CPA-4

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Q1)

Given,

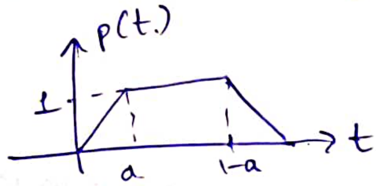
$$p(t) = \begin{cases} t/a & 0 \le t \le a \\ 1 & a \le t \le 1-a \\ (1-t)/a & 1-a \le t \le 1 \\ 0 & \text{else.} \end{cases} \qquad \text{where } 0 \le a \le \tfrac{1}{2}$$

In 0 to a → stright line with slope $\frac{1}{a}$

In. $a \le t \le 1-a$ → constant line with amplitude '1'.

In $1-a \le t \le 1$ → stright line with slope $= -\frac{1}{a}$ and y intercept as $\frac{1}{a}$

⇒ plot =



⇒ Now fourier transform :- $(p(t))$

$$p(f) = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$$

$$\Rightarrow p(t) = \int_0^a \frac{t}{a} e^{-j\omega t} + \int_a^{1-a} 1\, e^{-j\omega t} + \int_{1-a}^{1} \frac{1-t}{a} e^{-j\omega t}\, dt$$

$$= \frac{1}{a}\left[\frac{t e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{j^2\omega^2} \right]_0^a + \frac{e^{-j\omega t}}{-j\omega}\Bigg|_a^{1-a} + \frac{1}{a}\left[\frac{(1-t)e^{-j\omega t}}{-j\omega} + \frac{e^{-j\omega t}}{(-j\omega)^2} \right]_{1-a}^{1}$$

$$= \frac{1}{a}\left[\frac{a e^{-j\omega a}}{-j\omega} + \frac{e^{-j\omega a}}{\omega^2} + \frac{1}{-\omega^2} \right] + \frac{e^{-j\omega(1-a)}}{-j\omega} + \frac{e^{-j\omega a}}{j\omega} + \frac{1}{a}\left(\frac{-e^{-j\omega}}{\omega^2}\right)$$

$$- \frac{1}{a}\left[\frac{a e^{-j\omega(1-a)}}{-j\omega} + \frac{-e^{-j\omega(1-a)}}{\omega^2} \right]$$

$$= \frac{1}{a}\left[\frac{a e^{-jwa}}{-jw} + \frac{e^{-jwa}}{w^2} - \frac{1}{w^2}\right] + \frac{e^{-jw(1-a)}}{-jw} + \frac{e^{-jwa}}{jw} - \frac{1}{a}\frac{e^{-jw}}{w^2} - \frac{1}{a}\left[\frac{a e^{-jw(1-a)}}{-jw} + \frac{e^{-jw(1-a)}}{-w^2}\right]$$

$$= \cancel{\frac{-e^{-jwa}}{jw}} + \frac{e^{-jwa}}{aw^2} - \frac{1}{aw^2} + \cancel{\frac{-e^{-jw(1-a)}}{jw}} + \cancel{\frac{e^{-jwa}}{jw}} - \frac{e^{-jw}}{aw^2} + \cancel{\frac{e^{-jw(1-a)}}{jw}} + \frac{e^{-jw(1-a)}}{aw^2}$$

$$= \frac{e^{-jwa}}{aw^2} - \frac{1}{aw^2} - \frac{e^{-jw}}{aw^2} + \frac{e^{-jw(1-a)}}{aw^2} \longrightarrow (1)$$

$$= \frac{e^{-jwa}}{aw^2} + \frac{e^{jwa}}{aw^2} + \frac{e^{-jw}}{aw^2} - \frac{e^{-jw}}{aw^2} - \frac{1}{aw^2}$$

we know that $\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$, $\frac{e^{i\theta} - e^{-i\theta}}{2j} = \sin\theta$.

on keeping $w = 2\pi f$

$$\Rightarrow \frac{e^{-j2\pi f a}}{aw^2} - \frac{1}{aw^2} - \frac{e^{-j2\pi f}}{aw^2} + \frac{e^{-jw(1-a)}}{aw^2}.$$

