Mechatronics System Design-1
Assignment 1

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Question 1 :

A thermocouple used between 0 and 500°C has the following input-output characteristics:

Input T°C	0	100	200	300	500
Output $E\mu$V	0	5268	10777	16325	27388

(a) Find the equation of the ideal straight line.

(b) Find the non-linearity at 100°C and 300°C in μV and as a percentage of f.s.d.

NB: f.s.d. = full scale deflection

Solution :

(a) Given, to find equation of ideal straight line. When we observe the given data, it is not ideal as slope it is not same when we try to calculate it by taking any two pairs of points. Now let's find equation of ideal line. :
we know that in case of linearity (i.e ideal stright line) we find the line equation by joining minimum and maximum points.

$$\Rightarrow y - y_{\min} = \frac{y_{\max} - y_{\min}}{I_{\max} - I_{\min}} \cdot (I - I_{\min})$$

$$\begin{aligned} \text{where } I &= \text{input} \\ y &= \text{output} \\ \text{slope} &= \frac{y_{\max} - y_{\min}}{I_{\max} - I_{\min}} \end{aligned}$$

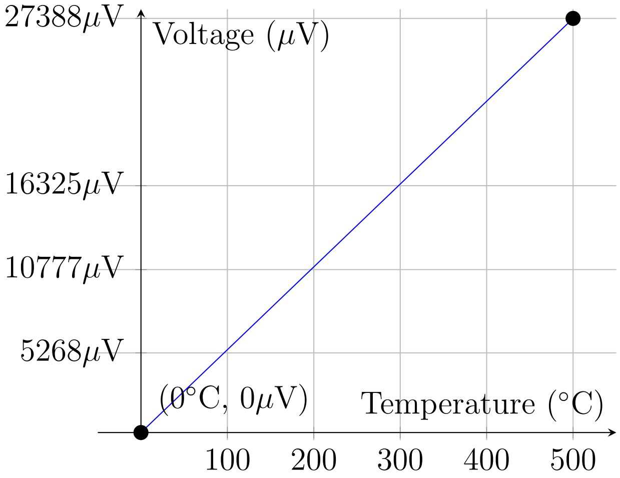


Figure 1: Temperature vs Voltage (Ideal straight line)

Now, according to the given data:

$$\begin{aligned}&\text{Temperature is input, and}\\&I_{\min} = 0°\text{C}, \quad I_{\max} = 500°\text{C}\\&y_{\min} = 0\mu V, \quad y_{\max} = 27388\mu V\end{aligned}$$

Therefore, the equation of ideal straight line is:

$$\begin{aligned}y - 0 &= \frac{27388 - 0}{500 - 0}(I - 0)\\ \Rightarrow y &= \frac{27388}{500} \cdot I\\ \Rightarrow y &= 54.776 \cdot I\end{aligned}$$

If we write in terms of x and y, then $\boxed{y = 54.776x}$, where y is the output and x is the temperature.

. . . .

(b) Given, to find nonlinearity at 100°C and 300°C

We know that nonlinearity $N(I) = O(I) - O_{\text{ideal}}(I)$

$$\Rightarrow N(I) = O(I) - (KI + a)$$

In our case, as the line passes through the origin $\Rightarrow a = 0$
$\Rightarrow$ Now by substituting equation of ideal line from part (a) we get:

$$N(I) = O(I) - 54.776I$$

Now finding non-linearity at 100°C:

$$\Rightarrow N\,(100°\text{C}) = 5268 - 54.776(100)$$
$$\Rightarrow N\,(100°\text{C}) = 5268 - 5477.6$$
$$\Rightarrow N\,(100°\text{C}) = -209.6$$

$$\Rightarrow \boxed{N\,(100°\text{C}) = -209.6\mu V}$$

Now finding non-linearity as % of full-scale deflection (fsd):
we know that fsd $= O_{\text{max}} - O_{\text{min}}$

$$\Rightarrow \text{Non-linearity} = \frac{|N(I)|}{O_{\text{max}} - O_{\text{min}}} \times 100$$
$$\Rightarrow \text{for } 100°\text{C}: \frac{209.6}{27388 - 0} \times 100 = \boxed{0.765\%}$$

At 300°C:

$$N\,(300°\text{C}) = O\,(300°\text{C}) - 54.776(300)$$
$$= 16325 - 16432.8 = -107.8$$
$$\Rightarrow N\,(300°\text{C}) = -107.8$$
$$\boxed{N\,(300°\text{C}) = -107.8\mu V}$$

Non-linearity as % of full-scale deflection:

$$f \cdot s \cdot d = O_{\text{max}} - O_{\text{min}}$$
$$= \frac{|N(I)|}{O_{\text{max}} - O_{\text{min}}} \times 100\%$$
$$\Rightarrow \quad \frac{107.8}{27388} \times 100 = 0.3936\%$$

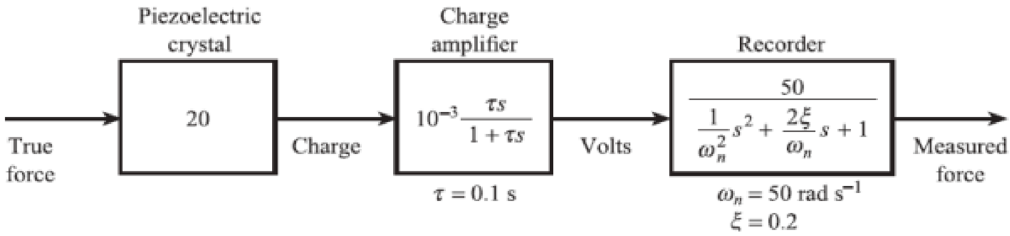
Therefore Non-linearity as %f.s.d for 300°C is $\boxed{0.3936\%}$

