

2025 edition

Deep Learning for Music Analysis and Generation

Fundamentals for Musical Audio

(audio → features)



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Outline

- Time and frequency representations for music
- Audio features for music
- Math in STFT

FMP Notebook

<https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1.html>

<https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2.html>

Part	Title	Notions, Techniques & Algorithms	HTML	IPYNB
B	Basics	Basic information on Python, Jupyter notebooks, Anaconda package management system, Python environments, visualizations, and other topics	[html]	[ipynb]
0	Overview	Overview of the notebooks (https://www.audiolabs-erlangen.de/FMP)	[html]	[ipynb]
1	Music Representations	Music notation, MIDI, audio signal, waveform, pitch, loudness, timbre	[html]	[ipynb]
2	Fourier Analysis of Signals	Discrete/analog signal, sinusoid, exponential, Fourier transform, Fourier representation, DFT, FFT, STFT	[html]	[ipynb]
3	Music Synchronization	Chroma feature, dynamic programming, dynamic time warping (DTW), alignment, user interface	[html]	[ipynb]

Part	Title	Notions, Techniques & Algorithms	HTML	IPYNB
4	Music Structure Analysis	Similarity matrix, repetition, thumbnail, homogeneity, novelty, evaluation, precision, recall, F-measure, visualization, scape plot	[html]	[ipynb]
5	Chord Recognition	Harmony, music theory, chords, scales, templates, hidden Markov model (HMM), evaluation	[html]	[ipynb]
6	Tempo and Beat Tracking	Onset, novelty, tempo, tempogram, beat, periodicity, Fourier analysis, autocorrelation	[html]	[ipynb]
7	Content-Based Audio Retrieval	Identification, fingerprint, indexing, inverted list, matching, version, cover song	[html]	[ipynb]
8	Musically Informed Audio Decomposition	Harmonic/percussive separation, signal reconstruction, instantaneous frequency, fundamental frequency (F0), trajectory, nonnegative matrix factorization (NMF)	[html]	[ipynb]

Outline

- **Time and frequency representations for music**
- Audio features for music
- Math in STFT

Audio Waveforms

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1S3_Waveform.html

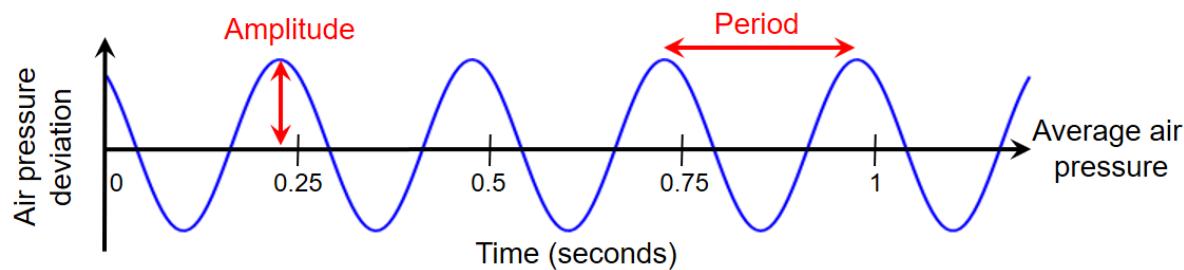
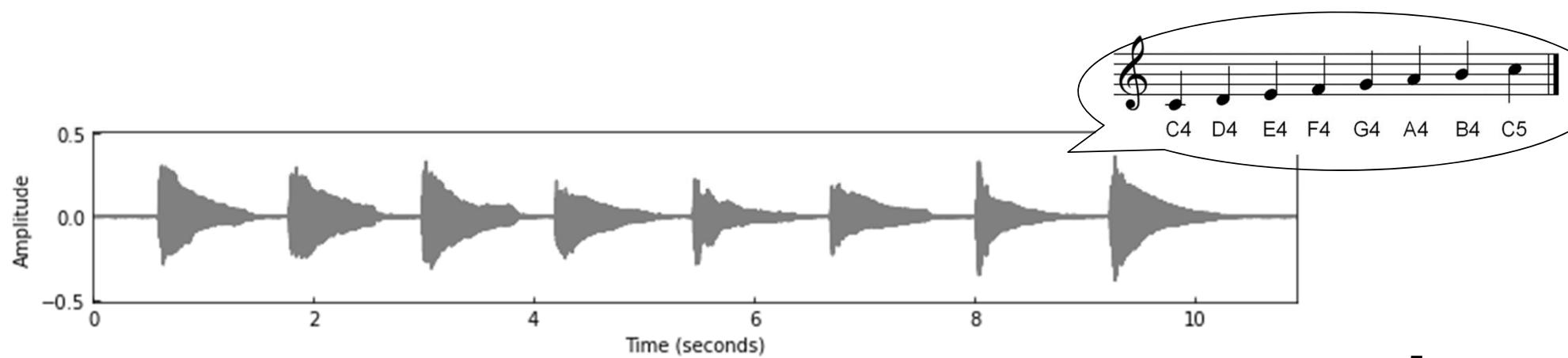


Figure 1.19 from [Müller, FMP, Springer 2015]



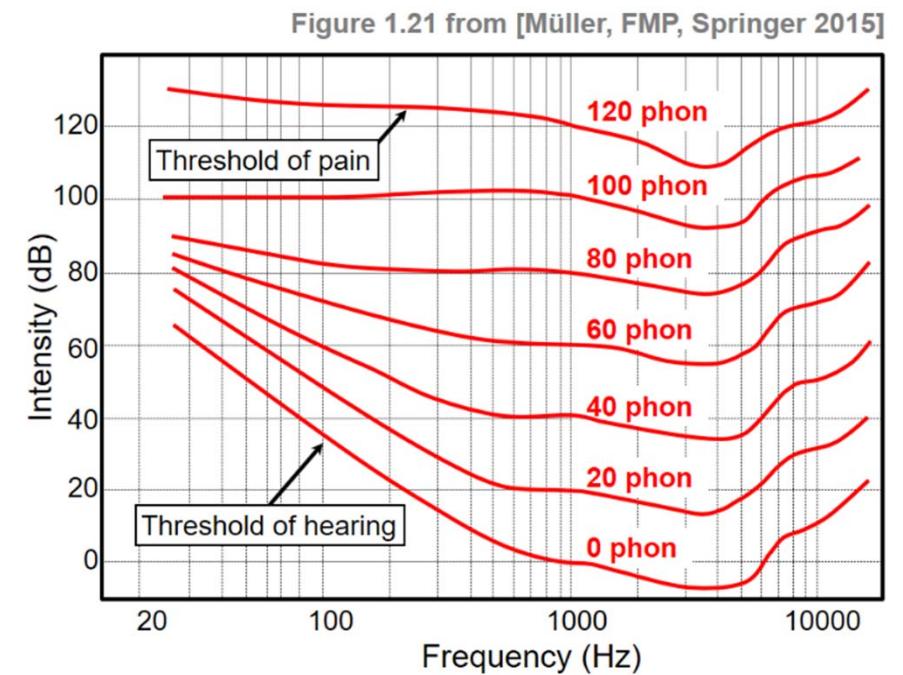
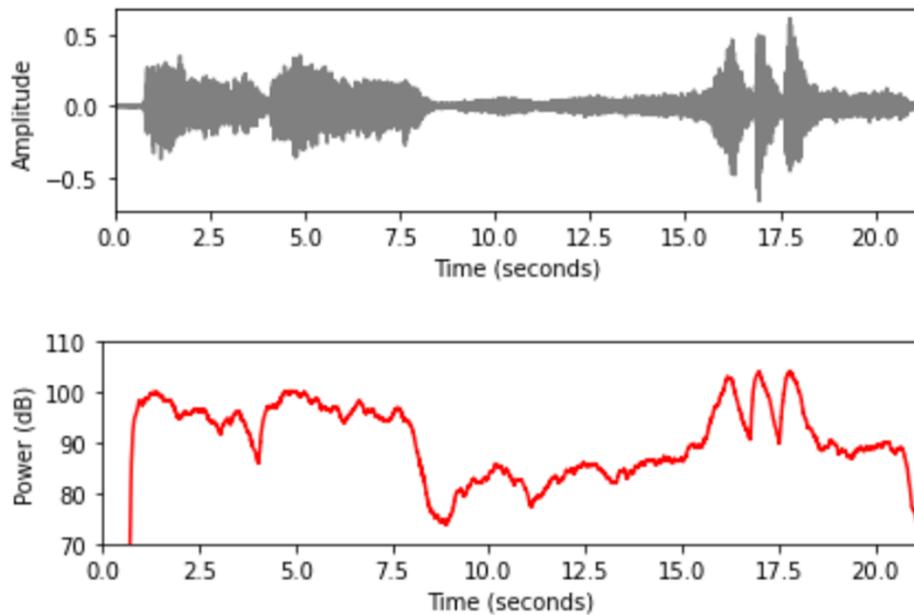
What Do We Perceive in Musical Audio?

- Loudness
- Timbre / instrument / singer identity
- Pitch / melody
- Chord / harmony
- Tempo / rhythm
- Style
- Emotion
- Spatial information

Dynamics, Intensity, and Loudness

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1S3_Dynamics.html

- **Intensity:** a physical property
 - Other names: power (dB); energy
- **Loudness:** a perceptual property
 - Less used in deep learning research



Timbre: Different Instruments

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1S3_Timbre.html

- **Timbre** (tone color; tone quality)
 - Allows a listener to distinguish the musical tone of a violin, an oboe, or a trumpet even if the tone is played at the same **pitch** and with the same **loudness**
 - Hard to grasp & *subjective*
 - May be described with words such as *bright*, *dark*, *warm*, and *harsh*

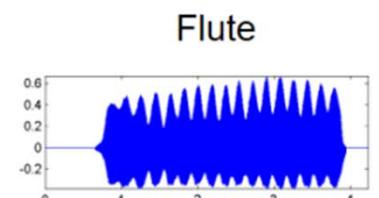
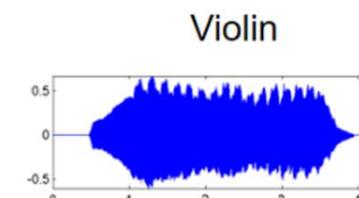
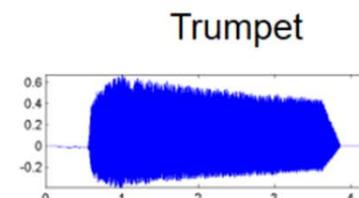
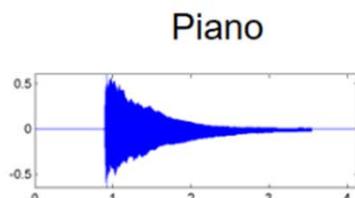


Figure 1.23 from [Müller, FMP, Springer 2015]