Now taking $e^{-j\pi f}$ common from all terms

$$\rightarrow \frac{e^{-j\pi f}\left[e^{-j2\pi f a} e^{j\pi f} - e^{j\pi f} - e^{-j\pi f} + e^{-j2\pi f(1-a)} e^{j\pi f}\right)}{aw^2}.$$

$$= e^{-j\pi f} \cdot (1-a)\,\mathrm{sinc}\,(1-a)\,\mathrm{sin}(af)$$

$$\text{as } e^{j\pi f(1-a)} - e^{-j\pi f(1-a)}$$

upon solving we get:-

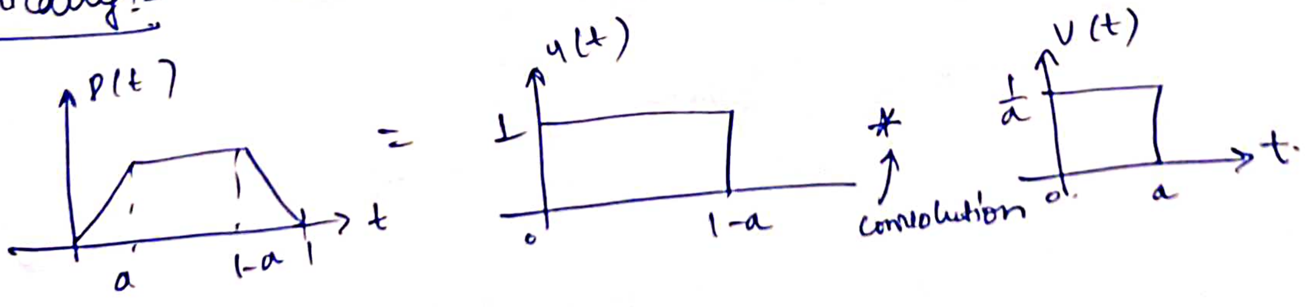
$$\frac{e^{-j\pi f}\left(e^{+j\pi f(1-a)} - e^{-j\pi f(1-a)}\right)\left(e^{j\pi f a} - e^{-j\pi f a}\right)}{a\,4\pi^2 f^2}$$

$$\text{as } \frac{e^{j\pi f(1-a)} - e^{-j\pi f(1-a)}}{2j} = \sin.(\pi f(1-a)) \quad \left\langle \frac{e^{i\theta} - e^{-i\theta}}{2j} = \sin\theta \right\rangle$$

$$\therefore = e^{-j\pi f}(1-a)\,\mathrm{sinc}((1-a)f)\,\mathrm{sinc}(af)$$

$$\Rightarrow \boxed{P(f) = (1-a)\,\mathrm{sinc}((1-a)f)\,\mathrm{sinc}(af)\,e^{-j\pi f}}$$

Graphically:-



$$p(t) = u(t) * v(t)$$

$$\Rightarrow \boxed{P(f) = U(f) \cdot V(f)}$$

$\therefore$ $u(f) = 1-a \operatorname{sinc}(1-a)f$

and $v(f) = \operatorname{sinc}(af)$

Now convolution will give centered version of $p(t)$

with $F.T = (1-a)\operatorname{sinc}((1-a)f)\operatorname{sinc} af$

But $p(t)$ is delayed by $\frac{1}{2}$ from its centered version $\tilde{p}(t)$

$\Rightarrow$ we get a phase of $e^{-j\pi f}$ in freq domain.

$$\boxed{\therefore P(f) = \tilde{P}(f)e^{-j\pi f} = (1-a)\operatorname{sinc}((1-a)f)\operatorname{sinc}(af)e^{-j\pi f}}$$

Ans.

(b) Average symbol energy of equiprobable 4-PAM is

$$\sigma_b^2 = \frac{(-1)^2 + (-3)^2 + 1^2 + 3^2}{4} = 5.$$

$\Rightarrow \sigma_b^2 = 5, \ T = 1$

PSD [Power spectral density for modulated signal

$$S_{u}(f) = \frac{\sigma_b^2}{T} |P(f)|^2$$

$$\Rightarrow S_u(f) = 5\left|(1-a)\,\text{sinc}((1-a)f)\,\text{sinc}(af)\right|^2$$

$$\boxed{S_u(f) = 5(1-a)^2\,\text{sinc}^2((1-a)f)\,\text{sinc}^2(af)}$$

(c) 95% of B.W = B satisfies the eq^n: dropping constants from PSD :-

$$\int_{-\frac{B}{2}}^{\frac{B}{2}} |P(f)|^2\,df = \frac{95}{100}\int_{-\infty}^{\infty} |P(f)|^2\,df.$$

