Not-So-Cleansing Recessions*

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Abstract

Recessions are periods in which the least productive firms in the economy exit, and as the economy recovers, they are replaced by new and more productive entrants. These *cleansing effects* imply that business cycles generate improvements in the average firm productivity. We argue that this is not sufficient to induce long-run gains in GDP and welfare. We show that these are driven by the intensity of *love-of-variety* in households' preferences. If the household has CES preferences, recessions do not bring about any improvement in GDP and welfare. If the economy features more *love-of-variety* than CES, the social planner finds it optimal to subsidize economic activity in recessions to avoid firm exit.

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1 Introduction

Recessions are often periods of increased reallocation of economic activity. The *liquidationist* view of business cycles has long posited that during downturns, unproductive economic units exit and, as the economy recovers, they are replaced by new and more productive firms.

In this paper, we revisit the role of recessions as moments of *cleansing* and creative destruction and, in particular, their effect on GDP and welfare. We consider a firm dynamics economy with heterogeneous firms à la Hopenhayn (1992)-Melitz (2003). We study the cleansing effect of business cycles driven by fluctuations in the fixed costs of production. In a partial equilibrium version of our model, we show that fixed cost recessions have cleansing effects through Schumpeterian forces: after a recession, fewer firms operate, and they are, on average, more productive than before the downturn. However, these gains need not translate into higher GDP and welfare. When products are differentiated, both households and firms might value the availability of varieties per se. If this is the case, they are not willing to trade off higher average firm productivity for fewer available products. In fact, the cleansing effects of recessions can result in higher average productivity of surviving firms, yet lower GDP and welfare.

In Proposition 1, we formalize this intuition. We show that when households have CES preferences, pre-cycle GDP and long-run post-cycle GDP and welfare are identical despite the composition of the economy being different: the economy features fewer but more productive firms. Whenever the household values varieties more than implied by CES preferences, a fixed cost cycle induces long-run GDP and welfare losses, despite its cleansing effect. We show that, in economies characterized by Dixit and Stiglitz (1975) preferences, the effect of recessions on long-run output can be decomposed into two terms: i) the effect on the output level of a CES economy; and ii) the effect on the number of available varieties, weighted by how much higher or lower than CES is love-of-variety (LoV). In partial equilibrium, the first effect is always zero, as the free entry equilibrium in CES is history-independent. However, since recessions trade off varieties for average productivity, an economy can be strictly worse or better off depending on the extent of love-of-variety. This element makes non-CES economies path-dependent since the composition of firms matters for welfare.

We show that this intuition extends to general equilibrium with a minor caveat: in GE, the cleansing effect frees up labor resources since fewer firms have to employ workers to pay the fixed costs of production. As a consequence, fixed cost cycles induce higher long-run welfare in CES economies. Nonetheless, we show in Proposition 2 that there exists a unique level of taste-for-variety, such that long-run GDP and welfare are identical to their pre-cycle levels.

We conclude our positive analysis by studying the role of recessions depth. We start by showing that, ranking economies by their love of variety, it is possible to partition them based on how cleansing recessions are. There are high LoV economies for which recessions are always welfare-reducing, independently of their magnitude. Similarly, low LoV economies always show cleansing effects. However, in economies with intermediate levels of love of variety some recessions are cleansing while others reduce long-run output and welfare. The intuition behind this result is that while the variety effect on welfare is constant in logs, the cleansing forces exhibit diminishing returns. As a consequence, while some recessions might increase welfare, others might reduce it. We conclude that for these economies, it is not possible to generally say whether recessions improve long-run outcomes or not, and such a statement has to be empirically evaluated recession by recession.

Finally, in Section 4, we study the social planner problem in the presence of fixed cost-induced recessions. We first extend results in Dixit and Stiglitz (1977) and Dhingra and Morrow (2019) on the constrained efficiency of CES economies with monopolistically competitive firms and free entry. We show that such results extend to economies that also consider the presence of incumbent firms. We then show that when the economy values varieties more or less than implied by CES, the market equilibrium features either too few or too many firms. The planner can correct this steady-state inefficiency with a combination of entry and fixed costs subsidies.

Importantly, we show that in economies in which households value varieties more than the CES benchmark, under mild assumptions on the productivity distribution, the social planner finds it optimal to increase fixed costs subsidies during recessions. This policy is motivated by inducing a smaller amount of exit during bad times than implied by the market equilibrium. This result speaks to frequent efforts by governments to reduce churn during bad times, as seen during the Covid-19 recession.[1]

Related Literature This paper draws insights from the literature on the optimal number of varieties (Dixit and Stiglitz, 1977; Dhingra and Morrow, 2019)[2] to discuss the cleansing effects of recessions. We build on Hopenhayn (1992), Grossman and Helpman (1993), and Melitz (2003) to study the effect of recessions in economies with love-of-variety and firm dynamics.

The notion that recessions can generate long-run benefits thanks to an acceleration of creative destruction dates back to Schumpeter (1939) and has been formalized in Caballero and Hammour (1994).They study an embedded capital model in which recessions induce firm exit, thereby accelerating the speed of modernization of installed capital. We study an economy in which there is no capital, but producers are heterogeneous. During recessions, the least productive firms exit and are subsequently replaced by entrants with higher average productivity. Importantly, we show that this is not enough to infer the behavior of output or welfare, as the equilibrium number of firms drops after a recession.

Related to our conclusion, Hamano and Zanetti (2017, 2022) highlight the existence, in a CES economy, of the welfare tradeoff between higher average productivity and fewer available varieties. We show that for a CES economy, the two effects perfectly offset each other in PE. In GE, the only welfare effect is the labor savings and the income windfall that this generates. The tradeoff between better firm selection and loss of variety becomes welfare-relevant only away from CES. In particular, when love-for-variety is larger than implied by the CES benchmark. Barlevy (2002) and Ouyang (2009) suggest that the cleansing effects may be lower than expected. Barlevy (2002) argues that job match quality is procyclical and that this effect quantitatively dominates the cleansing role of recessions. Ouyang (2009) considers a

[1] See Kozeniauskas et al. (2022) for an empirical evaluation of such programs.
[2] See Brakman and Heijdra (2001) for an extended review.

setting in which recessions may halt the entry of very productive firms and, thereby, reducing long-run growth. Our point is related but distinct: even when measured productivity improves during recessions, this may induce a loss of varieties that manifests in lower GDP and welfare.

Finally, our paper is closely related to Baqaee et al. (2023) and Ardelean (2006) who provide estimates for love-of-variety in production and in demand, respectively. Baqaee et al. (2023) find an elasticity of -.3% for marginal costs of firms adding suppliers. Ardelean (2006) finds a love-of-variety 42% lower than the one implied by CES.

2 An Industry Model of Entry and Exit

We begin by laying out the structure of production and consumption in the economy and then proceed to study the effects of business cycles.

2.1 Setup

Production We consider an industry in monopolistic competition, in which firms hire labor to produce their variety with constant returns to scale in labor l and heterogeneous productivity z: $y = zl$. Productivity is distributed according to some density function μ. Firms must pay a fixed cost f^c in units of labor to produce. In this partial equilibrium context, we assume that labor can be hired at an exogenous fixed wage rate, $w = 1$, which we set as the nominal anchor. Firms maximize their profits $\pi(z) = y(p(y) - 1/z) - f^c$ by choosing their price.

Industry Output A perfectly competitive intermediary combines varieties and sells the final composite good to households. The intermediary aggregates with a generalized CES production function à la Dixit and Stiglitz (1975), Ethier (1982), Benassy (1996)

$$Y = M^{q-\frac{1}{\sigma-1}} \left[\int y(z)^{\frac{\sigma-1}{\sigma}} \mu(z) dz \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Two parameters govern the aggregator: the elasticity of substitution, σ, which measures product similarity, and love-of-variety (LoV), q. LoV measures the additional productivity the intermediary derives from having a larger number of available varieties. This effect constitutes an aggregate production externality.[3] The classic Dixit and Stiglitz (1977) CES-aggregator is a special case where $q = q^{CES} := 1/(\sigma - 1)$. Hence, $q > 1/(\sigma - 1)$ implies stronger LoV than in the CES case and vice versa. A density function, μ, describes the distribution of firms. The total number of varieties M is given $\int \mu(z)\ \mathrm{d}z$. We stress that μ is not a probability density and hence does not integrate to 1.

[3] In an equivalent reading without intermediaries, the aggregator sits in the household utility function. The extra utility from having a larger number of available varieties is then an aggregate demand externality as in Blanchard and Kiyotaki (1987).

Demand There is a representative household with exogenous total income $\mathcal{I}$. We relax exogeneity when we consider a GE-economy in Section 3. Household utility is strictly increasing in Y, and households spend all their income on the composite good.

Entry Entry of ex-ante homogeneous firms grows the mass of available varieties. Entrants pay a fixed cost of entry f^e, also in terms of labor, to enter and draw a productivity level z from a baseline probability distribution μ^0. Potential entrants make entry decisions based on their expected post-entry profits.

2.2 Equilibrium

The household problem implies a demand schedule for each variety, which is taken into account by the corresponding monopolist. The monopolistic competition solution is independent of the aggregate production externality and implies the usual constant-markup pricing $p = \frac{\sigma}{\sigma-1}\frac{1}{z}$. At the optimum, firm profits, π, depend on the firm's productivity z and on competition, μ. Notably, μ only matters through an aggregate statistic: the index $\int z^{\sigma-1}\mu(z)\ \mathrm{d}z$, which we call *market intensity* and which is closely related to *aggregate productivity*, $\left(\int z^{\sigma-1}\mu(z)\ \mathrm{d}z\right)^{1/(\sigma-1)}$. Profits are given by

$$\pi(z,\mu) = \frac{\mathcal{I}}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}\mu(z)\ \mathrm{d}z} - f^c. \tag{2}$$

To characterize entry and exit, suppose that, at the onset, there are firms in the economy whose fixed cost of entry has been paid in the past and is thus sunk. These firms, which we call incumbents, can be arranged according to their productivity, giving rise to the incumbent distribution $z \sim \mu^I$. New firms enter if they expect a profit net of the fixed cost of entry, i.e. if $\mathbb{E}_0[\max\{\pi(z,\mu),0\}] - f^e > 0$, where the expectation is taken over the baseline probability distribution of productivity, μ^0. Firms understand that their entry decision is simultaneous to that of other potential entrants. As a consequence, they do not just consider the competition given by current incumbents but rather what the market will look like once all potential entrants have entered. We call this ex-post distribution μ. If a firm chooses to pay the entry cost and draw a productivity level, it may still find it unprofitable to produce due to the fixed production cost f^c. This fixed cost implies the existence of a threshold productivity $\underline{z}$ such that $\pi(\underline{z},\mu) = 0$. Entrants and incumbents alike produce if their productivity z lies above $\underline{z}$ and leave otherwise.

Denote the mass of entrants drawing from μ^0 by $E \geq 0$. Then, after entry, the distribution of firms, μ, is given by the sum of surviving entrants and incumbents. It is spelled out in eq. (5). Equilibrium is reached, if no additional entrepreneurs wish to enter (eq. 3) and no additional incumbents wish to quit (eq. 4); the two equilibrium objects—cutoff, $\underline{z}$ and mass of entrants, E—adjust to jointly satisfy eqs. (3) and (4):

$$f^e \geq \mathbb{E}_0[\max\{\pi(z,\mu),0\}], \tag{3}$$

$$0 = \pi(\underline{z},\mu), \tag{4}$$

$$\mu(z) = (\mu^I(z) + E\mu^0(z))\mathbb{I}_{\{z\geq\underline{z}\}} \;\; (\forall z \geq 0). \tag{5}$$

If there is scope for entry, then $E > 0$ and eq. (3) holds with equality. Combining (2), (4) and (3), we obtain the following equilibrium conditions:

$$E = \frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}} z^{\sigma-1}\mu^0(z)\,\mathrm{d}z} - \frac{\int_{\underline{z}} z^{\sigma-1}\mu^I(z)\,\mathrm{d}z}{\int_{\underline{z}} z^{\sigma-1}\mu^0(z)\,\mathrm{d}z}, \tag{6}$$

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty}\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1} - 1\right]\mu^0(z)dz. \tag{7}$$

Examining eq. (7), we note the following result:

Lemma 1 (Cutoff determination)
Whenever there is entry into the economy, the productivity cutoff $\underline{z}$ is independent of the incumbent distribution μ^I.

