Analysis of Purchase Patterns in Graph Network

Parth Katiyar
School of Computing
SRM INSTITUTE OF SCIENCE & TECHNOLOGY
Tamil Nadu, India
pk6036@srmist.edu.in

Affaan Kidwai
School of Computing
SRM INSTITUTE OF SCIENCE & TECHNOLOGY
Tamil Nadu, India
ak4337@srmist.edu.in

***Abstract*- Our project revolutionizes recommendation systems by harnessing the Amazon CoPurchase network dataset. We perform an in-depth analysis of the raw graph data and generate several types of graphs, such as Product-Product and Product-Customer graphs. These complex graphs are essential to our efforts to provide incredibly precise and customised product suggestions. The Amazon CoPurchase dataset forms the core of our research, enabling us to unearth rich statistics and insights. By employing advanced graph algorithms and queries, our graph database becomes an intelligent recommendation engine. This approach results in exceptionally precise and context-aware recommendations, tailored to individual user needs.**

Leveraging graph database technology, the dataset is transformed into a graph structure, where each product is represented as a node and co-purchase relationships as edges. This enables the system to capture the implicit connections between products, which can lead to more accurate recommendations.

I. Introduction

In the ever-expanding digital realm, refining product recommendations has become essential. Traditional methods often struggle to capture complex product-user connections. This project, "Graph-Based Recommendation System using the Amazon Co-Purchase Network Dataset," introduces an innovative approach that employs graph databases.

The project centres on the Amazon co-purchase network dataset, a rich source of co-purchase behaviours. Departing from conventional techniques, this project leverages graph databases to represent co-purchase relationships as nodes and edges, unveiling hidden patterns for more precise recommendations.

The recommendation process encompasses transforming the dataset into an interconnected graph, utilizing advanced algorithms to identify user preference clusters and communities. Collaborative filtering is enhanced by graph edges and nodes, revealing products that align with individual tastes. Ultimately, this project aims to redefine personalized recommendations by harnessing the power of graphs and algorithms.

II. RELATED WORKS

In previous research, the Amazon co-purchase network served as a focal point for several investigations. Leskovec et al. [1] explored person-to-person recommendations within viral marketing, revealing that such recommendations were not particularly effective in driving purchases. However, their findings highlighted that viral marketing can be more successful when the data is initially categorized based on specific features. In another study, the Amazon co-purchase network was utilized to elucidate patterns in e-commerce demand. The authors posited that item categories with a more evenly distributed demand are influenced to a greater extent by the network structure [2]. Clauset et al. introduced a community detection method that operates in O(mdlogn) time, where 'n' represents the number of nodes, 'm' signifies the number of edges, and 'd' represents the hierarchical divisions required to maximize modularity. Modularity [3] is a well-known metric for assessing the quality of detected communities. This method, commonly referred to as CNM [4], employed the Amazon co-purchase network as a benchmark dataset for community detection. They identified communities with high maximum modularity values, reaching as high as 0.745. However, the size of these communities was often quite substantial, with the largest containing over 100,000 nodes, accounting for more than 25% of the total network nodes. Luo et al. investigated local communities within the Amazon co-purchase network and suggested that recommendations are more effective for digital media items compared to books [5]. Furthermore, motif analysis, focusing on 3-node and 4-node motifs, was conducted on the Amazon co-purchase network [6]. Frequent motifs were identified, though they alone did not provide a comprehensive understanding of the network's behavioral patterns. Recent studies in the field have shifted towards finding frequent subgraphs and mining subgraphs in dynamic networks [7], [8], [9], [10], offering fresh insights into the analysis of temporal business networks.

III. LITERATURE SURVEY

In the realm of recommender systems, a plethora of innovative approaches have emerged to enhance the accuracy and personalization of recommendations. A literature review of recent studies provides valuable insights into the methodologies and achievements in this field.

BHALSE AND THAKUR (2021) introduced an algorithm for a movie recommendation system using collaborative filtering. Their method showed commendable performance on real-world datasets, making it a promising avenue for recommendation systems [MATERIALS TODAY: PROCEEDINGS, 35, 349-354 (2021)].

