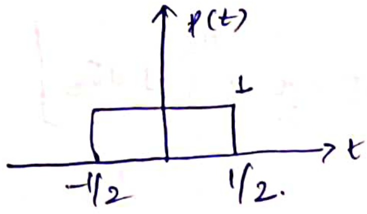
Assignment-3

Name :- Shaik Affan Adeeb
Roll No :- 2022102054

① Given,

$p(t) = I_{[-\frac{1}{2},\frac{1}{2}]}(t)$ denote rectangular pulse.



$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

$m(t) \longrightarrow$ FM modulator $\longrightarrow y(t) = 20\cos(2\pi f_c t + \phi(t))$

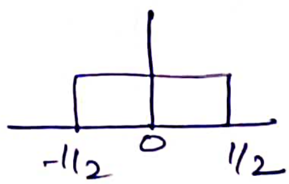
where $\phi(t) = 20\pi\int_{-\infty}^{t} m(\tau)\,d\tau + a$

where a is chosen such that $\phi(0)=0$

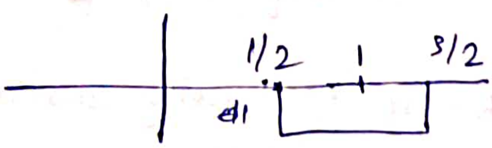
Now ~~m(t) = 1/2~~

for m(t) at n=0

$$m(t) = \sum_{n=-\infty}^{\infty} p(t-0) = p(t) =$$

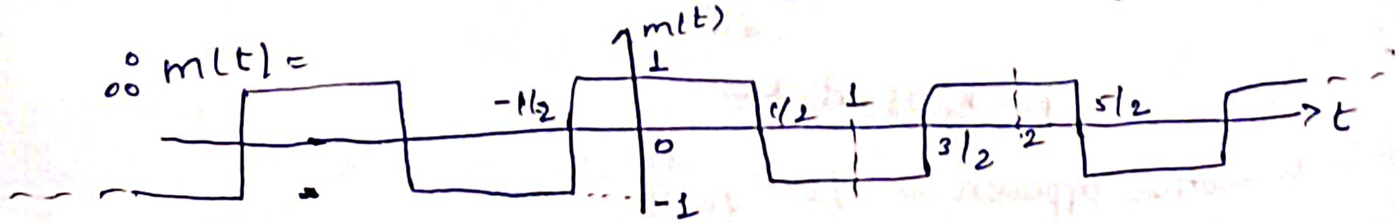


m(t) at n=1



similarly for n<0 we get mirror image w.r.t y axis

$\therefore$ m(t) =



Now $\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)d\tau + a$

We take $m(t)$ for 1 cycle & repeat it.

In $-1/2$ to $1/2$, $m(\tau)=1$

In $1/2$ to $3/2$, $m(\tau) = -1$

$$\Rightarrow \phi(t) = 20\pi\left[\int_{-1/2}^{1/2} 1\, d\tau + \frac{a}{20\pi} + \int_{1/2}^{3/2} -1\, d\tau\right]$$

$= 20\pi\left[t\right] + a$ for $-\frac{1}{2}$ to $1/2$

and $-20\pi(t) + a$ for $1/2$ to $3/2$

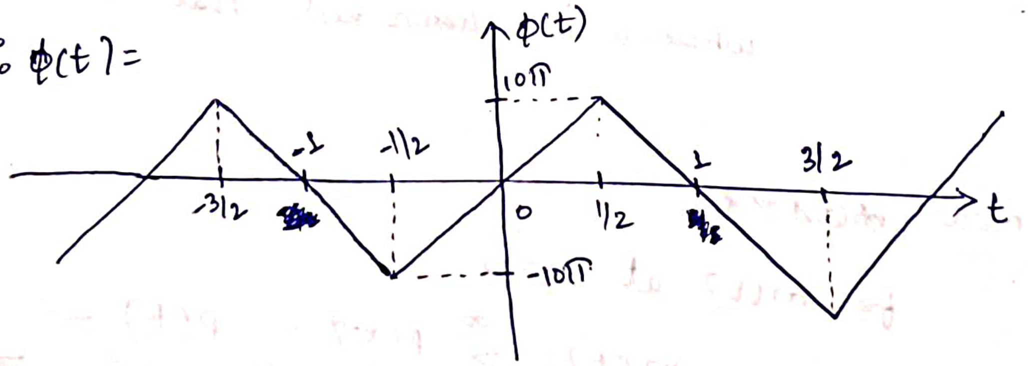
and this repeats as $m(t)$ is periodic.

$\Rightarrow$ and $\phi(0) = 20\pi(0) + a = 0$

$\Rightarrow a = 0$ $\quad\Rightarrow \phi(t) = 20\pi t$ for $-1/2$ to $1/2$

$= -20\pi t$ for $1/2$ to $3/2$

$\therefore \phi(t) =$



(b) Now, given B.W of $m(t) = 2$

B.W of $u(t)$ using Carsons formula $= 2(\Delta f + B)$

$$\Delta f = \frac{k_f m(t)_{max}}{2\pi}$$

but k_f is not known.

so another approach $\Rightarrow f(t) = \frac{1}{2\pi}\frac{d\phi}{dt} \Rightarrow$

Now from the plot of $\phi(t)$:-

$$\frac{d\phi}{dt} = \text{slope} = \pm 20\pi$$

$$\Rightarrow f(t) = \pm 10$$

$$\therefore \Delta f_{max} = 10$$

$$\Rightarrow \text{BW of } u(t) = 2(10+2) = \boxed{24\,Hz}$$

(c) we know that

$$\text{wide Band FM signal} = \phi_{FM}(t) = A\left[\cos w_c t - k_f a(t)\sin w_c t - \frac{k_f^2 a^2(t)\cos w_c t}{2!} + \cdots\right]$$

where $a(t) = \int_{-\infty}^{t} m(\tau)\, d\tau$ and B.W of $a(t)$ = B.W of $m(t)$