Pulse Energy is given by :-

$$\int_{-\infty}^{\infty} |P(f)|^2\,df = \int_{-\infty}^{\infty} |p(t)|^2\,dt \quad \langle \underline{\text{Parsevals thm}} \rangle$$

$$\Rightarrow 2\int_0^a \left(\frac{t}{a}\right)^2 dt + \int_a^{1-a} dt.$$

$$= \frac{2}{a^2}\left(\frac{a^3}{3}\right) + 1 - 2a$$

$$= 1 - 2a + \frac{2a}{3} = \boxed{1 - \frac{4a}{3}}$$

$$\Rightarrow \int_{-\infty}^{\infty} |P(f)|^2\,df = 1 - \frac{4a}{3}.$$

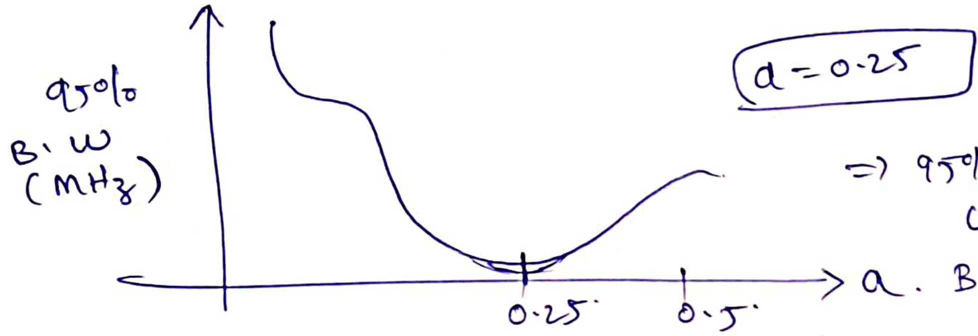
Now using symmetry of $|P(f)|^2$.

$$\int_0^{B/2} |P(f)|^2\,df = 2\int_0^{B/2} (1-a)^2\,\text{sinc}^2((1-a)f)\,\text{sinc}^2 af\,df.$$

$$= 0.95\left(1 - \frac{4a}{3}\right)$$

$$\Rightarrow \int_0^{B/2} \text{sinc}^2((1-a)f)\ \text{sinc}^2 af\ df = \frac{0.95(1-\frac{4a}{3})}{2(1-a)^2}.$$

Now on solving through matlab we get $\underline{\underline{a=0.25}}$



$\Rightarrow$ 95% power containment. B.W is minimized for $a \approx 0.25$

Unit time is 100ps

So unit B.W = 10MHz.

$\Rightarrow$ we plotted 10B with a. (10B MHz vs a)

Q2)) Given,

Linear modulation with signalling pulse

$$p(t) = \text{sinc}(at)\,\text{sinc}(bt)$$

a) we have to find a&b so that p(t) is Nyquist with 50% excess B.W for data rate of 40mbps using 16QAM.

we know: multiplication in time domain gives convolution in freq domain

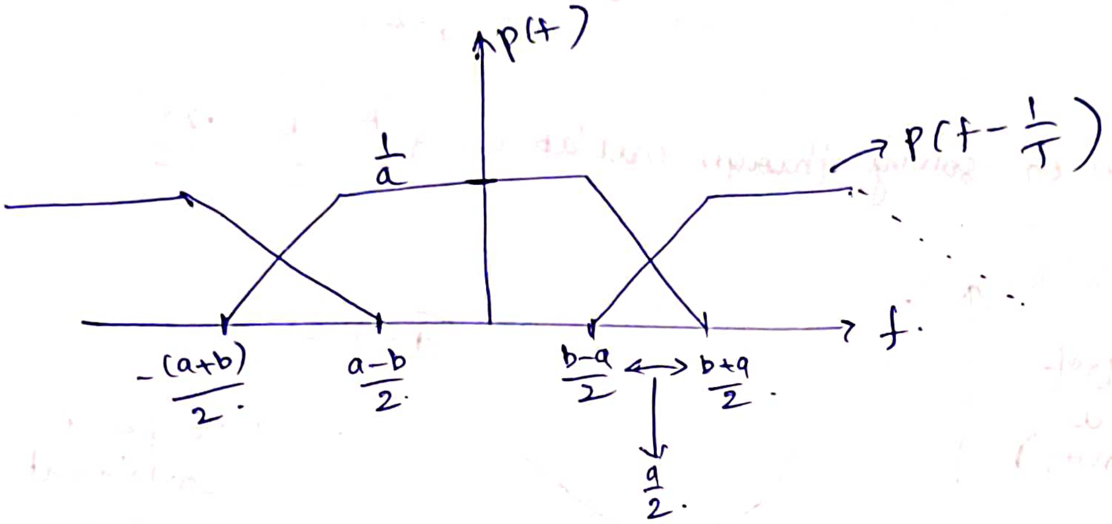
$$\Rightarrow p_1(t) = \text{sinc}(at), \quad p_2(t) = \text{sinc}(bt)$$