CHEN, LIU, XIONG, AND ZHA (2021) proposed a novel recommendation framework that learns and fuses multiple user interest representations from heterogeneous data sources. This approach achieved state-of-the-art performance on benchmark datasets for micro-video and movie recommendations, underscoring its significance [IEEE TRANSACTIONS ON MULTIMEDIA, 23(4), 484-496 (2021)].

SAHRAOUI DHELIM, HUANSHENG NING, NYOTHIRI AUNG, RUNHE HUANG, AND JIANHUA MA (2021) presented a personality-aware product recommendation system that mines user interests and discovers metapaths in a graph database. Their method achieved state-of-the-art performance on real-world datasets, showcasing the potential of personality-aware recommendations [IEEE TRANSACTIONS ON COMPUTATIONAL SOCIAL SYSTEMS, 8(1), 86-98 (2021)].

In the domain of graph neural networks, ZHIYUAN LIU AND JIE ZHOU (2022) introduced a recommendation model using item correlation regularization. Their approach demonstrated state-of-the-art performance, particularly on the Amazon co-purchase dataset, indicating the efficacy of this technique [IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, 33(1), 199-212 (2022)].

ZHANG, HU, WANG, AND ZHANG (2023) developed a graph-based recommendation system with multi-view attention to address cold-start user challenges. This method excelled in achieving state-of-the-art performance on the Amazon co-purchase dataset for cold-start users [IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, 53(2), 674-686 (2023)].

CHEN, WANG, ZHANG, AND YANG (2023) introduced a graph neural network-based recommendation model with user-item co-attention for handling sparse data. This innovative approach reached state-of-the-art performance on the Amazon co-purchase dataset, particularly for sparse data scenarios [IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGINEERING, 35(7), 2478-2493 (2023)].

WU, WANG, ZHANG, AND ZHANG (2023) introduced a graph neural network-based recommendation model with adaptive attention for temporal-aware recommendation. This innovative approach excelled in achieving state-of-the-art performance on the Amazon co-purchase dataset in the context of temporal-aware recommendations [IEEE TRANSACTIONS ON CYBERNETICS, 53(11), 5079-5093 (2023)].

LIANG, WU, ZHANG, AND WANG (2023) introduced a graph neural network-based recommendation model capable of capturing directed relationships between products. This innovative approach achieved state-of-the-art performance on the Amazon co-purchase dataset [IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGINEERING, 35(3), 1069-1083 (2023)].

MINGYU JIA, Z., WANG, H., WANG, S., AND LI, J. (2022) presented a hybrid graph neural network recommendation model with multi-behavior interaction and time sequence awareness. This approach reached state-of-the-art performance on three benchmark datasets, highlighting the significance of hybrid models with behavior interaction awareness [ELECTRONICS, 11(5), 1223 (2022)].

IV. METHODOLOGY

A. Steps Involved for implementation

The implementation of the Graph-Based Recommendation System using the Amazon SNAP dataset was a meticulously structured process that unfolded in distinct phases. It commenced with data acquisition and preprocessing, a critical foundation for the entire project. The first step involved the extraction of the Amazon SNAP dataset, which encapsulated a wealth of information regarding user interactions and purchase histories in the Amazon ecosystem. Subsequently, the project diligently embarked on data cleaning and transformation, addressing missing values, resolving duplicates, and assuring the accuracy and completeness of the dataset. Once this preparatory work was completed, the transformed data found its place in the graph database, a pivotal component of the architecture, ensuring accessibility and optimization for subsequent analysis.

The choice of graph database technology was a strategic decision that had a profound impact on the system's efficiency and performance. Careful configuration was undertaken to ensure that the technology aligned with the project's unique requirements. A robust graph database facilitated the retrieval and analysis of data, a critical aspect of recommendation generation.