▶ 0:04 / 0:04 - 🔊

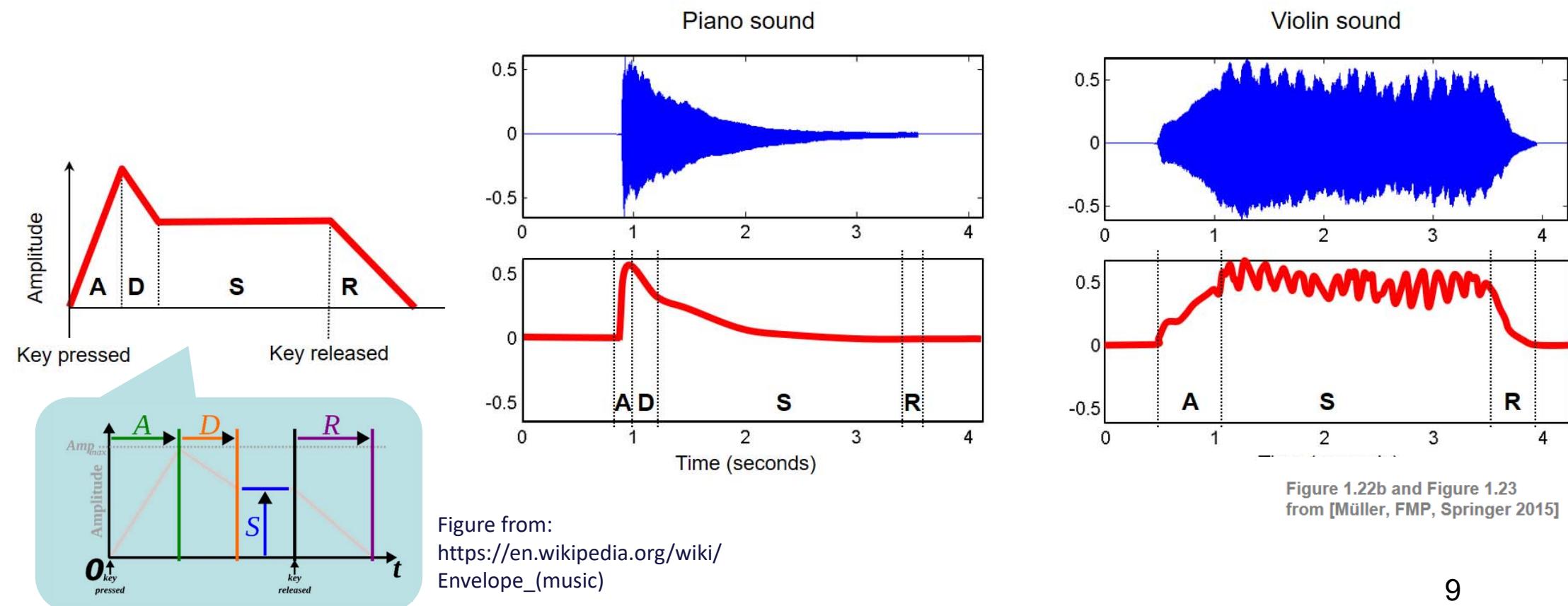
▶ 0:00 / 0:04 - 🔊

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Temporal Characteristics

- ADSR: attack (A), decay (D), sustain (S), and release (R)
 - Hard to analyze when there are concurrent notes



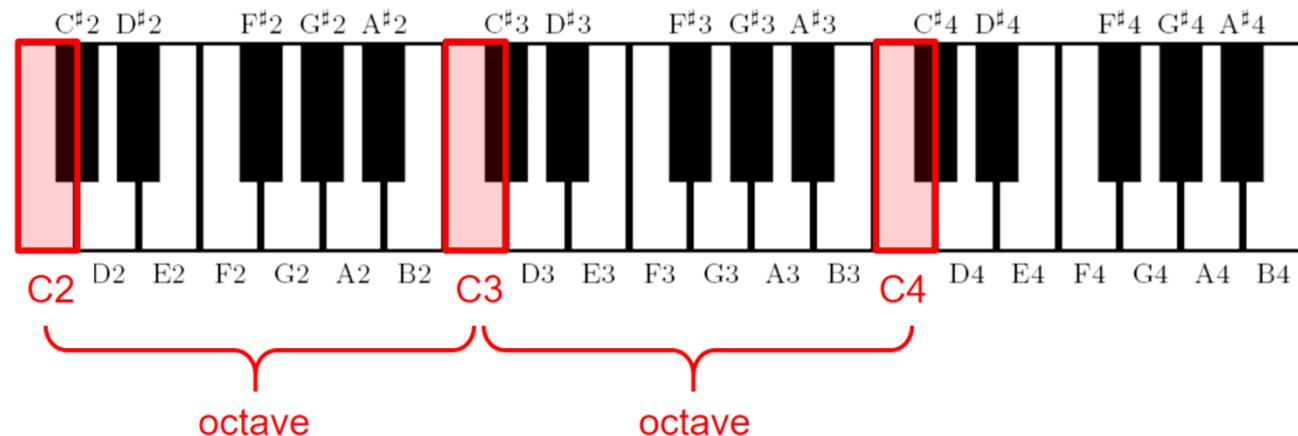
Notes and Pitches

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1S1_MusicalNotesPitches.html

- **Pitch Class**

- Two notes with *fundamental frequencies* in a ratio equal to *any power of two* (e.g., half, twice, or four times) are perceived as very **similar**
- All notes with this kind of relation can be grouped under the same *pitch class*
- Related to *chords/harmony*

Pitch class C = {..., C2, C3, C4, ...}



MIDI Note Numbers

https://en.wikipedia.org/wiki/MIDI_tuning_standard

$$f = 2^{(d-69)/12} \cdot 440 \text{ Hz} \quad d = 69 + 12 \log_2 \left(\frac{f}{440 \text{ Hz}} \right)$$

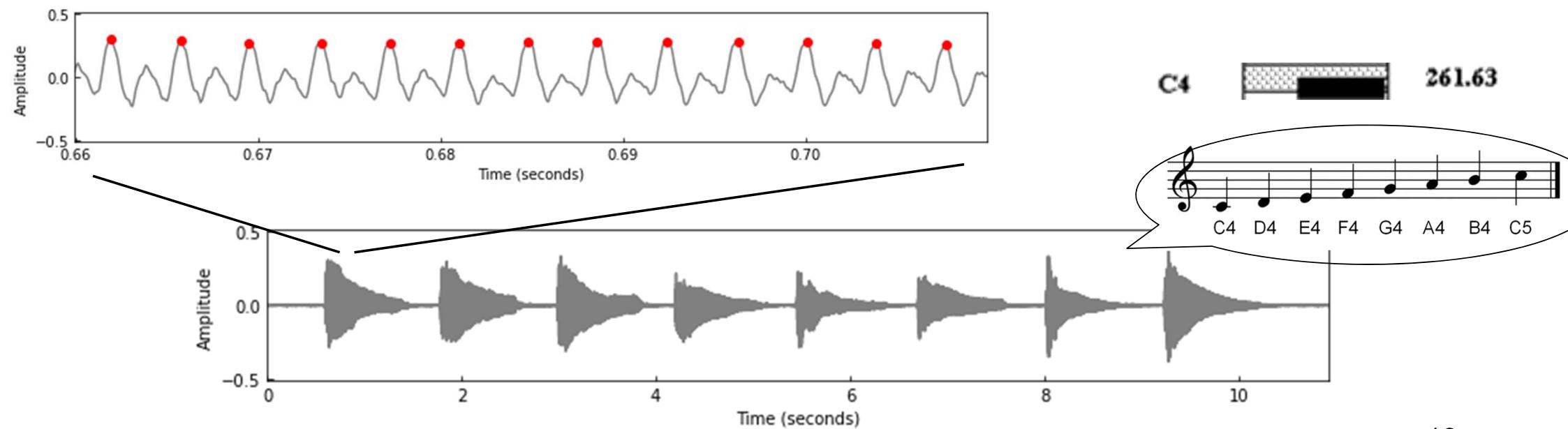
MIDI number	Note name	Keyboard	Frequency Hz	Keyboard	Frequency Hz
21	A0		27.500		
22	B0		30.868	29.135	
23	C1		32.703		
24	D1		36.708	34.648	
25	E1		41.203	38.891	
26	F1		43.654		
27	G1		48.999	46.249	
28			55.000	51.913	
29	A1		61.735	58.270	
30	B1		65.406		
31	C2		73.416	69.296	
32	D2		82.407	77.782	
33	E2		87.307		
34	F2		97.999	92.499	
35	G2		110.00	103.83	
36			123.47	116.54	
55				U3	
56				A3	220.00
57				B3	246.94
58				C4	261.63
59				D4	293.67
60				E4	329.63
61				F4	349.23
62				G4	392.00
63					369.99
64					
65					
66					
67					
68					
69				A4	440.00
70				B4	493.88
71				C5	523.25
72				D5	587.33
73				E5	659.26
74				F5	698.46
75				G5	783.99
76					739.99
77					
78					
79					
80					
81				A5	880.00
82				B5	987.77
83				C6	1046.5
84					

Audio Waveforms (Cont')

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C1/C1S3_Waveform.html

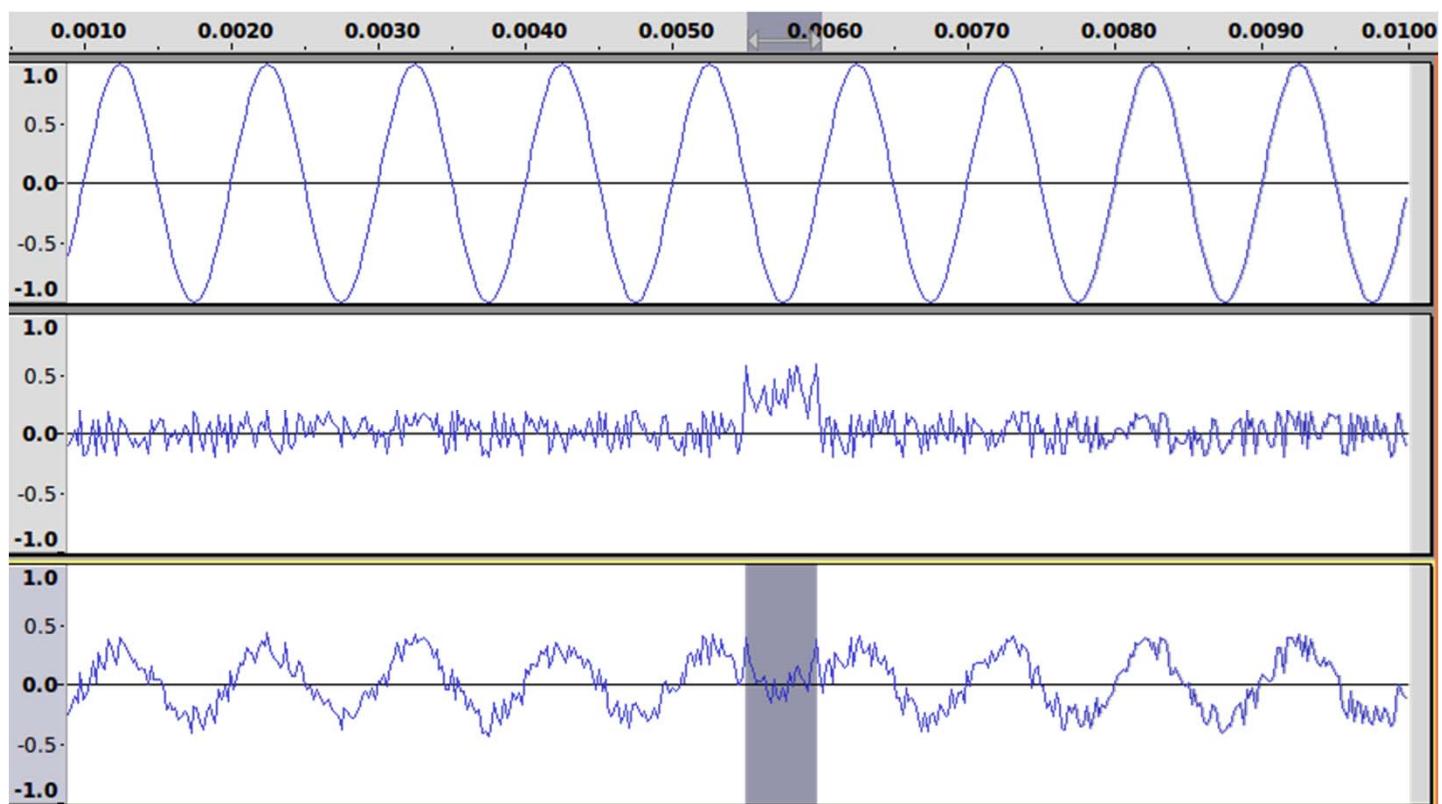
Zoom-in of the section between 0.66 and 0.71 seconds

