# C-T-A2

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(1) Given,

an AM (DSB-FC) signal : u(t):-

$$u(t) = [20\cos(2\pi f_c t) + 2\cos(3000\pi t)\cos(2\pi f_c t) + 10\cos(6000\pi t)\cos(2\pi f_c t)]$$

& $f_c = 10^5$ Hz.

$$\Rightarrow u(t) = [20\cos(2\pi \times 10^5 t) + \cos(203000\pi t) + \cos(197000\pi t) + \frac{10}{2}\cos(206000\pi t) + \frac{10}{2}\cos(194000\pi t)]$$

$$\Rightarrow u(t) = 20\cos(200000\pi t) + \cos(203{,}000\pi t) + \cos(197000\pi t) + 5\cos(206000\pi t) + 5\cos(194000\pi t)$$

In frequency domain:-

$$u(t) = 20\cos(2\pi(100000)t) + \cos(2\pi(101500)t) + \cos(2\pi(98500)t) + 5\cos(2\pi(103000)t) + 5\cos(2\pi(97000)t)$$

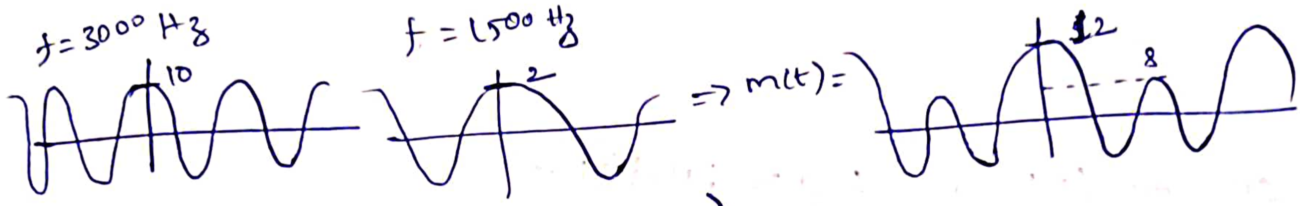
$$\Rightarrow U(f) = \frac{20}{2}\underbrace{[\delta(f-100{,}000) + \delta(f+100{,}000)]}_{1} + \frac{1}{2}\underbrace{[\delta(f-101{,}500) + \delta(f+101{,}500)]}_{2}$$

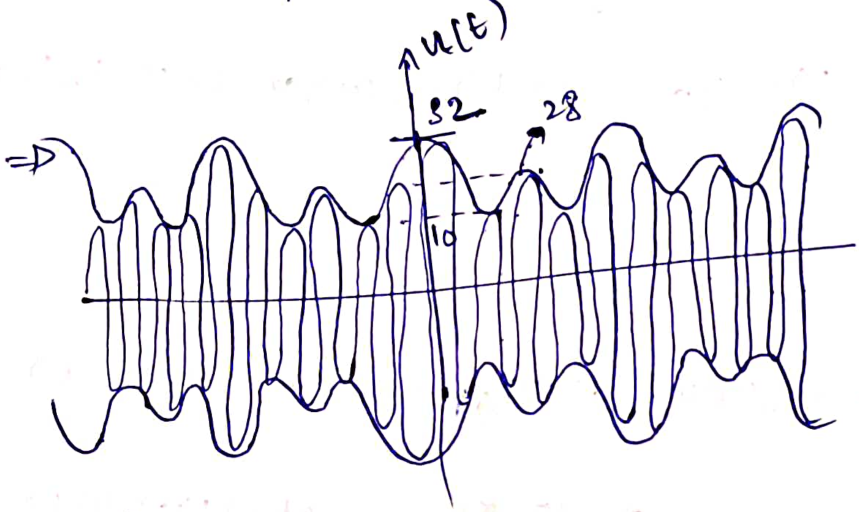
$$+ \frac{5}{2}\underbrace{[\delta(f-103{,}000) + \delta(f+103{,}000)]}_{3}$$

$$+ \frac{1}{2}\underbrace{[\delta(f-98{,}500) + \delta(f+98{,}500)]}_{4} + \frac{5}{2}\underbrace{[\delta(f-97{,}000) + \delta(f+97{,}000)]}_{5}$$

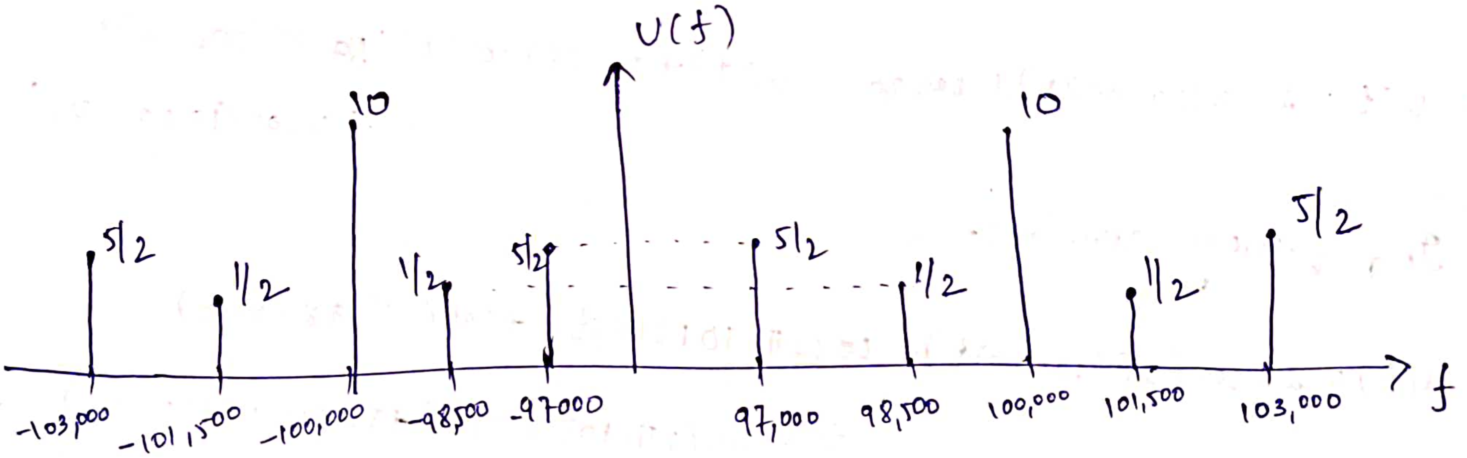
time domain diagram:-

at $t \rightarrow 0 \Rightarrow u(t) = 32$




frequency domain plot:-




(b) Power in each of frequency components:-

$$P_{10^5 Hz} = \frac{(20)^2}{2} = \frac{400}{2} = 200$$

$$P_{101500 Hz} = \frac{1^2}{2} = \frac{1}{2}$$

$$P_{103000 Hz} = \frac{5^2}{2} = \frac{25}{2} = 12.5$$

$$P_{98500 Hz} = \frac{1}{2}$$

$$P_{97000 Hz} = \frac{5^2}{2} = 12.5$$

(c) Modulation Index :-

Given DSB-FC signal is of form $(A+m(t))c(t)$

$\therefore$ $m(t) = 2\cos(3000\pi t) + 10\cos(6000\pi t)$

Now lets find whether max and min amplitudes of $m(t)$ are same or not.