Notably, the equilibrium conditions do not depend on q. Thus, firms produce independently of q, i.e., *as if* operating in a CES economy. If LoV changes, the downstream aggregating firm is able to produce more. However, market size $\mathcal{I}$ is fixed. Hence, this output expansion is accompanied by a proportional drop in the price of the aggregate good. Additionally, the relative importance of varieties is unchanged, making this shift irrelevant to the upstream industry.

To build intuition for the equilibrium conditions, we discuss the sufficiency of market intensity. First, note that the labor share $\frac{\sigma-1}{\sigma}$ of total income spent on the final product, $\mathcal{I}$, constitutes expenditures for production labor (as opposed to fixed cost payments). Therefore, the total number of hours spent on production is $\mathcal{I}\frac{\sigma-1}{\sigma}$, where w is normalized to 1. New entrants hence draw labor from existing firms. In a CES economy, this leads to decreasing marginal returns to entry through decreasing expected profits for the marginal entrant, and to decreasing returns to production for any given existing firm. Likewise, an increase in the productivity of existing firms decreases the expected profits of a potential entrant, because more productive firms take up more labor. Analogously, fixing an existing firm A, an increase in all other firms' productivity drives down A's profits. We learn that, for any given z, profits are affected at essentially two aggregate margins: number of competitors and productivity of competitors. This also implies that we can manipulate one of the margins (e.g., increase the number of firms by 1%) and adjust the other (e.g., decrease productivity of all firms by x_z%) to keep $\pi(z,\mu)$ unchanged. Essentially, in CES monopolistic competition, every firm's equally differentiated product is competing with the *same* aggregate of all other products, which suggests that the change in this product at one margin (the 1% increase) is undone by the *same* change at the other, such that $x_z\% = x\%$ for all z. These insights strongly suggest the existence of an aggregate statistic $s(\mu)$, which allows us to write $\pi(z,\mu) = \widetilde{\pi}(z, s(\mu))$; this statistic is precisely given by market intensity. Finally, since upstream firms behave as if they were living in a CES economy, this result carries over to $q \neq q^{CES}$ cases.

The intuition behind Lemma 1 is as follows. From eq. (2) we know that firm profits only depend on the competitive environment through the level of market intensity, $\int z^{\sigma-1}\mu(z)dz$, and the exact composition of μ is not payoff-relevant for firm production or entry choice. As a consequence, whether the competitors are other current entrants or incumbents is also irrelevant to firm decisions, and there is only one level of *market intensity* that solves eq. (4), which directly implies the result of the Lemma.[4] In contrast, if $E = 0$, then the cutoff relies heavily on the distribution of incumbents because if there is exit, then the marginal exiting firm is an incumbent. Equation (3) is slack and eq. (4) rearranges to $0 = \pi(\underline{z}, \mu^I)$.

We conclude the equilibrium characterization by noting that our results hold more generally than we have stated so far. We have defined the entry process as an instantaneous equilibrium in which all market participants anticipate the firm distribution after entry, μ. In an alternative equilibrium characterization without anticipation of μ, firms iteratively enter the market, only considering the ex-ante distribution of firms in each iteration. Given a distribution of incumbents, initially, a small mass of firms enters the market, and the low productivity subset folds immediately. This play repeats until no additional firm wishes to enter and no active firm wishes to quit. Remark 1 states that the two entry processes are equivalent.

Remark 1 (Entry game equivalence)
The instantaneous equilibrium has a unique solution $(E, \underline{z}, \mu)$*, which coincides with the limit point of the iterative entry game.*

This completes the characterization of the partial equilibrium of this economy. Next, we study the effect of business cycles.

2.3 Business Cycles

We define business cycles as MIT-shocks to the fixed costs of production, f^c. Such cycles could, for example, be driven by varying financing conditions in working capital constraints.

Let the economy run through three phases, capturing the dynamics of crises. In phase $\tau = 1$, fixed costs equal f_l^c, then unexpectedly increase to $f_h^c > f_l^c$ in $\tau = 2$, and subsequently revert to f_l^c in $\tau = 3$; subscripts l and h refer to low and high, respectively.[5] In phase 1, there is scope for entry, and the industry reaches the equilibrium determined by eqs. (6) and (7). In phase 2, the rise of fixed costs leaves no scope for entry and forces some of the incumbents, the firms that entered in phase 1, so $\mu_2^I \equiv \mu_1$, to exit. The productivity cutoff, determined

[4]This result relies heavily on the monopolistic competition assumption as all firms have infinitesimal size. In an oligopolistic context, it would be payoff-relevant for each individual firm whether, fixing a level of aggregate productivity, the competitive environment is composed of few productive or many less productive firms. See Ferrari and Queirós (2024) for the effect of firm heterogeneity on aggregate outcomes in oligopoly models.

[5]Note that considering the phase 3 reversion to f_l^c is equivalent to studying the limit of a slow-moving process of mean-reversion of the fixed cost to the long-run mean of f_l^c.

by the zero-profit condition alone (eq. 4), increases to $\underline{z}_2 > \underline{z}_1$. The incumbent productivity distribution of firms, μ_2^I, is truncated from below, generating the new distribution μ_2. We show this process in panels A and B of Figure 1. In phase 3, the fixed costs revert to f_l^c. Survivors of the crisis are now incumbents, so $\mu_3^I \equiv \mu_2$. Additionally, new firms enter. Equations (6) and (7) hold, and $E_3 > 0$. The post-crisis distribution of firms μ_3 is given by eq. (5), with $\mu^I = \mu_3^I$ and $E = E_3$. Panel C of Figure 1 displays the post-recession distribution μ_3. Note that the productivity of the least productive active firm is the same before and after the cycle, since by Lemma 1, the cutoff $\underline{z}_3$ equals the pre-crisis cutoff $\underline{z}_1$. Yet, the distribution of firms differs and $\mu_1 \neq \mu_3$ due to the presence of incumbents.

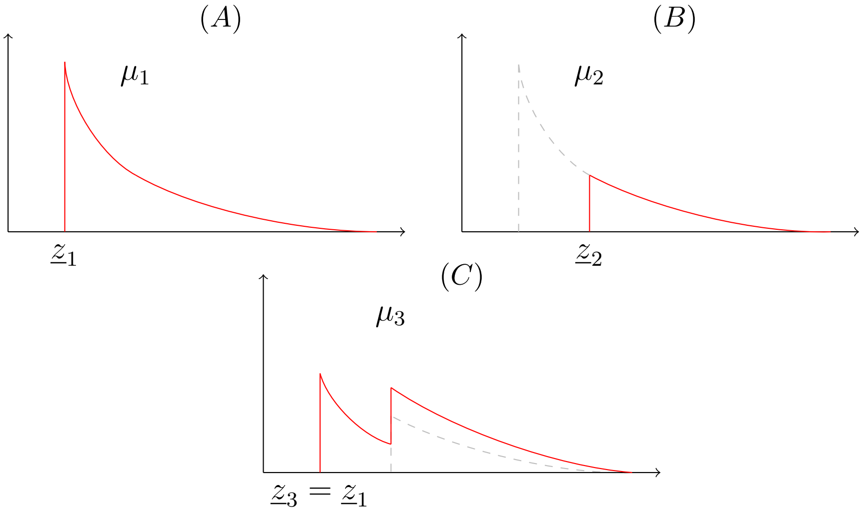

Figure 1: The figure shows the entry and exit dynamics over the business cycle. Panel (A) shows the distribution μ_1 before the shock hits. Upon impact, the left tail of firms with productivity less than $\underline{z}^2$ leave, creating distribution μ_2 (B). Finally, after fixed costs return to pre-shock levels and new firms drawn from the baseline distribution (μ^0) enter, μ_3 becomes the distribution of productivities in the market (C).

By substituting equilibrium production of each firm into the aggregator in eq. (1), we can write aggregate output (GDP) in equilibrium as

$$Y_\tau = M_\tau^{q-\frac{1}{\sigma-1}} L_\tau^d \left(\int z^{\sigma-1} \mu_\tau(z) \, \mathrm{d}z \right)^{\frac{1}{\sigma-1}}. \tag{8}$$

Here, L_τ^d is the amount of labor demanded for production at the fixed market wage. Proposition 1 characterizes how business cycles affect aggregate output relative to pre-cycle values.

Proposition 1

First, the aggregate demand for production labor by the industry, L^d, is independent of μ, f^c and f^e, and thus does not change through the cycle.

Second, the ratio of post-crisis output to pre-crisis output is given by

$$\frac{Y_3}{Y_1} = \left[\frac{M_3}{M_1} \right]^{q-\frac{1}{\sigma-1}} \left[\frac{\int z^{\sigma-1} \mu_3(z) \, \mathrm{d}z}{\int z^{\sigma-1} \mu_1(z) \, \mathrm{d}z} \right]^{\frac{1}{\sigma-1}}, \tag{9}$$

where

$$\frac{M_3}{M_1} < 1 \textit{ and } \frac{\int z^{\sigma-1}\mu_3(z)\ \mathrm{d}z}{\int z^{\sigma-1}\mu_1(z)\ \mathrm{d}z} = 1.$$

Furthermore,

$$\frac{Y_3}{Y_1} \gtreqqless 1 \Leftrightarrow q \lesseqqgtr q^{CES}. \tag{10}$$

Proposition 1 is our first main result: whether GDP and welfare are larger than before the recession depends uniquely on the LoV parameter q. It comes with several additional important observations. First, our result refers to long-run effects: any potential welfare loss is not driven by the transition in phase 2. When $q < q^{CES}$, the economy might still be worse off once transition costs are accounted for. What we show is that if $q > q^{CES}$, long-run welfare is lower even without the transition costs. Second, all the classic Schumpeterian forces of recessions are present: new entrants are, on average, more productive than exiters during the cycle. Recessions induce a *selection effect*, nonetheless, they may still result in long-run welfare losses. Third, we remark that the CES case is a knife-edge parametric restriction:

Remark 2 (CES Aggregation)
The special case of CES aggregation $\left(q = q^{CES}\right)$ *implies that* $Y_1 = Y_3$*: long-run output is unchanged pre- and post-recession.*

To build intuition for the above proposition and remark, it helps to look at the production of an individual firm with productivity z, which is given by

$$y_\tau(z) = z \cdot \underbrace{\mathcal{I}\frac{\sigma-1}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}\mu_\tau(z)\ \mathrm{d}z}}_{=:l^d(z),\text{ production labor demand}}. \tag{11}$$

We recall that firms produce as if they were living in a CES economy. This observation implies that any deviation of Y_τ from its CES counterpart is driven by the aggregation, not by individual production decisions. Therefore, we first discuss the mechanics of the business cycle on y_τ for a CES economy and then explain deviations from the CES aggregate as driven by the externality alone. Consider a firm with productivity z large enough to survive throughout the cycle. In phase 2, the presence of fewer competitors in the market lowers market intensity; hence, the firm expands its output: $y_2(z) > y_1(z)$. In phase 3, entrants start producing: facing an initially low competition, they drive up market intensity until the free entry condition holds. Since all costs have reverted back to pre-crisis levels, the only way to make potential entrants indifferent between entering and staying out is by adjusting aggregate productivity to pre-crisis levels.[6]

In a CES economy, output depends uniquely on aggregate TFP and aggregate demand for production labor, L^d_τ. The latter, however, does not depend on the fixed cost cycle but

[6]Note that, in our model, there is no strategic advantage from being an incumbent. In a different strategic setting, e.g., if incumbents were Stackelberg leaders and entrants followers, this may no longer be true.

exclusively on market size $\mathcal{I}$ and σ, which was already argued in Section 2.2. Since aggregate TFP and aggregate labor used in production are unchanged, so is output and, therefore, welfare.

For a nuanced deconstruction of effects, consider eq. (8). Taking logs and differencing pre- and post-crisis values yields

$$\Delta \log Y = q\Delta \log M + \Delta \log L^d + \Delta \log \bar{z}, \tag{12}$$

where we call $\bar{z}_\tau := \left(\int z^{\sigma-1}\frac{\mu_\tau(z)}{M}\,\mathrm{d}z\right)^{\frac{1}{\sigma-1}}$ *average productivity*. The first term, $q\Delta \log M$, is the *variety effect*, the second term, $\Delta \log L^d$, the *labor cleansing effect*, and the third term the *selection effect* of the crisis. Given that production labor is constant throughout the cycle, there is no labor cleansing effect in PE. The variety effect is driven by changes in M and amplified through LoV, q. The selection effect is channeled through changes in average productivity as exiting firms, on average, are less productive than entrants. In PE, the variety effect dominates the selection effect for $q > q^{CES}$, while for CES economies, the two cancel out exactly.