- Highly repetitive
- 13 high-pressure (red) points $\rightarrow 20 * 13 = 260$ Hz



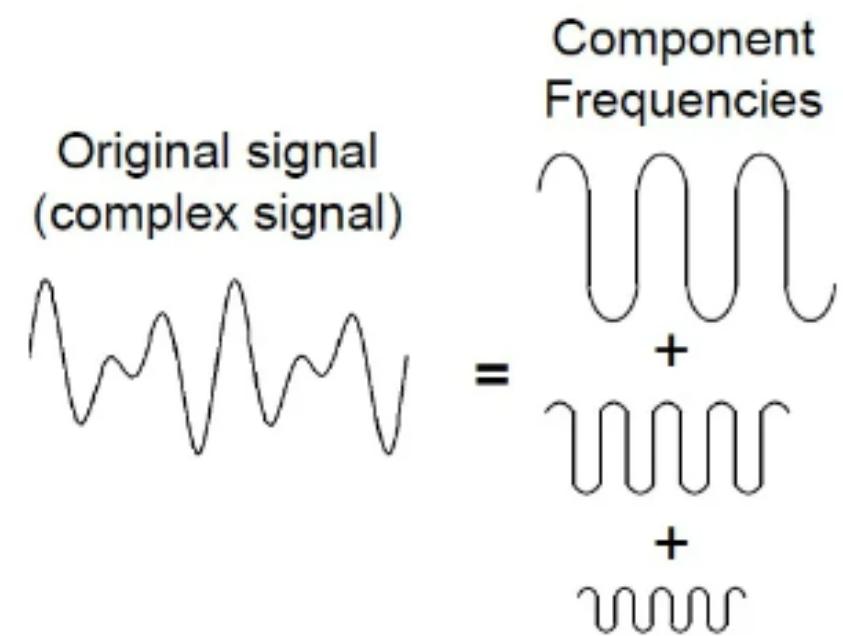
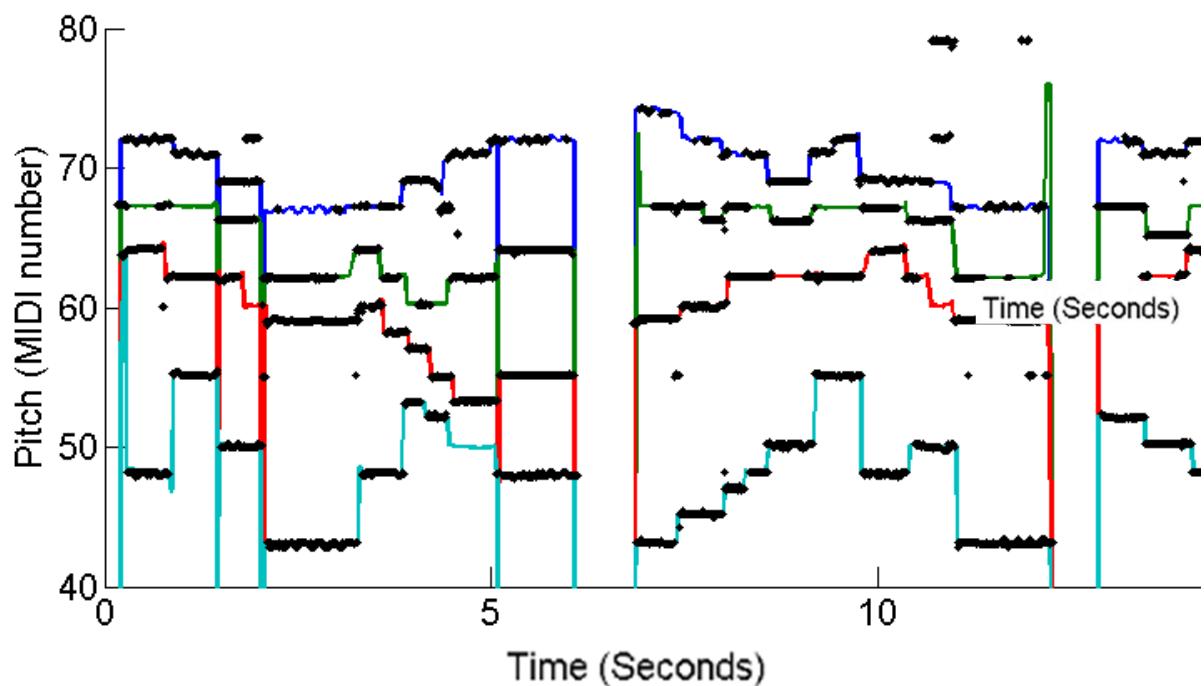
Temporal Characteristics

- **Zero-crossing rate**
 - How often a signal changes its sign
 - **Nosie**-like or not
 - Can be used in detecting the presence of vocals in speech signals
 - Cannot be used for estimating pitch due to noises and concurrent notes



<https://sandilands.info/sgordon/impact-of-noise-on-sine-wave-compared-to-square-wave>

Multiple Concurrent Pitches



<https://labsites.rochester.edu/air/projects/multipitch/multipitch.html>

<https://www.allaboutcircuits.com/technical-articles/an-introduction-to-filters/>

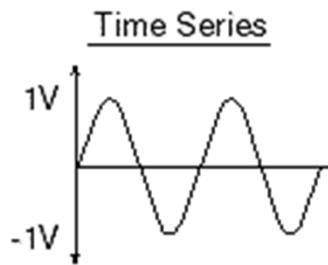
Frequency-domain Analysis

- Main subject of “Signals and Systems”
- Fourier Transform

Description

A pure 5kHz sine wave measuring 1 volt peak

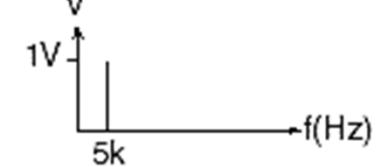
Time Series



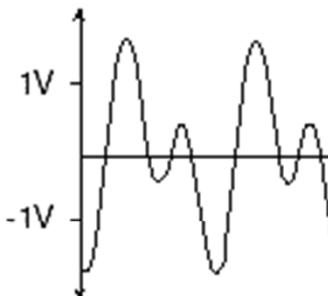
Fourier Expansion

$$v(t) = 1\sin(\omega_1)t$$
$$\omega_1 = 2\pi(5\text{kHz})$$

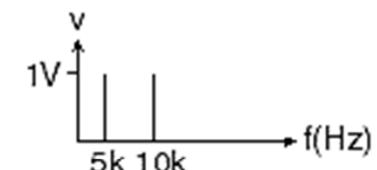
Power Spectrum



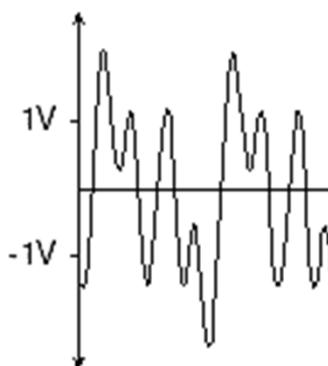
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together



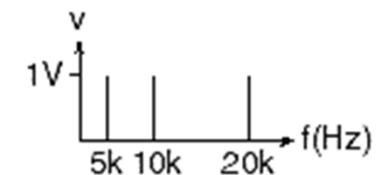
$$v(t) = 1\sin(\omega_1)t + 1\sin(\omega_2)t$$
$$\omega_1 = 2\pi(5\text{kHz})$$
$$\omega_2 = 2\pi(10\text{kHz})$$



A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together



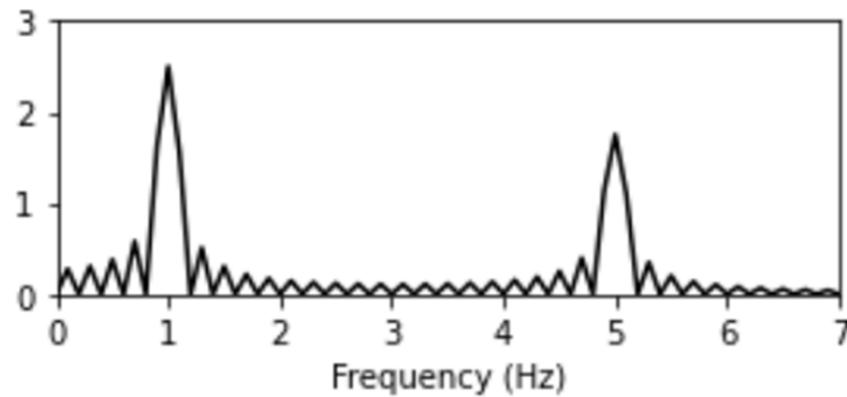
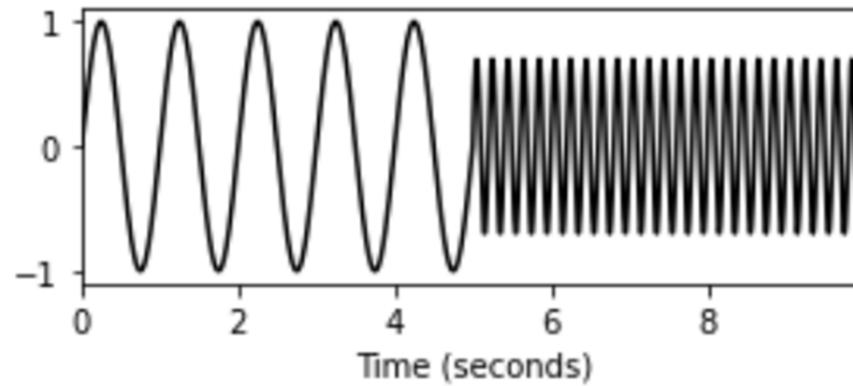
$$v(t) = 1\sin(\omega_1)t + 1\sin(\omega_2)t + 1\sin(\omega_3)t$$
$$\omega_1 = 2\pi(5\text{kHz})$$
$$\omega_2 = 2\pi(10\text{kHz})$$
$$\omega_3 = 2\pi(20\text{kHz})$$