As both $\cos(3000\pi t)$ and $\cos(6000\pi t)$ can't be $\boxed{-1}$ at a time so we differentiate to find min.

$$\frac{d(m(t))}{dt} = -2\sin(3000\pi t).3000\pi - 10\sin(6000\pi t)6000\pi = 0$$

$$\Rightarrow \quad -2\sin(3000\pi t)3000\pi = 10\sin(6000\pi t)6000\pi$$

$$-\cancel{2}\sin(3000\pi t) = 10\sin(6000\pi t)(\cancel{2})$$

$$\Rightarrow \frac{\sin(6000\pi t)}{\sin(3000\pi t)} = \frac{-1}{10}$$

$$\frac{2\sin(3000\pi t)\cos(3000\pi t)}{\sin(3000\pi t)} = \frac{-1}{10}$$

$$\Rightarrow \cos(3000\pi t) = \frac{-1}{20}$$

$$\Rightarrow \cos 2\theta = 2\cos^2\theta - 1$$

$$\Rightarrow \cos(6000\pi t) = 2\cos^2(3000\pi t) - 1$$

$$\Rightarrow \cos(6000\pi t) = 2\left(\frac{1}{400}\right) - 1$$

$$= \frac{1}{200} - 1 = \frac{-199}{200}$$

$\therefore$ minimum of $m(t) = 2\left(\frac{-1}{20}\right) + 10\left(\frac{-199}{200}\right)$

$$\frac{-1}{10} + \frac{-199}{20} = \frac{-2-199}{20} = \frac{-201}{20}$$

$\Rightarrow$ minimum amplitude $= -\frac{201}{20} = -10.05$

Now max when $\cos(3000\pi t)$ and $\cos(6000\pi t) = 1$

$\Rightarrow M_{max} = 10 + 2 = 12$

$\therefore$ Now modulation index when $m_{max}$ and $m_{min}$ are different is $\mu = \frac{m_{max} - \min}{2A + \max + \min} \Rightarrow \frac{12 - (-10.05)}{2(20) + 12 - 10.05}$

$$= \frac{22.05}{41.95} = 0.525625$$

$$\boxed{\therefore \mu = 0.525625}$$

(4) Power in side bands and total power :-

sidebands power :- $\frac{1}{2} |m(t)|^2$

side bands $= 2\cos(3000\pi t)\cos 2\pi f_c t + 10\cos(6000\pi t)\cos 2\pi f_c t$

$$P_{SB} = \frac{P\left(2\cos(3000\pi t) + 10\cos(6000\pi t)\right)}{2}$$

$$= \frac{\frac{4}{2} + \frac{100}{2}}{2} = \frac{52}{2} = \boxed{26W}$$

Total power = power of side bands + power of carrier $\Rightarrow P_{carrier} = \frac{A^2}{2} = \frac{(20)^2}{2} = 200W$

$\therefore$ Total power $= 200W + 26W = \boxed{226W}$

Q2)) SSB Modulation using Hilbert transform:-

Q2// Hilbert transform:-

$$x(t) \longrightarrow H\{x(t)\}$$

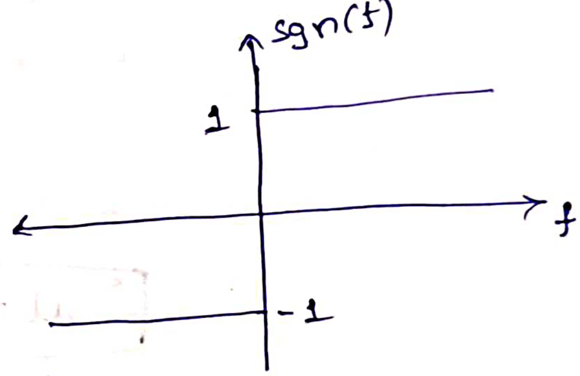
$$\Rightarrow X_h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\alpha)}{t-\alpha}\,d\alpha \Rightarrow x(t) * \frac{1}{\pi t}$$

Now $F(X_h(t)) = F\left(x(t) * \frac{1}{\pi t}\right) = \cancel{X(f)}$.

We know that from duality principle:

$$sgn(f) \overset{F.T}{\rightleftharpoons} \frac{1}{j\pi f}$$

$$\frac{1}{j\pi t} \overset{F.T}{\rightleftharpoons} sgn(-f)$$




as sgn is even function

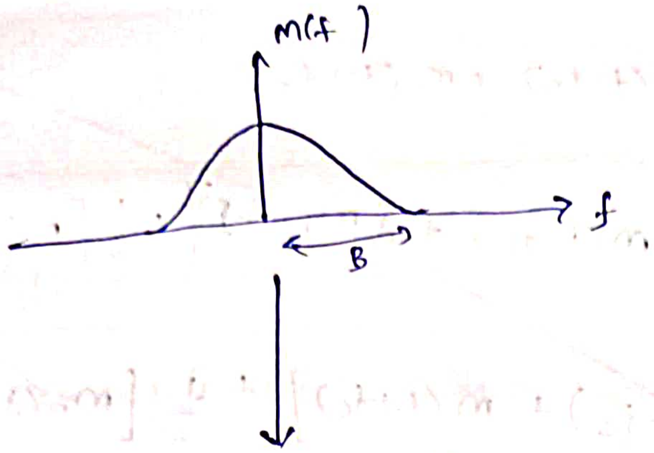
$$\Rightarrow \frac{1}{j\pi t} \overset{F.T}{\rightleftharpoons} -sgn(f)$$

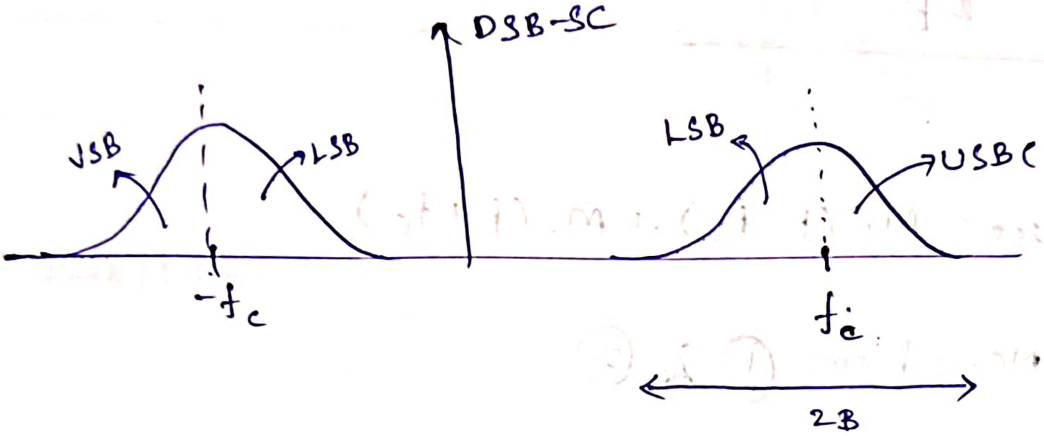
$\therefore$ Now $F\left(x(t) * \frac{1}{\pi t}\right) = X(f) \cdot (-j\,sgn(f))$

$$\Rightarrow F(X_h(t)) = \begin{cases} -j\,X(f), & f > 0 \\ j\,X(f), & f < 0 \end{cases}$$

$$= \begin{cases} X(f)e^{-j\pi/2}, & f > 0 \\ X(f)e^{j\frac{\pi}{2}}, & f < 0 \end{cases}$$