As a final discussion point, we consider output during the recession. If $q > q^{CES}$, it is immediate that Y_2 drops relative to Y_1 because both the number of varieties and market intensity in eq. (1) decrease. However, in the limit case of $q = 0$, we can write $Y_\tau = L^d \cdot \bar{z}_\tau$. In this extreme case, the intermediary is indifferent to the number of varieties and only cares about average productivity among active firms, $\bar{z}_\tau$, which *increases during the crisis*. The selection effect is positive, while the variety effect is zero. What may be perceived as a crisis within the industry turns out to be beneficial for aggregate output.[7]

Proposition 1 highlights another important aspect of our model, formalized in Remark 3.

Remark 3 (Path Dependence)
The stationary steady-state equilibrium is path-dependent.

We considered economies that, in phases 1 and 3, feature identical parameters. Nonetheless, they are characterized by different equilibrium allocations. This property is fully driven by the presence of incumbents. Our business cycles involve changes in the fixed costs of production, and the productivity cutoff is a direct function of these costs. Starting from a steady state with a certain level of f^c and distribution of active firms μ, the distribution of entrants $E\mu^0\mathbb{I}_{\{\bar{z}\geq \underline{z}\}}$ in μ is truncated at a new cutoff during the transition phase, i.e. phase 2. This alone already implies that the composition of firms in the new steady state, where fixed costs have reverted to f^c, cannot possibly be the same. In addition to this, we have that the mass of entrants when there is scope for entry and the cutoff when there is no scope for entry are both a function of the incumbents' distribution, creating another source of path dependency.

We conclude by noting that while our model setup and free-entry equilibrium are such that firms solve a static problem, the response of the measure of firms to business cycle fluctuations, and thus the steady-state equilibrium allocations, are identical in the case of forward-looking firms and large t. This is formalized in Remark 4.

[7] Such a scenario is even more likely in general equilibrium when the exit of low-productivity firms relaxes the resource constraint. For the rest of the paper, we abstract from such extremes and use the terms crisis and recession interchangeably.

Remark 4 (Forward-Looking Firms)
Suppose that:

1. *Firms know that the time-t path of $\{f_t^c\}_t$ is weakly decreasing.*

2. *Firms calculate the present discounted value of their profit stream.*

3. *Firms receive one-shot offers on whether to enter. If they take the offer, they pay the fixed costs of entry f^e, draw their productivity z, and can then delay production until they become profitable.*

For large t, the measure of firms in the economy is the same as in the case of myopic firms.

Therefore, our model is less restrictive than it might appear, and our results can be thought of as being generated by dynamic entry and exit decisions.

In Section 3, we close the model by assuming that consumers own the firms and supply labor. This generates a feedback between the competitive make-up of the economy and the expenditure on the composite good.

3 General Equilibrium

In GE, we impose two changes: (1) wages clear labor markets; (2) households earn labor and profit incomes. Labor market clearing dictates:

$$L^d + Mf^c + Ef^e = \bar{L}. \tag{13}$$

Therefore, in GE, we have endogenous wages and, thus, endogenous household income and total value of production. In comparison to PE, where the labor supply curve was perfectly elastic (any amount of labor could be bought at wage $w = 1$), it is now perfectly inelastic (the wage adjusts to fix labor demand at $\bar{L}$). Intuitively, labor market clearing becomes a binding resource constraint. We choose wages as the numeraire, $w = 1$.

Household income $\mathcal{I} = \bar{L} + \Pi$ is equal to expenditure $R = PY$, and firm profits become:

$$\pi(z, \mu) = \frac{R}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} \mu(z)\ \mathrm{d}z} - f^c. \tag{14}$$

Entry and exit are still pinned down by the *free-entry condition* and *zero-profit condition* (eqs. 3 and 4). When entry is strictly positive, eqs. (3) and (4) can be rearranged into a GE analog of eq. (6)

$$E = \frac{R}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^0(z)\ \mathrm{d}z} - \frac{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^I(z)\ \mathrm{d}z}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^0(z)\ \mathrm{d}z}, \tag{15}$$

and eq. (7), which is unchanged from PE. The equilibrium is fully characterized by eqs. (7), (13), and (15).

When there is no scope for entry, then $E = 0$, the *free-entry condition* is slack, and the *zero-profit condition*

$$\frac{R}{\sigma} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^I(z) \, \mathrm{d}z} - f^c = 0 \tag{16}$$

pins down the mass of active firms in the economy. The equilibrium is fully characterized by eqs. (13) and (16), and $E = 0$.

3.1 Business Cycles

In GE, entry and fixed costs divert labor resources away from production. However, entry costs are only temporary, and free up labor once paid. The income effect from these labor savings induces subsequential entry. Therefore, while we let the economy run through the same three phases as in PE, we cannot think of each phase as corresponding to one period anymore. In GE, each phase τ consists of an infinite number of periods $t \in \mathbb{N}$, with the labor savings effect vanishing as $t \to \infty$. The total number of entrants along this process in phase τ is called *steady state mass of entrants* and given by $E_\tau^{ss} \equiv \sum_{t=1}^{\infty} E_{\tau,t}$. Therefore, while in PE we compare the pre- and post-cycle static equilibrium, in GE we are comparing the pre- and post-cycle steady-state equilibrium. For a formal treatment of the steady-state equilibrium, see Definition 1 in the appendix. The following result extends Proposition 1 to the GE setting:

Proposition 2 (Cleansing Effects of Cycles)
The ratio of post-cycle steady-state output to pre-cycle steady-state output is given by

$$\frac{Y_3}{Y_1} = \left[\frac{M_3}{M_1}\right]^{q - \frac{1}{\sigma-1}} \left[\frac{L_3^d}{L_1^d}\right] \frac{\left(\int z^{\sigma-1} \mu_3 \, \mathrm{d}z\right)^{1/(\sigma-1)}}{\left(\int z^{\sigma-1} \mu_1 \, \mathrm{d}z\right)^{1/(\sigma-1)}}, \tag{17}$$

where

$$\frac{M_3}{M_1} < 1, \; \frac{L_3^d}{L_1^d} > 1, \; \frac{\int z^{\sigma-1} \mu_3 \, \mathrm{d}z}{\int z^{\sigma-1} \mu_1 \, \mathrm{d}z} > 1. \tag{18}$$

There exists a unique $q^\star > q^{CES}$ (which generally depends on the fixed cost cycle values f_l^c, f_h^c) for which $\frac{Y_3}{Y_1} = 1$. Furthermore,

$$\frac{Y_3}{Y_1} < 1 \iff q > q^\star. \tag{19}$$

General equilibrium introduces an additional effect of cleansing via recessions, the *labor cleansing effect*: as the long-run economy features fewer firms operating, fewer labor resources are used for fixed costs as opposed to production. As a consequence, the income obtained by the household, both in terms of labor payments and profits, increases. To see this, note that the fixed supply of labor $\bar{L}$ is used in production L^d and fixed costs payments Mf^c (recall that we are looking at steady-states and, thus, contemporary entry is zero). Furthermore, by the constant markup, total income is given by $\frac{\sigma}{\sigma-1}L^d$. Combining these two, it is immediate that the income obtained by the household is $\frac{\sigma}{\sigma-1}(\bar{L} - Mf^c)$. This GE income effect expands market size and creates additional entry in phase 3 relative to PE. In turn, the economy loses fewer varieties, and the variety effect is dominated by the selection effect in the CES case. Put differently, the relaxation of the resource constraint through the cycle has a direct labor cleansing effect on L^d and an income effect on market size, leading to increased market intensity and a dominating selection effect post-cycle.

The direct consequence of the combination of these forces is an output gain, even in the CES case, which is highlighted in Remark 5.

Remark 5 (CES Aggregation in General Equilibrium)
The special case of CES aggregation $\left(q = q^{CES}\right)$ *implies that* $\frac{Y_3}{Y_1}\left(q^{CES}\right) > 1$.

Based on the above intuition and remark, it is immediate that when $q > q^{CES}$, the economy values the loss of variety more than in the CES case. As a consequence, there exist a level of love-of-variety such that the household is indifferent between pre- and post-cycle outcomes as the three effects exactly cancel.

3.2 Depth of Crises

Until now, we have considered a fixed business cycle characterized by (f_l^c, f_h^c). Here, we study crises of varying intensity of the fixed cost increase, f_h^c. In a given economy, the effect of a fixed cost increase on long-run output is governed by the elasticity:

$$\frac{\partial \log Y_3}{\partial \log f_h^c} = \frac{\partial \log M_3}{\partial \log f_h^c} \times \left[\frac{\partial \log Y_3^{CES}}{\partial \log M_3} + (q - q^{CES})\right], \tag{20}$$

where $Y_\tau^{CES} := L_\tau^d(\int z^{\sigma-1}\mu_\tau(z)\ \mathrm{d}z)^{1/(\sigma-1)}$ is the phase τ output in the CES economy with $q = q^{CES}$. We study each element in turn. First, we analyze the effect of a marginal change in the depth of the crisis on the number of active firms in the long run, M_3. This is the first term of eq. (20). Second, we analyze the effect of marginal changes in the number of firms in the economy on long-run output. This is the term in the bracket of eq. (20).

Effects of f_h^c on the mass of active firms M_3. We have argued above that the long-run number of firms necessarily declines after a crisis. This effect is driven by selection, as the average entrant is strictly more productive than the average exiter. In particular, the successful entrants have an average productivity of $\mathbb{E}_0[z|z \geq \underline{z}_1]$, whereas the exiters have an average productivity of $\mathbb{E}_0[z|\underline{z}_1 \leq z \leq \underline{z}_2]$. As a consequence, the average firm in the economy becomes more productive, and all else equal, there is space for fewer firms, i.e., less entry is needed to drive market intensity to its equilibrium level. This is true for all recession intensities f_c^h and independently of LoV, q.

However, larger crises do not always imply marginally fewer firms in the long run: the sign of $\frac{\partial \log M_3}{\partial \log f_h^c}$ is ambiguous. This is driven by the selection effect: the magnitude of the selection force depends on how large the rise in the fixed cost f_c^h is. To understand this, compare the average successful entrant with the marginal exiter. The productivity of the marginal exiter is $\underline{z}_2$. When $\underline{z}_2$ is very low, $\underline{z}_2 < \mathbb{E}_0[z|z \geq \underline{z}_1]$, the selection effect is very strong, and less than one average entrant is needed to fill the gap left by the exiter: $\frac{\partial \log M_3}{\partial \log f_h^c} < 0$. However, when $\underline{z}_2$ is very high, $\underline{z}_2 > \mathbb{E}_0[z|z \geq \underline{z}_1]$, selection is very weak and the survivor lost at the margin creates space for more than one firm to enter the market: $\frac{\partial \log M_3}{\partial \log f_h^c} > 0$. We formalize this insight following in Lemma 2.

Lemma 2 (On $M_3(f_h^c)$)
The long run number of firms active in the economy $M_3(f_h^c) \leq M_1$ attains a unique minimum M_3^{min} at some crisis level, $f_h^{c,*} \in (f_l^c, \infty)$. For each crisis driven by $f_h^c \in (f_l^c, f_h^{c,*})$ there exists some crisis $f_h^{c\prime} \in (f_h^{c,*}, \infty)$, such that $M_3(f_h^c) = M_3(f_h^{c\prime})$.

The mechanism underlying this result is fully driven by the presence of incumbents. The marginal effect of the magnitude of the crisis on the long-run number of firms is given by

$$\begin{aligned}
\frac{\partial \log M_3}{\partial \log f_h^c} &= \frac{f_h^c}{M_3} \cdot \frac{\partial}{\partial f_h^c} \Bigg[\underbrace{M_1 p_0(\underline{z}_2)}_{\text{Surviving Firms in Phase 2}} + \underbrace{E_3 p_0(\underline{z}_1)}_{\text{Entrants in Phase 3}} \Bigg] \\
&= \frac{f_h^c}{M_3} \cdot \left[\underbrace{M_1 \frac{\partial p_0(\underline{z}_2)}{\partial \underline{z}_2}}_{<0} + \underbrace{p_0(\underline{z}_1) \frac{\partial E_3}{\partial \underline{z}_2}}_{>0} \right] \cdot \underbrace{\frac{\partial \underline{z}_2}{\partial f_h^c}}_{>0},
\end{aligned} \tag{21}$$

where $p_0(\underline{z}) = \int_{\underline{z}}^{\infty} \mu^0(z) dz$. As f_h^c increases, the crisis cutoff $\underline{z}_2$ shifts up. At low levels of f_h^c, the first term in the bracket of eq. (21) dominates: marginal exiters are less productive than entrants, and there is a marginal decline in the number of firms. At high levels of f_h^c, the marginal exiter is more productive than the average entrant, and, therefore, any additional increase in the fixed cost is associated with marginally more firms in the long run.