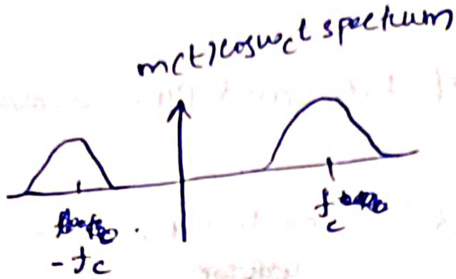
Now B.W of $m(t) = \frac{1}{2}$ Hz (as its period is 2 sec)

$\Rightarrow$ B.W of $a(t) = \frac{1}{2}$ Hz (Let this be B Hz)

and B.W of $a^2(t), a^3(t) \ldots\ldots = 2(\frac{1}{2}), 3(\frac{1}{2}), \ldots\ldots$

$(2B), (3B), \ldots\ldots$

Now when we take Fourier transform of $\phi_{FM}(t)$: (As we multiply with $\sin w_c t$ or $\cos w_c t$ the spectrum shifts by $\pm w_c$)

$m(t)\cos\omega_c t \xleftrightarrow{F.T}$ 

If Given $M(f) =$ 

similarly for $m(t)\sin\omega_c t$ also.

Now :- The major frequency components at at

$\omega_c, \omega_c \pm B, \omega_c \pm 2B, \omega_c \pm 3B \ldots$

$\Rightarrow$ spectrum has discrete components at (where $B = \frac{1}{2}$ Hz)

so :- $f_c, f_c \pm \frac{1}{2}, f_c \pm 1, f_c \pm \frac{3}{2} \ldots$

are main frequencies observed in spectrum
or discrete components

$\Rightarrow$ we get nonzero powers at $f_c + 0.5$, $f_c + 1$ only.

$\Rightarrow$ $\therefore$ when BPF of $f_c + 0.5$ is used then $k_f a(t) \sin w_c t$

we get as output and hence power will be nonzero.

when BPF of $f_c + 0.75$: There is no term in spectrum with $f_c + 0.75$

as major frequency component. so $\therefore$ There will no output & hence zero power at output.

when BPF of $f_c + 1$ is used then $\frac{k_f^2}{2} a^2(t) \cos w_c t$ term is

produced as output. so $\therefore$ power will be nonzero.

$\therefore$ for $\alpha = 0.5, 1$ power is non-zero.

(2) Given,

signal Bandwidth $= B_m = 10\,kHz$

we have to find minimum channel Bandwidth with $SNR = 10\,dB$

we know that :

$$\frac{S_o}{N_o} = \begin{cases} \dfrac{3L^2}{[\ln(1+\mu)]^2}, & \text{non-uniform} \\[2ex] \dfrac{3\,\widetilde{m^2(t)}\,L^2}{m_p^2}, & \text{uniform} \end{cases}$$

Lets assume $m(t) = 5\cos(2\pi f_m t)$ where $Pk\text{-}Pk = 10$

Let No of bits transfered per sample $= n$ $\hookrightarrow$ B.W of $m(t)$

$\Rightarrow$ channel bandwidth $= n(1.5)(2\frac{f_m}{2})$

$= n(1.5)(10k) = 15n$ kbps

Also for uniform, $SNR = \frac{3L^2 \widetilde{m^2}(t)}{m_p^2}$, power $= \widetilde{m^2}(t) = \frac{m_p^2(t)}{2}$.

$$\Rightarrow SNR = \frac{3L^2 \left(\frac{5^2}{2}\right)}{25} = \frac{3L^2}{2}.$$

$$\Rightarrow (SNR)_{dB} = 10\log_{10} c + 6n$$

$$\{ c = \frac{3}{2} \}$$

$$\Rightarrow 10\log\left(\frac{3}{2}\right) + 6n \geq 10 \quad \{\text{as asked to find minimum so } \geq 10\}$$

$$\Rightarrow 6n \geq 10 - 10\log_{10}(1.5)$$

$$\Rightarrow n \geq \frac{10}{6}(1 - \log(1.5))$$

$$\Rightarrow n \geq \frac{5}{3}(1 - \log_{10} 1.5) \quad \{\log 1.5 = 0.17\}$$

$$\Rightarrow \boxed{n \geq 1.383} \Rightarrow \text{we need } n = 2$$

$\boxed{\text{and min channel Bandwidth} = 15 \times 2 = 30 \text{ kbps}}$

Now for Non uniform:-.

$$SNR = \frac{3L^2}{[\ln(1+\mu)]^2} = \frac{3L^2}{(\ln(101))^2} = \frac{3L^2}{21.30} = \underbrace{0.14085}_{\text{'c'}} L^2$$

$$\Rightarrow SNR_{dB} = 10\log_{10}(0.14085) + 6n \geq 10 \Rightarrow 6n \geq 10(1 - \log_{10}(0.14085))$$

$$\Rightarrow n \geq \frac{5}{3}(1 - \log_{10}(0.14085))$$

$$\Rightarrow n \geq 3.0854$$

$$\Rightarrow \boxed{n = 4}$$

$\therefore$ For non uniform $= 15 \times 4 = \boxed{60 \text{ kbps}}$

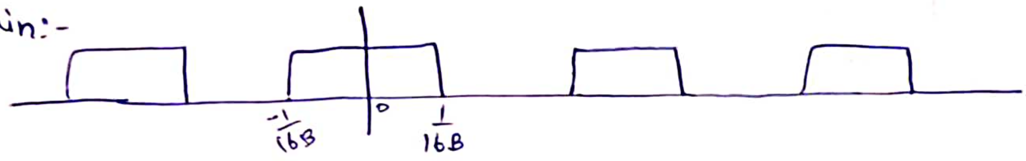
∴ Minimum channel Bandwidth for:
→ Uniform quantizer = 30kbps
→ Non-uniform quantizer = 60kbps