Question 2:

A force measurement system consisting of a piezoelectric crystal, charge amplifier and recorder is shown in Figure

Calculate the system dynamic error corresponding to the force input signal:

$$F(t) = 50\left(\sin 10t + \frac{1}{3}\sin 30t + \frac{1}{5}\sin 50t\right)$$



Solution :

Given,
Input force signal is:

$$F(t) = 50\left(\sin 10t + \frac{1}{3}\cdot\sin 30t + \frac{1}{5}\sin 50t\right)$$

we have to calculate the dynamic system error when $F(t)$ is passed through the above process.
If we consider the process as a cascade of three systems:

$$\text{Transfer function for piezoelectric crystal system : } G_1(s) = 20$$

$$\text{Transfer function for the charge amplifier} G_2(s) = \frac{10^{-3}\tau \mathrm{s}}{1+\tau s}, \tau = 0.1 \text{ s}$$

$$\text{Transfer function for the Recorder } G_3(s) = \frac{50}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}, \quad \omega_n = 50\,\mathrm{rad/s}, \quad \xi = 0.2$$

Now we know that the transfer function for the cascaded system is equal to the product of individual systems. Therefore, the transfer function of the cascaded system is given by:

$$H(s) = G_1(s)\cdot G_2(s)\cdot G_3(s)$$

$$H(s) = 20\cdot\frac{0.1s}{1+0.1s}\cdot 10^{-3}\cdot\frac{50}{\frac{1}{2500}s^2 + \frac{0.4s}{50} + 1}$$

$$\Rightarrow \frac{0.1s}{1+0.1s}\cdot\frac{1}{\frac{s^2}{2500}+\frac{4s}{500}+1}$$

$$\Rightarrow \frac{0.1s}{1+0.1s}\cdot\frac{2500}{s^2+20s+2500}$$

$$\Rightarrow \frac{250s}{(1+0.1s)(s^2+20s+2500)}$$

$$\Rightarrow H(s=j\omega) = \frac{250j\omega}{(1+0.1j\omega)(-\omega^2+20j\omega+2500)}$$

$$\text{Now, measured force :} F(t) = F(t)\cdot H(t)$$

We know that if the input is $Asin(\omega t)$, then steady-state output is given by:

$$|H(s=j\omega)|A\cdot\sin(\omega t+\phi_d)$$

$$where: \phi_d = \phi(H(s=j\omega)) \quad \text{(Reference: lecture-2, slide-11)}$$

Therefore, with the input $50\left(\sin 10t + \frac{1}{3}\sin 30t + \frac{1}{5}\sin 50t\right)$,
let's calculate the output of the cascaded system $F_m(t)$ by providing inputs $\sin 10t$, $\frac{1}{3}\sin 30t$, and $\frac{1}{5}\sin 50t$ separately and then adding them.

- Now calculating for $\sin(10t) \Rightarrow \omega = 10$

$$\Rightarrow |H(S=10j)| = \frac{250\times 10}{\sqrt{1+1}\sqrt{(2400)^2+40000}} = \frac{2500}{\sqrt{2}\sqrt{58,00,000}}$$
$$= \frac{2500}{3405.8772} = \boxed{0.73402}$$

$$\text{Now as } \arg\left(\frac{z_1}{z_2z_3}\right) = \arg(z_1) - [\arg(z_2)+\arg(z_3)]$$

$$\Rightarrow \phi_d = \arg(H(10j)) = \tan^{-1}\left(\frac{2500}{0}\right) - \left[\tan^{-1}(1)+\tan^{-1}\left(\frac{200}{2400}\right)\right]$$
$$= \frac{\pi}{2} - \left[\frac{\pi}{4}+\tan^{-1}\left(\frac{1}{12}\right)\right]$$
$$= \frac{\pi}{4} - \tan^{-1}\left(\frac{1}{12}\right)$$
$$= 45 - 4.763$$
$$= \boxed{40.2364 \text{ degrees}}$$

Therefore when sin(10t) is passed through the whole process the output is:

$$\boxed{0.73402\sin(10t+40.2364)}$$

- Now calculating for $\sin(30t) \Rightarrow \omega = 30$

$$H(S=30j) = \frac{250\times 30j}{(1+3j)(-900+20j(30)+2500)}$$
$$\Rightarrow |H(30j)| = \frac{7500}{\sqrt{10}\sqrt{2560000+360000}} = \frac{7500}{\sqrt{10}\sqrt{2920000}}$$
$$= \frac{7500}{\sqrt{29200000}} = \frac{7500}{5403.702} = \boxed{1.38793}$$

$$\phi_d = \arg(H(s=30j)) = \tan^{-1}\left(\frac{7500}{0}\right) - \left[\tan^{-1}(3) + \tan^{-1}\left(\frac{600}{1600}\right)\right]$$
$$= \frac{\pi}{2} - \left[71.565^\circ + \tan^{-1}\left(\frac{3}{8}\right)\right]$$
$$= 90^\circ - [71.565^\circ + 20.556^\circ]$$
$$= 90 - 92.121^\circ$$
$$= \boxed{-2.121^\circ}$$

Therefore when $\frac{1}{3}sin(30t)$ is passed through whole process the output is:

$$\boxed{\frac{1.38793}{3}\sin(30t - 2.121)}$$

- Calculating for $\sin(50t) \Rightarrow \omega = 50$

$$H(S=50j) = \frac{250j(50)}{(1+5j0(-2500+1000j+2500)}$$

$$\Rightarrow |H(50j)| = \frac{12500}{\sqrt{26}\times 1000} = \frac{12500}{5.09901 \times 1000} = \frac{12500}{5099.01} = \boxed{2.451456}$$

$$\phi_d = \arg(H(s=50j)) = \frac{\pi}{2} - \left[\tan^{-1}(5) + \frac{\pi}{2}\right] = -\tan^{-1}(5) = \boxed{-78.6900^\circ}$$

