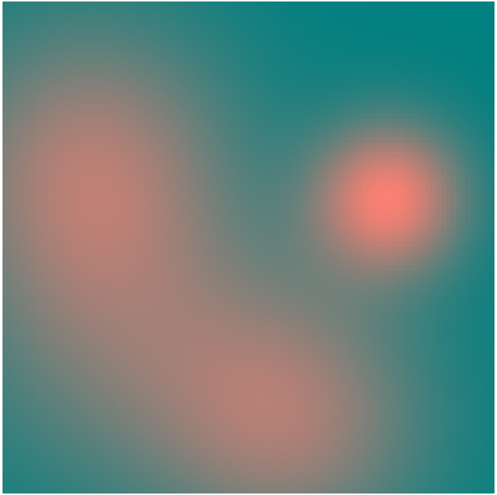
Root mean square loss evolution:



Example: reconstructing the original density

Final estimate:





Hidden Poisson process parametric model

Assume now that demands come from a marked Poisson process

$$\Phi = \sum \delta_{(x_k, v_k)}$$

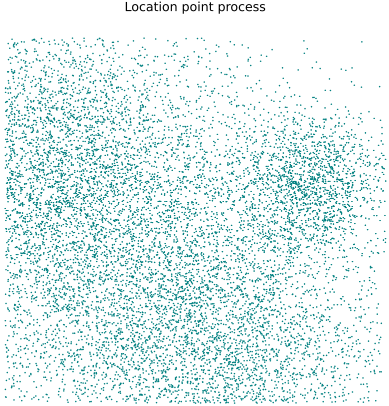
with:

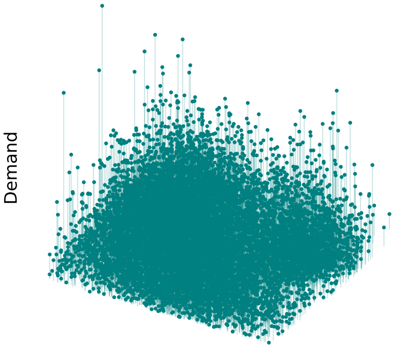
- Spatial intensity $\Lambda(dx)$, $x \in \mathcal{X}$, modeled through an RBF density $\lambda(x;\theta)dx$.
- Individual marks (customer demands) exponentially distributed with rate ν.

Hidden Poisson process parametric model

Example

A realization of the process Φ:





Observation model

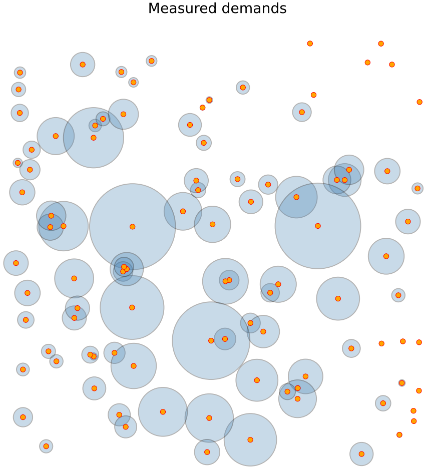
- We observe the total cell demands:

$$Y_i = \int_{V_i} v\,\Phi(dx, dv) = \sum_k v_k \mathbf{1}_{\{x_k \in V_i\}}$$

- For each cell:

$$\begin{aligned} N_i &\sim \quad \mathrm{Poisson}(\lambda_i(\theta)) \\ Y_i &= \quad \textstyle\sum_{k=1}^{N_i} V_k \end{aligned}$$

with $V_k \sim \exp(\nu)$ and cells are independent.



Problem: the number of points acts as a *hidden variable*.

Maximum likelihood approach

- Ideally one would like to maximize $p(y;\theta)$. Difficult to compute.
- Expectation-Maximization approaches fail (a posteriori distribution also difficult).
- Consider maximizing the combined likelihood:

$$p(n,y) = \prod_{i=1}^{m} e^{-\lambda_i(\theta)} \frac{\lambda_i(\theta)^{n_i}}{n_i!} p(y_i \mid n_i), \quad \text{with } \lambda_i(\theta) = \int_{V_i} \lambda(x;\theta)\,dx$$