③ Given,

Q3/ $g(t) \rightarrow$ Band limited to B Hz is sampled by periodic pulse $P_{T_s}(t)$ made of rectangular pulse of width $\frac{1}{8B}$ second, repeating at 2B pulses/sec.

we have to show that $\bar{g}(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos(4n\pi Bt)$

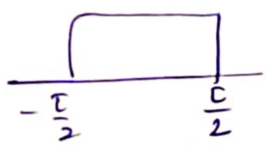
pulse train:-



Now lets find its fourier series coefficients:-

Lets find C_n:

$$C_n = \frac{1}{T_0} \int_{T_0} x(n)\, e^{-jnw_0 t}$$



$$= \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-jnw_0 t}\, dt.$$

Let $-jnw_0 t = \theta$

$\Rightarrow dt = \frac{-d\theta}{jnw_0}$

$$= \frac{1}{T_0} \int_{-\tau/2}^{\tau/2} \frac{-e^{\theta}\, d\theta}{jnw_0}$$

when $t = -\tau/2 \Rightarrow \theta = jnw_0 \tau/2$

$t = \tau/2 \Rightarrow \theta = -jnw_0 \tau/2.$

$$= \frac{1}{T_0} \int_{jnw_0\tau/2}^{-jnw_0\tau/2} e^{\theta} \left(\frac{-1}{jnw_0}\right) d\theta$$

$$\Rightarrow \frac{-1}{(jnw_0)T_0} \left[e^{-jnw_0\tau/2} - e^{jnw_0\tau/2} \right] = \frac{-1}{jnw_0 T_0} \left[-2j\sin\left(nw_0 \frac{\tau}{2}\right) \right]$$

$$\Rightarrow C_n = \frac{\tau}{T_0} \operatorname{sinc}\left(\frac{nw_0\tau}{2}\right)$$

$$\Rightarrow \boxed{c_n = \frac{\tau}{T_0} \operatorname{sinc}\left(\frac{n w_0 \tau}{2}\right)}$$

Here $\tau = \frac{1}{8B}$ and $T_0 = \frac{1}{2B}$

$w_0 = 2\pi f_0$, $f_0 = 2B$.

$$\Rightarrow c_n = \frac{1}{4} \operatorname{sinc}\left(2n B\pi \frac{1}{8B}\right)$$

$$\Rightarrow \boxed{c_n = \frac{1}{4} \operatorname{sinc}\left(\frac{n\pi}{4}\right)}$$

Now pulse train $= \sum_{n=-\infty}^{\infty} c_n e^{jnw_0 t}$

$x(t)$

In polar form :

$$x(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(nwt)$$

$\left\langle d_n = \sqrt{A_n^2 + B_n^2} \right\rangle$

$= \sqrt{d_n^2 \cos^2\phi + d_n^2 \sin^2\phi}$

$= d_n$

on substitution:

$$d_0 = c_0 \quad , d_n = 2c_n$$

$$c_0 = \frac{1}{4} \quad \langle \text{as } \operatorname{sinc}(0) = 1 \rangle$$

$(w_0 = 2\pi f_0 , f = 2B)$

$$\Rightarrow x(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1}{2} \operatorname{sinc}\left(\frac{n\pi}{4}\right) \cos(n(2\pi \cancel{f}(2B))t)$$

$$\Rightarrow x(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1}{2} \operatorname{sinc}\left(\frac{n\pi}{4}\right) \cos(2\pi n(2B)t)$$

pulse train

$$\therefore \bar{g}(t) = \frac{g(t)}{4} + \sum_{n=1}^{\infty} \frac{1}{2} \operatorname{sinc}\left(\frac{n\pi}{4}\right) \cos(4\pi nBt)\, g(t)$$

$\langle \text{as } \bar{g}(t) = x(t) g(t) \rangle$

$$\Rightarrow \bar{g}(t) = \frac{g(t)}{4} + \sum_{n=1}^{\infty} \frac{1}{\cancel{2}} \left(\frac{\overset{2}{\cancel{4}}}{n\pi}\right) \sin(n\pi) \cos(4n\pi Bt)\, g(t)$$

$$\Rightarrow \boxed{\bar{g}(t) = \frac{g(t)}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(n\pi) \cos(4\pi nBt)\, g(t)}$$

Hence proved

Now we have to show that $g(t)$ can be recovered by passing $\bar{g}(t)$ through LPF of B.W = B and gain = 4.

Now in $\bar{g}(t) = \underbrace{\frac{1}{4} g(t)}_{\text{B.W = B component}} + \underbrace{\sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos(4\pi B n t) g(t)}_{\text{has B.W = 2B component}}$

$\therefore$ while passing this through ideal LPF of B.W = B $\Rightarrow$ freq component of 2B is restricted

$\therefore$ we get only that component which has freq B

$\therefore$ at out we get $\frac{1}{4} g(t)$ multiplied by gain 4

$\therefore$ output of filter $= \frac{1}{4} \times g(t) \times 4 = \boxed{g(t)}$

Hence proved.