Effect of M_3 on long-run output Y_3. Fix LoV at some arbitrary level, q. In this q-economy, as highlighted by eq. (20), the elasticity of output Y_3 with respect to M_3 decomposes into i) the elasticity of Y_3^{CES}—output if the economy had LoV of level q^{CES}—and ii) the elasticity of the variety effect in the q-economy relative to a CES economy. The latter is constant and its sign depends on whether LoV in the economy is higher or lower than in the CES benchmark. The elasticity of Y^{CES} with respect to M_3 is always negative. A higher number of firms M_3 affects Y^{CES} in two different ways: i) more firms directly imply more fixed cost payments and, therefore, less labor used in production, and ii) more firms in the economy are associated with weaker selection. Both of these effects reduce Y_3^{CES}.

To complete the characterization, note that $\frac{\partial \log Y_3^{CES}}{\partial \log M_3}$ is globally negative but not constant across different crisis intensities, f_h^c. Instead, it is decreasing in absolute terms. Consider $\frac{\partial Y_3}{\partial M_3} \frac{M_3}{Y_3}$ at $M_3 = M_1$. To build intuition, assume that the case holds because $f_h^c = f_l^c$ and not because the crisis is infinitely large. To decrease the number of firms by 1%, one needs to eliminate substantially more firms during the crisis, otherwise, entrants in phase 3 would be too numerous. The reason is as follows: killing the least productive percent of firms leads to large savings in fixed costs relative to losses in production. Therefore, income increases in phase 3, which expands equilibrium market intensity and the number of entrants, largely undoing the removal of firms during the crisis. Thus, one requires large labor cleansing and income effects in the crisis, begetting a large relative increase in output and, therefore, a high elasticity. Alternatively, if we started from a somewhat smaller M_3, Y_3 has already reaped the benefits of labor cleansing and selection and is large compared to Y_1. To decrease M_3 by 1% now only requires removing a few more firms during the crisis since now the marginal survivor is much closer in productivity to the marginal entrant. This, in turn, implies a smaller increase through labor cleansing over (an already large) Y_3. In effect, the elasticity is low.

We can now return to the total effect of having more firms in the post-cycle steady state. On the one hand, additional firms generate one-to-one gains from varieties, provided that $q > q^{CES}$. On the other hand, additional firms generate more-than-linear reductions in the labor-saving and selection gains from the crisis, and thus in Y_3^{CES}. We conclude that reducing firm and hence product variety M_3 has always a negative effect on long-run output if q is large, and always positive if q is small. In between these two extremes, reducing firm and hence product variety M_3 has a negative effect on long-run output for small values of M_3 (where the variety effect is stronger than cleansing and selection) and positive for large values of M_3. Next, we discuss this region of ambiguity and its implication for the effect of crises of varying magnitude, f_h^c, on total output.

Effect of f_h^c on Y_3. Taking stock, we know that as the crisis intensity increases, the mass of varieties available in phase 3 first decreases then increases, and that this mass of varieties can have unambiguously positive, negative, or indeterminate effects on total output. We assemble these insights about the effect of f_h^c on Y_3 in Proposition 3:

Proposition 3 (Interaction of cycle depth and LoV)
Consider the sequence of economies indexed by their love-of-variety q. Then there exists a unique interval $(q_\circ, q^\circ)$ with $q_\circ > q^{CES}$ such that:

1. $q \geq q^\circ \implies Y_3/Y_1 \leq 1$ *for all* f_h^c.
2. $q \leq q_\circ \implies Y_3/Y_1 \geq 1$ *for all* f_h^c.
3. *for all* $q \in (q_\circ, q^\circ)$*, larger crises can be welfare improving or welfare reducing depending on their intensity* f_h^c.

Proposition 3 provides three results. First, there exists a level of LoV $q_\circ$ such that any economy with $q \geq q^\circ$ faces GDP and welfare drops for any crisis intensity f_h^c. These are economies that value the presence of varieties so much that they experience long-run welfare losses, even for the smallest crises. Symmetrically, there is a subset of q-economies that do not value varieties as much and find any recession *cleansing*, independently of its intensity. Finally, the set of economies indexed by $q \in (q_\circ, q^\circ)$ is such that small and extremely large recessions can be cleansing while medium recessions reduce welfare. This follows by the decreasing returns to labor cleansing and selection effects as the number of varieties shrinks. We sketch these behaviors out in Figure 2.

Panel (A) shows how CES output, Y^{CES}, and the total mass of firms, M_3, respond to different increases in the fixed costs, f_h^c. Both quantities are independent of q. Relative to before the crisis, the mass M_3 drops, reaching a minimum at depth $f_h^{c,*}$, where all the incumbents with productivity less than the average successful entrant have been replaced. CES output always increases relative to before the cycle, and reaches its highest level when no further returns from labor cleansing and selection effects can be extracted, i.e., at $f_h^{c,*}$. As the fixed cost rises toward eliminating every firm phase 2, all quantities return to their pre-crisis levels.

Next, panel (B) shows the evolution of the love-of-variety effect for different levels of q. When $q > q^{CES}$, this effect mirrors the path of the number of firms in panel (A). When $q < q^{CES}$, the LOV externality is such that the economy is better off when fewer firms are active, so the effect tracks that of M_3/M_1.

Finally, panel (C) shows the behaviour of output for different $q-$economies. When q is large, recessions are unambiguously welfare reducing and, vice versa, when q is small they are always cleansing. In the intermediate range, some recessions can induce welfare gains while others generate welfare losses depending on their depth.

This set of intermediate $q-$economies is characterized by an inference problem. In fact, past data on recessions cannot inform on whether future recessions will have cleansing effect or not.

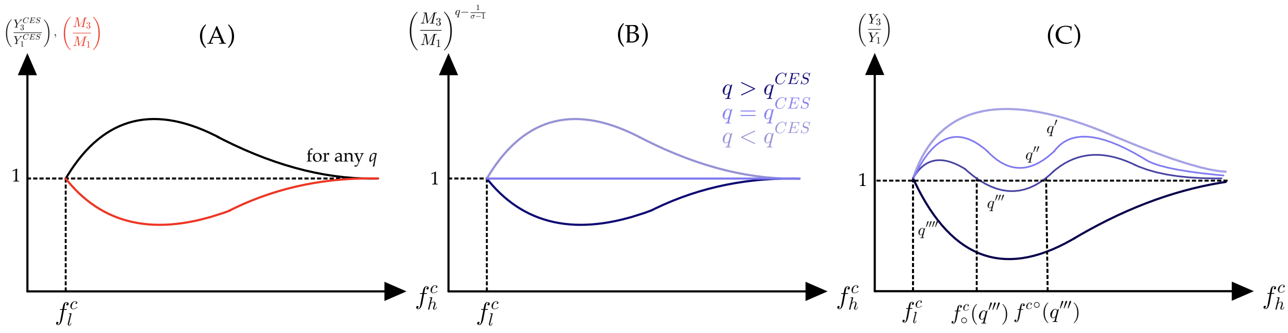

Figure 2: Decomposition of the output ratio for varying levels of LOV, q and crisis, f_h^c. Panel (A) shows the how Y_3^{CES}/Y_1^{CES} and M_3/M_1 vary with f_h^c. Panel (B) shows how LOV contributes to the output ratio through the factor $(M_3/M_1)^{q-q^{CES}}$. Panel (C) sketches the curve of Y_3/Y_1 for values of q, $q' < q'' < q''' < q''''$, whereby only $q''' \in (q_\circ, q^\circ)$ holds (cf. Proposition 3). Correspondingly, at q''', the effect of crises is ambiguous, and there are two crises $f_\circ^c(q''') < f^{c\circ}(q''')$ at which the output ratio returns equals 1. A proof of the 'wiggly' shape when LOV is q'' is provided in the proof of Proposition 3.

We conclude by noting that in our economy, recessions are path-dependent and that the depth of the current crisis has implications for output and welfare in subsequent downturns. This is formalized in Remark 6.

Remark 6 (Path Dependence of Recessions)
Let a q-economy experience two cycles of the same intensity f_c^h. Then: a) the post-crisis distribution, output, and welfare are different across the two cycles; b) the recessions generate different degrees of cleansing, captured by $\underline{z}_2^1 \leq \underline{z}_2^2$, where $i \in \{1, 2\}$ denote the cycle.

Having experienced the first recession, the q-economy starts the second crisis with a different incumbent population. During downturns, the composition of incumbents determines the strength of selection effects for a given increase in the fixed cost f_c^h (see eq. 16). This directly implies a different long-run distribution of firms and, therefore, output and welfare.

4 Policy

In this section, we study a social planner problem for economies with incumbents. The planner chooses the allocation of labor between entry costs, fixed costs, and each individual firm's production, controlling the entry mass E^{SP}, the cutoff $\underline{z}^{SP}$. Since taste-for-variety and incumbencies do not distort the firms' decision problem, a result by Dhingra and Morrow (2019) applies, which says that a given amount of production labor is split efficiently between firms and therefore E^{SP} and $\underline{z}^{SP}$ are sufficient tools for the social planner to steer the economy.[8] The social planner makes decisions sequentially, in period t and, possibly, phase of cycle, τ. Wherever it matters, we therefore write $E_{\tau,t}^{SP}$ and $\underline{z}_{\tau,t}^{SP}$.

We start by showing that the government can use subsidies to achieve the first-best allocation even when $q \neq 1/(\sigma - 1)$. The easiest way to achieve this is to subsidize or tax f^e and f^c in eq. (7) and (15); when firms and entrants face costs $\delta_{\tau,t}^c f_\tau^c$ and $\delta_{\tau,t}^e f^e$ instead, the government can shift $E_{\tau,t}$ and $\underline{z}_{\tau,t}$ to efficient levels in general equilibrium, by solving the corresponding equations for $(\delta_{\tau,t}^e, \delta_{\tau,t}^c)$, while financing the intervention using a lump-sum tax paid by the households. In the knife-edge case of CES it turns out no intervention is necessary.

4.1 Planner Entry and Exit Choices

We build the optimal behavior of the planner over the cycle by studying in isolation the cases of pure entry and pure exit. We combine these insights to study the socially optimal policy over the cycle. The social planner chooses $(E^{SP}, \underline{z}^{SP})$ to maximize the objective

$$\begin{aligned} \max_{E^{SP}, \underline{z}^{SP}} \quad & Y^{SP} = (M^{SP})^{q - \frac{1}{\sigma-1}} L^{d,SP} \left[\int z^{\sigma-1} \mu^{SP}(z) \, \mathrm{d}z \right]^{\frac{1}{\sigma-1}} \qquad \text{s.t.} \qquad (22) \\ & \bar{L} \geq L^{d,SP} + (p_0(\underline{z}^{SP}) f^c + f^e) E^{SP} + f^c I, \\ & \mu^{SP}(z) = (\mu^I(z) + E^{SP} \mu^0(z)) \mathbb{I}_{\{z \geq \underline{z}^{SP}\}}(z), \\ & \underline{z}^{SP}, E^{SP} \geq 0. \end{aligned}$$

[8] This observation uses the fact that if the social planner wanted to keep unprofitable firms in the economy, she would always choose the most productive unprofitable firms.

The objective, Y^{SP}, is current output, the a social planner would want to maximize if she does not care about long term production, and corresponds to a Ramsey planner problem if $\beta = 0$. One can expect the solution to this tractable static problem to be a reasonable approximation to that of a dynamic Ramsey problem. The reason is that the trade-off which a social planner makes is not as stark as a classical consumption-savings decision. This follows from the dual role of allocating labor to entry. Not only does a unit of labor allocated to entry today rather than tomorrow free up f^e resources tomorrow, but also entry today may increase today's output through the presence of an additional firm. If the economy lacks many establishments, there is no trade-off between entry today versus entry tomorrow, and it pays off to rush establishments into the market.