- Now, since given n_i, the demands are independent exponentials we have:

$$p(y_i \mid n_i) = \frac{1}{(n_i-1)!} \nu^{n_i} y_i^{n_i-1} e^{-\nu y_i}.$$

Maximizing the counts likelihood

Given an estimate of $\Lambda(dx)$ and ν, we can maximize each term over n_i, thus decoding the hidden variable:

$$\max_{n_i} e^{-\lambda_i(\theta)} \frac{\lambda_i(\theta)^{n_i}}{n_i!} \frac{1}{(n_i - 1)!} \nu^{n_i} y_i^{n_i - 1} e^{-\nu y_i}.$$

The maximum is attained for:

$$n_i^*(n_i^* + 1) = \lambda_i(\theta)\nu y_i$$

That is:

$$n_i^* \approx \sqrt{\lambda_i(\theta)\nu y_i}.$$

Estimating the RBF parameters

With the hidden variables estimated, the joint likelihood as a function of θ is:

$$\ell(\theta; n, y) = \sum_{i=1}^{m} -\lambda_i(\theta) + n_i \log(\lambda_i(\theta)) + n_i \log(\nu) - \nu y_i + (n_i - 1)\log(y_i) - \log(n_i!(n_i-1)!).$$

The new estimate for ν follows immediately by differentiation:

$$\hat{\nu} = \frac{\sum_i n_i}{\sum_i y_i}.$$

As for the RBF parameters, we perform a gradient approach as before:

$$\frac{\partial \ell}{\partial \theta} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \theta} = \sum_{i=1}^{m} \left[\frac{n_i}{\lambda_i(\theta)} - 1 \right] \frac{\partial \lambda_i(\theta)}{\partial \theta}.$$

The derivatives of $\lambda_i(\theta)$ are similar to the preceding ones.

Maximum likelihood algorithm

Given a suitable initial condition $\theta^{(0)} = (\{w_j^{(0)}\}, \{\mu_j^{(0)}\}, \{\sigma_j^{2\,(0)}\}, \nu^{(0)})$, at each step k, iterate until convergence:

1. Sample N uniformly distributed random points in $\mathcal{X}$.
2. Estimate $\lambda_i(\theta)$ ant the moment integrals to compute its gradient.
3. Decode the hidden variables $n_i = \sqrt{\lambda_i(\theta^{(k)})\nu^{(k)} y_i}$
4. Perform a gradient step on nodes and bandwidths:

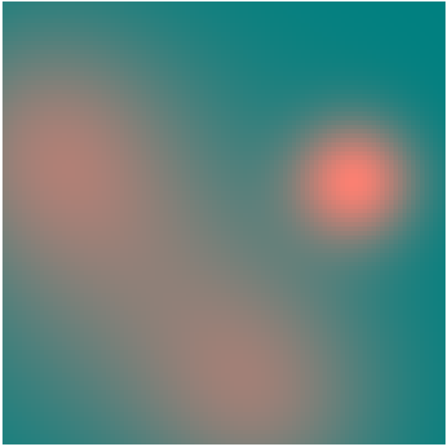
$$w_j^{(k+1)} = w_j^{(k+1)} + \alpha_k \nabla \ell(\theta^{(k)})_{w_j},$$
$$\mu_j^{(k+1)} = \mu_j^{(k)} + \alpha_k \nabla \ell(\theta^{(k)})_{\mu_j}, \quad (\sigma_j^2)^{(k+1)} \leftarrow (\sigma_j^2)^{(k)} + \alpha_k \nabla \ell(\theta^{(k)})_{\sigma_j^2}.$$

5. Update $\hat{\nu}^{(k+1)} = \frac{\sum_i n_i}{\sum_i y_i}$.

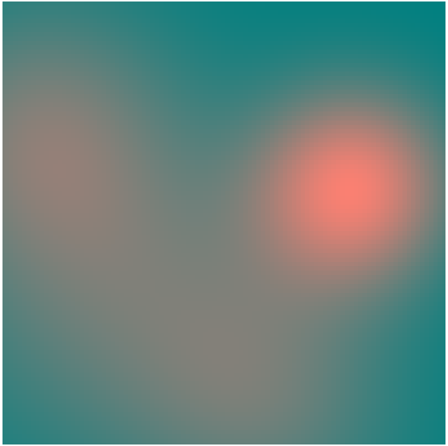
Example

Maximum likelihhod reconstruction

Original density



Reconstructed density



Take home messages...

- EVs are popping up, we have to prepare the infrastructure.
- We can estimate spatial energy demand based on current gas consumption measurements.
- We analyzed two different approaches with different properties.
- We would like to expand on the mathematical analysis, in particular the connection with stochastic gradient descent and more general transport measures and problems.

Thank you!

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