$$P(f) = P_1(f) * P_2(f)$$

which results in trapezoid

(p.T.O)

assuming $a \le b$



$R_b = \frac{\text{bit rate}}{\text{no of bits}}$ (symbol rate)

No of bits of 16 QAM $= \log_2 16 = 4$

Bit rate $= 40$ mbps $\quad R_b = \frac{40 \text{ mbps}}{4} = 10$ mbps.

Hence $a = 10$ MHz

p(t) has 50% extra B.W.

$\Rightarrow B_T = \frac{(a+b)}{2} = \frac{R_b}{2}\left(1+\frac{50}{100}\right) = \frac{R_b}{2}\left(\frac{3}{2}\right)$

$\Rightarrow 10 + b = 15 \Rightarrow \boxed{b = 5} \Rightarrow \boxed{b = 5 \text{ MHz}}$

$\frac{a}{2} = \frac{1}{2T}$. & pulse has zeros at integer multiple of T, so we minimise ISI and get this so

$\Rightarrow \frac{1}{2T}(1+\alpha) = \frac{a+b}{2}$.

$\boxed{b = \frac{\alpha}{T}}$

$\frac{1}{T} = 10M$ symbols/sec.

$\Rightarrow \frac{a}{2} = \frac{1}{2} \times 10 \Rightarrow \boxed{a = 10MHz}$

$b = \frac{\alpha}{T} = \frac{1}{2} \times 10 = \boxed{5MHz.}$

$\alpha = 50\% = \frac{1}{2}$

$\Rightarrow \boxed{\text{occupied B.W} = 15 MHz}$

(b) Given,

Data rate for 16QAM = 40 mbps.

" " " 8 PSK = 18 Mbps

No of symbols $= \log_2^{16} = 4$

$\Rightarrow$ symbol rate for 16QAM $= \frac{1}{T_1} = \frac{40}{\log_2 16} = 10^M$ symbols/sec

$\Rightarrow$ No of symbols $= \log_2^8 = 3.$

symbol rate for 8PSK $= \frac{1}{T_2} = \frac{18}{\log_2^8.} = 6.M$ symbols/sec.

So the conditions should be L

a = 10, b = 10

a = 10, b = 6

a = 6, b = 10

a = 6, b = 6

we have to choose for b > a

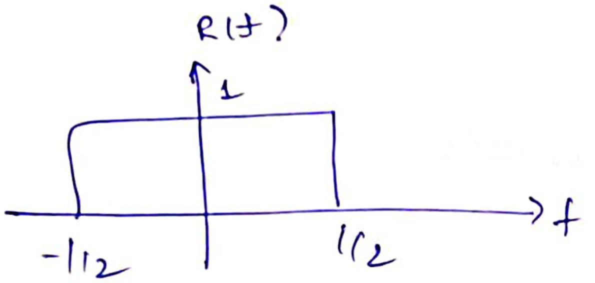
so a = 6, b = 10 is the best option.

Q3) Given,

$R(f) = I_{[-\frac{1}{2}, \frac{1}{2}]}(f)$.