For characterizations of social planner solutions through the cycle, one thinks of (τ, t)-subscripts being attached to the choice variables of (22), and a law-of-motion of the incumbent measure $\mu^I_{\tau,t} = (\mu^I_{\tau,t-1} + E_{\tau,t-1}\mu^0)\mathbf{1}_{\{\bar{z} \geq \underline{z}^{SP}_{\tau,t-1}\}}$, given some $\mu^I_{\tau,0}$. From the first order conditions on E^{SP} and $\underline{z}^{SP}$, we characterize how entrants, cutoff, and steady-state mass of entrants $E^{ss,SP}$ change through presence of the aggregation externality ($q \neq q^{CES}$) and incumbents.

We characterize optimal policy in Proposition 4, where we distinguish two cases. In case [A], the social planner faces an economy with only a few incumbents all of which are highly productive. For example, this is the state of an economy treated in Section 3 which stands at the beginning of phase 3. Such an economy is in a state of entry, and the cutoff chosen by the social planner, $\underline{z}^{SP}$, forces unproductive entrants but none of the previous incumbents to leave. Second, in [B], we treat the case of an economy which features *too many* incumbents, like the economy in phase 2 of the cycle. Specifically, we assume that incumbents were drawn from the baseline distribution, such that $\mu^I(z) = \mu^0(z)\mathbb{I}_{\{z \geq \underline{z}\}}(z)$ holds for some $\underline{z}$.

Additionally, we make the distributional assumption (A1) that $\mathbb{E}_0\left[(z/\underline{z})^{\sigma-1}\,\middle|\, z \geq \underline{z}\right]$ is weakly decreasing in the cutoff throughout, $\underline{z}$, which guarantees that points [A]-4 and [B] of Proposition 4 (monotonicity of $\underline{z}$ in q) hold. Distributions satisfying the assumption are for example those in the Pareto family.[9]

Proposition 4 (Social Planner Solutions)
The interior solutions $E^{SP}, \underline{z}^{SP}$ of the social planner problem given by eq. (22) have the following properties:
[A] Case with entry: $E^{SP} > 0$ (assuming $\mu^I(z) = 0\ \forall z \leq \underline{z}^{SP}$).

1. *$E^{SP}, \underline{z}^{SP}, E^{ss,SP}$ coincide with general equilibrium allocations $E, \underline{z}, E^{ss}$ if and only if $q = q^{CES}$.*

2. *Number of entrants E^{SP} and total mass of entrants $E^{ss,SP}$ are both strictly increasing in q, and $(E^{SP}, M^{SP}) = (E, M)$ if and only if $q = q^{CES}$.*

[9] A sufficient condition for $\mathbb{E}_0\left[(z/\underline{z})^{\sigma-1}\,\middle|\, z \geq \underline{z}\right]$ to be decreasing is that the elasticity of the anticumulative corresponding to μ^0 with respect to the truncation $\underline{z}$, $\varepsilon_{1-F}(\underline{z})$, is globally larger than -1, i.e. $\varepsilon_{1-F}(\underline{z}) \geq -1,\ \forall \underline{z} \geq 0$.

3. *The presence of incumbents μ^I with higher average productivity than entrants in the market decreases all $E^{SP}, E^{SP,ss}, E$ and E^{ss}. It also decreases the final number of firm $M^{SP,ss} = E^{SP,ss} + I$.*

4. *Incumbents do not affect the social planner steady-state cutoff.*

5. *Under (A1): Without incumbents before entry, the cutoff $\underline{z}^{SP}$ is decreasing in q, i.e. the social planner discards fewer of the entering firms.*

[B] Under (A1): Case with exit, $\mu^I(z) \propto \mu^0(z)\mathbb{I}_{\{z \geq \underline{z}\}}(z)$ holds for some $\underline{z}$

In this case, $E^{SP} = 0$ and $\underline{z}^{SP}$ depends again negatively on q, furthermore, $\underline{z}^{SP} = \underline{z}^{GE}$ if and only if $q = q^{CES}$.

The result in Proposition 4 extends previous insights from Spence (1976); Dixit and Stiglitz (1977); Mankiw and Whinston (1986); Parenti et al. (2017) and Dhingra and Morrow (2019) to economies with firm heterogeneity and incumbents. When the planner is constrained by the presence of already existing incumbent firms μ^I, it optimally chooses the same cutoff and mass of entrants as in the market equilibrium if and only if the intensity of love-of-variety is that of CES preferences. If LoV is larger, there are inefficiently few firms operating in the market. These potential inefficiencies represent steady-state distortions. For any LoV, the optimal number of entrants is decreasing in the incumbent mass. In our setting, where incumbents are, on average, more productive than successful entrants, the planned economy eventually contains fewer firms the larger the incumbent mass, I, when approaching a steady state. Intuitively, the social planner internalizes the trade-off between allocating more labor to the most productive firms versus producing more varieties through a less productive entry margin.

Without incumbents, LoV affects the cutoff in the expected fashion: A higher q leads the social planner to decrease $\underline{z}$, e.g., by subsidizing fixed costs, such that lower-productivity firms can survive. While ambiguous in the short run, a general distribution of incumbents, μ^I, does *not* affect the cutoff as the economy approaches its steady state. The reason is that the social planner problem is, just like the equilibrium conditions, fundamentally agnostic regarding the precise firm heterogeneity at hand and only balances average productivity post entry with number of available varieties.

Next, we characterize the optimal policy intervention during business cycles.

Remark 7 (Ramsey Planner)
One can compare the short-run planner solution to the steady state solution of a Ramsey planner problem facing a discount factor β in eq. (23):

$$E^{ss,Ramsey}(\underline{z}; q) = \bar{L}\frac{q}{q+1}\frac{1}{f^c p_0(\underline{z}) + f^e \frac{1}{(1+q)(1+\beta)}}. \tag{23}$$

Starting from an empty economy, the Ramsey solution approaches the cumulative sum of entrants of the social planner as $\beta \to 0$. Notably, the role of β is rather small. Even for values of β close to 1, the effect on total entry in the long run is moderate as long as f^c/f^e is large. We present the Ramsey problem in more detail in the appendix.

4.2 Optimal Policy through the Cycle

To contrast planner solutions and market allocations, consider a cycle like in Section 2.3, starting from an empty economy. We show that the social planner solution dominates the equilibrium allocation throughout the cycle with phases $\tau \in \{1, 2, 3\}$, even if the social planner cannot anticipate changes in fixed cost. For sake of this analysis, assume that $q > q^{CES}$, so large even that $\underline{z}_{0,0}^{SP} = 0$ almost binds. Note that $\underline{z}_{1,t}^{SP} = \underline{z}_1^{SP}$ and $\underline{z}_{2,t}^{SP} = \underline{z}_2^{SP}$ are constant throughout, and that $\underline{z}_{3,t}^{SP}$ converges to some $\underline{z}_{\tau}^{SP}$ (this is shown in the proof of Proposition 4). Figure 3 sketches the evolution of the distribution of active firms throughout the cycle.

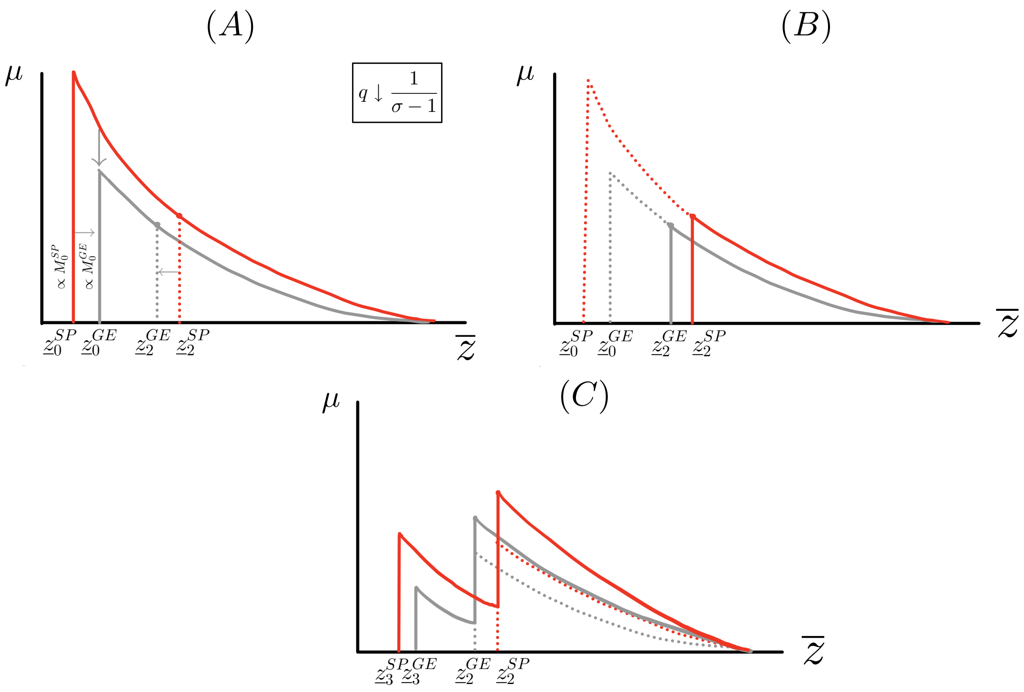

Figure 3: Evolution of μ through the business cycle for the market and social planner allocations. Assume that $q >> q^{CES}$.

Panel (A) displays the pre-crisis steady-state allocation ($\tau = 1, t \to \infty$). Because q is large, the social planner sets $\underline{z}_1^{SP}$ small and adds entrants to the economy. It holds that $\underline{z}_1^{SP} < \underline{z}_1$ and $M_1^{SP} > M_1$. Once fixed costs go up, the social planner chooses $\underline{z}_2^{SP}$. For large q, $\underline{z}_2^{SP} > \underline{z}_2$ must also hold, since the large mass $E_1^{ss,SP}$ chosen beforehand now requires extensive dropout so that fixed costs can still be paid with positive production. It is true that the crisis in the social planner economy kills more firms than in the GE-economy. This, however, is exclusively caused by the fact that the social planner economy carries a lot more firms into the crisis than the GE economy. Were the social planner to suddenly leave the SP economy to market forces in phase 2, even more firms would perish. The gray arrows indicate the direction of convergence as q approaches $q^{CES} = \frac{1}{\sigma-1}$ from above, and the social planner solution moves towards the equilibrium allocation.

In Panel (B), we see the firm distribution during the crisis ($\tau = 2$). The social planner solution pre-cycle has caused a deepening of the crisis: an economy that sustains a larger number of varieties becomes more vulnerable to an increase in fixed cost. The relative decrease in the number of varieties and total productivity (as given by the productivity integral) is much larger in the centrally planned economy than in the equilibrium case. Note, however, that welfare is still larger for the social planner economy: the large stock of firms caused by a large $E_1^{ss,SP}$ allows the social planner to feasibly choose some $\underline{z}'$ which equates total number of firms to the number of firms in the GE distribution μ_2, while creating a distribution which dominates μ_2. Hence, $Y_2^{ss,SP} \geq Y_2^{ss}$.

Finally, in panel (C), the distribution has settled on the post-crisis firm distribution. The market cutoff level has reverted back to its original level $\underline{z}_1$, whereas $\underline{z}_3^{SP}$ may lie right of $\underline{z}_1^{SP}$ due to the presence of incumbents. Overall, the social planner solution clearly dominates the general equilibrium distribution in terms of output since the social planner inherits a distribution of incumbents with larger mass and higher cutoff. We summarize that the short-run social planner achieves consistently higher output but with exacerbated volatility through the cycle.