Frequency-domain Analysis

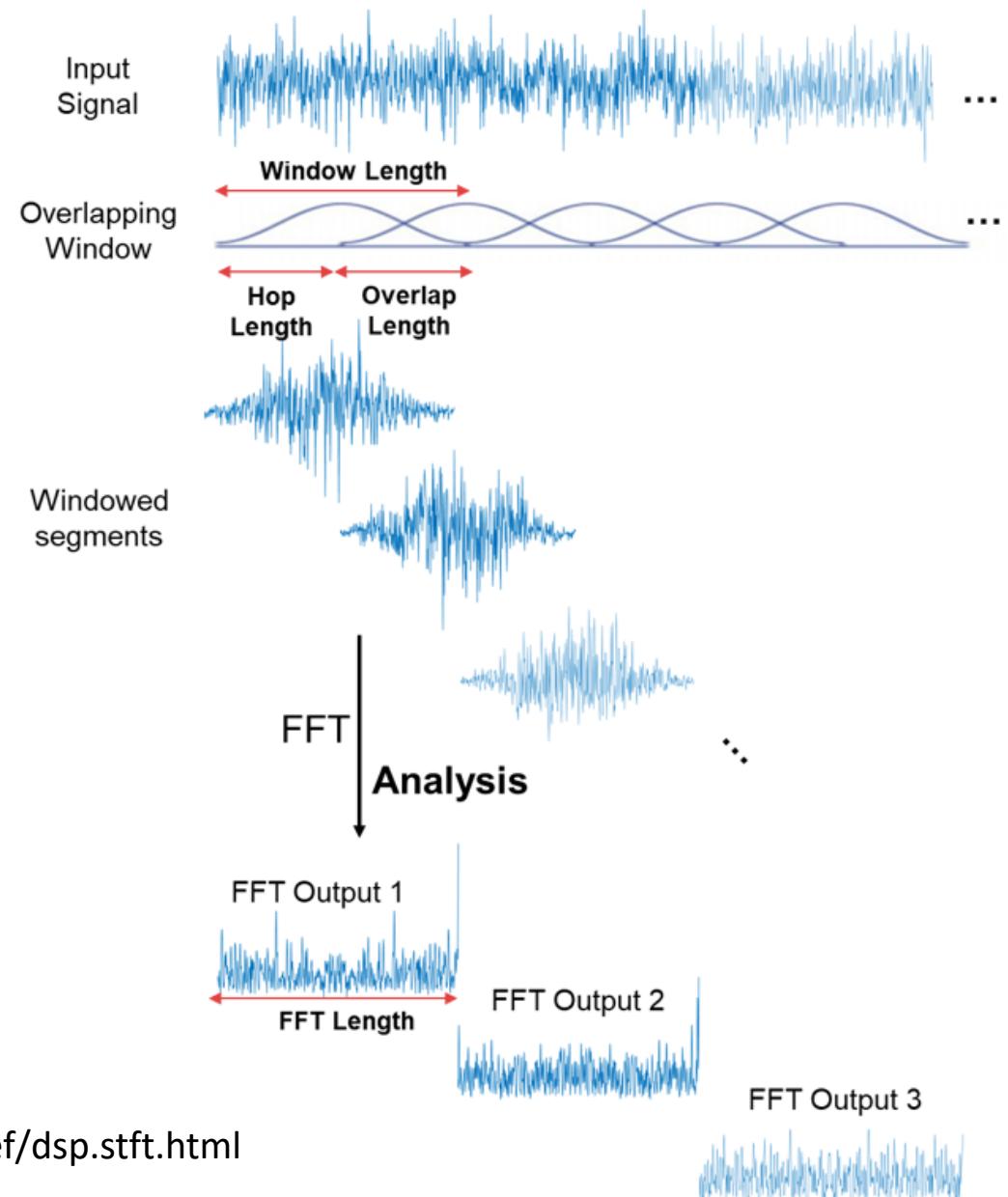
https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html

- Fourier Transform cannot localize temporal events



Frequency-domain Analysis

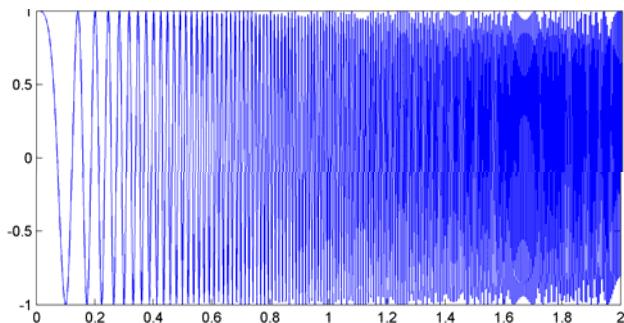
- Fourier Transform (FFT) cannot localize temporal events
- **Short-Time Fourier Transform (STFT)**
 - *Time-frequency representation* of signals
 - Slide **short windows** over the input signal and then compute the FFT locally for each windowed signal



<https://ww2.mathworks.cn/help/dsp/ref/dsp.stft.html>

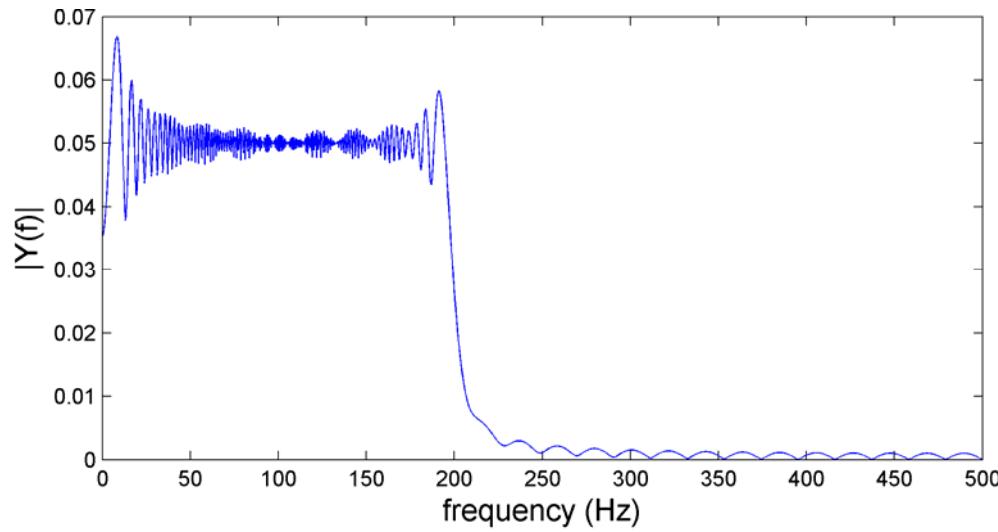
FFT vs STFT

Time-domain waveform

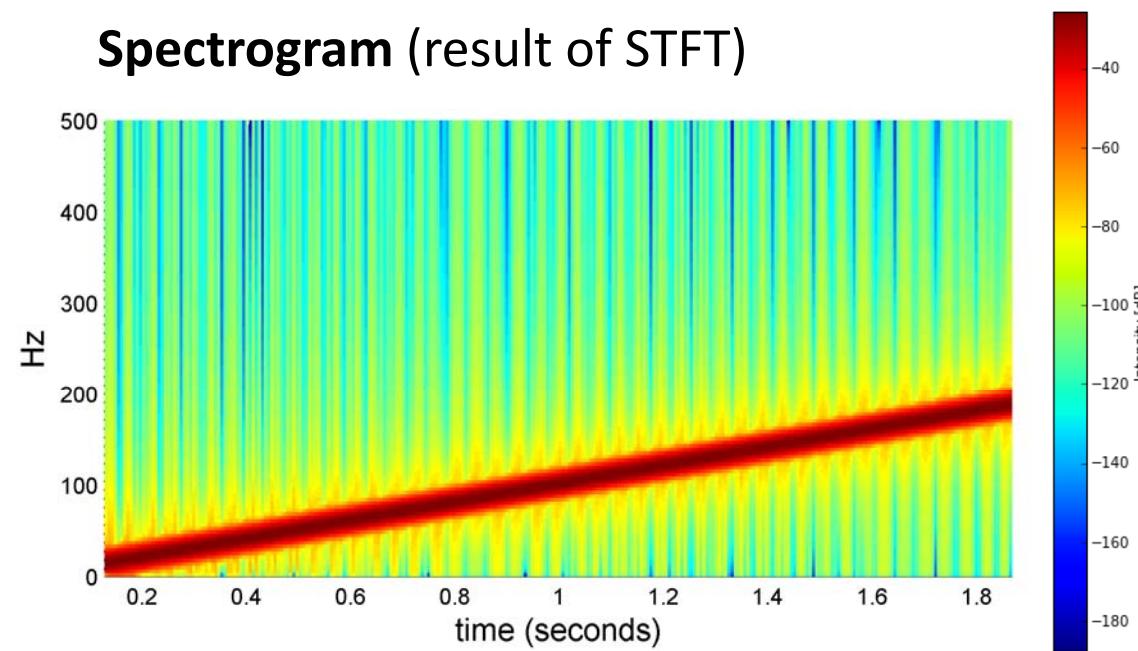


- **Waveform:** amplitude vs time
- **Spectrum:** amplitude vs frequency
- **Spectrogram:** time vs frequency; use *color* for amplitude

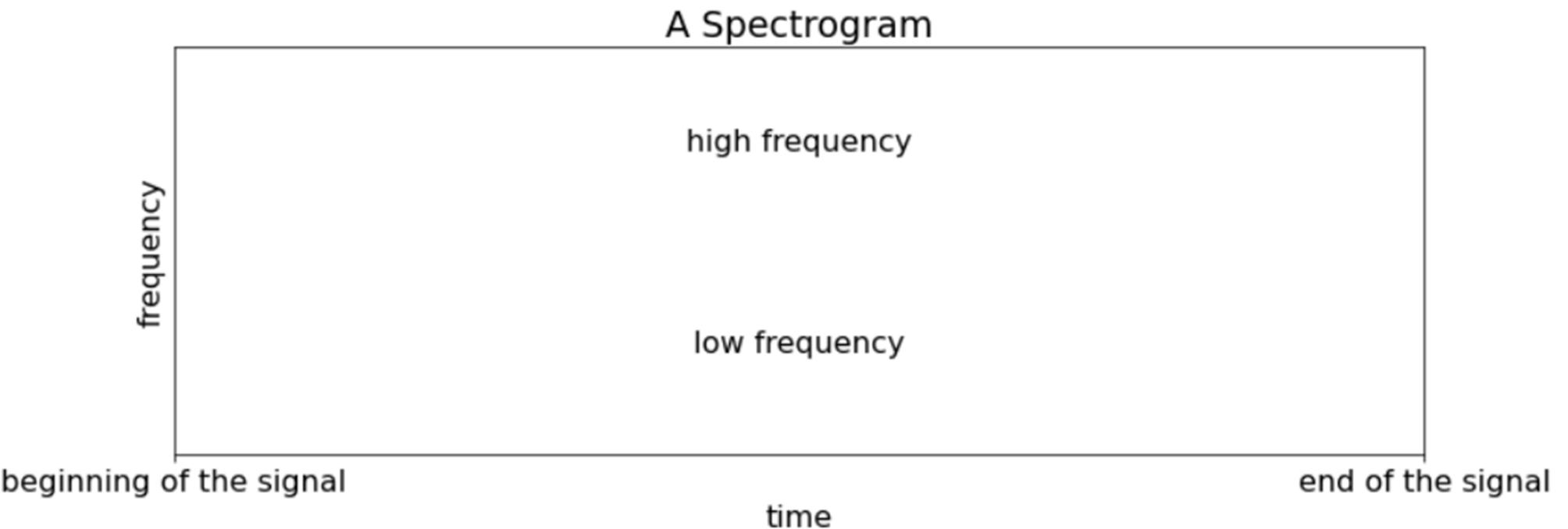
Spectrum (result of FFT)



Spectrogram (result of STFT)