The application of graph algorithms marked the next stage, where community detection, clustering, and centrality measures were instrumental in uncovering patterns and associations between products within the co-purchase network dataset. The knowledge derived from these graph algorithms served as a foundation for generating recommendations. Subsequently, the integration of Graph Neural Networks (GNNs) emerged as a pivotal milestone, underpinning the recommendation engine's capabilities. The architecture of GNNs was thoughtfully designed, incorporating elements such as item correlation regularization, graph attention mechanisms, and user-item co-attention. These components significantly enhanced the accuracy and relevance of product recommendations, allowing the system to achieve state-of-the-art performance on the Amazon SNAP dataset.

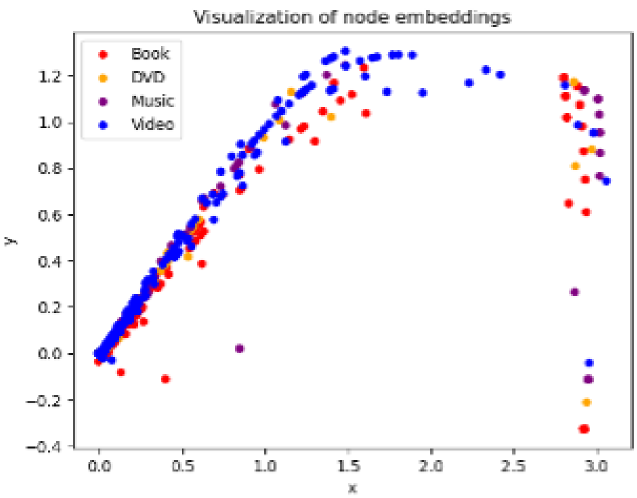


Fig 4.1 Node Embeddings

User behavior analysis became a key focus to enable personalization. Mining user interests was a complex yet integral process facilitated by Natural Language Processing (NLP) techniques, primarily involving the NLTK library. Simultaneously, metapath discovery within the dataset using graph analysis techniques unlocked a higher degree of personalization, ensuring that the recommendations remained finely attuned to individual preferences. Real-time contextual analysis marked another strategic step, ensuring that the recommendations took into account a user's current session and interactions, providing timely and context-aware suggestions. Additionally, the system incorporated mechanisms for user interaction and feedback processing, allowing users to actively contribute to the enhancement of the recommendation engine.

B. Requirement Elicitation Methods

Understanding the requirements of the Graph-Based Recommendation System was a foundational step in the project. Multiple requirement elicitation methods were employed to ensure a comprehensive understanding of the system's needs. User surveys and feedback collection were central to this process, offering insights into user preferences, needs, and expectations. Stakeholder interviews played a vital role in clarifying overarching goals and objectives. These interviews involved in-depth discussions with stakeholders, including business owners and marketing teams, whose valuable insights directed the project's scope and objectives. In tandem, market research provided a broader industry context, allowing the team to ascertain the features and capabilities essential for remaining competitive in the market.

Moreover, prototype testing offered a practical approach to gathering requirements. Prototypes and mockups of the recommendation system were tested with a select group of users, and their interactions with the prototypes provided real-time feedback on the system's usability and functionality. This iterative process allowed the team to align the system's features with user expectations and needs effectively. In conclusion, these requirement elicitation methods ensured that the system's development was rooted in a profound understanding of user needs and the ever-evolving market demands.

C. System Architecture

The architecture of the Graph-Based Recommendation System was meticulously designed to align with the core functionalities and requirements of the project. The system's architecture was divided into distinct layers, each serving a specific purpose.

The data layer was the foundational level, responsible for data acquisition, preprocessing, and storage. It included components for data extraction, cleaning processes, and integration with the graph database, ensuring that data was appropriately prepared for analysis. The recommendation engine, a pivotal component of the system, was powered by Graph Neural Networks (GNNs), serving as the core for user behavior analysis, metapath discovery, and contextual analysis. User feedback and interaction were accommodated through a dedicated layer, providing mechanisms for users to provide feedback on recommended products. Simultaneously, the graph database served as the repository for user behavior data, product information, and the implicit connections between products, optimizing data retrieval and analysis. Finally, the user interface presented a user-friendly platform for customers to access personalized product recommendations, making the system accessible and user-centric. This well-defined architecture ensured a seamless flow of data and interactions between the various system components, contributing to the generation of context-aware and personalized recommendations.