When there is entry, the optimal values for $\delta^c_{\tau,t}, \delta^e_{\tau,t}$ change with the state of the economy and converge to values δ^c_τ and δ^e_τ as the entry dynamics subside. For the sake of this discussion, we compare these steady-state policies when considering phases of the cycle. Additionally, an equilibrium with $\underline{z} = 0$ is not actually implementable as it would require an infinite ratio $\frac{\delta^e f^e}{\delta^c f^c}$. We thus consider a situation in which $\frac{\delta^e_t f^e}{\delta^c_t f^c_t}$ is large yet finite. For a given small $\underline{z}_1^{SP}$ in the above discussion, firms must face a small fixed cost of production $\delta^c_1 f^c_1$ (by eq. (15)) and a much larger fixed cost of entry, $\delta^e_1 f^e_1$ (potentially a tax). The value of $\delta^c_{\tau,t}$ is determined jointly by the zero-profit-condition and the cutoff equation (eq. (4), (7)); however, once the steady-state has been reached, δ^c_τ satisfies eq. (4) with zero entry and the steady-state entrants as incumbents while fixing $\underline{z}$ at the social planner value. By evaluating eq. (4) at the $\underline{z}^{SP}$ which satisfies the first-order-condition of the social planner, we immediately obtain the following proposition:

Proposition 5 (Optimal Policy)
The optimal subsidy for production fixed-cost when the social planner economy attains its steady state after entry, or when the social planner raises $\underline{z}^{SP}$ to provoke exit, i.e., whenever $E^{SP} = 0$, is given by

$$\delta^c(\underline{z}^{SP}) = \left[[q(\sigma - 1) - 1] \left(\mathbb{E}_0 \left[\left(\frac{z}{\underline{z}^{SP}} \right)^{\sigma-1} \mid z \geq \underline{z}^{SP} \right] \right)^{-1} + 1 \right]^{-1}. \tag{24}$$

Furthermore, under (A1), fixed-cost subsidies are counter-cyclical and in the cycle discussed in Section 4.2 we have $\delta^c_1 \geq \delta^c_2$ and $\delta^c_3 \geq \delta^c_2$.

Recall that $\underline{z}_1^{SP} < \underline{z}_2^{SP}$. Using the first part of Proposition 5, we see that $\delta^c_1 \geq \delta^c_2$, i.e. the government should increase subsidies precisely if (A1) holds, which yields the second part.[10] Since the cutoff reverts back below $\underline{z}_2^{SP}$ at the end of the cycle, we deduce that, under the monotonicity assumption made above, subsidies decrease again, and thus $\delta^c_3 \geq \delta^c_2$ holds, too.

We conclude that the social planner in an economy with $q > q^{CES}$ finds it optimal to increase the fixed cost subsidy during recessions. This policy change is motivated by having

[10] In the special case of a Pareto baseline distribution, it turns out that $\delta^c_1 = \delta^c_2$.

fewer firms exit the market.

5 Conclusions

We study the cleansing effect of recessions in an economy where households value the consumption of multiple differentiated varieties. We show that recessions driven by rising fixed costs of production generate *cleansing* in the sense that exiting firms are replaced by more productive entrants. Nonetheless, we show that this need not translate into long-run gains in GDP and welfare. Whether the household is better off in the long run fundamentally depends on the extent of *love-of-variety* in their preferences. We show that when *love-of-variety* is stronger than in the benchmark CES economy, a social planner finds it optimal to increase fixed-cost subsidies during recessions to reduce the extent of firm exit.

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A Definitions

Definition 1 (Steady-State Equilibrium After Entry)
The allocation $(L^{d,ss}, M^{ss})$ and measure of active firms μ^{ss} constitute a steady-state equilibrium after entry, given an initial measure of incumbents μ^I, if

1. *the cutoff $\underline{z}$ is determined by eq. (7),*
2. *the steady-state distribution of firms is given by $\mu^{ss} = (\mu^I + E^{ss}\mu^0)\mathbf{1}_{\{z \geq \underline{z}\}}$, where $E^{ss} \equiv \sum_{t=1}^{\infty} E_t$ denotes the total entry mass across time,*
3. *eq. (15) holds with $E = 0$, and μ^{ss} as incumbent measure,*
4. *the mass of active firms is given by $M^{ss} = \int_{\underline{z}}^{\infty} \mu^I \, \mathrm{d}z + E^{ss} p_0(\underline{z})$,*
5. *labor markets clear $\bar{L} = L^{d,ss} + M^{ss} f^c$,*

and if the economy before entry (given μ^I) is in a state of positive entry.

B Proofs

In this section, note the following alterations and definitions:

- Density objects are treated as measures to allow for more generality.
- The definition applies:
$$Z(\underline{z}, \mu) := \int_{(\underline{z},\infty)} z^{\sigma-1} \, \mathrm{d}\mu.$$
- p_0 is the anticumulative of μ^0 (i.e., $1 - CDF$).

Proof of Remark 1. Let there be a baseline P-measure μ^0 from which productivity is drawn. We then have the following iterative entry procedure:

1. At iteration $k \geq 0$, a mass of firms, $M^{(k)}$, enter initially until:
$$0 = \mathbb{E}_0[\max\{\pi(z, \mu^I, \mu^{(k-1)}, M^{(k)}), 0\}] - f^e = \int_{\underline{z}^{(k)}}^{\infty} \left[\frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} d\mu' + \int z^{\sigma-1} d\mu^I} - f^c \right] d\mu^0 - f^e$$
where $\mu' := M^{(0)}\mu^0$ in the first iteration and $\mu' := \mu^{(k-1)} + M^{(k)}\mu^0$ in the kth iteration.
2. Currently active firms check their profitability, and all firms with productivity less than $\underline{z}^{(k)}$ leave the market. The cut-off $\underline{z}^{(k)}$ is determined by:
$$\pi(\underline{z}^{(k)}, \mu^I, \mu^{(k-1)}, M^{(k)}) = \frac{\mathcal{I}}{\sigma} \frac{(\underline{z}^{(k)})^{\sigma-1}}{\int z^{\sigma-1} d\mu' + \int z^{\sigma-1} d\mu^I} - f^c = 0$$
$$(\underline{z}^{(k)})^{\sigma-1} = f^c \frac{\sigma}{\mathcal{I}} \left[\int z^{\sigma-1} d\mu' + \int z^{\sigma-1} d\mu^I \right]$$
3. After exit, the measure of firms left in the market is relabeled as the new incumbent measure, and given by:
$$\mu^I \leftarrow \mu^I + \int_{\underline{z}^{(k)}}^{\infty} 1 \, d\mu' = \mu^I + \mu^{(k)}$$
where μ^I is not affected by the truncation since scope for entry implies that the threshold is weakly lower than the one previously incurred by the incumbents.

Steps 1 to 3 are iterated until convergence.

Equivalence of threshold $\underline{z}$

By step 1, we have that at iteration k the following holds:

$$f^e = \int_{\underline{z}^{(k)}}^{\infty} \left[\frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} d\mu^{(k-1)} + M^{(k)} \int z^{\sigma-1} d\mu^0 + \int z^{\sigma-1} d\mu^I} - f_c \right] d\mu^0 \tag{25}$$
$$= \frac{\mathcal{I}}{\sigma} \frac{\mu^0(z \geq \underline{z}^{(k)}) \, \mathbb{E}_0[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{\int z^{\sigma-1} d\mu^{(k-1)} + M^{(k)} \mathbb{E}_0[z^{\sigma-1}] + \int z^{\sigma-1} d\mu^I} - \mu^0(z \geq \underline{z}^{(k)}) f_c \tag{26}$$

Solving for $M^{(k)}$, we obtain:

$$M^{(k)} = \frac{1}{\mathbb{E}_0[z^{\sigma-1}]} \left[\frac{\mathcal{I}}{\sigma} \frac{\mu^0(z \geq \underline{z}^{(k)}) \, \mathbb{E}_0[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c \mu^0(z \geq \underline{z}^{(k)})} - \int z^{\sigma-1} d\mu^{(k-1)} - \int z^{\sigma-1} d\mu^I \right] \tag{27}$$

Substituting the derived $M^{(k)}$ in the cut-off equation of step 2, we obtain:

$$\underline{z}^{(k)} = f^c \frac{\mu^0(z \geq \underline{z}^{(k)}) \, \mathbb{E}_0[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c \, \mu^0(z \geq \underline{z}^{(k)})} \tag{28}$$

$$\frac{f^e}{f^c} = \mu^0(z \geq \underline{z}^{(k)}) \left[\frac{\mathbb{E}_0[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{\underline{z}^{(k)}} - 1 \right] \tag{29}$$

First, note that clearly none of this is dependent on having started at iteration k. We would have obtained 29 also by having derived $M^{(0)}$ in 27. Second, note that $\underline{z}^{(k)}$ is a constant sequence as it depends only on f^c and f^e, which don't change with k. Hence, we can write $\underline{z}_k = \underline{z} \; \forall k \in \mathbb{N}$. Third, note that the equation pinning down the threshold in the iterative entry procedure is the same as in the simultaneous entry game (7).

Equivalence of entry mass E

With $\underline{z}^{(k)} = \underline{z} \; \forall k \in \mathbb{N}$, it is clear that all we are doing is stacking scaled versions of the baseline measure truncated at $\underline{z}$. Therefore, 27 can be written as:

$$M^{(k)} = \frac{1}{\mathbb{E}_0[z^{\sigma-1}]} \left[\frac{\mathcal{I}}{\sigma} \frac{\underline{z}}{f^c} - (M^{(0)} + ... + M^{(k-1)}) \int_{\underline{z}}^{\infty} z^{\sigma-1} d\mu^0 - \int z^{\sigma-1} d\mu^I \right] \tag{30}$$

where we used 28 to substitute in $\frac{\underline{z}}{f^c}$.
This shows that $\{M^{(k)}\}$ is a decreasing sequence. Since $M^{(k)} \geq 0$ holds $\forall k$, $\{M^{(k)}\}$ has a real limit point, and this limit point must be 0. Hence, taking the limit as $k \to \infty$, we obtain:

$$0 = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}}{f^c} - \sum_{i=0}^{\infty} M^{(i)} \int_{\underline{z}}^{\infty} z^{\sigma-1} d\mu^0 - \int z^{\sigma-1} d\mu^I \tag{31}$$

$$\sum_{i=0}^{\infty} M^{(i)} = E = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} d\mu^0} - \frac{\int z^{\sigma-1} d\mu^I}{\int_{\underline{z}}^{\infty} z^{\sigma-1} d\mu^0} \tag{32}$$

which is clearly the same entry mass as in the simultaneous entry game (see 6).

Equivalence of measure μ

Assuming a bounded μ^0, it is clear that $\mu^{(k)} := \sum_{i=0}^{k} M^{(i)} \int_{\underline{z}}^{\infty} 1 d\mu^0$ converges to $\mu^E := E \int_{\underline{z}}^{\infty} 1 d\mu^0$ as $k \to \infty$, given that the partial sums of the entry mass series are a Cauchy sequence.
The limit measure $\mu = \mu^I + E \int_{\underline{z}}^{\infty} 1 d\mu^0$ is identical to the measure of active firms in the simultaneous entry game equilibrium. ∎

Proof of Lemma 1. We note that the existence of the incumbent measure μ^I does not matter for the determination of the cutoff. To see this, let

$$
\begin{aligned}
f^e &= \mathbb{E}_0[\max\{\pi(z,\mu^P),0\}] && (33)\\
&= \int_{\{z\geq \underline{z}\}} \frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{E\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\,\mathrm{d}\mu_0 + \int_{\{z\geq \underline{z}\}} z^{\sigma-1}\,\mathrm{d}\mu^I} - f^c\,\mathrm{d}\mu^0 && (34)\\
&= \frac{\mathcal{I}}{\sigma} \frac{\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\,\mathrm{d}\mu^0}{E\int_{\{z\geq \underline{z}\}} z^{\sigma-1}\,\mathrm{d}\mu_0 + \int_{\{z\geq \underline{z}\}} z^{\sigma-1}\,\mathrm{d}\mu^I} - f^c\mu^0(\{z\geq \underline{z}\}) && (35)\\
&= \frac{\mathcal{I}}{\sigma} \frac{Z(\underline{z},\mu^0)}{EZ(\underline{z},\mu^0)+Z(\underline{z},\mu^I)} - f^c\mu^0(z\geq \underline{z}) && (36)\\
&= \frac{\mathcal{I}}{\sigma} \frac{Z(\underline{z},\mu^0)}{\left(\frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}^{\sigma-1}}{Z(\underline{z},\mu^0)} - \frac{Z(\underline{z},\mu^I)}{Z(\underline{z},\mu^0)}\right)Z(\underline{z},\mu^0)+Z(\underline{z},\mu^I)} - f^c\mu^0(z\geq \underline{z}) && (37)
\end{aligned}
$$

Here, we see that $Z(\underline{z},\mu^I)$ cancels, and using the fact that $Z(\underline{z},\mu^0) = \mathbb{E}_0[z^{\sigma-1} \mid z\geq \underline{z}]\mu^0(z\geq \underline{z})$ we obtain

$$
\frac{f^e}{f^c} = \mu^0(z\geq \underline{z})\left(\frac{\mathbb{E}_0[z^{\sigma-1}\mid z\geq \underline{z}]}{\underline{z}^{\sigma-1}} - 1\right). \tag{38}
$$