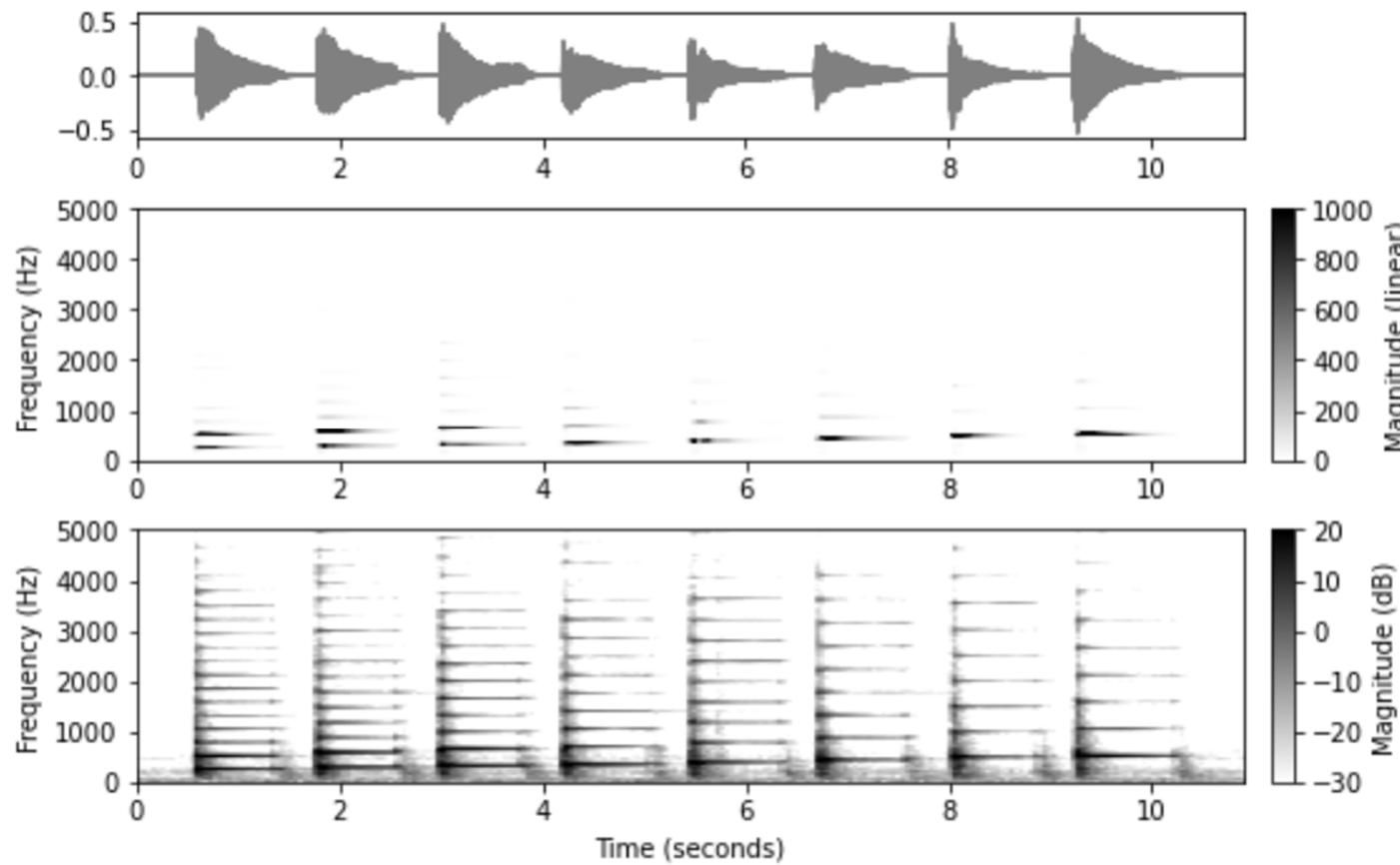
Time-Frequency Analysis via STFT



https://music-classification.github.io/tutorial/part2_basics/input-representations.html

Time-Frequency Analysis via STFT

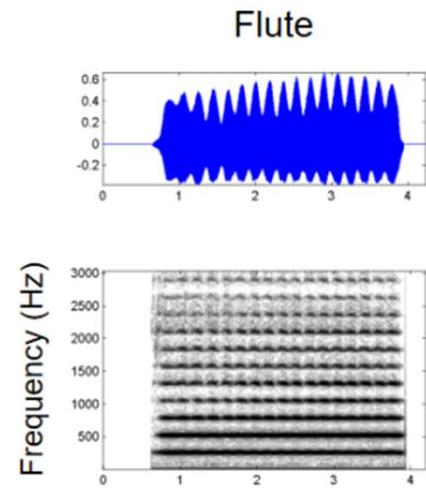
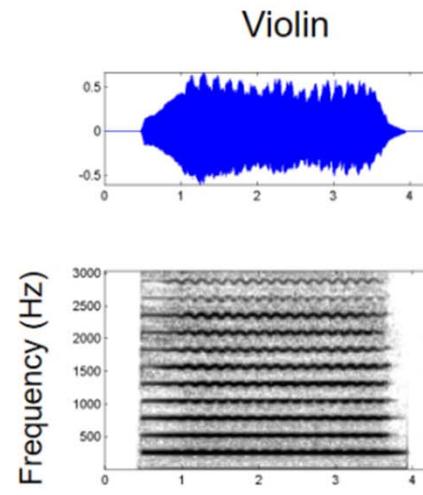
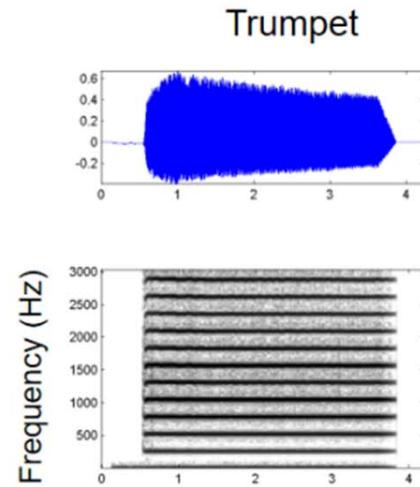
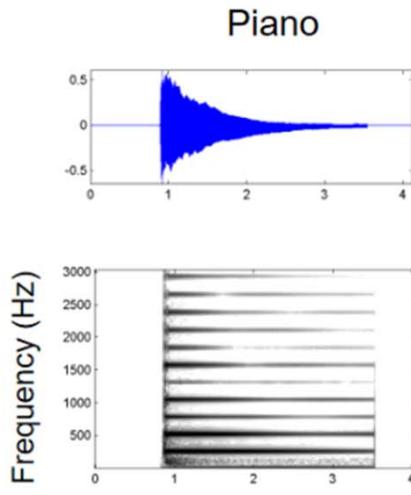
https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html



- **Time-frequency representation of audio**
 - Linear frequency, linear magnitude
 - Linear frequency, logarithmic magnitude (dB)

Spectral Characteristics

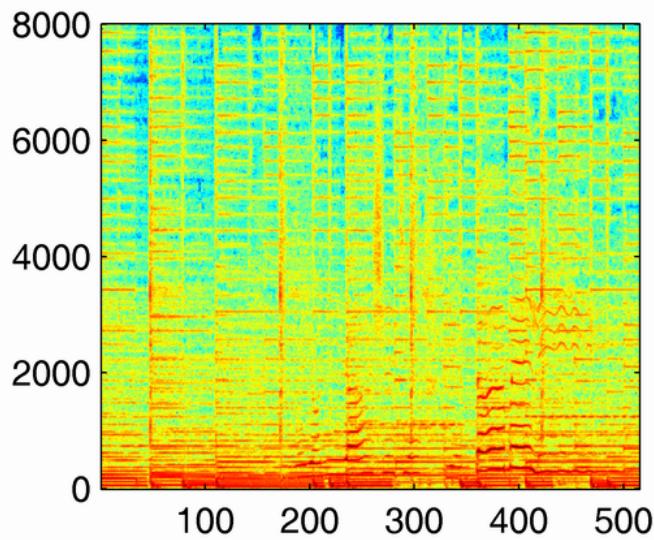
- Fundamental frequency (**F0**)
- **Partials** (harmonics): usually at frequencies which are **integer multiples** of F0
 - The partials in different instruments may exhibit **relative strengths**
 - The frequency may *deviate* from the ideal harmonics (this is called **inharmonicity**)



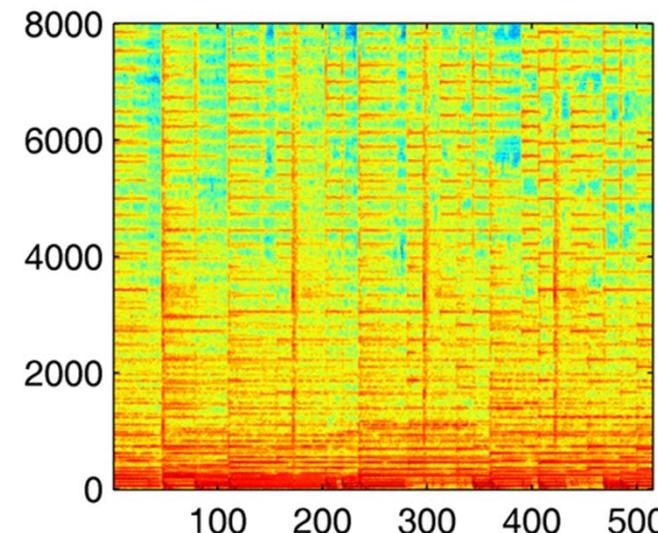
Time-Frequency Analysis via STFT

- For tasks such as classification/detection and source separation

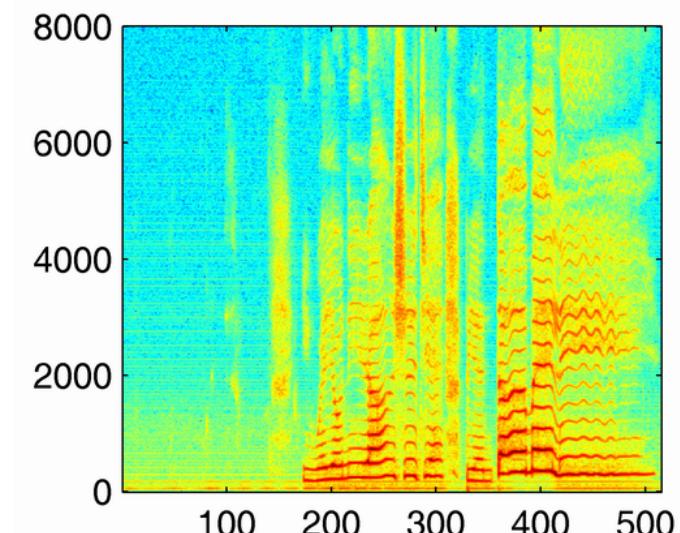
mixture



instrumental (clean)

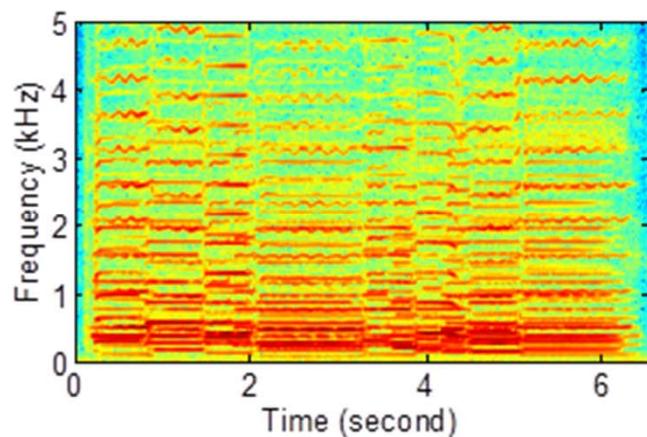


vocal (clean)

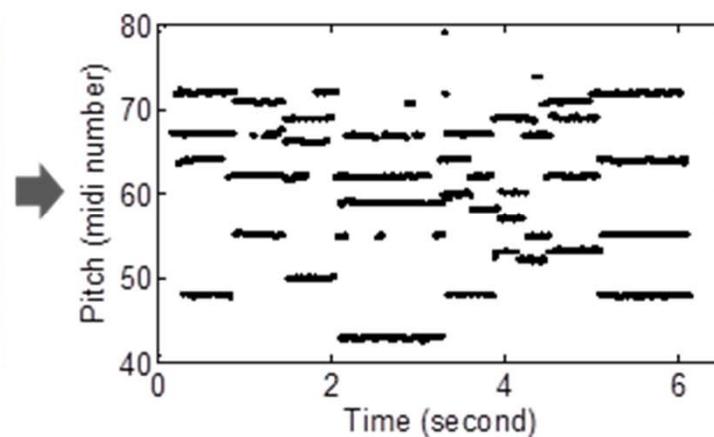


Time-Frequency Analysis via STFT

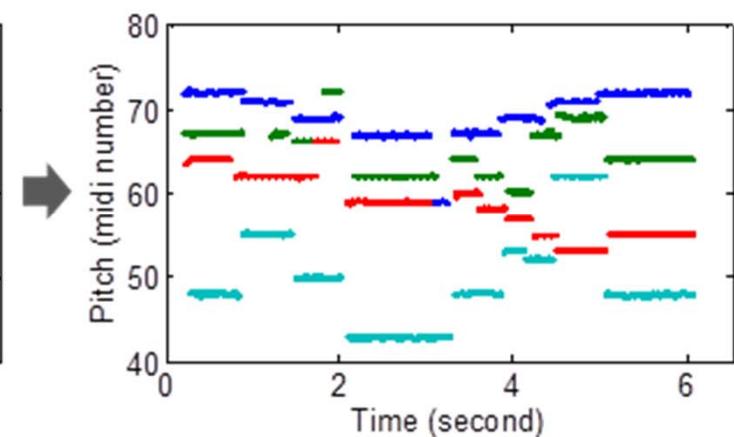
- For tasks such as multi-pitch estimation (MPE) and transcription