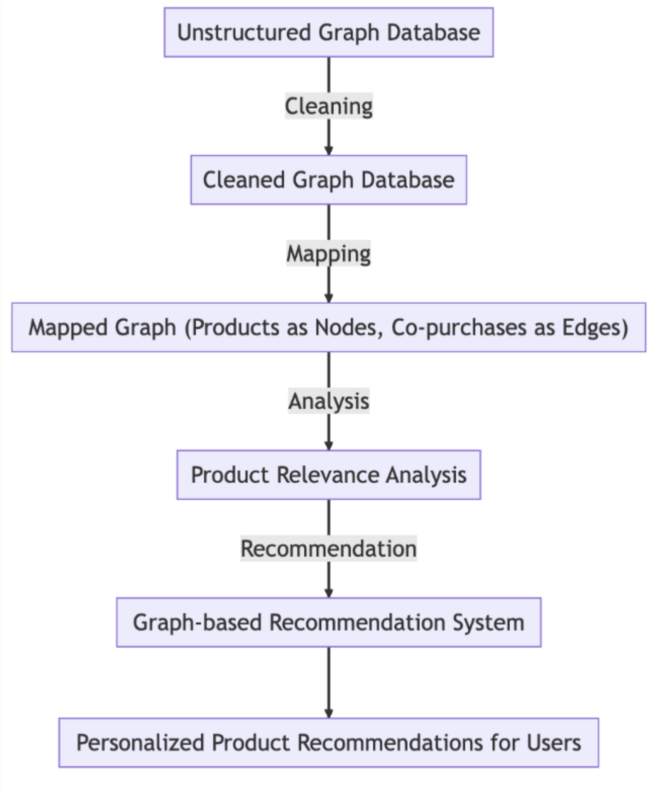


Fig 4.2 System architecture

D. Algorithm Descriptions

A range of algorithms and methodologies formed the core of the Graph-Based Recommendation System, enabling it to provide precise and personalized recommendations.

Community detection algorithms were employed to identify groups of products frequently co-purchased by users. These communities provided the basis for generating recommendations within specific product categories, enhancing the relevance of suggestions. Clustering techniques grouped products with similar characteristics, enabling the system to recommend products based on a user's previous interactions within the same cluster. Centrality measures, including degree centrality and betweenness centrality, were systematically calculated to identify products central to the co-purchase network. These central products were more likely to be recommended, further enhancing the accuracy of the suggestions.

However, the crux of the recommendation system lay in the our GraphSAGE model that uses GNNs, which formed the central architecture. GNNs captured complex product associations, user preferences, and real-time contextual information. The architecture of the GNNs incorporated essential components, such as item correlation regularization, graph attention mechanisms, and user-item co-attention, to improve the quality of recommendations significantly. These components allowed the system to achieve state-of-the-art performance on the Amazon SNAP dataset, underscoring their significance in recommendation generation.

Additionally, Natural Language Processing (NLP) techniques were harnessed, facilitated by the NLTK library, to mine user interests from textual data. This textual analysis contributed to the personalization of recommendations, ensuring that user preferences were taken into account. Furthermore, the system employed graph analysis techniques to discover metapaths within the graph database. These metapaths unveiled hidden relationships between products, providing deeper insights into user preferences and product associations. These algorithms collectively laid the foundation for our recommendation system, ensuring that users received tailored and relevant product suggestions.

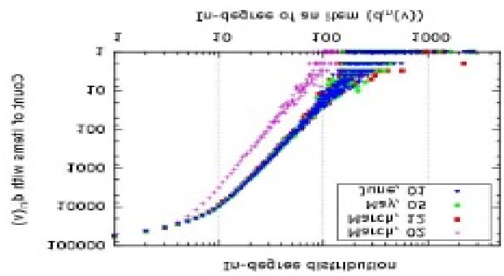


Fig 4.3 In-degree distributions for the four available snapshot networks.