Therefore when $\frac{1}{5}sin(50t)$ is passed through the whole process the output is:

$$\boxed{\frac{2.451456}{5}\sin(50t - 78.690)}$$

- Now after cascading outputs of individual blocks, measured force is :-

$$F_m = 50[0.73402\sin(10t + 40.2364) + \frac{1.38793}{3}\sin(30t - 2.121) + \frac{2.451456}{5}\sin(50t - 78.690)\Big]$$

Now therefore dunmaic error = F(t) - $F_m(t)$

$$\Rightarrow 50\left[0.73402\sin(10t+40.2364) + \frac{1.38793}{3}\sin(30t - 2.121)\right.$$
$$\left. + \frac{2.451456}{5}\sin(50t - 78.690) - \sin 10t - \frac{1}{3}\sin 30t - \frac{1}{5}\sin 50t\right]$$

Question 3:

The following results were obtained when a pressure transducer was tested in a laboratory under the following conditions:

I Ambient temperature 20°C, supply voltage 10 V (standard)

II Ambient temperature 20°C, supply voltage 12 V

III Ambient temperature 25°C, supply voltage 10 V

Input (barg)	0	2	4	6	8	10
I (mA)	4	7.2	10.4	13.6	16.8	20
II (mA)	4	8.4	12.8	17.2	21.6	28
III (mA)	6	9.2	12.4	15.6	18.8	22

(a) Determine the values of K_M, K_I, a and K associated with the generalised model equation $O = (K + K_M I_M) I + a + K_I I_I$.

(b) Predict an output value when the input is 5barg, $V_S = 12$ V and ambient temperature is 25°C.

Solution :

The general form of the model equation is given as:

$$O = (K + K_M I_M) I + a + K_I I_I$$

In this equation:

- O is the observed output,
- I is the input (barg),
- I_M is the main effect corresponding to changes in I,
- I_I is the interactive effect corresponding to changes in I due to changes in the supply voltage or ambient temperature,
- K is the static sensitivity,
- K_M is the main effect sensitivity,

- a is the bias, and
- K_I is the interactive effect sensitivity.

The observed output (O) is related to the input (I), supply voltage (V), and ambient temperature (T) through the following equation:

$$O = (K + K_M I_M)I + a + K_I I_I$$

Now, let's find the values of K_M, K_I, a, and K using the provided data.

- For case I: When ambient temp = 20°C and $V = 10$ V

As Given standard $\Rightarrow I_m = I_I = 0$

$$\begin{aligned} &\Rightarrow \text{ equation: } O = kI + a \\ &\text{when } I = 0 \Rightarrow O = 4 \\ &\Rightarrow \boxed{a = 4} \end{aligned}$$

Now

$$\begin{aligned} k = \text{ slope } &= \frac{7.2 - 4}{2 - 0} = \frac{3.2}{2} = 1.6 \\ \Rightarrow \boxed{k = 1.6} \end{aligned}$$

- For case II: When ambient temp = 20°C and $V = 12$ V

$$\begin{aligned} O = k'I + a' \Rightarrow O = (k + k_m I_m)\, I + a + k_I I_I \\ \Rightarrow K' = \frac{0_{\text{max}} - 0_{\text{min}}}{I_{\text{max}} - I_{\text{min}}} = \frac{28 - 4}{10 - 0} = 2.4 \end{aligned}$$

From data of $I, II \Rightarrow I_m = 12 - 10 = 2 \text{ V} \Rightarrow \boxed{I_m = 2 \text{ V}}$

$$\begin{aligned} \text{Now} \quad k' &= k + k_m I_M \\ &\Rightarrow 2.4 = 1.6 + k_M(2) \\ &\Rightarrow \frac{0.8}{2} = k_M \Rightarrow \boxed{k_M = 0.4} \end{aligned}$$

- For case III: When ambient temp = 25°C and $V = 10$ V

$$O = (k + I_m k_m)\, I + a'$$
$$\text{at } I = 0 \Rightarrow O = 6 \Rightarrow a' = 6$$

Now from data of I and III

$$I_I = 25 - 20 = 5$$

$$\begin{aligned}\text{Now as}\quad a' &= a + k_I I_I \\ \Rightarrow 6 &= 4 + k_I(5) \\ \Rightarrow & \boxed{k_I = \frac{2}{5} = 0.4}\end{aligned}$$

$$\text{Therefore}\quad \boxed{k_M = 0.4, K_I = 0.4, k = 1.6, a = 4}$$

And generalised model equation :

$$O = (K + K_M I_M)I + a + K_I I_I \Rightarrow \boxed{O = 2.4I + 6}$$

....