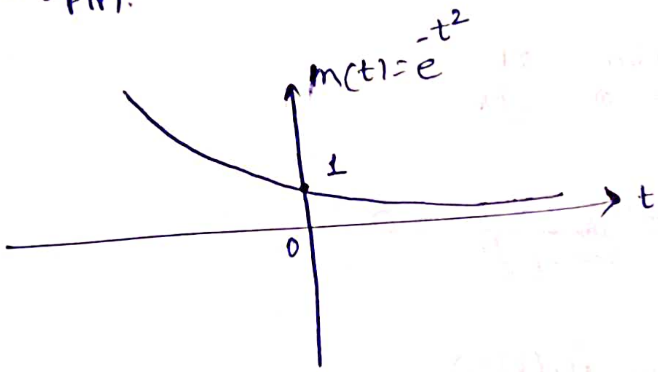
Q4) 1)

Given,

$m(t) = e^{-t^2}$, $f_c = 10^4$ Hz. (told to take this as B.W of m(t)) (in wattsapp grp)

$k_f = 6000\pi$, $k_p = 8000\pi$

(a) Δf_{FM} & Δf_{PM}.



→ assuming signal starts from t = 0

$\Rightarrow m_{max} = 1$

$m_{min} = 0$

$$\therefore \Delta f_{FM} = \frac{k_f(1-0)}{2(2\pi)} = \frac{6000\pi}{4\pi} = 1500 \text{ Hz}.$$

$$\Rightarrow \boxed{\Delta f_{FM} = 1500 \text{ Hz}}$$

$$\Delta f_{PM} = k_p\left[\frac{\dot{m}(t)_{max} - \dot{m}(t)_{min}}{2\cdot 2\pi}\right] \Rightarrow \dot{m}(t) = -2te^{-t^2}$$

Now we have to find max & min of $-2te^{-t^2}$

Lets differentiate it for finding points of max & min :-

$$\Rightarrow \frac{d}{dt}(-2te^{-t}) = 0 \Rightarrow -2\left[t(-2t)e^{-t^2} + e^{-t^2}\right] = 0 \Rightarrow e^{-t^2}\left[1 - 2t^2\right] = 0$$

$$e^{-t^2}(1-2t^2)=0$$

$\Rightarrow$ one root is at $t=\infty$ & one is at $t=\pm\frac{1}{\sqrt{2}}$

as we are assuming $t>0 \Rightarrow t=\frac{1}{\sqrt{2}}$ is another root

$\therefore$ at $t=\infty$, $\dot{m}(t) \Rightarrow \lim_{t\to\infty} -2t\,e^{-t^2}$

$$\Rightarrow \lim_{t\to\infty} \frac{-2t}{e^{t^2}}$$

LH Rule ! $\lim_{t\to\infty} \frac{-2}{2t \cdot e^{t^2}} = 0$

$\therefore$ at $t=\infty$, $\dot{m}(t)=0$

at $=\frac{1}{\sqrt{2}} \Rightarrow \dot{m}(t) = -2\left(\frac{1}{\sqrt{2}}\right) e^{-\frac{1}{2}}$

$$= \frac{-2}{\sqrt{2}} \frac{1}{\sqrt{e}}$$

$$= \frac{-2}{\sqrt{2e}}$$

$\Rightarrow \dot{m}(t) = \frac{-2}{\sqrt{2e}}$ at $t=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \dot{m}(t)_{max}=0$ & $\dot{m}(t)_{min}=\frac{-2}{\sqrt{2e}}$

$\therefore \Delta f_{PM} = \frac{8000\cancel{\pi}}{4\cancel{\pi}}\left(\frac{2}{\sqrt{2e}}\right) = \frac{2000\times 2}{\sqrt{2e}} = \frac{4000}{\sqrt{2e}}\,Hz = \frac{4000}{\sqrt{5.4365}} = \frac{4000}{2.331}$

$$\boxed{\therefore \Delta f_{PM} = \frac{4000}{\sqrt{2e}} = 1716.001\ Hz}$$

Now Bandwidths of FM and P.M waves:-

$$BW = 2(\Delta f + B) \text{ (considering wide Band)}$$

$\Rightarrow$ FM B.W $= 2(1500 + 10^4) = 2(11500) = \boxed{23{,}000\,Hz}$

$\Rightarrow$ PM B.W $= 2(1716.001 + 10^4) = 2(11716.001) = \boxed{23432.002\,Hz}$

~~Now In many cases we~~ consider

Now considering signal from ~~m~~ $-\infty$ to ∞ for PM calculations (as we are getting finite max & min values for $\dot{m}(t)$)

$$\Rightarrow \dot{m}(t)_{max} = \dot{m}\left(\frac{-1}{\sqrt{2}}\right)$$

$$= -2\left(\frac{-1}{\sqrt{2}}\right)e^{-1/2}$$

$$= \frac{2}{\sqrt{2e}}$$

$$\Rightarrow \Delta f_{pm} = \frac{8000\pi}{2\pi}\left(\frac{\dot{m}(t)_{max} - \dot{m}(t)_{min}}{2}\right)$$

Now as $\dot{m}(t)_{max} = \dot{m}(t)_{min}$

$$\Rightarrow \Delta f_{pm} = \frac{8000\pi}{2\pi}\,\frac{\dot{m}_p}{1}$$

$$= 4000\left(\frac{2}{\sqrt{2e}}\right)$$

$$= \frac{8000}{\sqrt{2e}} = \frac{8000}{\sqrt{5.4365}} = \frac{8000}{2.331} = 3432.003\ Hz$$

$$\Rightarrow \boxed{\Delta f_{pm} = 3432.003\ Hz = \frac{8000}{\sqrt{2e}}\ Hz}$$