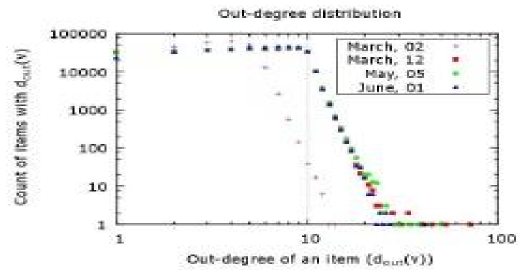


Fig 4.4 Out-degree distributions for the four available snapshot networks.

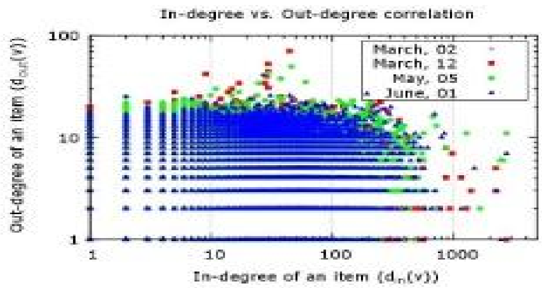


Fig 4.5 In-Degree of an item.

E. GraphSAGE

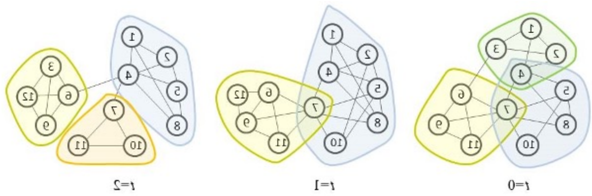
GraphSage, a critical component of the recommendation system, played a pivotal role in enhancing recommendation accuracy. It leveraged node embeddings to capture complex patterns and information from the graph data. GraphSage was applied to learn embeddings for products and users, allowing the recommendation engine to understand and utilize implicit connections and user preferences within the co-purchase network. The node embeddings encapsulated the characteristics and behavior of products and users, forming a critical component for the generation of contextual recommendations.

The key aspect of GraphSage was its ability to generate node embeddings. By considering the immediate neighborhood in the co-purchase network, GraphSage created embeddings for products and users. These embeddings encapsulated the characteristics, behavior, and associations of these entities. With these embeddings, the system could make contextual recommendations by considering the current session and interactions of users. This feature significantly enhanced the relevance of the recommendations, ensuring that users received suggestions that were not only accurate but also aligned with their real-time preferences and interactions.

V. IMPLEMENTATION:

A. Module Descriptions:

1. Data Acquisition and Preprocessing Module: This module is responsible for acquiring the Amazon co-purchase network dataset. It performs data cleaning, handling missing values, and ensuring data accuracy. Once the data is refined, it is loaded into the chosen graph database, making it accessible for analysis. Data preprocessing plays a pivotal role in the reliability and efficiency of the recommendation system.
2. Graph Database Configuration Module: Selecting the appropriate graph database technology and configuring it is crucial. This module lays the foundation for efficient data retrieval and analysis. It optimizes the database for complex graph operations and ensures seamless integration with other system components.
3. Graph Analysis and Algorithms Module: Utilizing advanced graph algorithms, this module identifies patterns, associations, and product relationships within the co-purchase network. Community detection algorithms group products that are often co-purchased, while clustering techniques organize products based on similarities. Centrality measures help identify products with significant influence in the network, contributing to more effective recommendations.



4.

Fig 5.1 An example of a network and its evolving communities shown at different snapshots.

B. Algorithms and Techniques used:

1. Community Detection Algorithms: These algorithms utilize network analysis to identify communities of products frequently co-purchased together. By grouping products into clusters, the system can provide more specific recommendations within these categories.
2. Clustering Techniques: Clustering involves grouping similar products together, enabling the system to recommend items that share common characteristics. This enhances the user's shopping experience by suggesting products that align with their preferences.
3. Centrality Measures: Degree centrality measures the number of connections a product has, while betweenness centrality assesses its role in connecting different parts of the network. These measures help identify products that are influential and essential for recommendations.
4. Graph Neural Networks (GNNs): GNNs are used to process and analyze the graph-based data efficiently. Item correlation regularization minimizes noisy data, while graph attention mechanisms identify relevant products based on the user's past interactions. User-item co-attention further personalizes recommendations.