Spectrogram



Multi-pitch estimation results



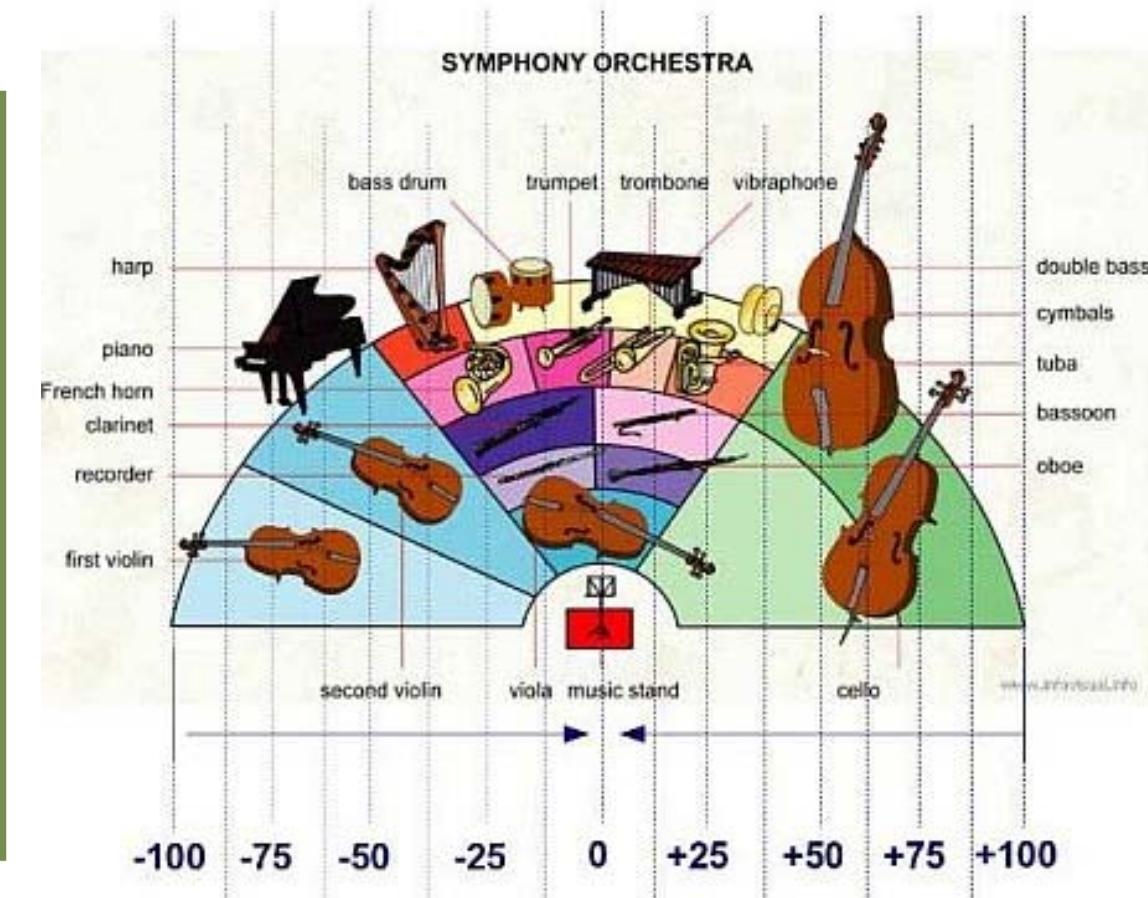
MPE + instrument detection

<https://labsites.rochester.edu/air/projects/multipitch/multipitch.html>

Spatial Information



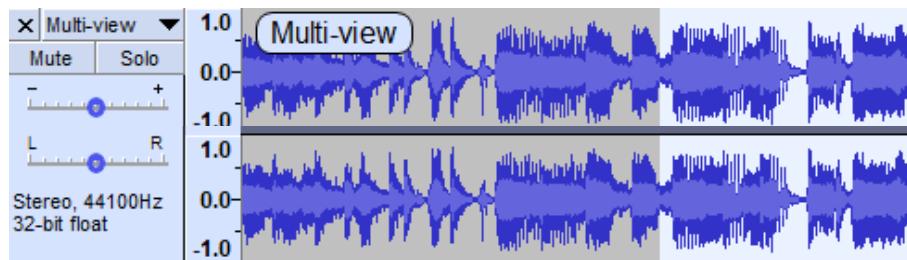
<https://www.renegadeproducer.com/audio-panning.html>



<https://www.audiorecording.me/symphony-orchestra-panning-and-reverb-settings.html>

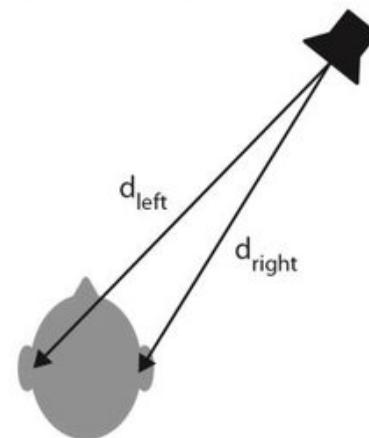
Spatial Information: Mono vs Stereo

- Monaural
- Binaural

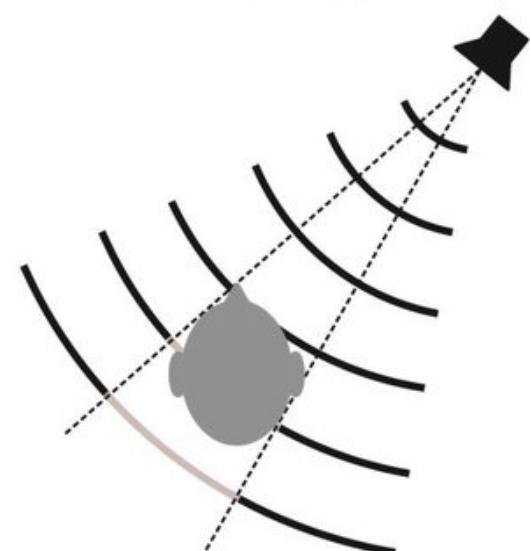


- Stereo-to-mono conversion
 - By taking the average
- Mono-to-stereo conversion
 - Need research
https://www.youtube.com/watch?v=aWxmQKm_s8Q

a) Binaural localization cue:
interaural time difference (ITD)



b) Binaural localization cue:
interaural intensity difference (IID)



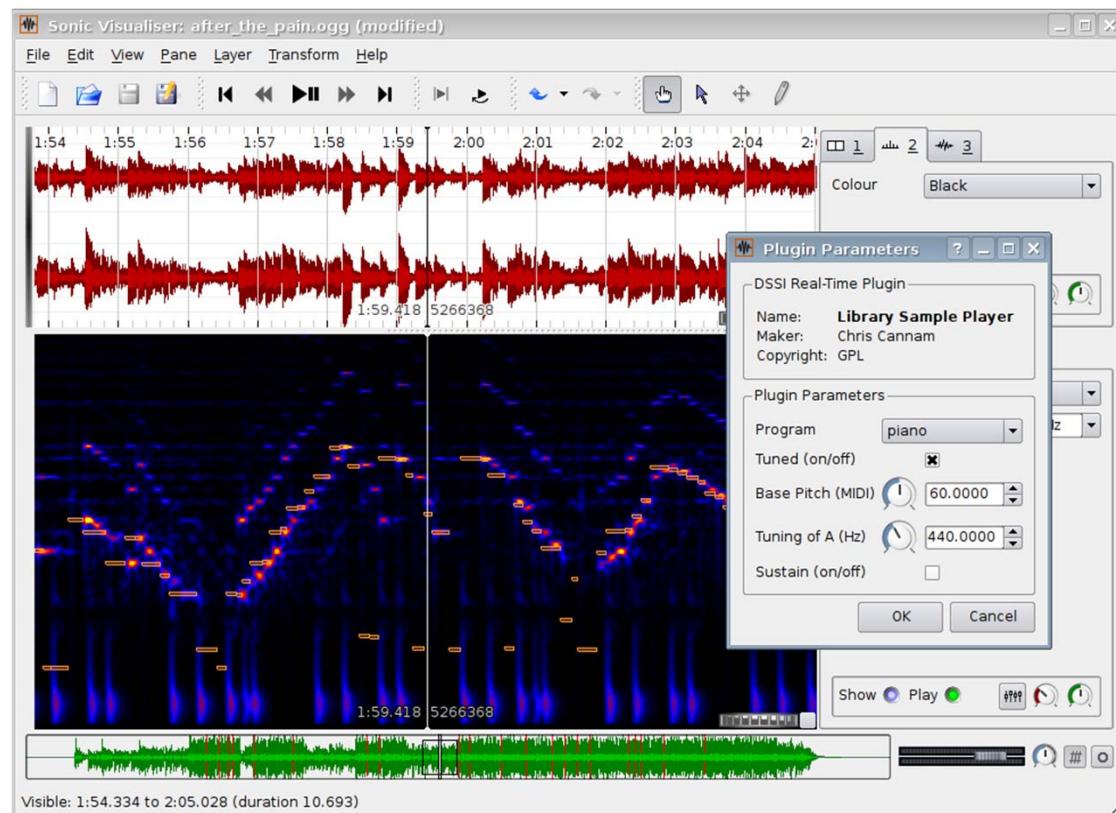
Source: https://www.researchgate.net/figure/Binaural-and-monaural-cues-used-for-sound-localization-a-For-sound-sources-off-the_fig1_299281975

