

ASSIGNMENT - 3

(MY UNDERSTANDING)

By Analyzing the algorithm it takes $O(n)$ times as it requires a constant time to find the elements and get a median from the 5 elements. Then we further divide them $n/5$ for each and find the median for each set of 5.

a) How many baby medians are less than or equal to the chosen pivot? How many greater than or equal to the pivot?

Solution - we can say that it can have

$$\frac{3}{10} \leq \text{pivot} \quad \& \quad \frac{3}{10} \geq \text{pivot}$$

$\therefore \frac{3}{10}$ of the elements which is 30% of the elements are less than or equal to and 30% are greater than or equal to the pivot.

Example :- sets - $\{40, 4, 37, 44, 15\}, \{50, 14, 29, 22, 31\}, \{28, 2, 25, 31, 19\}$

From the following sets the medians are $\{37, 29, 19\}$ and the pivot is 29 which is the baby median if we see there are only 8 elements less than the pivot $\{2, 3, 4, 14, 15, 19, 22, 28\}$ and further 7 elements are greater than the pivot value. We can say that if we randomly select the value we get an element n .

b) The elements other than elements of its set are less than or equal to the pivot can be if we consider from the equation question as $s = n/5$ where each set of elements is divided the median from that set is taken out as $s/2$ and three of 5 elements are less than or equal to the pivot.

\therefore we can say that the other element which are less than or equal to chosen pivot is

$$3\left[\frac{s}{2}\right] \geq \frac{3e}{10} \quad \text{where } e = n \text{ numbers.}$$

and if it is greater than or equal to the chosen pivot then $3\left[\frac{s}{2}\right] \geq \frac{3e}{10}$

c) I have given above that the algorithm used in determining the approach takes $O(n)$ time complexity as partitions are used. To prove we can use trial & error method. If we have a pile of books and we consider to put them in a recursion tree then the tree will have books from the root to its children and it goes on. The amount of time used in this tree can be a constant c times $2n$.

\therefore The partition takes $O(n)$ to arrange the books recursively.

d) The recurrence equation relation is the following -

$$T(n) \leq cn + T(n/5) + T(7n/10)$$

and the time complexity is $O(n)$

So the one to find the median of the baby medians is $T(n/5)$

and to recur on the larger of L and G is $T(7n/10)$ which recursively goes on of elements.

e) By induction step

Let us consider the above recurrence of the pile of books. We can say for a constant number.

$$cn + \left[1 + \left(\frac{19}{20}\right) + \left(\frac{19}{20}\right)^2 + \left(\frac{19}{20}\right)^3 + \left(\frac{19}{20}\right)^4 + \dots + \left(\frac{19}{20}\right)^n \right]$$

This is almost equal to $20cn$ which proves

that the run time is $O(n)$ time complexity.

OR - $T(n) = O(n)$ if $T(n) \leq c \leq kn$ then $k \geq c$

$$\begin{aligned} T(n) &\leq T(n/5) + T(7n/10) + cn \\ &\leq k(n/5) + k(7n/10) + cn \\ &\leq k(n/5) + k(7n/10) + cn \\ &= k(9n/10) + cn \end{aligned}$$

$\therefore 9k/10 + c \leq k$ the $T(n) \leq kn$ holds. if we pick $k=60c$ or $20c$

the $T(n) \leq kn$, Induction proved.

f) The time complexity of $T(n)$ if $r=3$?

$$T(n) \leq T(n/3) + T(2n/3) + cn$$

is the recurrence relation

$$\Rightarrow O(n \log_{3/2} n)$$

g) The recurrence relation of $r=7$ is

$$T(n) \leq T(n/7) + cn$$

time complexity of this is $O(n \log_7 n)$

The comparing part is done in the code and written below.

h) Quicksort if compared with selection sort Quicksort runtime where $r=7$ runs faster than selection sort algo where the K th smallest is selected with time complexity of

if we compare the time complexity of

$r=3$ which gives time $2E-05$ (sec)

$r=7$ which gives time $8.8E-06$ (sec)

$r=5$ which gives time $1.8E-05$ (sec)

I have also coded the time complexities of every input you give. The output can be compared in the end of the code.