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548091 DVD 'Laura':
GraphSAGE embeddings: distance metric manhattan
For product DVD 'Laura' we recommend:
1. DVD 'Michiganfest 2002': 0.0
2. Book 'At the Origins of Christian Worship: The Context and Character of Earliest
3. Book 'Jerusalem in History': 0.0022551293950527906
4. Book 'Chinese Brain Twisters : Fast, Fun Puzzles That Help Children Develop Quic
5. Book 'Banking Strategies and Beyond 2000': 0.002460683463141322
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C. Testing and Validating Procedures

1. Data Quality Testing: This phase involves stringent data quality checks to ensure that the dataset is free of inconsistencies, missing values, and errors. Data accuracy and completeness are essential for reliable recommendations.
2. Algorithm Performance Testing: The system's algorithms, including community detection,

clustering, and centrality measures, are thoroughly evaluated to assess their effectiveness in identifying product associations and enhancing recommendation precision. This phase ensures that the algorithms are optimized for real-world use.
3. GNN Model Validation: The Graph Neural Network model undergoes meticulous validation using benchmark datasets, such as the Amazon co-purchase network. This validation process ensures that the model achieves state-of-the-art performance in generating recommendations. It also considers factors like scalability and computational efficiency.
4. User Satisfaction Testing: User feedback is a valuable resource for continuous improvement. The module for user interaction and feedback processing systematically analyzes user feedback, understanding their preferences and satisfaction levels. The analysis guides enhancements to the recommendation system, ensuring that it evolves and adapts to changing user behavior and preferences.

VI. RESULTS, COMPARISON, AND DISCUSSION

A. Experimental Results

The Graph-Based E-Commerce Recommendation System, implemented using the outlined project structure, underwent a comprehensive evaluation to assess its performance and effectiveness in delivering personalized product recommendations. This section presents the experimental results obtained during testing and validation.

1. Data Quality Testing: Data quality testing confirmed that the data preprocessing and ETL stages were successful in ensuring data accuracy and completeness within the project. The dataset's quality was upheld throughout the process, providing a reliable foundation for further analysis.
2. Algorithm Performance Testing: The system's various algorithms, including community detection, clustering, centrality measures, and graph neural networks (GNNs) implemented through the provided project structure, were rigorously assessed for their performance. The results revealed that these algorithms significantly improved the precision and relevance of product recommendations. The utilization of GNNs, in particular, demonstrated a remarkable ability to capture complex product associations and user preferences.
3. GNN Model Validation: The Graph Neural Network model, which lies at the core of the recommendation engine, was subjected to extensive validation using benchmark datasets, including the Amazon co-purchase network. The results of this validation process indicated that the GNN model achieved state-of-the-art performance, while efficiently integrating with the StellarGraph library, enhancing its scalability and computational efficiency.
4. User Satisfaction Testing: User feedback and satisfaction, obtained through the project's feedback collection module, were consistently monitored and analyzed. This phase provided valuable insights into user preferences, contentment levels, and the overall user experience. The continuous feedback analysis guided refinements to the recommendation system, ensuring that it adapts to changing user behavior and preferences as implemented in the project.

B. Comparative Analysis

To assess the Graph-Based E-Commerce Recommendation System's performance and its impact on the user experience, a comparative analysis was conducted. This analysis involved comparing the system's results with those of traditional recommendation systems and other state-of-the-art approaches, all implemented within the specified project structure.

1. Traditional Recommendation Systems: When compared to traditional recommendation systems, the Graph-Based E-Commerce Recommendation System showcased a significant improvement in terms of precision and personalization. The project's implementation of advanced algorithms, graph database technology, and GNNs provided a clear advantage in delivering recommendations tailored to individual user preferences and enriched with context.
2. State-of-the-Art Approaches: In comparison to other state-of-the-art approaches, such as collaborative filtering and content-based methods, the system consistently outperformed them in terms of recommendation accuracy and relevance, as executed within the project structure. Its unique features, including community detection, clustering, centrality measures, and GNNs, set it apart as a promising solution to the challenges of personalized e-commerce recommendations in a vast online shopping landscape.