■

Proof of Proposition 1. Start by noticing that

$$
\begin{aligned}
\int z^{\sigma-1}\,\mathrm{d}\mu_3 &= \int z^{\sigma-1}\,\mathrm{d}\big(E_1\mu^0\mathbf{1}_{\{z\geq \underline{z}_2\}} + E_3\mu^0\mathbf{1}_{\{z\geq \underline{z}_3\}}\big) && (39)\\
&= E_1Z(\underline{z}_2,\mu^0) + E_3Z(\underline{z}_3,\mu^0) && (40)\\
&= E_1Z(\underline{z}_2,\mu^0) + E_1\left(1-\frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)Z(\underline{z}_3,\mu^0) && (41)\\
&= E_1Z(\underline{z}_1,\mu^0) && (42)\\
&= \int z^{\sigma-1}\,\mathrm{d}\mu_1, && (43)
\end{aligned}
$$

Therefore, the factor $\int z^{\sigma-1}\,\mathrm{d}\mu_t$ remains unchanged in before and after the cycle. Next, consider some $a \geq \underline{z}_2 > \underline{z}_1$, then

$$\frac{\mu_3((a,\infty])}{\mu_3(\Omega)} = \frac{E_1\mu^0(a,\infty] + E_3\mu^0(a,\infty]}{\mu_3(\Omega)} \tag{44}$$

$$= \frac{E_1\mu^0(a,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(a,\infty]}{\mu_3(\Omega)} \tag{45}$$

$$= \frac{E_1\mu^0(a,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(a,\infty]}{E_1\mu^0(\underline{z}_2,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,\infty]} \tag{46}$$

$$> \frac{E_1\mu^0(a,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(a,\infty]}{E_1\mu^0(\underline{z}_1,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,\infty]} \tag{47}$$

$$= \frac{\mu^0(a,\infty]}{\mu^0(\underline{z}_1,\infty]} \tag{48}$$

$$= \frac{\mu_1(a,\infty]}{\mu_1(\Omega)}. \tag{49}$$

Likewise, for $\underline{z}_1 \leq a < \underline{z}_2$

$$1 - \frac{\mu_3((a,\infty])}{\mu_3(\Omega)} = \frac{\mu_3((0,a])}{\mu_3(\Omega)} = \frac{E_1\mu^0(a,a] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,a]}{E_1\mu^0(\underline{z}_2,\infty] + E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,\infty]} \tag{50}$$

$$\leq \frac{E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,a]}{E_1\left(1 - \frac{Z(\underline{z}_2,\mu^0)}{Z(\underline{z}_1,\mu^0)}\right)\mu^0(\underline{z}_1,\infty]} \tag{51}$$

$$= \frac{\mu_1((0,a])}{\mu_1(\Omega)} = 1 - \frac{\mu_1((a,\infty])}{\mu_1(\Omega)} \tag{52}$$

so that again $\frac{\mu_3((a,\infty])}{\mu_3(\Omega)} \geq \frac{\mu_1((a,\infty])}{\mu_1(\Omega)}$. Thus, the probability measure $\frac{\mu_3}{\mu_3(\Omega)}(\cdot)$ strictly first order dominates the probability measure $\frac{\mu_1}{\mu_1(\Omega)}(\cdot)$. Therefore their respective expectations, which can be calculated as follows, are related by

$$\int \frac{\mu_1}{\mu_1(\Omega)}[a,\infty)\ \mathrm{d}a < \int \frac{\mu_3}{\mu_3(\Omega)}[a,\infty)\ \mathrm{d}a.$$

However, this is equivalent to

$$\frac{\int \mu_1[a,\infty)\ \mathrm{d}a}{\mu_1(\Omega)} = \frac{\int z^{\sigma-1}\ \mathrm{d}\mu_1}{\mu_1(\Omega)} < \frac{\int z^{\sigma-1}\ \mathrm{d}\mu_3}{\mu_3(\Omega)} = \frac{\int \mu_3[a,\infty)\ \mathrm{d}a}{\mu_3(\Omega)},$$

and combined with the first part of this proof, we obtain $\mu_1(\Omega) > \mu_3(\Omega)$. ■

Proof Proposition 2. Note first that in the absence of entry fixed cost, f^e, Equation 15 determines E^{ss} directly: There is no additional labor freed up from lower entry expenses, hence the entry process terminates immediately and jumps to E^{ss}. Rewrite Equation 15 now as

$$E_3 = \frac{\mathcal{I}_3}{\sigma f^c} \frac{\underline{z}}{Z(\underline{z}, \mu^0)} - E_1 \frac{Z(\underline{z}, \mu^I)}{Z(\underline{z}, \mu^0)} \tag{53}$$

$$= \frac{\mathcal{I}_1}{\sigma f^c} \frac{\underline{z}}{Z(\underline{z}, \mu^0)} - E_1 \frac{Z(\underline{z}, \mu^I)}{Z(\underline{z}, \mu^0)} + (\mathcal{I}_3 - \mathcal{I}_1) \frac{1}{\sigma f^c} \frac{\underline{z}}{Z(\underline{z}, \mu^0)} \tag{54}$$

$$= E_1 \left(1 - \frac{Z(\underline{z}_2, \mu^0)}{Z(\underline{z}_1, \mu^0)}\right) + (\mathcal{I}_3^{GE} - \mathcal{I}_1^{GE}) \frac{1}{\sigma f^c} \frac{\underline{z}}{Z(\underline{z}, \mu^0)} \tag{55}$$

where $\mathcal{I}_t = R_t$ is steady-state GE income (i.e. without fixed cost of entry). Note that

$$(\mathcal{I}_3 - \mathcal{I}_1) = \frac{\sigma}{\sigma - 1} f^c (E_1 p_0(\underline{z}_1) - (E_1 p_0(\underline{z}_2) + E_3 p_0(\underline{z}_1))) \tag{56}$$

$$= \frac{\sigma}{\sigma - 1} f^c (M_1 - M_3). \tag{57}$$

Suppose that $(\mathcal{I}_3 - \mathcal{I}_1) \leq 0$, then $M_3 \geq M_1$. Since $E_3 \leq E_1 \left(1 - \frac{Z(\underline{z}_2, \mu^0)}{Z(\underline{z}_1, \mu^0)}\right)$, following the steps of the proof of Proposition 1, we know that $\int z^{\sigma-1} \, \mathrm{d}\mu_3 \leq \int z^{\sigma-1} \, \mathrm{d}\mu_1$. But then, it also holds that $\int z^{\sigma-1} \, \mathrm{d}\frac{\mu_3}{M_3} \leq \int z^{\sigma-1} \, \mathrm{d}\frac{\mu_1}{M_1}$, i.e. the average productivity of a firm active in the market is lower post cycle. By construction of the exit and entry process (clipping the left tail of the productivity distribution in period 2 and allowing entry of average firms in period 3) this cannot be. Hence, $(\mathcal{I}_3 - \mathcal{I}_1) > 0$ and thus

$$M_3 < M_1, \text{ and } \int z^{\sigma-1} \, \mathrm{d}\mu_3 > \int z^{\sigma-1} \, \mathrm{d}\mu_1. \tag{58}$$

Consequently, $L_3^d > L_1^d$ holds, too. The output ratio between pre- and post crisis steady-state depending on q is then given by

$$\frac{Y_3}{Y_1}(q) = \left[\frac{M_3}{M_1}\right]^{q - \frac{1}{\sigma-1}} \left[\frac{L_3^d}{L_1^d}\right] \frac{(\int z^{\sigma-1} \, \mathrm{d}\mu_3)^{1/(\sigma-1)}}{(\int z^{\sigma-1} \, \mathrm{d}\mu_1)^{1/(\sigma-1)}}, \tag{59}$$

and none of the equilibrium quantities on the right-hand-side depend on q. By inspection, it follows that $\frac{Y_3}{Y_1}\left(\frac{1}{\sigma-1}\right) > 1$ for the CES case, and there exists a unique $q^* > \frac{1}{\sigma-1}$ for which $\frac{Y_3}{Y_1}(q^*) = 1$. ■

Proof of Proposition 3 and Figure 2. We prove the characteristics of Figure 2, panels A and B, first. To this end, note that $\underline{z}_2$ is a strictly increasing, monotonic function of f_h^c defined by the zero profit condition (4), with $\underline{z}_2 \to \infty$ as $f_h^c \to \infty$, and any analysis in terms of $\underline{z}_2$ carries over in terms of f_h^c. Now note that

$$M_3 = E_1 p_0(\underline{z}_2) + E_3 p_0(\underline{z}_1) \text{ and}$$

$$E_3 = \left(1 + \frac{\underline{z}_1}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}_1](\sigma - 1)}\right)^{-1} E_1 \left[1 - \frac{Z(\underline{z}_2, \mu^0)}{Z(\underline{z}_1, \mu^0)} + (p_0(\underline{z}_1) - p_0(\underline{z}_2)) \frac{\underline{z}_1}{Z(\underline{z}_1, \mu^0)(\sigma - 1)}\right] \tag{60}$$

which follows from substituting eq. (57) into eq. (55) and rearranging for E_3. Using eq. (60), we calculate

$$\frac{\partial}{\partial \underline{z}_2} M_3 = E_1 f(\underline{z}_2) \left(\frac{\frac{\underline{z}_2 + \frac{1}{\sigma-1}\underline{z}_1}{\mathbb{E}_0[z^{\sigma-1}|z \geq \underline{z}_1]}}{1 + \frac{1}{\sigma-1}\frac{\underline{z}_1}{\mathbb{E}_0[z^{\sigma-1}|z \geq \underline{z}_1]}} - 1 \right). \tag{61}$$

If $\underline{z}_2 = \underline{z}_1$, one verifies that the term in parentheses is negative, while for large enough $\underline{z}_2$, it is positive. By continuity, there exists a $\underline{z}_2^*$, such that

$$\frac{\partial M_3}{\partial \underline{z}_2} \begin{cases} < 0 \text{ if } \underline{z}_2 < \underline{z}_2^* \\ = 0 \text{ if } \underline{z}_2 = \underline{z}_2^* \\ > 0 \text{ if } \underline{z}_2 > \underline{z}_2^*. \end{cases} \tag{62}$$

Now, partial derivatives are

$$\frac{\partial [M_3/M_1]^{q-q^{CES}}}{\partial \underline{z}_2} = (q - q^{CES}) \left[\frac{M_3}{M_1}\right]^{q-q^{CES}-1} \frac{1}{M_1}\frac{\partial M_3}{\partial \underline{z}_2}, \tag{63}$$

$$\frac{\partial (L_3^d/L_1^d)}{\partial \underline{z}_2} = \frac{-f_l^c}{L_1^d}\frac{\partial M_3}{\partial \underline{z}_2}, \tag{64}$$

$$\frac{\partial}{\partial \underline{z}_2}\left[\frac{\int z^{\sigma-1}\, \mathrm{d}\mu_3}{\int z^{\sigma-1}\, \mathrm{d}\mu_1}\right]^{1/(\sigma-1)} = \left[\frac{\int z^{\sigma-1}\, \mathrm{d}\mu_3}{\int z^{\sigma-1}\, \mathrm{d}\mu_1}\right]^{1/(\sigma-1)-1} \frac{1}{\sigma-1}\frac{\underline{z}_1}{Z(\underline{z}_1,\mu^0)}\frac{-1}{E_1(\sigma-1)}\frac{\partial M_3}{\partial \underline{z}_2}, \tag{65}$$

from which we see that eq. (64, 65) have the reverse sign order of (62), while eq. (63) has the same sign order, iff $q > q^{CES}$. Combined with the facts that each factor is 1 at f_l^c (no crisis in phase 2) and at $f_h^c \to \infty$, the graphs in panel A and B of Figure 2 follow. Furthermore, if $q \leq q^{CES}$, then the graph for q' in panel C follows, and $(Y_3/Y_1)(q, f_h^c) \geq 1$ for all $f_h^c \geq f_l^c$.