(b) When the input is 5 barg, Vs=12v and ambient temperature is 25°C the output values can be predicted as:
As the modifying and interfering inputs are the same so equation remains same :

$P = 5\,\text{barg}, \quad V_S = 12\,\text{V at } 25°C$

$O = (K + K_M I_M)I + a + K_I I_I$

Substituting the values from part a, we get:

$O = (1.6 + (0.4 \times 2)) \cdot (5) + 4 + (0.4 \times 5)$

$O = 12 + 4 + 2 = \boxed{18\,\text{mA}}$

Question 4 :

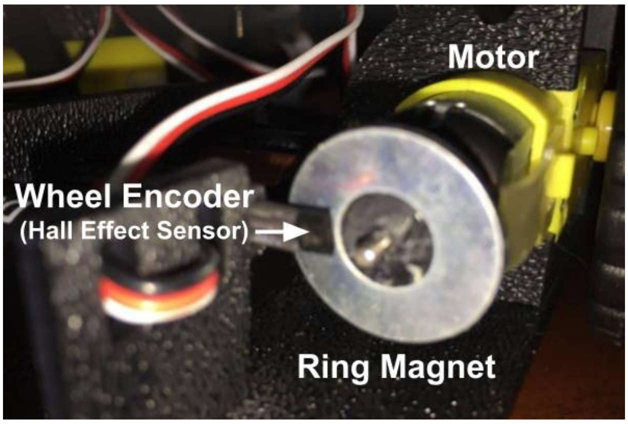
With appropriate diagrams and equations, explain the working of a wheel encoder?

Solution :

Wheel Encoder is made of two parts:
1) Hall effect sensor
2) Ring magnet

This device is commonly employed to tally how many times a motor turns, specifically when the motor turns a wheel connected to a ring magnet. As the ring rotates, a sensor detects changes in the magnetic field and keeps track of the number of rotations. This counting mechanism allows us to measure the distance traveled or the turns made by the robot. In simpler terms, it helps in determining how far or how much the robot has moved or turned by keeping tabs on the rotations of the wheel. In brief:

- The Hall Effect sensor is a device that can detect the presence of a magnetic field.
- It is positioned near the rotating part of the system, typically close to the edge of the wheel.
- As the wheel rotates, the Ring Magnet attached to it generates a changing magnetic field.



The rubber, ring magnet, and the connecting shaft all spin together at the same speed. There's a gear ratio between the motor axle and the ring magnet.

If this ratio is, let's say, 'N to 1', it means that for every N turns of the magnet, the wheel completes one full rotation. Now, if there are 'p' poles in the ring magnet, it indicates that the Hall effect sensor near the ring feels 'p' changes in the magnetic field for each rotation of the magnet. With this info, we can use it to estimate how far the vehicle has traveled.

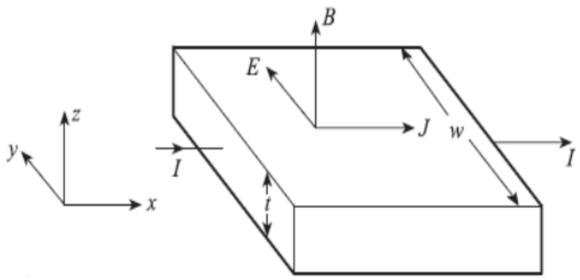
Working :

- When the wheel or shaft turns, it makes the ring magnet spin, and this brings its changing magnetic field close to the Hall Effect sensors.

- As this happens, the sensors generate signals. If a sensor comes across a magnetic pole, it produces a high voltage because of the Hall effect. On the other hand, when it lines up with a gap between the poles, the voltage drops to a low level.

- These signals from the Hall effect sensors can be processed in two ways: either as digital signals (high or low) or as analog signals, where the strength of the magnetic field is represented.

Hall Effect Sensor : We know,



$$
\begin{aligned}
&F_q = q(\vec{v} \times \vec{B}) \\
&I = neAv_\alpha \\
&\vec{v}_d = \text{drift velocity} \\
&n = \text{carrier density} \\
&A = \text{cross-sectional area of the wire} \\
&\varepsilon = \text{voltage (emf)}
\end{aligned}
$$

Therefore, form flemings right hand thumb rule charge moving in a current flow experiences a force perpendicular to V_d and B. By this charges accumulate at one side and creates a potential difference :

$$V = \frac{R_H}{t} IB \quad R_H = \text{ Hall coefficient}$$

$$\begin{aligned}
&\frac{F_B}{q} = E_B = \vec{v}_d \times \vec{B} = \frac{IB}{neA} \\
&I = nAev_d \\
&E_B = \frac{IB}{newt} \quad \text{As A=w*t}
\end{aligned}$$

$$\begin{aligned}
&\frac{V}{W} = \frac{IB}{newt} \quad \left\{ \text{voltage } V = \frac{E}{W} \right\} \\
&R_H = \frac{1}{ne} = \text{Hall coefficient} \\
&\Rightarrow v = R_H \frac{IB}{t} \\
&R_H = \text{ Hall Coefficient } \left(\mathrm{m}^3/c \right) \\
&t = \text{ thickness } (m) \\
&1 = \text{Current}(A) \\
&B = \text{ magnetic flux density } (T)
\end{aligned}$$

The voltage changes with the change in magnetic field due to poles of the magnet

$$D = \frac{2\pi r}{N}$$

- r = radius of wheel
- D = distance traveled in one ring magnet cycle
- N = motor to gear ratio

Distance traveled per pulse:

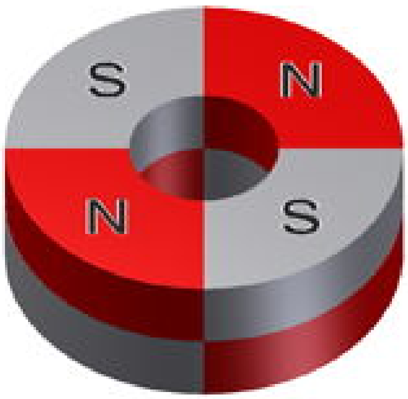
$$\begin{aligned}
&D = \frac{2\pi r}{NP}, \quad P = \text{No. of poles of the ring magnet} \\
&D = \frac{2\pi r}{NP} N_p, \quad N_p = Numberof \text{ pulses per second}
\end{aligned}$$