Now assuming B.W of m(t) to be negligible

$$\Rightarrow B.W_{FM} = 2(\Delta f_{fm}) = 2(1500) = 3\ kHz$$

$$B.W_{PM} = 2\Delta f_{pm} = 2(3432.003) = 6864.006\ Hz.$$

and if we consider B.W of $m(t) = 10^4$ Hz

$$\Rightarrow B.W_{FM} = 2(10^4 + 1500) = 23\ kHz$$

$$B.W_{PM} = 2(10^4 + 3432.003) = 26864.006\ Hz = 26.864\ kHz$$

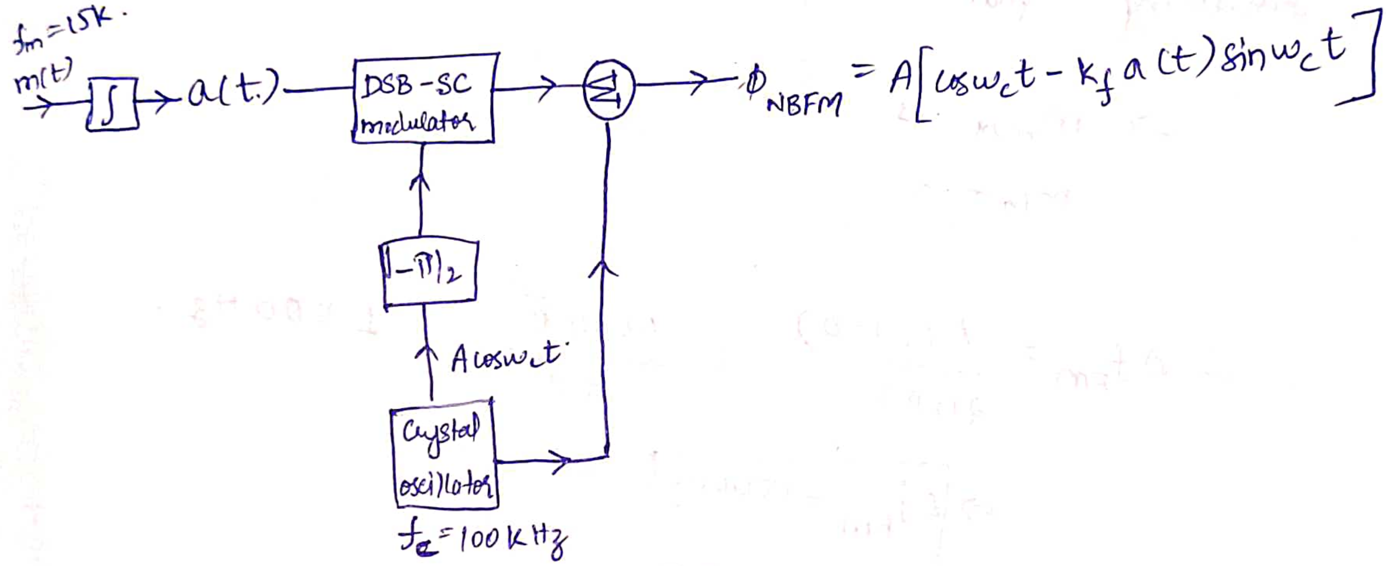
Q5)) Given,

FM signal has maximum angular deviation = 0.1 radians

$\Rightarrow \Delta f = \beta \cdot B = 0.1 \times 15$

$= 1.5K$

(1) B.W of msg signal = 15kHz.



Now we have to generate an FM signal at $f_c = 104\,MHz$ and $\Delta f = 75\,KHz$.

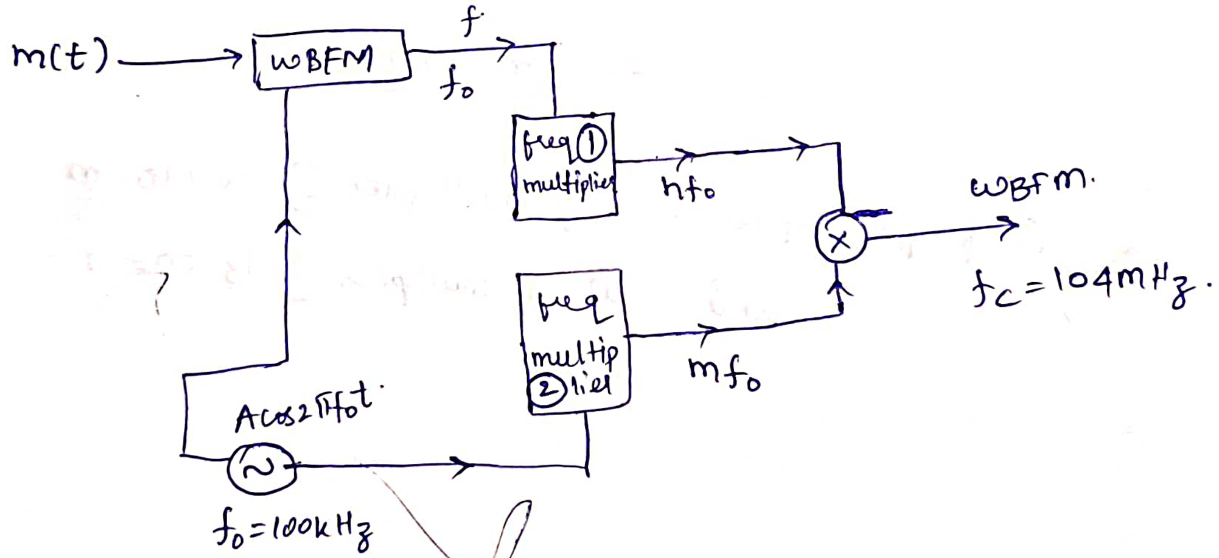
Method 1):