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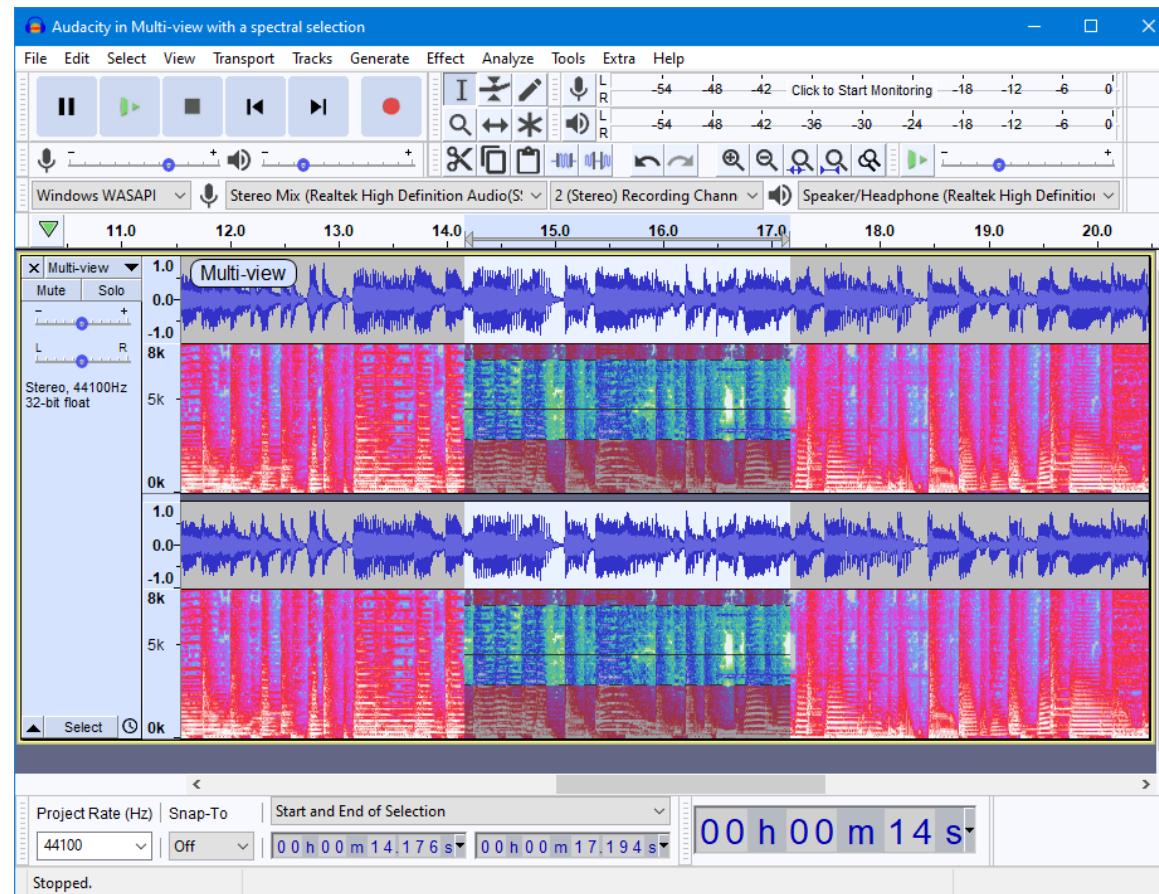
Software: Sonic Visualiser

<http://www.sonicvisualiser.org/>



Software: Audacity

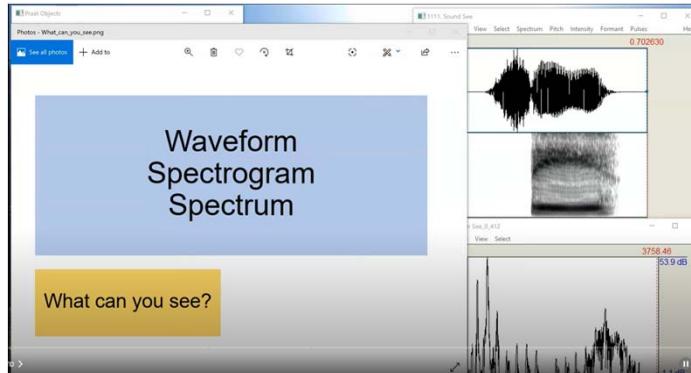
<https://www.audacityteam.org/>



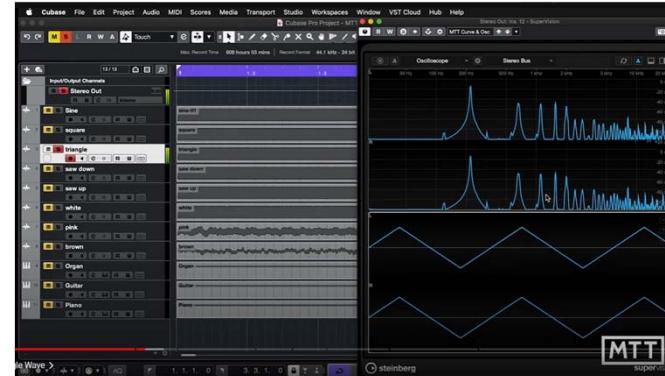
Let's Find Some Audio Recordings

- <https://www.freesound.org/>
 - <https://www.freesound.org/people/acclivity/sounds/22347/>
 - https://www.freesound.org/people/Rudmer_Rotteveel/sounds/316915/
 - <https://www.freesound.org/people/Jaylew1987/sounds/321112/>
 - <https://www.freesound.org/people/mickel11/sounds/90803/>

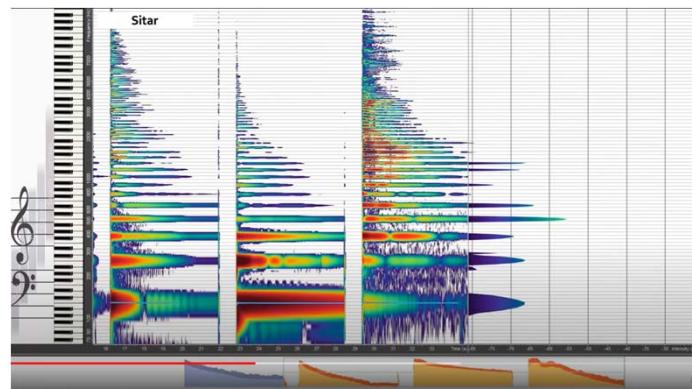
Related Videos



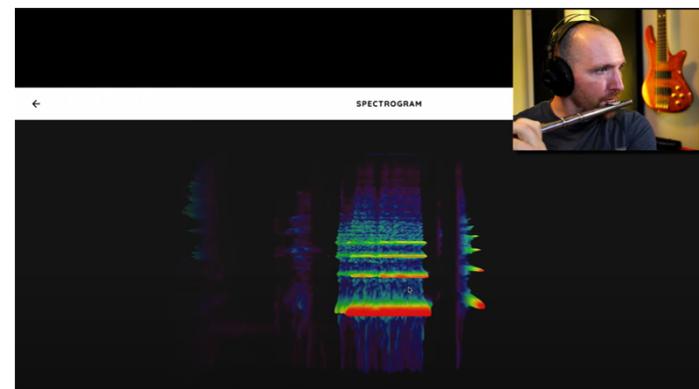
<https://www.youtube.com/watch?v=2Hj1kAWVjLo>



<https://www.youtube.com/watch?v=fZ18C5RXDcc>



<https://www.youtube.com/watch?v=VRAXK4QKJ1Q>

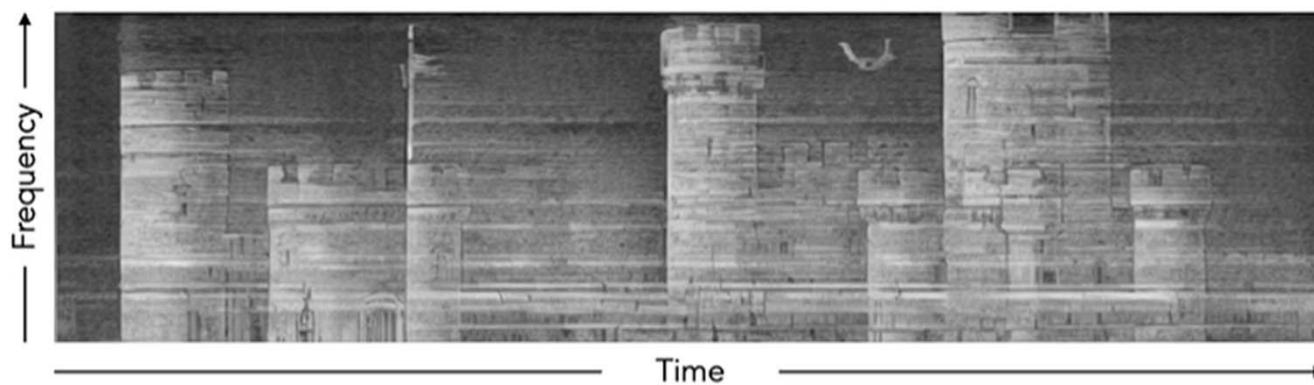


<https://www.youtube.com/watch?v=jEO0GHU3xsU>

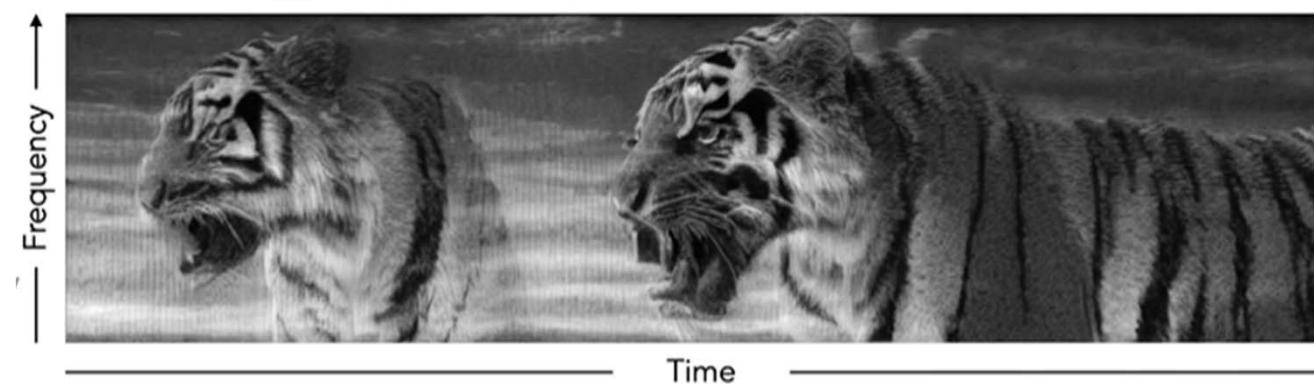
Fun Stuff: “Images that Sound”

<https://ificl.github.io/images-that-sound/>

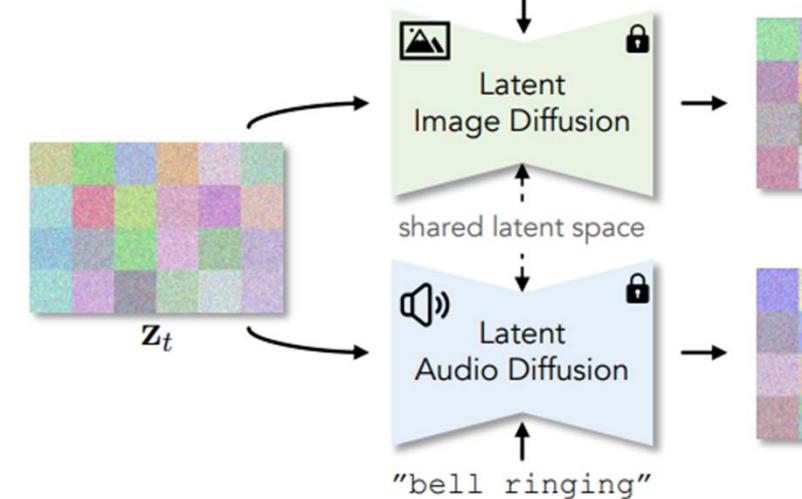
“castle with bell towers, grayscale, lithograph style”



“tiger, grayscale, black background”



“castle with bell towers, grayscale”