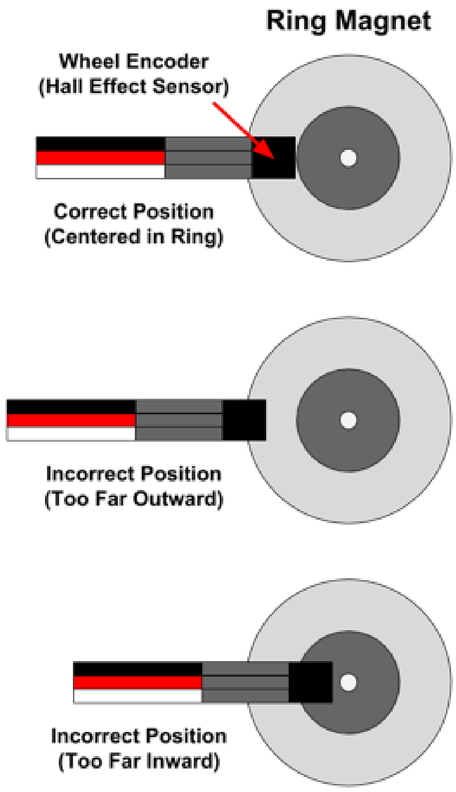
where D=Distance traveled per second

Position Odometry :

$$\frac{dx}{dt} = v_x = v\cos\psi$$
$$x(t) = \int_0^t v\cos\psi dt + x(0)$$
$$\frac{dy}{dt} = v_y = v\sin\psi$$
$$y(t) = \int_0^t v\sin\psi dt + y(0)$$

Ring Magnet :




Ring Magnet
Wheel Encoder
(Hall Effect Sensor)
Correct Position
(Centered in Ring)
Incorrect Position
(Too Far Outward)
Incorrect Position
(Too Far Inward)

Question 5(a) :

Simulate the following systems on MATLAB and plot the output.
Refer supporting documents (Tutorial documentation) for explanations.
a) Mass Spring Damper:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

Change :

- System Parameters (mass, spring const, damping coefficient)
- Initial values (position, velocity)
- Input (ideally assumed 0)

Solution :

```
clc;
clear all;

n = input('Enter the number of systems (n): ');

m = zeros(1, n);
c = zeros(1, n);
k = zeros(1, n);

y = zeros(n, 2);

for l = 1:n
    helper = sprintf('Enter the mass %d value: ', l);
    m(l) = input(helper);

    helper = sprintf('Enter damping coefficient %d value: ', l);
    c(l) = input(helper);

    helper = sprintf('Enter spring constant %d value: ', l);
    k(l) = input(helper);
end

```

```
tspan = [0 30];

f = @(t) 0;
%f = @(t) sin(t);


for l = 1:n
    x0 = input(['Enter initial x position for system ', num2str(l), ': ']);
    v0 = input(['Enter initial y velocity for system ', num2str(l), ': ']);

    odeSystem = @(t, y) [y(2); (1/m(l)) * (f(t) - c(l)*y(2) - k(l)*y(1))];

    [t_system, sol] = ode45(odeSystem, tspan, [x0, v0]);

    disp(['Results for system ', num2str(l), ':']);

    disp(['Final Position: ', num2str(sol(end, 1))]);
    disp(['Final Velocity: ', num2str(sol(end, 2))]);

    figure;
    subplot(2, 1, 1);
    plot(t_system, sol(:, 1), 'LineWidth', 2);
    title(['System ', num2str(l), ' - Position Response']);
    xlabel('Time (s)');
    ylabel('Position');
    grid on;

    annotation('textbox', [0.7, 0.8, 0.1, 0.1], 'String', ['m = ', num2str(m(l))]);
    annotation('textbox', [0.7, 0.75, 0.1, 0.1], 'String', ['c = ', num2str(c(l))]);
    annotation('textbox', [0.7, 0.7, 0.1, 0.1], 'String', ['k = ', num2str(k(l))]);

    subplot(2, 1, 2);
    plot(t_system, sol(:, 2), 'LineWidth', 2);
    title(['System ', num2str(l), ' - Velocity Response']);
    xlabel('Time (s)');
```

```
    ylabel('Velocity');
    grid on;
end
```

Approach for above code :

As we can't solve second order differential equation in matlab directly so we divide it into two linear differential equations and solve:

$$
\begin{aligned}
&m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)\\
&m\ddot{x} + c\dot{x} + kx = f(t)\\
&\ddot{x} = \frac{1}{m}(f(t) - c\dot{x} - kx)
\end{aligned}
$$

Let $x = q_1$, consider $\frac{dx}{dt} = \frac{dq_1}{dt} = dq_2$:

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{d^2q_1}{dt^2} = \frac{d}{dt}q_2$$