$\Rightarrow$ Hilbert transform is an ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$

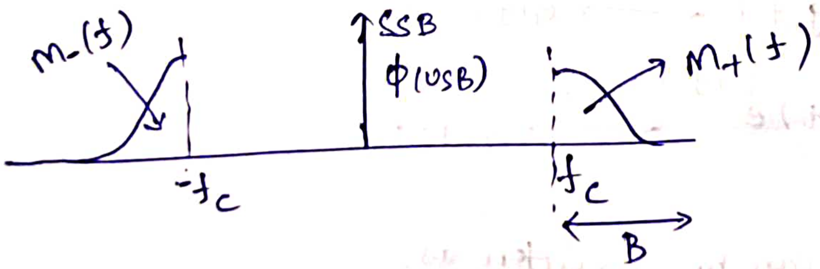



In DSB-SC 2 times of bandwidth of message signal is required to transmit. So we have introduced an SSB-modulation technique in which ~~uses~~ ~~only~~ the same bandwidth as of message signal is required to transmit.

Lets show how we actually do this:-

Firstly, I am taking ~~to~~ only USB for transmittion :-




$$M_+(f) = M(f)\,u(f) = M(f)\tfrac{1}{2}[1+sgn(f)]$$

$$= \boxed{\tfrac{1}{2}[M(f) + \underbrace{M(f)sgn(f)}_{jM_h(f)}]} \quad (1)$$

as $u(f)=\frac{1}{2}[1+sgn(f)]$

$u(-f)=\frac{1}{2}[1-sgn(f)]$

And $M_-(f) = M(f)\,u(-f) = M(f)\frac{1}{2}[1-sgn(f)] = \boxed{\frac{1}{2}[M(f) - jM_h(f)]}$ (2)

Now $\phi_{USB} = M_+(f-f_c) + M_-(f+f_c)$

Now from 1 & 2

$$\phi_{USB} = \frac{1}{2}\left[M(f-f_c) + jM_h(f-f_c)\right] + \frac{1}{2}\left[M(f+f_c) - jM_h(f+f_c)\right]$$

$$= \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] + \underbrace{\frac{j}{2}\left[M_h(f-f_c) - M_h(f+f_c)\right]}_{3}$$

Now from frequency shifting property:-

$$g(t) \xleftrightarrow{F.T} G(f)$$

$$g(t)e^{j2\pi f_0 t} \xleftrightarrow{F.T} G(f-f_0)$$

$\Rightarrow$ 3 can be written as:

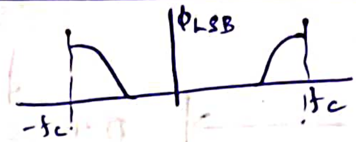
$$m(t)\underbrace{\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right]}_{\cos(2\pi f_c t)} - \frac{m_h(t)}{2j}\underbrace{\left[e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right]}_{\sin(2\pi f_c t)}$$

$$= m(t)\cos(2\pi f_c t) - m_h(t)\sin(2\pi f_c t)$$

$$\therefore \boxed{\phi_{USB} = m(t)\cos(2\pi f_c t) - m_h(t)\sin(2\pi f_c t)}$$

Similarly:- $\phi_{LSB} = \frac{1}{2}[M(f+f_c) + M(f-f_c)] - \frac{j}{2}[M_h(f+f_c) - M_h(f-f_c)]$

$$\boxed{\phi_{LSB} = m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)}$$




SSB Demodulation:-

$$\phi_{USB}(t) \longrightarrow \otimes \longrightarrow \text{LPF} \longrightarrow m(t)$$

$$\uparrow$$

$2\cos(2\pi f_c t)$
carrier signal

$$\phi_{USB}\, 2\cos(\omega_c t) = m(t)\cos(\omega_c t)2\cos(\omega_c t) - 2m_h(t)\sin(\omega_c t)\cos(\omega_c t)$$

$$= m(t)\left[1 + \cos(2\omega_c t)\right] - m_h(t)\sin(2\omega_c t)$$

$\Rightarrow$

as $\cos 2\theta = 2\cos^2\theta - 1$ & $\sin 2\theta = 2\sin\theta\cos\theta$

$$= m(t) + \underbrace{m(t)\cos(2\omega_c t)}_{\text{high freq}} - \underbrace{m_h(t)\sin(2\omega_c t)}_{\text{high freq}}$$

$$\big\downarrow \text{LPF}$$

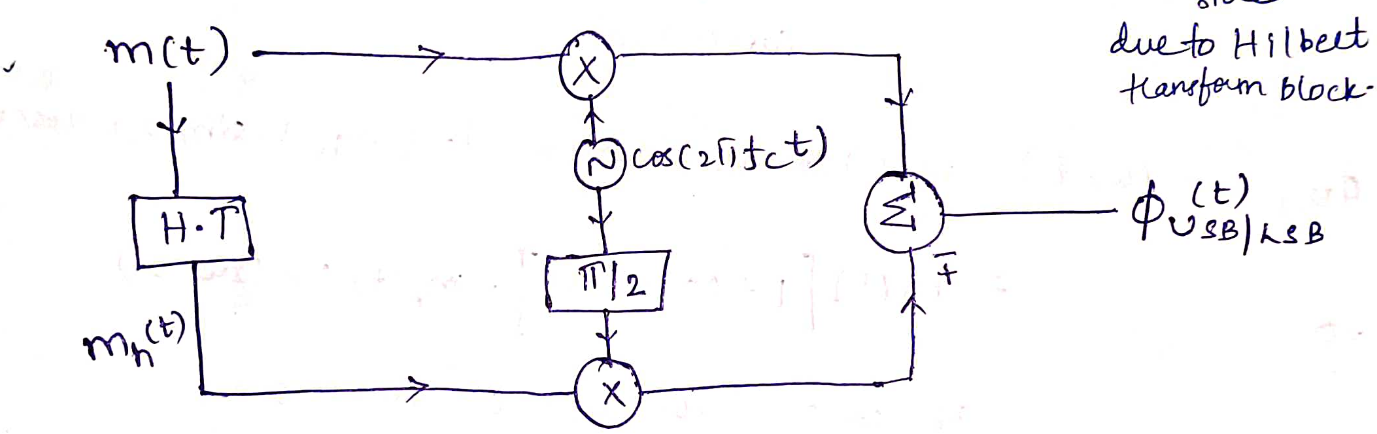
$$\boxed{m(t)}$$

$\therefore$ we have got $m(t)$.

