Assignment 4
Communication Theory

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Question 1 :

Consider the pulse

$$p(t) = \begin{cases} t/a, & 0 \leq t \leq a \\ 1, & a \leq t \leq 1-a \\ (1-t)/a, & 1-a \leq t \leq 1 \\ 0, & \text{else} \end{cases}$$

$$where \quad 0 \leq a \leq \frac{1}{2}$$

(a) Sketch $p(t)$ and find its Fourier transform $P(f)$.
(b) Consider the linearly modulated signal $u(t) = \sum_n b[n]p(t-n)$, where $b[n]$ take values independently and with equal probability in a 4 -PAM alphabet $\{\pm 1, \pm 3\}$. Find an expression for the PSD of u as a function of the pulse shape parameter a.
(c) Numerically estimate the 95% fractional power containment bandwidth for u and plot it as a function of $0 \leq a \leq \frac{1}{2}$. Assume the unit of time is 100 picoseconds and specify the units of bandwidth in your plot.

Solution :

(a)

Given,
To sketch $p(t)$ and find its Fourier transform $P(f)$:

- In o to $a \rightarrow$ straight line with slope $\frac{1}{a}$

- In $a \leq t \leq 1-a \rightarrow$ constant line with amplitude ' 1 '.
- In $1-a \leq t \leq 1 \rightarrow$ slight line with slope $= -\frac{1}{a}$ and y interecpt as $\frac{1}{a}$

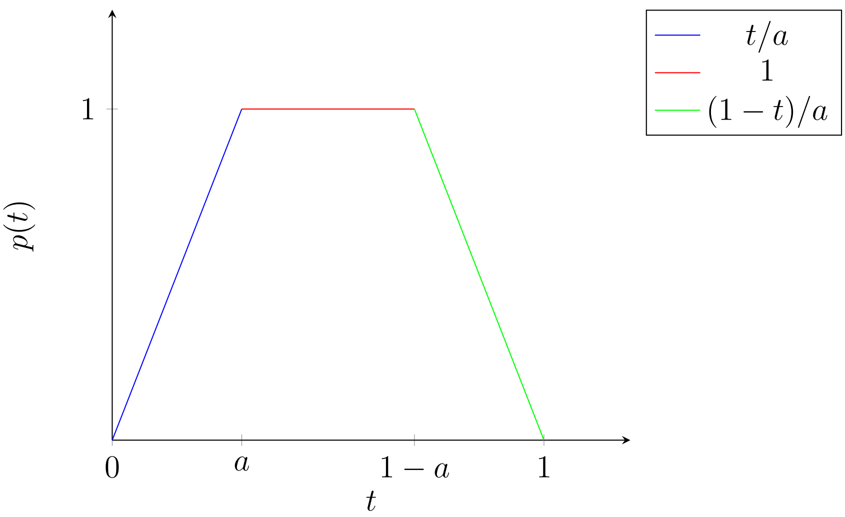

Figure 1: Plot of pulse $p(t)$

Now finding Fourier transform $P(f)$: $\Rightarrow$ Now FT of $(p(t)) = P(f)$

$$p(f) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt$$

$$\Rightarrow P(t) = \int_0^a \frac{t}{a}e^{-j\omega t} + \int_a^{1-a} 1e^{-j\omega t} + \int_{1-a}^{1} \frac{1-t}{a}e^{-j\omega t}dt$$

$$= \frac{1}{a}\left[\frac{te^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{j^2\omega^2}\right]_0^a + \left.\frac{e^{-j\omega t}}{-j\omega}\right|_a^{1-a} + \frac{1}{a}\left[\frac{(1-t)e^{-j\omega t}}{-j\omega} + \frac{e^{-j\omega t}}{(-j\omega)^2}\right]_{1-a}^{1}$$

$$= \frac{1}{a}\left[\frac{ae^{-j\omega a}}{-j\omega} + \frac{e^{-j\omega a}}{\omega^2} + \frac{1}{-\omega^2}\right] + \frac{e^{-j\omega(1-a)}}{-j\omega} + \frac{e^{-j\omega a}}{j\omega} + \frac{1}{a}\frac{(-e^{-j\omega})}{\omega^2} - \frac{1}{a}\left[\frac{ae^{-j\omega(1-a)}}{-j\omega} + \frac{-e^{-j\omega(1-u)}}{\omega^2}\right]$$

Remaining is continued in hand written solutions