$$\Rightarrow m\frac{d^2q_1}{dt} + c\frac{dq_1}{dt} + kq_1 = f(t)$$

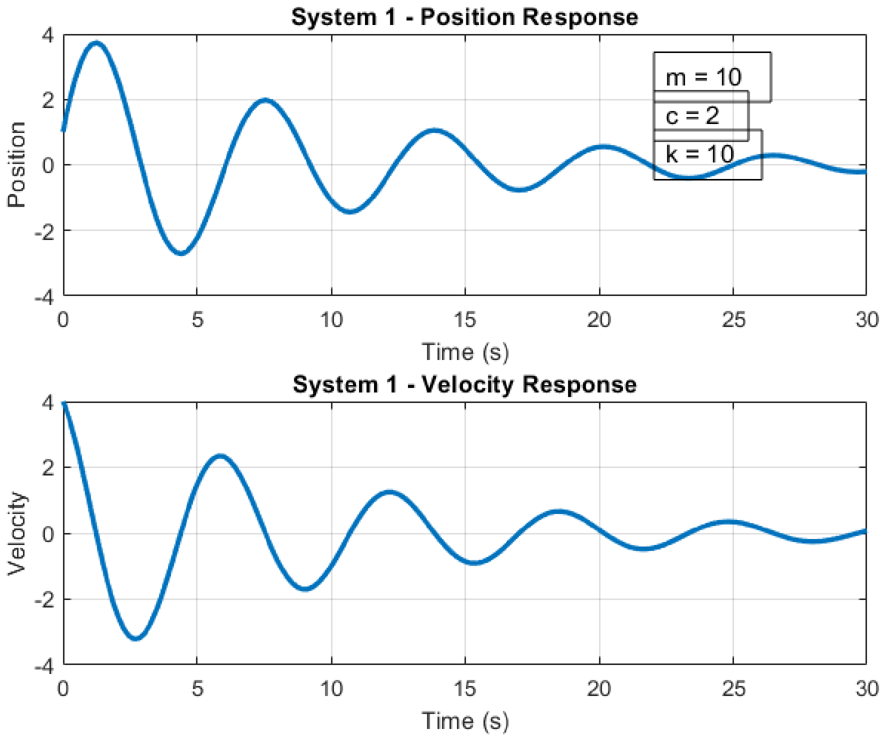
$$m\frac{dq_2}{dt} + cq_2 + kq_1 = f(t)$$

$$\frac{dq_2}{dt} = \frac{(f(t) - kq_1 - cq_2)}{m}$$

Equations of motion are:

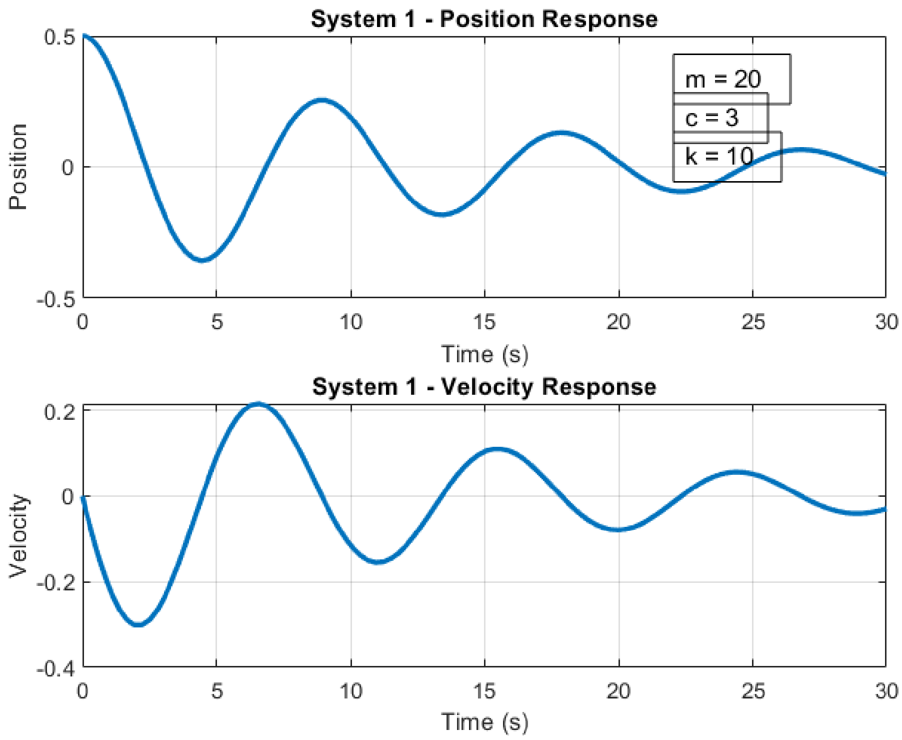
$$
\begin{aligned}
&\frac{dq_1}{dt} = q_2 \quad \& \quad \frac{dq_2}{dt} = \frac{(f(t) - kq_1 - cq_2)}{m}\\
&\frac{d}{dt}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} q_2 \\ \frac{(f(t)-kq_1-cq_2)}{m} \end{bmatrix}
\end{aligned}
$$

Plots :(c=damping constant, k=spring constant)



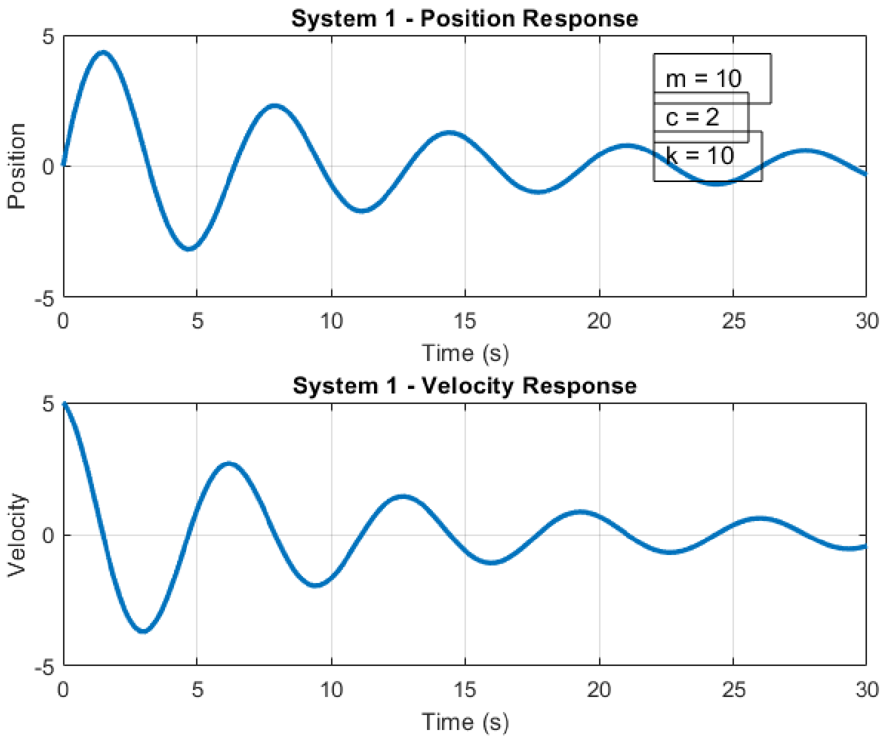
f(t)=0

```
Command Window
Enter the number of systems (n): 1
Enter the mass 1 value: 10
Enter damping coefficient 1 value: 2
Enter spring constant 1 value: 10
Enter initial x position for system 1: 1
Enter initial y velocity for system 1: 4
Results for system 1:
Final Position: -0.20423
Final Velocity: 0.071203
fx >>
```