This is the entire SSB modulation & demodulating using Hilbert Transform.

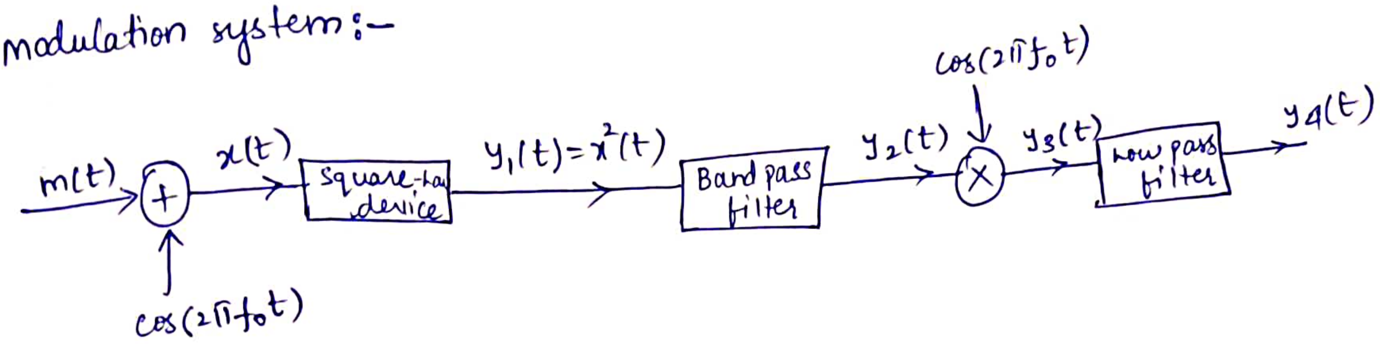
Block diagram :-

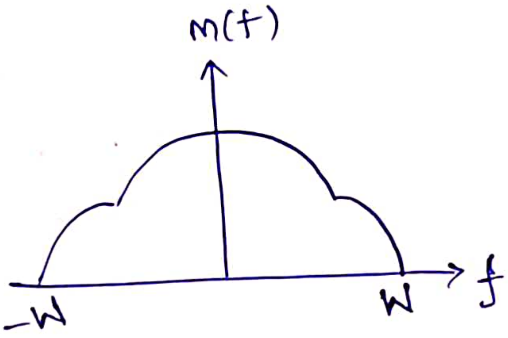
Bandwidth ⇒ $B_S = B_m$, Power = $P_S = P_m/4$, high complexity at transmitter side due to Hilbert transform block.



Q3) Given,

modulation system:-

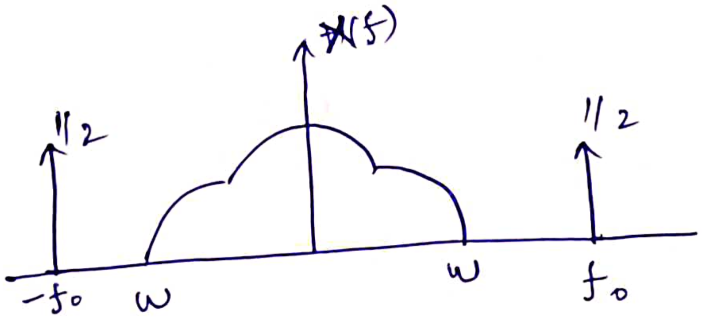



Now $x(t) = m(t) + \cos(2\pi f_0 t)$

$$X(f) = M(f) + \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

$\Rightarrow X(f) =$



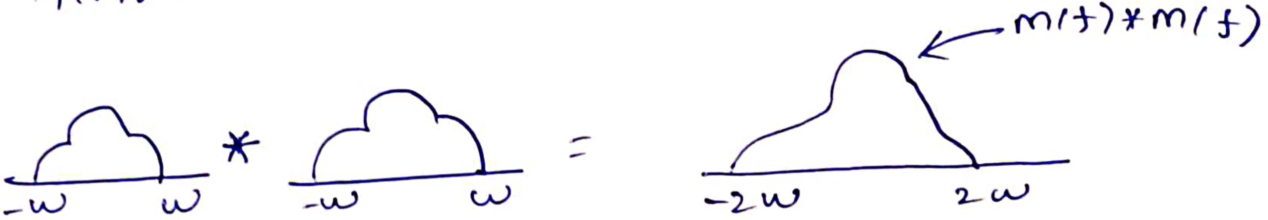

Now $y_1(t) = [m(t) + \cos(2\pi f_0)]^2$

$$= m^2(t) + \cos^2(2\pi f_0) + 2m(t)\cos(2\pi f_0 t)$$

$$= m^2(t) + 2m(t)\cos(2\pi f_0 t) + \frac{1}{2} + \frac{\cos(4\pi f_0 t)}{2}$$

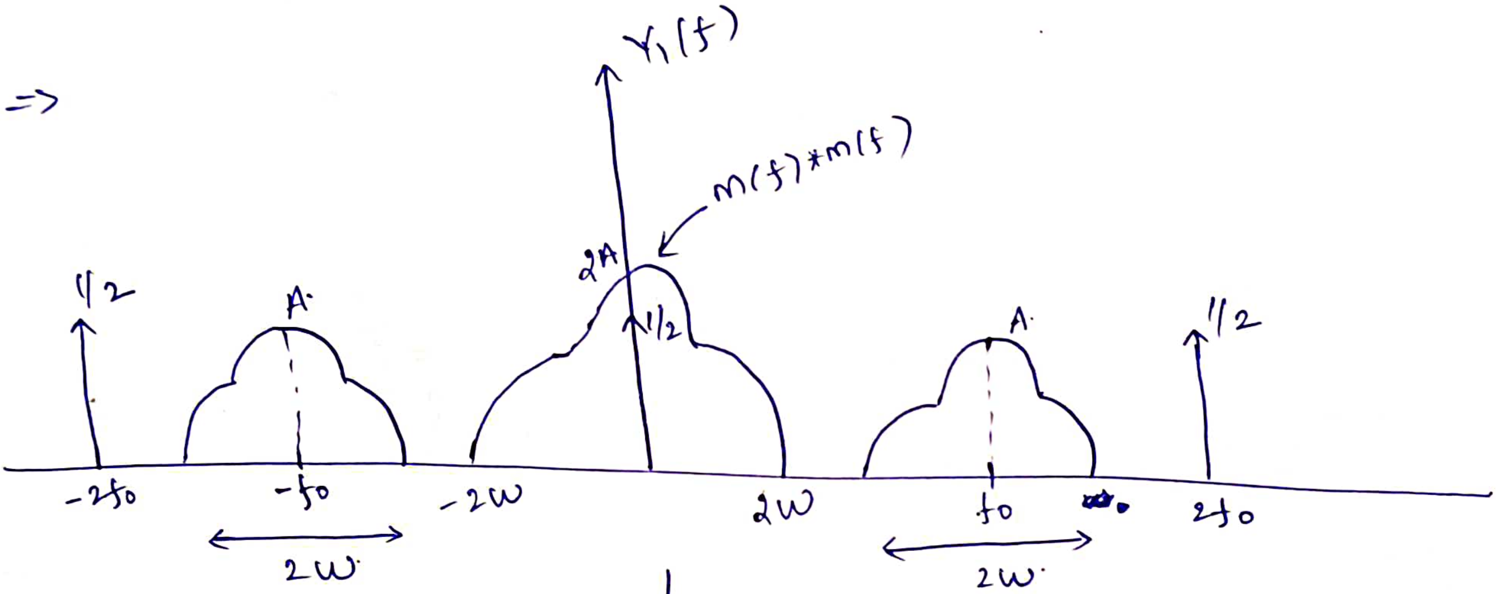
Now $Y_1(f) = M(f) * M(f) + \frac{2}{2}\left[M(f - f_0) + M(f + f_0)\right] + \frac{\delta(f)}{2} + \frac{\delta(f + 2f_0) + \delta(f - 2f_0)}{2}$