FM wave equation is given by:

$$S(t) = A\cos(2\pi f_0 t + \beta_{NBFM} \sin 2\pi f_m t)$$

$$= A\cos(2\pi(100k)t + 0.1 \sin(2\pi(15\times10^3)t))$$

$$\beta_{WBFM} = \frac{\Delta f_{WBFM}}{f_m} = \frac{75k}{15k} = 5$$



After passing through multiplier ① (freq):-

$$S_x(t) = A\cos[2\pi f_0 t + n\beta_{NBFM} \sin 2\pi f_m t]$$

$$n = \frac{\beta_{WBFM}}{\beta_{NBFM}} = \frac{5}{0.1} = \underline{50}$$

$\Rightarrow$ output of multiplier ① $= A\cos(2\pi\times5\times10^6 t + 50\times0.1 \sin(2\pi\times15\times10^3 t))$

$$= A\cos(2\pi\times5\times10^6 t + 5\sin(2\pi\times15\times10^3 t))$$

Ans

Now seeing output of freq multiplier ②:-

Let freq gets multiplied by factor of 'm'

$\Rightarrow 10^5 \cdot m \, Hz \rightarrow$ output freq of m2

output of mixer = $5MHz \pm m \cdot 10^5$

output freq of multiplier 1 ($f_o \cdot n \Rightarrow 50 \times 100K = 5MHz$)

$\Rightarrow \quad 5M + 0.1mM = 104M$. (all in Hz)

$\Rightarrow \quad 5 + 0.1m = 104$

$\Rightarrow \quad \boxed{m = 990}$

$\therefore$ freq multiplier factor of multiplier ② is $990 = m$

and that of multiplier ① is $50 = n$

Method 2:-

Lets assume output of narrowband FM modulator = $u(t) = A\cos(2\pi f_o t + \phi(t))$

then output of freq multiplier ① is

$$u_1(t) = A\cos(2\pi n_1 f_o t + n_1 \phi(t))$$

Similarly output of freq multiplier ② is

$$u_2(t) = A\cos(2\pi m f_o t)$$

After mixing:

$y(t) = u_1(t) u_2(t)$

$= A^2 [\cos(2\pi n_1 f_o t + n_1 \phi(t)) \cdot \cos(2\pi m f_o t)$

$= \frac{A^2}{2} [\cos(2\pi(n_1 + m) f_o t + n_1 \phi(t)) + \cos(2\pi(n_1 - m) f_o t + n_1 \phi(t)]$

Now as Given,

B.W of $m(t) = 15\text{KHz}$

$\Rightarrow$ max freq deviation $= \Delta f_{max} = \beta_f(BW)$

$= 0.1 \times 15k = 1.5\text{KHz}$

Now, we have to achieve $\Delta f = 75\text{KHz}$.

$\Rightarrow$ output of wide band modulator. and freq multiplier (1) should be same (i.e. n_1)

$$\Rightarrow n_1 = \frac{f}{\Delta f} = \frac{75}{1.5} = \boxed{50}$$

Now then using up-converter, freq modulated signal is :

$$y(t) = \frac{A^2}{2} \cos(2\pi(n_1+n_2)f_0 t + n_1\phi(t))$$

Now carrier freq : $f_c = (n_1 + m)f_0 = 104\,\text{MHz}$

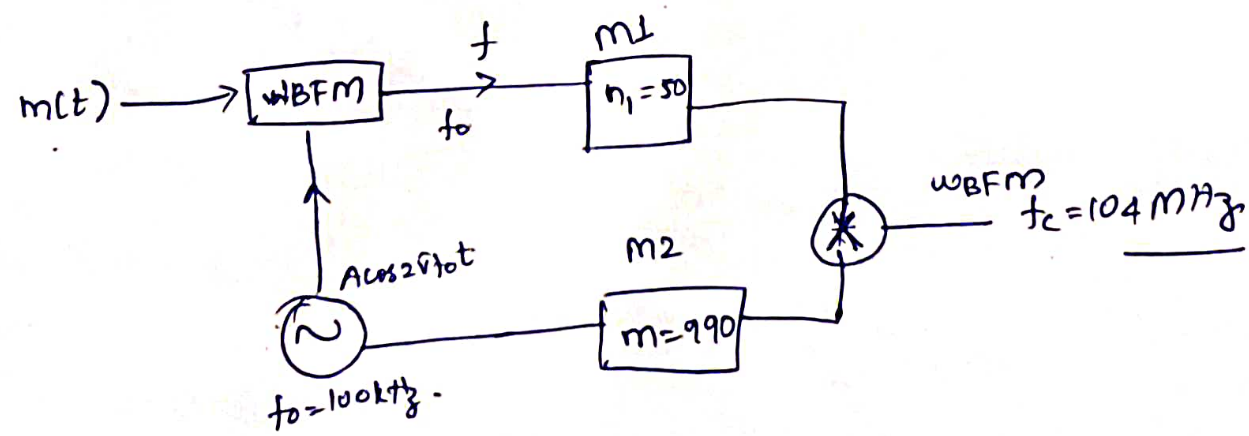
$$\Rightarrow (n_1+m)\,100 \times 10^3 = 104 \times 10^6$$

$$\Rightarrow n_1 + m = 1040$$

$$\Rightarrow m = 1040 - 50$$

$$\Rightarrow \boxed{m = 990}$$

$\Rightarrow$



Q5)
(2) If carrier frequency for wideband FM signal is to be within ± 2 Hz we have to find maximum drift of 100kHz oscillation.

$$f_c + \Delta f_c = 50(f_o \pm d) + 990(f_o \pm d) \quad \text{where } d \to \text{max drift}$$

$$\Rightarrow \Delta f_c = n f_o + m f_o \qquad n = 50, \quad m = 990$$

$$2 = n f_o + m f_o$$

$$\Rightarrow \Delta f_o = \frac{2}{n+m} = \frac{2}{1040} \quad \left(\frac{2}{50+990}\right) = 0.001923 \text{ Hz}$$

$$\Rightarrow \Delta f_o = 0.0019 \text{ Hz} \Rightarrow 0.001923 = d$$

$\therefore$ maximum allowable drift of 100kHz oscillation is $\boxed{\pm 0.0019 \text{ Hz}} \Rightarrow 0.001923$ Hz.