$f(t)=0$

```
Command Window
Enter the number of systems (n): 1
Enter the mass 1 value: 20
Enter damping coefficient 1 value: 3
Enter spring constant 1 value: 10
Enter initial x position for system 1: 0.5
Enter initial y velocity for system 1: 0
Results for system 1:
Final Position: -0.028465
Final Velocity: -0.029219
fx >>
```



For f(t)=sin(t)

Command Window

```
Enter the number of systems (n): 1
Enter the mass 1 value: 10
Enter damping coefficient 1 value: 2
Enter spring constant 1 value: 10
Enter initial x position for system 1: 0
Enter initial y velocity for system 1: 5
Results for system 1:
Final Position: -0.32933
Final Velocity: -0.44204
>>
```

. . . .

b) Simple Pendulum:

$$\frac{d^2\theta}{dt^2} + \frac{b}{m}\frac{d\theta}{dt} + \frac{g}{L}\sin\theta = 0$$

Change :

- System Parameters (mass, length, damping coefficient)
- Initial Values (position, velocity)
- Input Force

Matlab Code :

```
clc;
clear all;

n = input('Enter the number of systems (n): ');

m = zeros(1, n);
b = zeros(1, n);
L = zeros(1, n);

y = zeros(n, 2);

for l = 1:n
    helper = sprintf('Enter the mass %d value: ', l);
    m(l) = input(helper);

    helper = sprintf('Enter damping coefficient %d value
       : ', l);
    b(l) = input(helper);

    helper = sprintf('Enter length %d value: ', l);
    L(l) = input(helper);
end

tspan = [0 30];

f = @(t) 0;
```

```
for l = 1:n
    theta0 = input(['Enter initial theta position for system ', num2str(l), ': ']);
    omega0 = input(['Enter initial omega velocity for system ', num2str(l), ': ']);

    odeSystem = @(t, y) [y(2); -(b(l)/m(l)) * y(2) - (9.81/L(l)) * sin(y(1))];

    [t_system, sol] = ode45(odeSystem, tspan, [theta0, omega0]);

    disp(['Results for system ', num2str(l), ':']);

    disp(['Final Theta Position: ', num2str(sol(end, 1))]);
    disp(['Final Omega Velocity: ', num2str(sol(end, 2))]);

    figure;
    subplot(2, 1, 1);
    plot(t_system, sol(:, 1), 'LineWidth', 2);
    title(['System ', num2str(l), ' - Theta Position Response']);
    xlabel('Time (s)');
    ylabel('Theta Position');
    grid on;

    annotation('textbox', [0.7, 0.8, 0.1, 0.1], 'String', ['m = ', num2str(m(l))]);
    annotation('textbox', [0.7, 0.75, 0.1, 0.1], 'String', ['b = ', num2str(b(l))]);
    annotation('textbox', [0.7, 0.7, 0.1, 0.1], 'String', ['L = ', num2str(L(l))]);

    subplot(2, 1, 2);
    plot(t_system, sol(:, 2), 'LineWidth', 2);
    title(['System ', num2str(l), ' - Omega Velocity Response']);
    xlabel('Time (s)');
    ylabel('Omega Velocity');
    grid on;
```

```
end
```

Calculations :

As we can't solve second order differential equation in matlab directly so we divide it into two linear differential equations and solve similarly as we have done in part a:

$$\frac{d^2\theta}{dt^2} + \frac{b}{m}\frac{d\theta}{dt} + \frac{g}{L}\sin\theta = 0$$

