Problem Set 03: Propositional Logic and Equivalences

CS/MATH 113 Discrete Mathematics

Spring 2024

1. Show that the following are logically equivalent without using truth tables:
   (a) $(p \Rightarrow r) \vee (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

   Solution:
   $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional disjunction
   $(\neg p \vee \neg q) \vee (r \vee r)$ Associative law
   $\neg(p \wedge q) \vee r$ Idempotent and demorgan's law
   $(p \wedge q) \rightarrow r$ conditional disjunction

   (b) $\neg\left[\neg\left[(p \vee q) \wedge r\right] \vee \neg q\right] \equiv q \wedge r$

   Solution:
   $\neg\left[\neg(p \vee q) \vee \neg r \vee \neg q\right]$ Demorgan's law
   $(p \vee q) \wedge r \wedge q$ Demorgan's law
   $[q \wedge (p \vee q)] \wedge r$ Associative law
   $[(q \wedge p) \vee (q \wedge q)] \wedge r$ Distributive law
   $[q \vee (q \wedge p)] \wedge r$ Idempotent law
   $q \wedge r$ Absorption law

   (c) $(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r) \equiv p \vee [r \wedge (t \vee \neg q)]$

   Solution:

2. Use Truth tables to see if the following statements are logically equivalent:
   (a) $p \Rightarrow (q \vee r) \equiv (q \Rightarrow p) \wedge (p \Rightarrow r)$

   Solution:

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$q \rightarrow p$	$p \rightarrow r$	$(q \rightarrow p) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	T	T	F	T	F
F	T	F	T	T	F	T	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

since the truth value of the statements donot match, they are not equivalent.

(b) $(p \vee q) \Rightarrow r \equiv (p \Rightarrow r) \wedge (q \Rightarrow r)$

Solution:

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Since the truth values for both Propositional statements are same, they are logically equivalent.

(c) $p \Rightarrow (q \vee r) \equiv \neg r \Rightarrow (p \Rightarrow q)$

Solution:

p	q	r	$q \vee r$	$\neg r$	$p \to q$	$p \to (q \vee r)$	$\neg r \to (p \to q)$
T	T	T	T	F	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	T	T
F	F	F	F	T	T	T	T

Since the truth values of the two Propositional are both the same, they are logically Equivalent.

3. Express the negation of each of the following statements in natural language using De Morgan's laws.

(a) Graduates take a job in industry or go to graduate school or start their own ventures.

Solution: Graduates will not take a job in industry and won't go to graduate school and won't start their own ventures.

(b) First year students know python and calculus.

Solution: First year students don't know python or they don't know calculus

(c) Horizon is new and bright.

Solution: Horizon is not new or not bright.

4. Determine whether each of the following compound propositions is satisfiable.

(a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

Solution: When both the truth value of both p and q are False, the compound proposition is True. Hence it is satisfiable because of the statement being a contingency.

(b) $(p \implies q) \wedge (p \implies \neg q) \wedge (\neg p \implies q) \wedge (\neg p \implies \neg q)$

Solution: This compound proposition can be also be written as
$[(\neg p \vee q) \wedge (\neg p \vee \neg q) \wedge (\ p \vee \neg q)] \wedge (p \vee q)$
The statement to left is the same as the one in last part for which we reasoned that it is only True when both p and q are False, whereas $p \vee q$ is True when atleast of one of them is True. This causes the conditions to be contradictory thus the whole compound proposition is a contradiction, hence unsatisfiable.

(c) $(p \iff q) \wedge (\neg p \iff q)$

Solution: There is no assignment of truth values to p and q that makes the whole propositional statement True, making this statement a contradiction. Thus it is not satisfiable.