Plot of $Y_1(f)$:-




(some result of convolution we will get but it will be of 2W bandwidth)

$\Rightarrow$




Now passing this through bandpass filter:- (of bandwidth 2W, centered at fo)

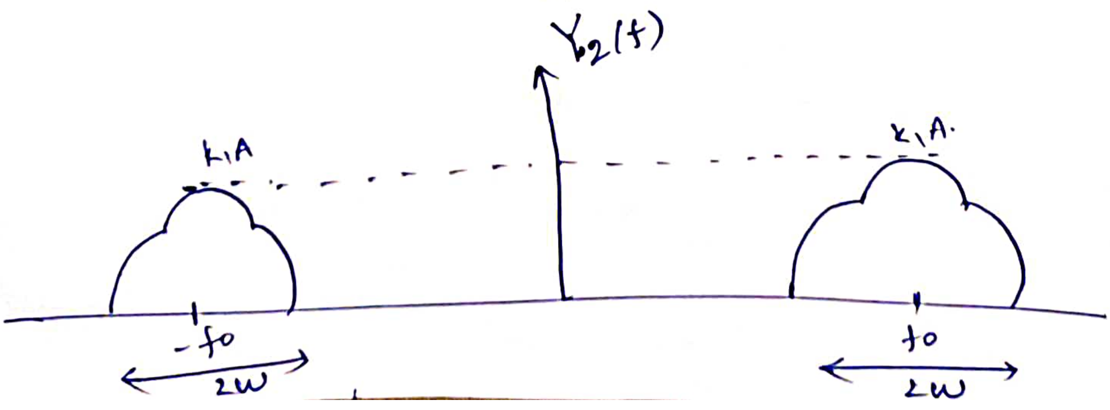
BPF

$\Rightarrow$ Middle ($M(f) * M(f)$) component and side impulse at $2f_0$ will be removed.

$\Rightarrow$


$\Rightarrow y_2(t) = 2m(t)\cos(2\pi f_0 t)$ if $k_1 = 1$

$\Downarrow$



$\Rightarrow y_2(t) = 2k_1\, m(t) \cos(2\pi f_0 t)$ {where $k_1$ = amplitude of BPF}

$y_3(t) = 2k_1 m(t) \cos(2\pi f_0 t) \cdot \cos(2\pi f_0 t)$

assuming $k_1 = 1$

{$\cos 2\theta = 2\cos^2\theta - 1$}

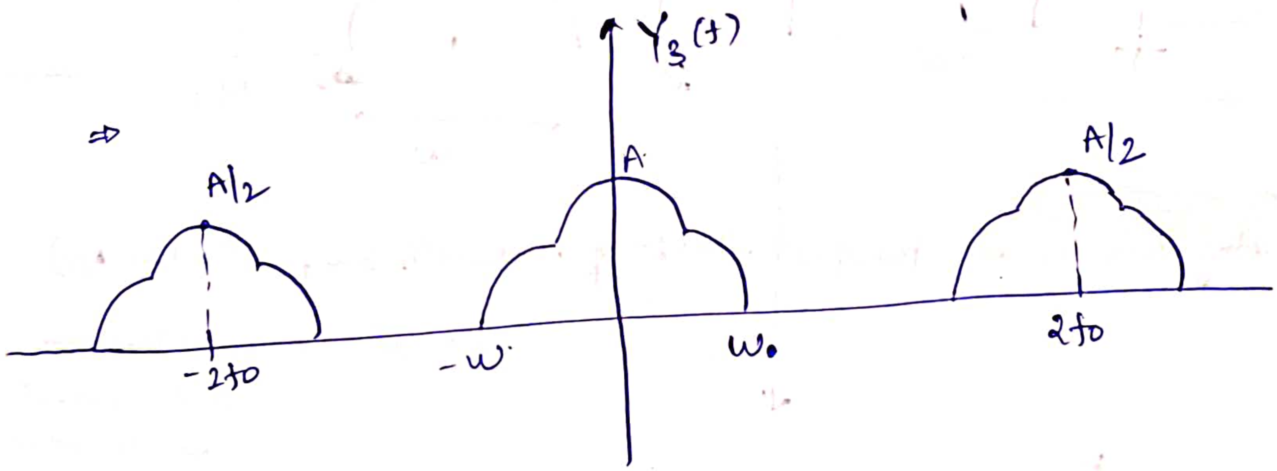
$\Rightarrow y_3(t) = 2m(t)\left[\frac{1+\cos(4\pi f_0 t)}{2}\right]$

$\Rightarrow y_3(t) = m(t)(1+\cos(4\pi f_0 t))$

$y_3(t) = m(t) + m(t)\cos(4\pi f_0 t)$

$Y_3(f) = M(f) + \frac{1}{2}\left[M(f-2f_0) + M(f+2f_0)\right]$

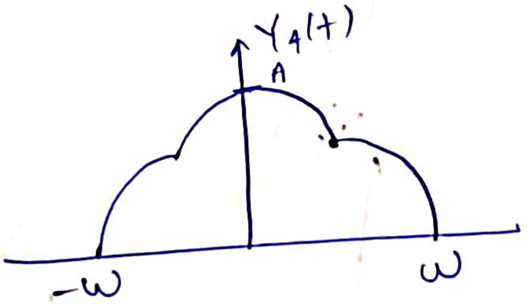
$\Rightarrow$




Now $y_3(t)$ is passed through LPF (assuming amplitude is 1) of B.W = W

$\Rightarrow m(t)\cos(4\pi f_0 t)$ component will not be visible after passing through LPF.