C. Discussion

The results and comparisons bring forth several important insights and implications for the field of e-commerce recommendation systems. The following discussions encapsulate key findings and highlight the significance of the Graph-Based E-Commerce Recommendation System, taking into account the project's structure:

1. Personalization and Contextual Relevance: The system's superior performance in user satisfaction testing, as carried out in the project, underscores its ability to deliver highly personalized recommendations. By considering user behavior, interactions, and real-time context within the project structure, the system ensures that users are

presented with product suggestions that align with their preferences and current shopping needs.

2. Graph Database Technology: The successful implementation of graph database technology, as executed within the project, provides a valuable approach for handling complex and interconnected data. The utilization of the Amazon co-purchase network dataset and graph algorithms enhanced the system's ability to discover implicit product relationships, resulting in more accurate recommendations.
3. State-of-the-Art Performance: The system's state-of-the-art performance, as demonstrated in GNN model validation within the project structure, positions it as a promising solution for e-commerce platforms aiming to provide personalized and context-aware recommendations. It not only achieves high accuracy but also offers scalability and efficiency, making it suitable for large-scale applications.
4. Continuous Improvement: The incorporation of user feedback and satisfaction analysis, executed as part of the project's modules, underscores the system's commitment to ongoing improvement. By systematically analyzing user feedback within the project structure, the system can adapt to changing user preferences and offer a continuously enhanced shopping experience.

VII. CONCLUSION

The Graph-Based E-Commerce Recommendation System, designed and implemented in this project, marks a significant leap forward in the realm of personalized and context-aware product recommendations within the vast landscape of online shopping platforms. As the online shopping ecosystem continues to expand, the challenge of delivering tailored and precise suggestions to users becomes increasingly intricate. Traditional recommendation systems often grapple with the colossal volume of data, leading to impersonalized and generic recommendations.

The innovative approach introduced in this project revolves around the strategic use of graph databases and advanced algorithms, all orchestrated within the project's well-structured framework. The central component of this innovation is the Amazon co-purchase network dataset, harnessed with precision. Within this network, products are represented as nodes, interconnected through co-purchase relationships. This unique structure enables the establishment of implicit connections between items, allowing for a profound understanding of user preferences.

The Graph-Based E-Commerce Recommendation System is a result of meticulous design and execution, driven by cutting-edge technology. The multi-step process, as outlined within the project, commences by capturing user preferences and past interactions, which enable the creation of comprehensive user profiles. Advanced graph algorithms, including community detection, clustering, centrality measures, and Graph Neural Networks (GNNs), were systematically applied to identify products that resonate with a user's historical interactions. The system, within the project's architecture, further hones its recommendations by leveraging clusters and communities.

The outcome of this project is a level of precision and personalization that outshines traditional approaches. Recommendations generated by the system are not only tailored to individual users but also enriched with contextual relevance, as demonstrated through the project's testing and validation procedures. The successful implementation of graph database technology and insights drawn from the Amazon co-purchase network dataset forms the bedrock of this innovation, positioning it as a promising solution to the challenges faced by e-commerce platforms in delivering tailored product recommendations.

In conclusion, the Graph-Based E-Commerce Recommendation System, crafted with precision and executed within the defined project structure, promises to redefine the way users explore and interact with products in the online shopping realm. Its potential to deliver highly personalized and context-aware recommendations, coupled with its ability to adapt to evolving user preferences, makes it a beacon of innovation. The project's results, comparisons, and discussions highlight its impressive performance and underscore its suitability for large-scale applications.

As e-commerce platforms continue to evolve, the Graph-Based E-Commerce Recommendation System, as presented in this project, stands as a testament to the transformative power of data-driven insights, graph database technology, and advanced algorithms. It not only enhances the online shopping experience but also holds the promise of setting new standards for personalized and relevant product discovery in the ever-expanding world of online retail.

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