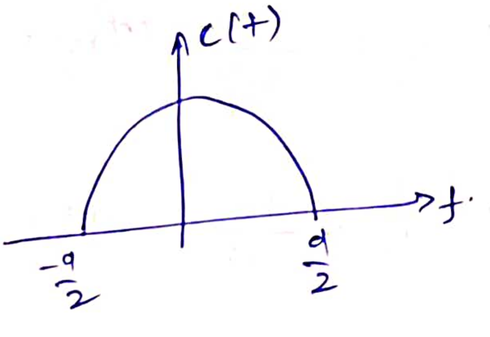
(a) $R(f)$ & $c(f)$ for $0 < a < 1$

$R(f) = I_{[-\frac{1}{2}, +\frac{1}{2}]}(f)$



$c(f) = \frac{\pi}{2a} \cos\left(\frac{\pi f}{a}\right) I_{[-\frac{a}{2}, \frac{a}{2}]}$

$c(f)$ is a cosine function with amp $\frac{\pi}{2a}$ from $-\frac{a}{2}$ to $\frac{a}{2}$



(b) we have to show Freq domain raised cosine pulse $s(f)$

$S(f) = R(f) * c(f)$

$S = R * C$ is symmetric as R, C are symmetric

$S(f) = \int R(f-v)\, c(v)\, dv$

$= R(v-f)\, I(v)\, dv.$

Using symmetry L

$$f \le \left(\frac{1-a}{2}\right)$$

$$R(v-f) = 1 \,,\, C(v) > 0$$

$$\Rightarrow S(f) = \int_{-\frac{a}{2}}^{\frac{a}{2}} C(v)\, dv = 1$$

for $\frac{1-a}{2} < f < \frac{1+a}{2}$

$$S(f) = \int_{f-\frac{a}{2}}^{\frac{a}{2}} C(v)\, dv.$$

$$= \frac{\frac{\pi}{2a}}{\frac{\pi}{a}} \left[\sin\left(\frac{\pi v}{a}\right)\right]_{f-\frac{1}{2}}^{a/2}$$

$$= \frac{1}{2}\left[1 - \sin\left(\frac{\pi}{a}\left(f-\frac{1}{2}\right)\right)\right]$$

Lets check it is raised cosine or not; -

$$f = \frac{\pi}{a}\left(f - \left(\frac{1-a}{2}\right)\right)$$

$$\frac{\pi}{a}\left(f-\frac{1}{2}\right) = f - \frac{\pi}{2}.$$

for $\frac{1-a}{2} < f < \frac{1+a}{2}$, $0 < f < \frac{\pi}{2}$

$$S(f) = \frac{1}{2}\left(1 - \sin\left(f - \frac{\pi}{2}\right)\right) =$$

$$\boxed{= \frac{1}{2}(1 + \cos f)}$$

for $F > \left(\frac{1+a}{2}\right)$ $S(f) = 0$.

(c) $s(t) = x(t)\,c(t)$

$$C(f) = \int c(t)\, e^{j2\pi f t}\, dt.$$

$$= \frac{\pi}{2a} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{2}\left(e^{\frac{j\pi t}{a}} + e^{\frac{-j\pi t}{a}}\right) e^{j2\pi f t}\, dt$$

$$= \frac{\pi}{4a}\left[\frac{e^{j\left(\frac{\pi}{a} + 2\pi f\right)t}}{j\left(\frac{\pi}{a} + 2\pi f\right)} + \frac{e^{j\left(\frac{-\pi}{a} + 2\pi f\right)t}}{j\left(-\frac{\pi}{a} + 2\pi f\right)}\right]_{t=-\frac{a}{2}}^{\frac{a}{2}}$$

$$= \frac{\cos \pi a f}{1 - 4a^2 f^2}.$$

Time domain $= s(t) = \operatorname{sinc}(t)\left(\frac{\cos \pi a t}{1 - 4a^2 t^2}\right)$

Raised cosine L -

$s(t)$ decays as $1/t^3$

$$g(t) = s(t/T)$$

$$x(t) = \sum_{n} b(n)\, g(t - nT)$$

$$= \sum_{n} b(n)\, s\left(\frac{t}{T} - n\right)$$

So by fixing t we realize that

$s\left(\frac{t}{T} - n\right)$ decays roughly as $\frac{1}{t^3}$.

The convergence of $\sum_{n} \frac{1}{n^3}$ implies that sum is the

preceding eq^n.

$\therefore$ $s(t/T)$ is used for BPSK signalling at rate $1/T$

then magnitude of the transmitted waveform is always finite.

$g(t) = s(t/T)$

$$x(t) = \sum_n b(n) g(t - nT)$$

$$= \sum_n b(n) s\left(\frac{t}{T} - n\right)$$

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