$\therefore Y_4(f) =$




Bandwidths :-

B.W of $x(t) = \boxed{f_0}$, B.W of $y_1(t) = \boxed{2f_0}$, B.W of $\boxed{y_2(t) = 2W}$, B.W of $\boxed{y_3(t) = f_0 + W}$, B.W of $\boxed{y_4(t) = W}$

| | |
|---|---|
| Signal | Bandwidth |
| $x(t)$ | $f_0$ |
| $y_1(t)$ | $2f_0$ |
| $y_2(t)$ | $2W$ |
| $y_3(t)$ | $f_0+W$ |
| $y_4(t)$ | $W$ |

Q4)) Given,

$$M(f) = \begin{cases} j2\pi f & , |f| < 1 \\ 0 & , \text{otherwise} \end{cases}$$

we know that $w_i(t) = \frac{d\theta_i(t)}{dt}$

$$\Rightarrow \int d\theta_i(t) = \int w_i(t)\, dt$$

$$\theta_i(t) = \int_{-\infty}^{t} w_c + k_f m(t) \,) dt$$

$$= \int_{-\infty}^{t} w_c\, dt + \int_{-\infty}^{t} 1\, m(t)\, dt \quad \langle \text{as } k_f = 1 \rangle$$

$$= \int_{0}^{t} w_c\, dt + \int_{-\infty}^{t} m(t) dt \quad \langle \text{assuming causal} \rangle$$

$$\boxed{\theta_i(t) = w_c t + \int_{-\infty}^{t} m(t) dt}$$

(a) Now: $m(t) = \int_{-\infty}^{\infty} M(f)\, e^{j2\pi f t}\, df$

$$= \int_{-1}^{1} j2\pi f\, e^{j2\pi f t}\, df$$

$$= 2\pi j \left[ f \frac{e^{j2\pi f t}}{j2\pi t} - \int \frac{e^{j2\pi f t}}{j2\pi t}\, df \right]$$

$$= 2\pi j \left[ f \frac{e^{j2\pi f t}}{2\pi t} - \frac{e^{j2\pi f t}}{(j2\pi t)^2} \right]_{-1}^{1}$$

$$= 2\pi j \left[ \frac{e^{j2\pi t} + e^{-j2\pi t}}{j2\pi t} + \left( \frac{e^{j2\pi t} - e^{-j2\pi t}}{(2\pi t)^2} \right) \right]$$

$$= 2\pi j \left[ \frac{\cos(2\pi t)}{j\pi t} + \frac{j \sin(2\pi t)}{2\pi^2 t^2} \right]$$

On solving above Integral we get :

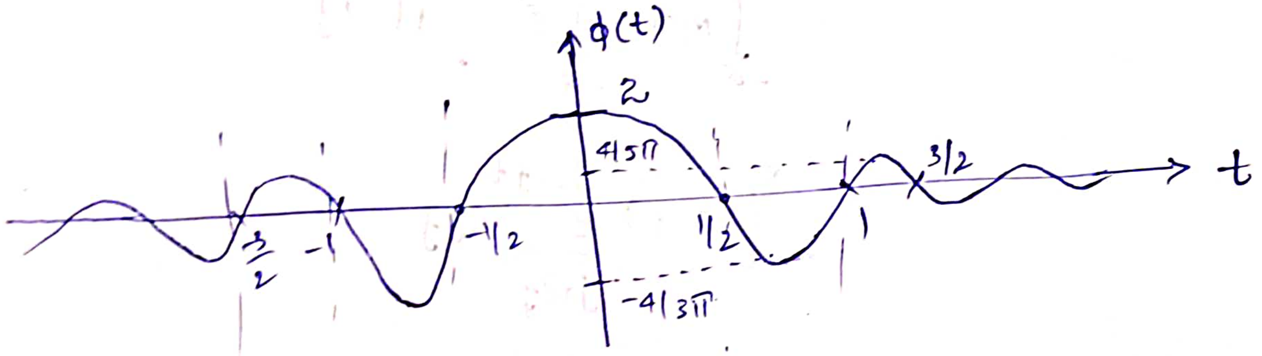
$$\frac{\sin(2\pi t)}{\pi t} = 2\,\text{sinc}(2\pi t)$$

$$\Rightarrow \phi(t) = 2\,\text{sinc}(2\pi t)$$

at $t = 0 = 2$

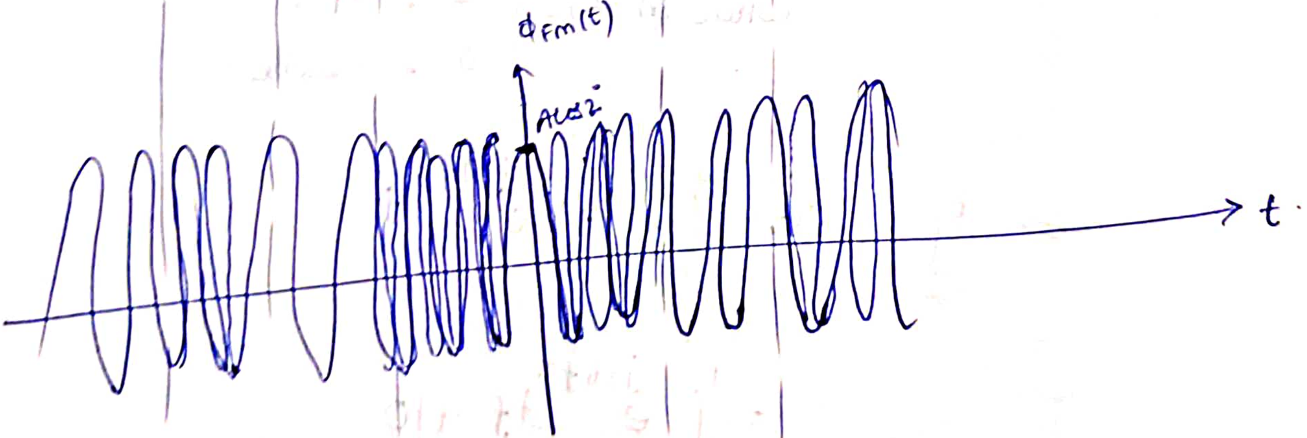
$t = 3/4 = \frac{-4}{3\pi}$

$t = \frac{5}{4} = \frac{4}{5\pi}$




(b) $\phi_{FM}(t) = A\cos(2\pi f_c t + \phi(t))$

$= A\cos(2\pi f_c t + 2\,\text{sinc}(2\pi t))$




when $\text{sinc}(2\pi t)$ has high amplitude then $\phi_{FM}$ has high frequency.