- m: mass
- g: gravity
- b: damping
- L: length

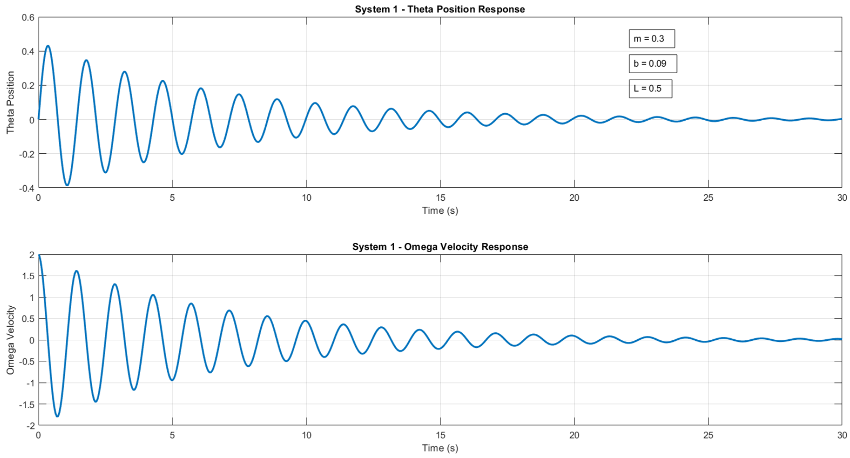
$$\begin{aligned}
&\text{Let } \theta = \theta_1, \\
&\text{Consider } \frac{d\theta}{dt} = \frac{d\theta_1}{dt} = \theta_2. \\
&\Rightarrow \frac{d^2\theta}{dt^2} = \frac{d^2\theta_1}{dt^2} = \frac{d}{dt}\left(\frac{d\theta_1}{dt}\right) = \frac{d}{dt}(\theta_2). \\
&\Rightarrow \frac{d^2\theta}{dt^2} + \frac{b}{m}\frac{d\theta}{dt} + \frac{g}{L}\theta = 0. \\
&\frac{d^2\theta_1}{dt^2} + \frac{b}{m}\frac{d\theta_1}{dt} + \frac{g}{L}\sin\theta_1 = 0. \\
&\frac{d\theta_2}{dt} + \frac{b}{m}\theta_2 + \frac{g}{L}\theta_1 = 0.
\end{aligned}$$

Equations of motion:

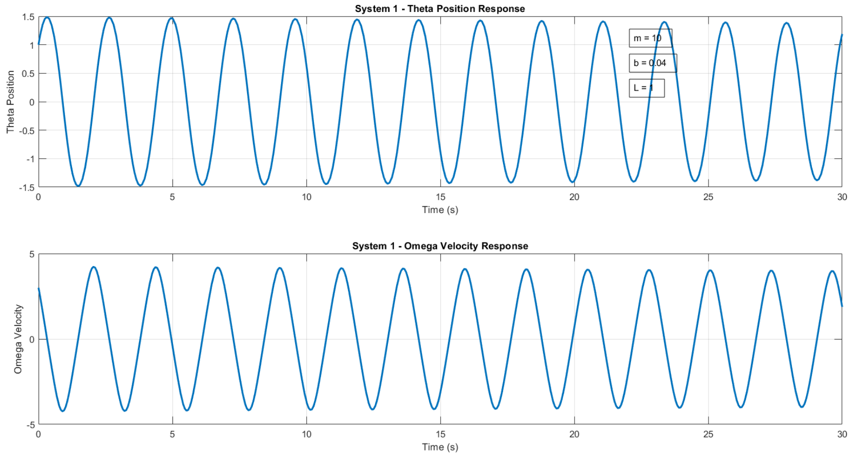
$$\begin{aligned}
\frac{d\theta_1}{dt} &= \theta_2 \\
\frac{d\theta_2}{dt} &= -\frac{b}{m}\theta_2 - \frac{g}{L}\sin\theta_1 - \frac{10}{L}\sin(\theta_1)
\end{aligned}$$

$$\frac{d}{dt}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \theta_2 \\ -\frac{b}{m}\theta_2 - \frac{g}{L}\sin\theta_1 - \theta_1 \end{bmatrix}$$

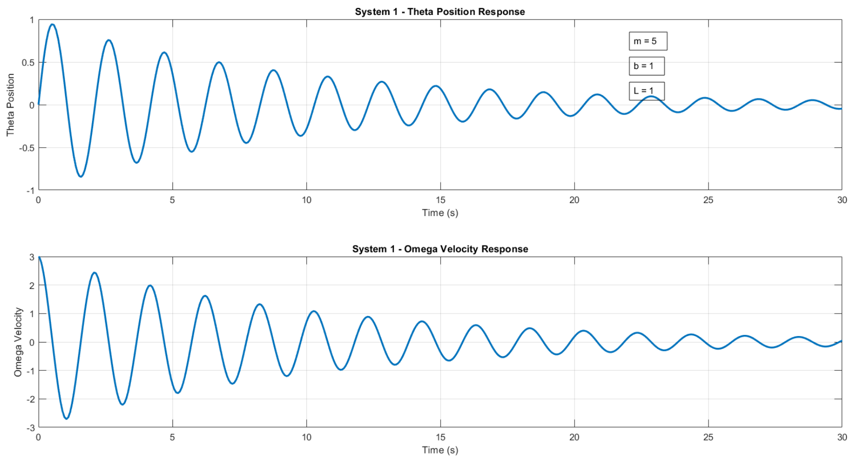
Plots :(B=damping coefficient,l=length)




```
Command Window
Enter the number of systems (n): 1
Enter the mass 1 value: 0.3
Enter damping coefficient 1 value: 0.09
Enter length 1 value: 0.5
Enter initial theta position for system 1: 0
Enter initial omega velocity for system 1: 2
Results for system 1:
Final Theta Position: 0.0031549
Final Omega Velocity: 0.016699
fx >>
```




```
Command Window
Enter the number of systems (n): 1
Enter the mass 1 value: 10
Enter damping coefficient 1 value: 0.04
Enter length 1 value: 1
Enter initial theta position for system 1: 1
Enter initial omega velocity for system 1: 3
Results for system 1:
Final Theta Position: 1.1905
Final Omega Velocity: 1.8773
fx >>
```



```
Command Window
Enter the number of systems (n): 1
Enter the mass 1 value: 5
Enter damping coefficient 1 value: 1
Enter length 1 value: 1
Enter initial theta position for system 1: 0
Enter initial omega velocity for system 1: 3
Results for system 1:
Final Theta Position: -0.045916
Final Omega Velocity: 0.050057
fx >>
```