Q 6)) Given,

PM broadcasting B.W of $W = 15KHz$

$f_0 = 2100Hz$

$\beta = 5$, Avg to pk-pk ratio $= 0.5$

we have to find improvement in output SNR of PM with pre-emphasis & de-emphasis filtering compared to a baseband system.

Now:-

$$\left(\frac{S}{N}\right)_0 = 3\beta^2 m_p \left(\frac{S}{N}\right)_b$$

Baseband SNR

output SNR.

$$= 3(5)^2(0.5)\left(\frac{S}{N}\right)_b$$

$$= 12.5 \times 3 \left(\frac{S}{N}\right)_b$$

$$= 37.5 \left(\frac{S}{N}\right)_b$$

In terms of dB:-

$$\left(\frac{S}{N}\right)_0 = 10\log_{10} 37.5 + \left(\frac{S}{N}\right)_b$$

$$\Rightarrow \left(\frac{S}{N}\right)_0 \approx 15.7403 + \left(\frac{S}{N}\right)_b \text{ dB} \longrightarrow (1)$$

$\therefore$ FM without pre & de emphasis filtering perform 15.7dB better than a baseband system.

Now

$$\left(\frac{S}{N}\right)_{PD} = \frac{\frac{1}{3}\left(\frac{w}{f_0}\right)^3}{\frac{3w}{f_0} - \arctan\left(\frac{w}{f_0}\right)} \left(\frac{S}{N}\right)_0$$

SNR output with pre emphasis and deemphasis

$$= \frac{\frac{1}{3}\cdot\left(\frac{15000}{2100}\right)^3}{3\left(\frac{15000}{2100}\right) - \tan^{-1}\left(\frac{15000}{2100}\right)} \times \left(\frac{S}{N}\right)_0$$

$$= \left(\frac{S}{N}\right)_{PD} = \frac{\left(\frac{50}{7}\right)^3 \cdot \frac{1}{3} \cdot \left(\frac{S}{N}\right)_0}{3\left(\frac{50}{7}\right) - \tan^{-1}\left(\frac{50}{7}\right)}$$

$$\left(\frac{S}{N}\right)_{PD} = 21.3\left(\frac{S}{N}\right)_0$$

In terms of dB:-

$$\left(\frac{S}{N}\right)_{PD} = 10\log_{10} 21.3 + \left(\frac{S}{N}\right)_0 \text{ dB}$$

$$\left(\frac{S}{N}\right)_{PD} \approx 13.2837 + \left(\frac{S}{N}\right)_0 \text{ dB}$$

$$\Rightarrow \left(\frac{S}{N}\right)_{PD} = 13.283 + \left(\frac{S}{N}\right)_0 \text{ dB}$$

$$= 13.283 + 15.7403 + \left(\frac{S}{N}\right)_b \text{ dB} \ \{\text{from } ①\}$$

$$\boxed{\left(\frac{S}{N}\right)_{PD} = 29.024 + \left(\frac{S}{N}\right)_b \text{ dB}}$$

$\therefore$ Overall improvement when using pre-emphasis and de-emphasis filtering rather than baseband system is <u>29.024 dB</u>.

Q7)) Given,

$B.W \text{ of } w = 8\ kHz.$

$\text{Attenuation} = 40\,dB$

$\text{PSD of } \frac{\eta}{2} = 10^{-12}\ W/Hz$

$\text{available } B.W = 60\ kHz$

we have to find:-

① Min required transmitter power & corresponding modulation index

$$\left(\frac{S}{N}\right)_{input} dB = (\text{Attenuation})\,dB + \left(\frac{S}{N}\right)_{output} dB$$

$$= 40\,dB + 40\,dB = 80\,dB$$

$$\Rightarrow \left(\frac{S}{N}\right)_{input} dB = 80\,dB$$

$$10\log(?) = 80$$

$$\Rightarrow \boxed{10^{8}} = \left(\frac{S}{N}\right)_{input}$$

↳ not in dB

$$\Rightarrow \text{min. transmitted power} = 10^{8} \times 2 \times 10^{-12} \times 60k \quad \{10^{8} \times N_0 \times 2B\}$$

$$= 120 \times 10^{-4}\,k = \boxed{\cancel{800\,\mu W}}$$

$$= \boxed{12\,W}$$

Now considering BW of fm signal = channel BW

$$2(\beta+1)W = B$$

$$\Rightarrow \beta = \frac{B}{2W} - 1 \Rightarrow \frac{60k}{2\times 8k} - 1 = \frac{15}{4} - 1 = \frac{11}{4} = 2.75$$

$$\Rightarrow \boxed{\beta = 2.75}$$

(b) Now if we take, SNR as 60dB

$$\left(\frac{S}{N}\right)_{input} dB = (\text{Attenuation})_{dB} + \left(\frac{S}{N}\right)_{output} dB$$

$$= 40dB + 60dB$$

$$= 100\,dB$$

$$10\log(?) = 100 \qquad = 10^{10} \text{ (After converting from dB)}$$

$$? = 10^{10}$$

$\therefore$ Minimum transmitted Power $= 10^{10} \times N_0 \times 2B$

$$= 10^{10} \times 10^{-12} \times 2 \times 60 \times 10^{3}$$

$$= 12 \times 10^{2} = \boxed{1200W}$$

(c)

$$2(\beta+1)W = B$$

$$\Rightarrow \beta = \frac{B}{2W} - 1 = \frac{60k}{2\times 8k} - 1 = \boxed{2.75}$$

$\Rightarrow$ Modulation index remains same.