M-2 proof of Integral in (a) :-

M-2) $m(t) = \int_{-\infty}^{\infty} M(f) e^{j2\pi f t} df$

Now $\int_{-\infty}^{t} m(t)\, dt = \int_{-\infty}^{t} \int_{-\infty}^{\infty} M(f) e^{j2\pi f t} df\, dt$

$= \int_{-\infty}^{\infty} M(f) \int_{-\infty}^{t} e^{j2\pi f t} dt \cdot df$

$= \int_{-\infty}^{\infty} \underbrace{M(f) \frac{e^{j2\pi f t}}{j2\pi f}}_{M^*(f)} df$

Now we know that :

$\int_{-\infty}^{t} m(t) dt \xrightarrow{F.T} M^*(f)$ let $M^*(f) = \frac{M(f)}{j2\pi f}$

where $M^*(f) = \begin{cases} 1, & |f| < 1 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow \int_{-\infty}^{t} m(t) dt = \int_{-\infty}^{\infty} M^*(f) e^{j2\pi f t} df$

$= \int_{-1}^{1} e^{j2\pi f t} df + \int 0$

$= \left. \frac{e^{j2\pi f t}}{j2\pi t} \right|_{-1}^{1} = \frac{1}{2\pi j t} \left[ e^{j2\pi t} - e^{-j2\pi t} \right]$

$= \frac{\sin(2\pi t)}{\pi t} = \boxed{2\operatorname{sinc}(2\pi t)}$

(c) magnitude of instantenous frequency deviation from carrier at $t = 1/4$

$$\Rightarrow \omega_i(1/4) = \frac{d\theta}{dt}\Big|_{t=1/4}$$

and $$\frac{d\theta}{dt} = \frac{d}{dt}\left(2\pi f_c t + \frac{\sin(2\pi t)}{\pi t}\right)$$

$$= 2\pi f_c + \frac{2\pi^2 t\cos(2\pi t) - \sin(2\pi t)\cdot\pi}{\pi^2 t^2}$$

$$= 2\pi f_c + \frac{2\pi^2 t\cos(2\pi t) - \pi\sin(2\pi t)}{\pi^2 t^2}$$

Now $$\frac{d\theta}{dt}\Big|_{t=1/4} = 2\pi f_c + \frac{\frac{\pi^2}{2}\cos\left(\frac{\pi}{2}\right) - \pi\sin\left(\frac{\pi}{2}\right)}{\frac{\pi^2}{16}}$$

$$= 2\pi f_c - \frac{\pi\times 16}{\pi^2}$$

$$= 2\pi f_c - \frac{16}{\pi}$$

Now deviation $= f_i - f_c$

$$\omega_i - \omega_c = 2\pi f_c - \frac{16}{\pi} - 2\pi f_c = \boxed{\frac{-16}{\pi} = -5.0955 \text{ rad/sec}}$$

$$\Rightarrow \text{magnitude of freq deviation at } t = \frac{1}{4} = \boxed{\frac{16}{\pi} = 5.095 \text{ rad/sec}}$$

$$\Rightarrow \Delta f = \frac{8}{\pi^2} \text{ Hz}$$

(d) Bandwidth of $u(t)$

from Carson's rule :-
(approximation) $= 2(\Delta f + B)$

from signal we know that $B=1$

and $\Delta f = \frac{8}{\pi^2}$

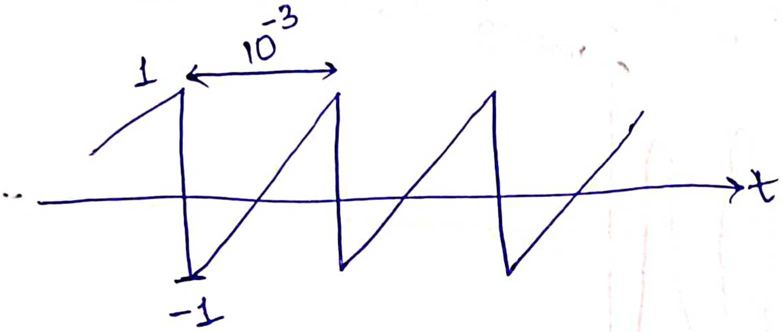
$= 2\left(\frac{8}{\pi^2} + 1\right) = \frac{16}{\pi^2} + 2 = 1.6227 + 2$

$= 3.622$

$\Rightarrow$ $\boxed{\text{Bandwidth of } u(t) = 3.622 \text{ Hz}}$

Q5)) Given,

modulating signal $m(t)$ :-




ⓐ we have to sketch $\phi_{FM}(t)$ and $\phi_{PM}(t)$ for signal $m(t)$ if :

$\omega_c = 2\pi \times 10^6$ , $k_f = 2000\pi$ and $k_p = \frac{\pi}{2}$

As $\omega_i = \frac{d\theta}{dt}$ and $\phi_{FM} = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(t)\, dt]$

$= d[\omega_c + k_f ]_{\infty}^{t} m(t) dt$

$\Rightarrow \omega_i = \omega_c + k_f m(t)$

$\Rightarrow 2\pi f_i = \omega_c + k_f m(t)$

$\Rightarrow f_i = \frac{2\pi \times 10^6 + 2000\pi m(t)}{2\pi}$

$\Rightarrow \boxed{f_i = 10^6 + 1000 m(t)}$

Now from given plot :

$$f_{i(min)} = 10^6 + 1000\,m(t) \quad , \quad m(t) \in [-1, 1]$$

$$\Rightarrow f_{i(min)} = 10^6 - 1000$$

$$= (1000 - 1)10^3$$

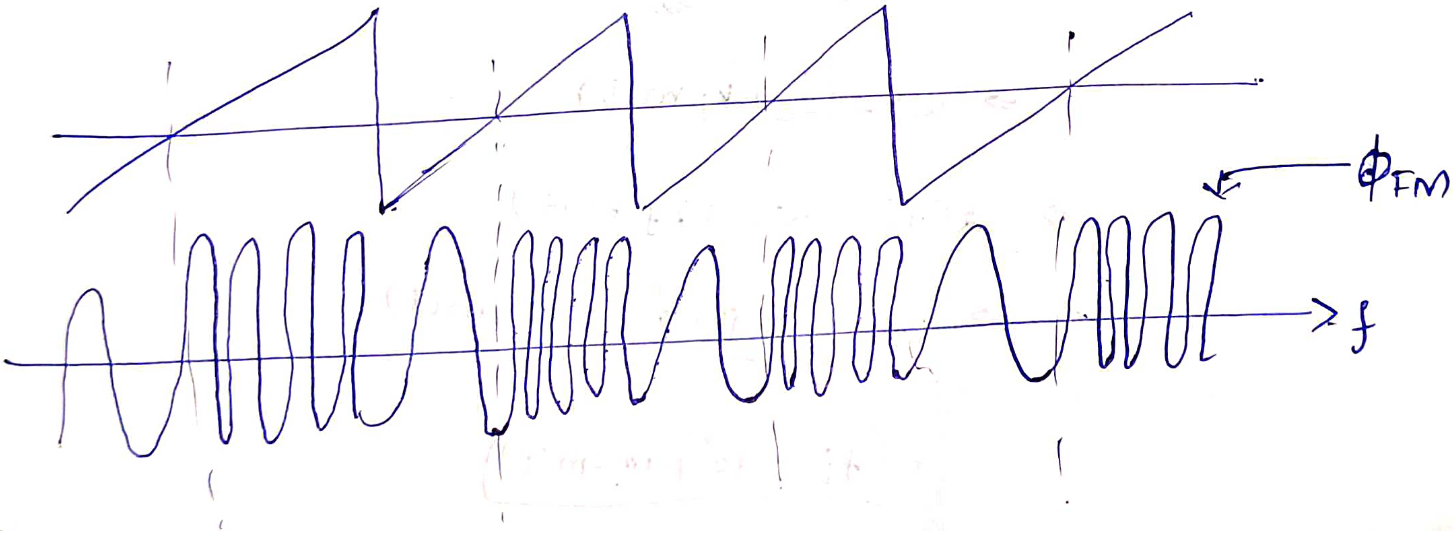
$$\Rightarrow \boxed{f_{i(min)} = 999 \text{ KHz}}$$

and $f_{i(max)} = 10^6 + 1000 = 10^3 (1000 + 1)$

$$\Rightarrow \boxed{f_{i(max)} = 1001 \text{ KHz}}$$

The Instantenous frequency of FM wave increases from 999 kHz to 1001 kHz in $10^{-3}$ sec.