Assume now that $q > q^{CES}$. For reference, note that

$$\frac{\int z^{\sigma-1}\, \mathrm{d}\mu_1}{\int z^{\sigma-1}\, \mathrm{d}\mu_3} = 1 + \frac{M_1 - M_3}{E_1(\sigma-1)}\frac{\underline{z}_1}{Z(\underline{z}_1,\mu^0)} \tag{66}$$

is the ratio of productivity integrals. The output ratio only depends on f_h^c through $\underline{z}_2$ and it depends on $\underline{z}_2$ only through M_3. Consider the output ratio now as a function of M_3 given q, i.e. $\frac{Y_3}{Y_1}(M_3;q)$, and note that the domain of $M_3(\underline{z}_2)$ is $[M_{min}, M_1]$ for some M_{min}, and that M_3 runs from M_1 monotonically down to $M_{min} := M_3(\underline{z}_2^*)$ and back up to M_1 in the limit. We write

$$\frac{Y_3}{Y_1}(M_3;q) = F_1(M_3,q)F_2(M_3)F_3(M_3),$$

corresponding to its three contributing factors, and take the derivative

$$\frac{\partial}{\partial M_3}\frac{Y_3}{Y_1} = (F_1'(M_3,q)/F_1(M_3,q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3))\frac{Y_3}{Y_1}(M_3;q) \tag{67}$$

$$=: g(M_3,q)\frac{Y_3}{Y_1}(M_3;q). \tag{68}$$

The derivative vanishes iff (substituting in)

$$0 = (F_1'(M_3,q)/F_1(M_3,q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3)) \tag{69}$$

$$= \frac{(q - q^{CES})}{M_3} \underbrace{-\frac{f_l^c}{L_3^d(M_3)} - \frac{\int z^{\sigma-1}\,\mathrm{d}\mu_1}{\int z^{\sigma-1}\,\mathrm{d}\mu_3}\frac{1}{\sigma-1}\frac{\underline{z}_1}{Z(\underline{z}_1,\mu^0)}\frac{1}{E_1(\sigma-1)}}_{:=-s(M_3)/M_3} \tag{70}$$

$$= v(M_3,q), \tag{71}$$

which is strictly decreasing in M_3 (to see that the third summand is decreasing cf. eq. 66). Therefore, if M_3^* satisfying eq. (69) exists, it is unique. Since $M_{min} \leq M_3 \leq M_1$, it can only exist if $v(M_{min},q) \geq 0 \geq v(M_1,q)$. Clearly, if q is large enough, then v must be globally positive and if q gets close to q^{CES}, then v is negative for some, and eventually all $M_3 \in [M_{min}, M_1]$. One checks that an interior critical point exists iff $q \in (\underline{q}, \overline{q})$, where $\underline{q} = q^{CES} + s(M_1)$ and $\overline{q} = q^{CES} + s(M_{min})$. On one hand, if $q \leq \underline{q}$, even though there is LOV relative to CES, the output ratio is decreasing in the number of post crisis varieties and any crisis is cleansing. On the other hand, if $q \geq \overline{q}$, then the output ratio is globally increasing in the number of varieties and all crises come at an output loss. For the region in between, $q \in (\underline{q}, \overline{q})$, there is an extremum at some M_3^*. This is a maximum because Y_3/Y_1 satisfies second-order conditions at M_3^* for such q. See this as follows. As argued, $g(M_3;q)$ is decreasing, so $g' < 0$. The second derivative of Y_1/Y_3 evaluated at M_3^* is

$$g'(M_3^*;q)\frac{Y_3}{Y_1}(M_3^*;q) + g(M_3^*;q)\frac{Y_3}{Y_1}'(M_3^*;q) = g'(M_3^*;q)\frac{Y_3}{Y_1}(M_3^*;q) + g(M_3^*;q) \times 0 < 0. \tag{72}$$

Consequently, there is an optimal number of firms in phase 3, $M_3^* \in (M_{min}, M_1)$, created by an "optimally cleansing business cycle". Because the mapping between f_h^c and M_3 is parabola-shaped, its inverse image always has two elements, and any optimal post crisis number of firms can be created by two different values of f_h^c. This implies the graph displayed in panel C for q''. It is easy to check that the output ratio uniformly decreases in q, which explains why the graph for q''' lies below. Therefore, as the curve of Y_3/Y_1 shifts down, there must be an interval $(q_\circ, q^\circ) \subseteq (\underline{q}, \overline{q})$ where the output ratio crosses 1 multiple times, creating two disparate intervals in which crises are cleansing.

The partial equilibrium result follows immediately since F_2 and F_3 are independent of f_h^c and of q, by Proposition 1. ■

Proof Proposition 4. We start by characterizing the optimal policy in the case of entry and then study the one of pure exit.

Entry: The interior first-order condition on entry is

$$E^{SP}(\underline{z};\mu^I,q)=\frac{\bar{L}-If^c}{p_0(\underline{z})f^c+f^e}\frac{1}{\sigma}-\frac{Z(\underline{z},\mu^I)}{Z(\underline{z},\mu^0)}\frac{\sigma-1}{\sigma}+\Delta E^{SP}, \tag{73}$$

which equals the CES GE-entry plus a difference term, ΔE^{SP}. The latter is given by

$$\Delta E^{SP}=\left(q-\frac{1}{\sigma-1}\right)L^d\frac{\sigma}{\sigma-1}(p_0(\underline{z})f^c+f^e)^{-1}Q(\mu^I,\underline{z}), \tag{74}$$

where $Q(\mu^I,\underline{z})$ is the ratio between the average productivity of firms post-entry and the average productivity of (successful) entrants. It is immediate that if the household has CES preferences ($q=1/(\sigma-1)$, then $\Delta E^{SP}=0$, and the market outcome is constrained efficient. Over time, a stock of firms builds up (see discussion in Section 3.1), so $\sum_{t\geq 0}E_t^{SP}\to E^{SP,ss}$, where we call $E^{SP,ss}$ the (steady-state) total mass of entrants for the social planner problem. As E_t^{SP} tends to 0, Q tends to 1, and the incumbent mass building up is, eventually, $\widetilde{I}=I+E^{SP,ss}$. (Similarly, the measure of incumbents is stacked up.) Substituting these into eq. (73) and rearranging, we find

$$E^{SP,ss}=\frac{(\bar{L}-If^c)q}{f^cp_0(\underline{z})(1+q)+f^e}-\frac{Z(\underline{z},\mu^I)}{Z(\underline{z},\mu^0)}\frac{f^cp_0(\underline{z})+f^e}{f^cp_0(\underline{z})(1+q)+f^e}. \tag{75}$$

The socially optimal number of entrants is increasing in taste-for-varieties, q, and decreasing in the mass and productivity of active incumbents. Fixed cost of entry, f^e, stop to matter if and only if $q\to\infty$. Comparing this to eq. (15), it so follows that the general equilibrium steady-state mass converges to $E^{SP,ss}$ if and only if $q=\frac{1}{\sigma-1}$. It also follows that in an economy with high $q>\frac{1}{\sigma-1}$, the equilibrium number of entrants is too small compared to the short-run optimal number, so $E^{GE}<E^{SP}$. Conversely, for small q, the equilibrium number of varieties is larger than the short-run efficient number.

Using the first order condition on E^{SP}, the interior first order condition on $\underline{z}^{SP}=\underline{z}^{SP}(E;q,\mu^I)$ yields:

$$\begin{aligned}&(\bar{L}-If^c)Z(\underline{z},\mu^0)\left(\frac{f^e}{f^cp_0(\underline{z})+f^e}\frac{1}{\sigma}\frac{J}{K}-\left[1-\frac{\underline{z}^{\sigma-1}}{\mathbb{E}_0[z^{\sigma-1}\mid z\geq\underline{z}]}\right]\frac{1}{\sigma}\frac{\sigma-\frac{J}{K}}{\sigma-1}\right)\\&=Z(\underline{z},\mu^I)(f^cp_0(\underline{z})+f^e)\left(\frac{f^e}{f^cp_0(\underline{z})+f^e}\left(1-\frac{\sigma-1}{\sigma}\frac{1}{K}\right)+\frac{1}{\sigma}\frac{1}{K}\left[1-\frac{\underline{z}^{\sigma-1}}{\mathbb{E}_0[z^{\sigma-1}\mid z\geq\underline{z}]}\right]\right).\end{aligned} \tag{76}$$

The fraction J/K and J and K individually take value 1 precisely when $q=\frac{1}{\sigma-1}$. Only in this case, are the terms in the large parentheses both equivalent to $\dfrac{f^eZ(\underline{z},\mu^0)}{\delta_f}-(Z(\underline{z},\mu^0)-\underline{z}^{\sigma-1}p_0(\underline{z}))$. eq. (76) is then equivalent to eq. (7), the GE cutoff.

In general, we need to unpack J and K to see how the cutoff is determined in a non-CES world. Here, note that $J = 1 + [(\sigma - 1)q - 1]Q$ and $K = 1 + \frac{1}{\sigma}[(\sigma - 1)q - 1]Q$. If we approach a steady state (or drop the incumbents), $Q \to 1$. Then eq. (76) simplifies:

$$\frac{f^e}{f^c p_0(\underline{z}) + f^e}(\sigma - 1)q - \left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_0[z^{\sigma-1} \mid z \geq \underline{z}]}\right] = 0, \tag{77}$$

and under mild regularity conditions on μ^0, we can conclude that $\frac{\partial \underline{z}}{\partial q} < 0$. Intuitively, if the economy has a strong preference for varieties, the entry barriers for new establishments should be lower. Eq. (77) also holds with entrants, if all entrants were drawn from the same distribution, i.e. if $\mu^I \propto \mathbb{I}(z \geq \underline{z})\mu^0$.

Entry: Suppose incumbent firms are distributed according to $\mu^I = M_0\mu^0\mathbb{I}(z^{\sigma-1} \geq \underline{z}_0)$, and that fixed costs f^c are such that the optimal cutoff $\underline{z}$ is larger than $\underline{z}_0$, and that optimal entry, therefore, is nil. Hence, the relevant production function to maximize is

$$Y = (M_0)^q\, p_0(\underline{z})^{q - \frac{1}{\sigma-1}} \left(\bar{L} - f^c(M_0 p_0(\underline{z}))\right) Z(\underline{z}, \mu^0)^{\frac{1}{\sigma-1}}, \tag{78}$$

where taking first order conditions yields

$$f^c M_0 p_0(\underline{z}^{SP}) = \frac{\bar{L} - f^c M_0 p_0(\underline{z}^{SP})}{\sigma - 1}\left([q(\sigma - 1) - 1] + \frac{(\underline{z}^{SP})^{\sigma-1}}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}^{SP}]}\right), \tag{79}$$

which is equal to the market equilibrium outcome only if $q = \frac{1}{\sigma-1}$. For larger q, the cutoff is again optimally set lower than in the equilibrium allocation. ■

Proof of Proposition 5. In the steady-state targeted by the social planner, eq. (79) must hold once entry has subsided, i.e., with $E^{SP} = 0$ and some $\underline{z}^{SP}$. We now evaluate eq. (4) at this $\underline{z}^{SP}$, whereby we have swapped f^c for $\delta^c f^c$ beforehand. Solving for δ^c yields the result. ■

C Ramsey Planner

To further motivate working with the short run social planner, we take a look at an economy starting with no incumbents through the eyes of a Ramsey planner. She takes a (household) discount factor β and utility function u as given, and maximizes $\sum_{t\geq 0}\beta^t u(Y_t)$ under the constraint that $Y_t = Y(M, E)$. The term $Y(M, E)$ describes total output if a measure of $M\mu^0\mathbf{I}(z^{\sigma-1} \geq \underline{z})$ former entrants, now incumbents, are present and a mass of E entrepreneurs newly flows into the market. She splits labor between new entrants and production, and allocates production labor to each variety. Since the split of production labor remains efficient in equilibrium even in presence of the aggregate externality (the proof in Dhingra and Morrow (2019) still applies), it suffices to consider a planner who chooses E_t and $\underline{z}_t$ to steer the entire economy while respecting the resource constraints. For tractability, we consider a social planner who takes the cutoff as given and constant. Taking the cutoff to be a parameter allows us to analyze the recursive formulation of social planner problem given by

$$V(M;\underline{z}) = \max_{E\geq 0} u\big(Y(M,E)\big) + \beta V(M+E;\underline{z}), \text{ where} \tag{80}$$

$$Y(M,E) = L^d(E) Z(\underline{z},\mu^0)^{\frac{1}{\sigma-1}} (M+E)^q p_0(\underline{z})^{q-\frac{1}{\sigma-1}} \tag{81}$$

and $L^d = \bar{L} - (p_0(\underline{z}) f^c + f^e)E - (M p_0(\underline{z})) f^c \geq 0$ is production labor.

We differentiate with respect to E, use the envelope theorem and impose that in the steady state, $E = E' = 0$ holds. The steady state cumulative sum of entrants is then given by Equation 23.