(c) Considering pre-emphasis & de-emphasis filter with $\tau = 75\,\mu s$

Modulation Index formula $= 2(\beta+1)(W + \frac{1}{\tau}) = 60\,kHz$

$$\Rightarrow 2(\beta+1)\left(8k + \frac{10^6}{75}\right) = 60k$$

$$\Rightarrow 2(\beta+1)(8\times10^3 + 13.33\times10^3) = 60k.$$

$$= 2(\beta+1)(8 + 13.33) = 60$$

$$\Rightarrow \beta+1 = \frac{60}{2\cdot(21.33)}$$

$$\Rightarrow \beta = 1.4064 - 1$$

$$\Rightarrow \boxed{\beta = 0.4064}$$

Modulation index = 0.4064

Q8))

Given,

a signal s(t) sampled at rate $\frac{1}{T_s}$

$$s(t) \xleftrightarrow{F.T} S(f)$$

$$B(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} S\left(f + k\frac{1}{T_s}\right)$$ denotes the sum of translates of the spectrum.

we have to prove:-

(a) B(f) is periodic with period $\frac{1}{T_s}$:-

To prove that B(f) is periodic with $\frac{1}{T_s}$, we need to show that

$$B\left(f+\frac{1}{T_s}\right) = B(f)$$

$$\Rightarrow B\left(f+\frac{1}{T_s}\right) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} S\left(f + \frac{k}{T_s} + \frac{1}{T_s}\right)$$

Now as we know that S(f) is periodic with period $\frac{1}{T_s}$ as when we ~~s~~ do fourier transform of an Impulse trains or sampled then

we get a periodic pulse with ~~period~~ its period equal to sampling rate in frequency spectrum.

$$\therefore S\left(f+\frac{1}{T_s}\right) = S(f)$$

$$\Rightarrow \frac{1}{T_s}\sum_{k=-\infty}^{\infty} S\left(f+\frac{k}{T_s}+\frac{1}{T_s}\right) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} S\left(f+\frac{k}{T_s}\right)$$

$$\Rightarrow B\left(f+\frac{1}{T_s}\right) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} S\left(f+\frac{k}{T_s}\right)$$ which is equal to B(f)

$\boxed{\therefore B\left(f+\frac{1}{T_s}\right) = B(f)}$ $\therefore$ B(f) is periodic with period $\frac{1}{T_s}$ is proved.

(b) The samples $s(nT_s)$ are Fourier Series for $B(f)$ satisfying

$$s(nT_s) = T_s \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} B(f) e^{j2\pi f n T_s} df$$

$$B(f) = \sum_{n=-\infty}^{\infty} s(nT_s) e^{-j2\pi f n T_s}$$

To prove we apply Inverse fourier transform:

$$s(nT_s) = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi f n T_s} df.$$

writing the integrals as infinite sum of integrals over segments of length $\frac{1}{T}$:

$$\Rightarrow s(nT_s) = \sum_{k=-\infty}^{\infty} \int_{\frac{k-\frac{1}{2}}{T_s}}^{\frac{k+\frac{1}{2}}{T_s}} S(f) e^{j2\pi f n T_s} df.$$

we then make substitution: $\alpha = f - \frac{k}{T_s}$

$\Rightarrow f = \alpha + \frac{k}{T_s}$, when $f = \frac{k-\frac{1}{2}}{T_s} \Rightarrow \alpha = -\frac{1}{2T_s}$

$\left\{ f = \frac{k+\frac{1}{2}}{T_s} \Rightarrow \alpha = \frac{1}{2T_s}\right.$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \int_{\frac{k-\frac{1}{2}}{T_s}}^{\frac{k+\frac{1}{2}}{T_s}} S(f)\, e^{j2\pi f n T_s} = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} S\left(\alpha + \frac{k}{T_s}\right) e^{j2\pi\left(\alpha + \frac{k}{T_s}\right) n T_s} d\alpha$$

$$= \sum \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} S\left(\alpha + \frac{k}{T_s}\right) e^{j2\pi \alpha n T_s} \cdot \cancel{e^{j2\pi k n}}^{\,1}\, d\alpha.$$

$$= \sum \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} S\left(\alpha + \frac{k}{T_s}\right) e^{j2\pi \alpha n T_s} d\alpha.$$

$$\Rightarrow s(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} s\left(\alpha + \frac{k}{T_s}\right) e^{j2\pi\alpha n T_s} \, d\alpha$$

Now as limits are Independent of k so, we can move k inside the integral.

$$\Rightarrow s(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} \left(\sum_{k=-\infty}^{\infty} s\left(\alpha + \frac{k}{T_s}\right) \right) e^{j2\pi\alpha n T_s} \, d\alpha$$

$$\Rightarrow s(nT_s) = T_s \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} B(\alpha) \, e^{j2\pi\alpha n T_s} \, d\alpha \quad \left\{ \text{as } B(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} s\left(f + \frac{k}{T_s}\right) \right\}$$

$$\therefore \; s(nT_s) = T_s \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} B(\alpha) \, e^{j2\pi\alpha n T_s} \, d\alpha \quad \text{is proved} \longrightarrow (1)$$

Proving 2^{nd} eq^n :-

$$s(nT_s) = s(t) \text{ at } t = kT_s$$

$$s(nT_s =) = s(t) \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

$$\Rightarrow F\cdot T\,(s(nT_s)) = S(f) * \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_s}\right)$$

(or)

DTFT

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} S\left(f - \frac{k}{T_s}\right)$$

$$= B(f)$$

$\therefore$ DTFT of $s(nT_s) = B(f)$

$$\Rightarrow \boxed{\sum_{n=-\infty}^{\infty} s(nT_s)\, e^{-j2\pi f n T_s} = B(f)}$$

Hence proved