→ clearly as $f_i = 10^6 + 1000m(t)$ ⇒ when $m(t)$ is increasing, frequency is increasing and when $m(t)$ is decreasing, frequency is decreasing.



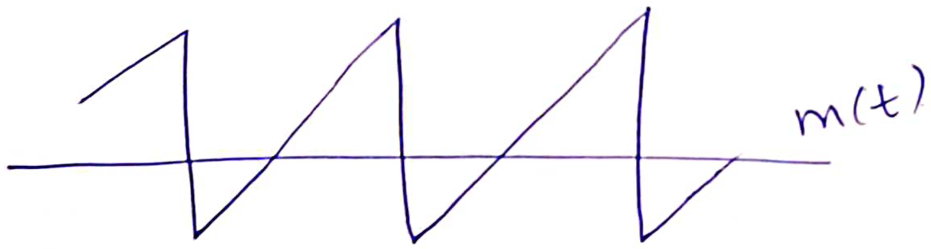
PM :- m(t) has jump discontinuity over $\frac{10^{-3}}{2}$ to $\frac{10^{3}}{2}$

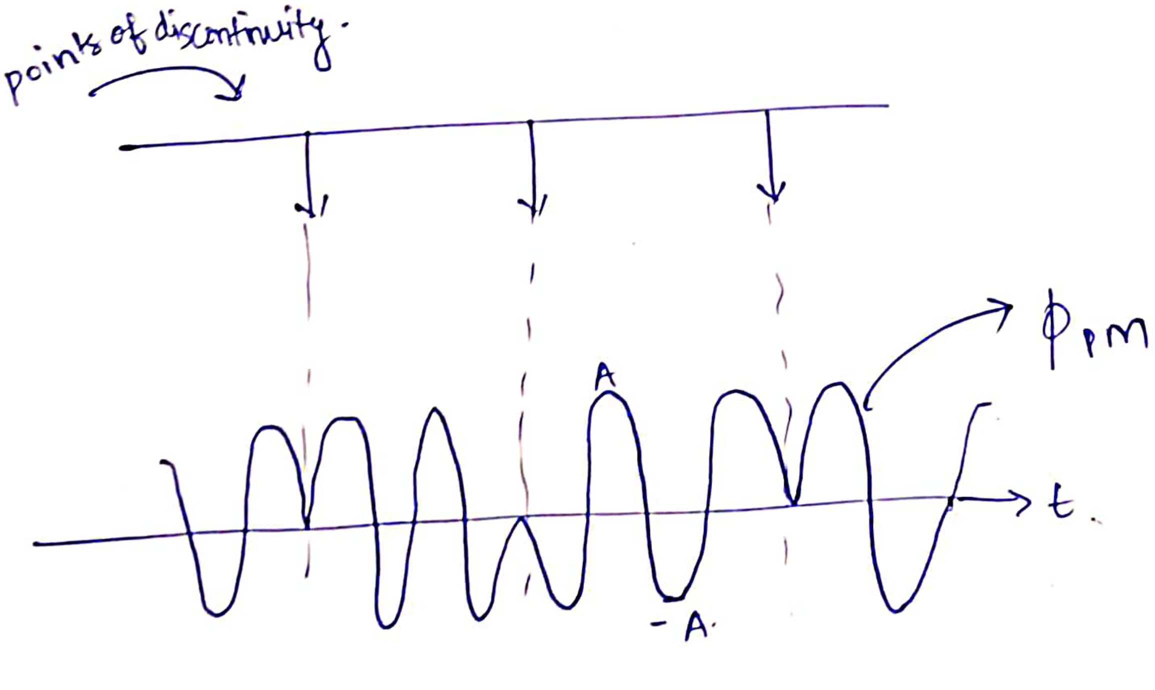
$$\psi_{PM} = \cos\left(2\pi(10^6)\pi + \frac{\pi}{2} m(t)\right] = \cos\left(2\pi(10^6)t + 1000\pi t\right]$$

At the discontinuity, jump is md. ⇒ $k_p m_d = \pi$ ⇒ phase discontinuity

∴ Carrier frequency is constant at $10^6 + 500$ Hz. But at points of discontinuity we observe a phase discontinuity of $2\pi$.

For $k_p > \pi$, phase discontinuity will be greater than $2\pi$ by which we dont get a proper demodulated signal.



Reason for $k_p < \pi$ :-

In phase Modulation :

$k_p m(t) \in [-\pi, \pi]$, as exceeding this Limit will give multiple phase for same value of t.

$\Rightarrow k_p m(t) \in [-\pi, \pi]$ is must required

and $m(t) \in [-1, 1]$ for given signal

$\Rightarrow k_p m(t) \in [-k_p, k_p]$ is Valid always

$\boxed{\therefore k_p < \pi}$ is the condition required in this case.

In brief :-

For FM :-

as $m_p = \pm 1$

$|k_f m_p| = 2000\pi$

as $|k_f m_p| >> 1$ [WBFM]

$\Rightarrow 2000\pi >> 1$.

For PM :- max freq deviation $= \Delta\omega = k_p \dot{m}_p$

for WBFM $\Rightarrow k_p \dot{m}_p < 4\pi B$

$\Rightarrow k_p << \frac{4\pi B}{2\dot{m}_p}$

$\Rightarrow k_p << \frac{4\pi B}{2\left(\frac{2}{10^{-3}}\right)} \Rightarrow k_p << \frac{4\pi \times 10^{3} \times 10}{4}$

$\Rightarrow \boxed{k_p << \pi}$

(b) Base band signal bandwidth $\Rightarrow B = 5000\,Hz$

For FM :- $\Delta f = \dfrac{k_f m_p}{2\pi} = \dfrac{2000\pi}{2\pi} = 1\,kHz$

$\Rightarrow B_{FM} = 2(\Delta f + B) = 2(1+5) = 12\,kHz$

$\Rightarrow \boxed{B_{FM} = 12\,kHz}$

$B_{pm} = \dfrac{\Delta f}{B_m} = \dfrac{k_p m_p}{B_m \cdot 2\pi} = \dfrac{\cancel{\pi}/2}{2\cancel{\pi}\cdot 5\times10^{3}} = \dfrac{1}{20}\times10^{-3}$

$= 0.5\times10^{-4}$

$\Rightarrow B_{pm} = 2B_m(1+\beta)$

$= 2\times5\times10^{3}(0.5\times10^{-4}+1)$

$\Rightarrow \boxed{B_{pm} \approx 10\,kHz}$