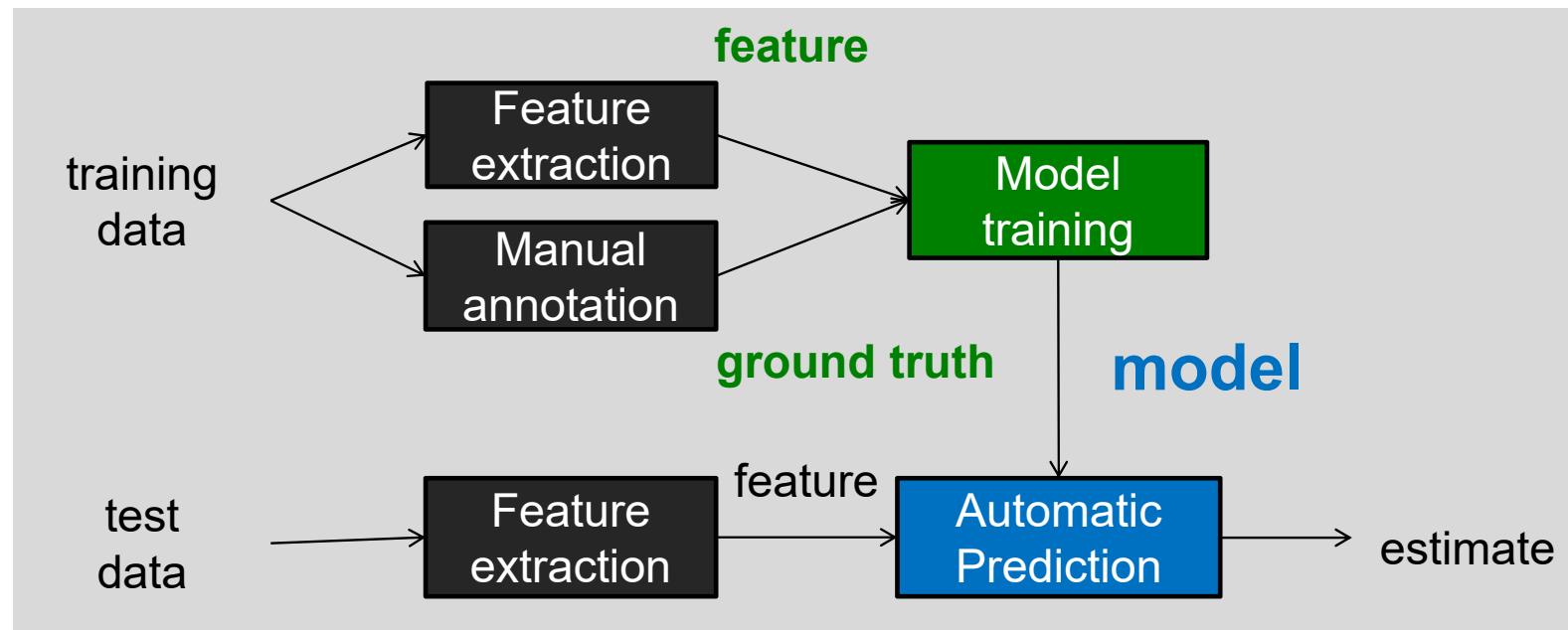


Outline

- Time and frequency representations for music
- **Audio features for music**
- Math in STFT

Representing the Audio as Features is Needed in ML/DL

- Given: $\{(x_1, y_1), \dots, (x_N, yN)\}$
 - $x_i \in \mathbb{R}^M$: feature representation
 - $y_i \in \{-1, +1\}$: class label



Library: Torchaudio

https://pytorch.org/audio/0.11.0/tutorials/audio_feature_extractions_tutorial.html

```
waveform, sample_rate = get_speech_sample()

n_fft = 1024
win_length = None
hop_length = 512

# define transformation
spectrogram = T.Spectrogram(
    n_fft=n_fft,
    win_length=win_length,
    hop_length=hop_length,
    center=True,
    pad_mode="reflect",
    power=2.0,
)
# Perform transformation
spec = spectrogram(waveform)

print_stats(spec)
plot_spectrogram(spec[0], title="torchaudio")
```

Library: LibROSA

<https://librosa.org/doc/latest/index.html>

https://colab.research.google.com/github/stevetjoa/musicinformationretrieval.com/blob/gh-pages/ipython_audio.ipynb

```
[ ] import librosa  
x, sr = librosa.load('audio/simple_loop.wav')  
  
[ ] X = librosa.stft(x)  
Xdb = librosa.amplitude_to_db(abs(X))  
plt.figure(figsize=(14, 5))  
librosa.display.specshow(Xdb, sr=sr, x_axis='time', y_axis='hz')
```

Library: LibROSA

<https://librosa.org/doc/latest/index.html>

Audio loading

```
load (path, *[sr, mono, offset, duration, ...])  
stream (path, *, block_length, frame_length, ...)  
to_mono (y)  
resample (y, *, orig_sr, target_sr[, ...])  
get_duration (*[y, sr, S, n_fft, ...])  
get_samplerate (path)
```

Time-domain processing

```
autocorrelate (y, *[max_size, axis])  
lpc (y, *, order[, axis])  
zero_crossings (y, *[threshold, ...])  
mu_compress (x, *[mu, quantize])  
mu_expand (x, *[mu, quantize])
```

Signal generation

```
clicks (*[times, frames, sr, hop_length, ...])  
tone (frequency, *[sr, length, duration, phi])  
chirp (*, fmin, fmax[, sr, length, duration, ...])
```

Magnitude scaling

```
amplitude_to_db (S, *[ref, amin, top_db])  
db_to_amplitude (S_db, *[ref])  
power_to_db (S, *[ref, amin, top_db])  
db_to_power (S_db, *[ref])  
perceptual_weighting (S, frequencies, *[kind])  
frequency_weighting (frequencies, *[kind])  
multi_frequency_weighting (frequencies, *[...])  
A_weighting (frequencies, *[min_db])  
B_weighting (frequencies, *[min_db])
```

Library: LibROSA

<https://librosa.org/doc/latest/index.html>

Time unit conversion

`frames_to_samples` (frames, *[, hop_length, n_fft])
`frames_to_time` (frames, *[, sr, hop_length, ...])
`samples_to_frames` (samples, *[, hop_length, ...])
`samples_to_time` (samples, *[, sr])
`time_to_frames` (times, *[, sr, hop_length, n_fft])
`time_to_samples` (times, *[, sr])
`blocks_to_frames` (blocks, *, block_length)
`blocks_to_samples` (blocks, *, block_length, ...)
`blocks_to_time` (blocks, *, block_length, ...)

Frequency unit conversion

`hz_to_note` (frequencies, **kwargs)
`hz_to_midi` (frequencies)
`midi_to_hz` (notes)
`midi_to_note` (midi, *[, octave, cents, key, ...])
`note_to_hz` (note, **kwargs)
`note_to_midi` (note, *[, round_midi])

Music notation

`key_to_notes` (key, *[, unicode])
`key_to_degrees` (key)

Library: LibROSA

<https://librosa.org/doc/latest/index.html>

Spectral representations

```
stft (y, *[, n_fft, hop_length, win_length, ...])  
istft (stft_matrix, *[, hop_length, ...])  
cqt (y, *[, sr, hop_length, fmin, n_bins, ...])  
icqt (C, *[, sr, hop_length, fmin, ...])
```

Harmonics

```
interp_harmonics (x, *, freqs, harmonics[, ...])  
salience (S, *, freqs, harmonics[, weights, ...])  
f0_harmonics (x, *, f0, freqs, harmonics[, ...])  
phase_vocoder (D, *, rate[, hop_length, n_fft])
```

Spectral features

```
chroma_stft (*[, y, sr, S, norm, n_fft, ...])  
chroma_cqt (*[, y, sr, C, hop_length, fmin, ...])  
chroma_cens (*[, y, sr, C, hop_length, fmin, ...])  
chroma_vqt (*[, y, sr, V, hop_length, fmin, ...])  
melspectrogram (*[, y, sr, S, n_fft, ...])  
mfcc (*[, y, sr, S, n_mfcc, dct_type, norm, ...])  
rms (*[, y, S, frame_length, hop_length, ...])  
spectral_centroid (*[, y, sr, S, n_fft, ...])
```

Pitch and tuning

```
pyin (y, *, fmin, fmax[, sr, frame_length, ...])  
yin (y, *, fmin, fmax[, sr, frame_length, ...])  
estimate_tuning (*[, y, sr, S, n_fft, ...])
```

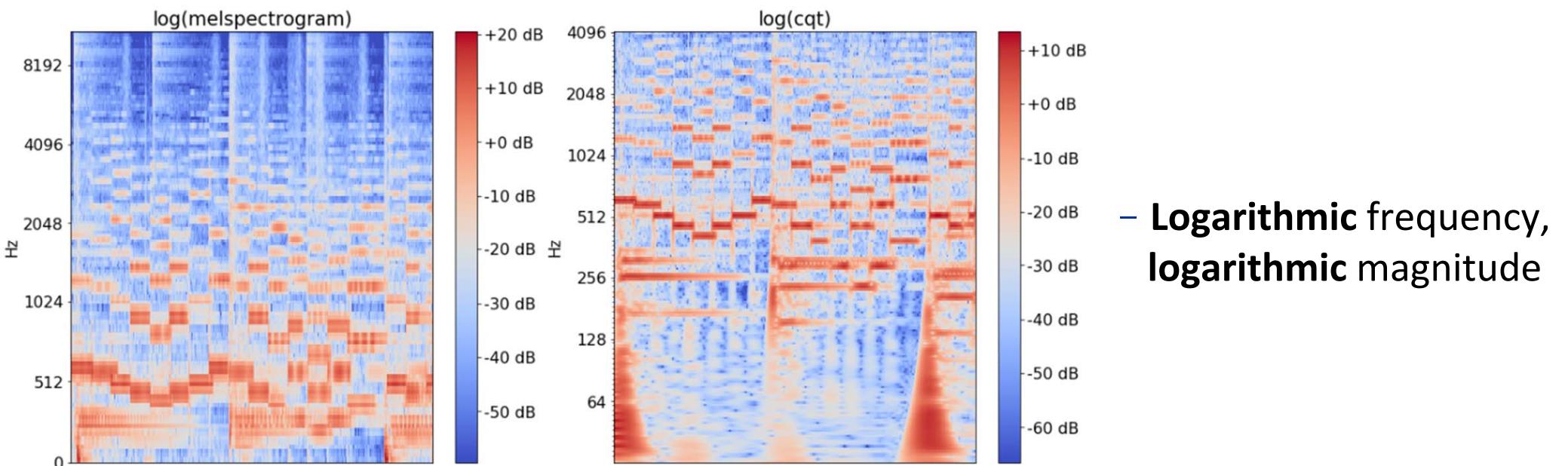
Rhythm features

```
tempo (*[, y, sr, onset_envelope, tg, ...])  
tempogram (*[, y, sr, onset_envelope, ...])
```

Constant-Q Transform (CQT)

https://music-classification.github.io/tutorial/part2_basics/input-representations.html

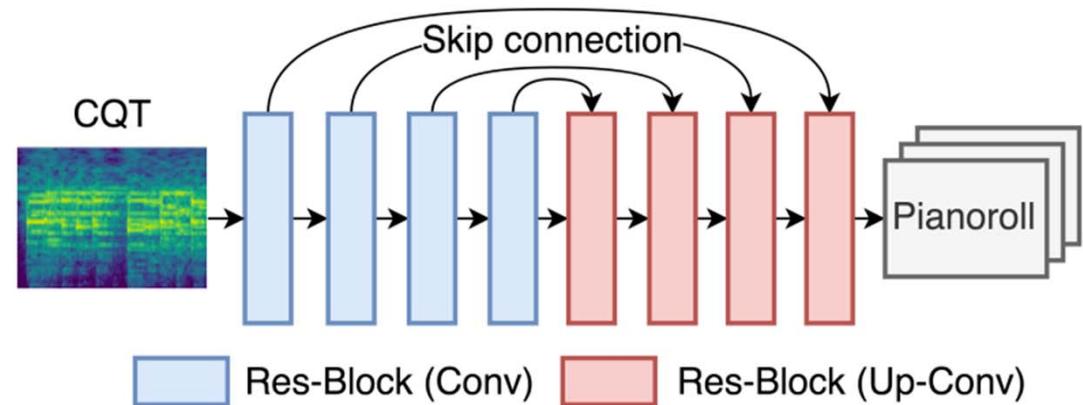
- **STFT** (*linearly*-spaced frequencies)
- **CQT** (*logarithmically*-spaced, closer to human auditory perception)



Constant-Q Transform (CQT)

- Good for **pitch-related** tasks (e.g., multi-pitch estimation or transcription)

Ref: Hung et al, “Multitask learning for frame-level instrument recognition,” ICASSP 2019



CQT - PyTorch ↗

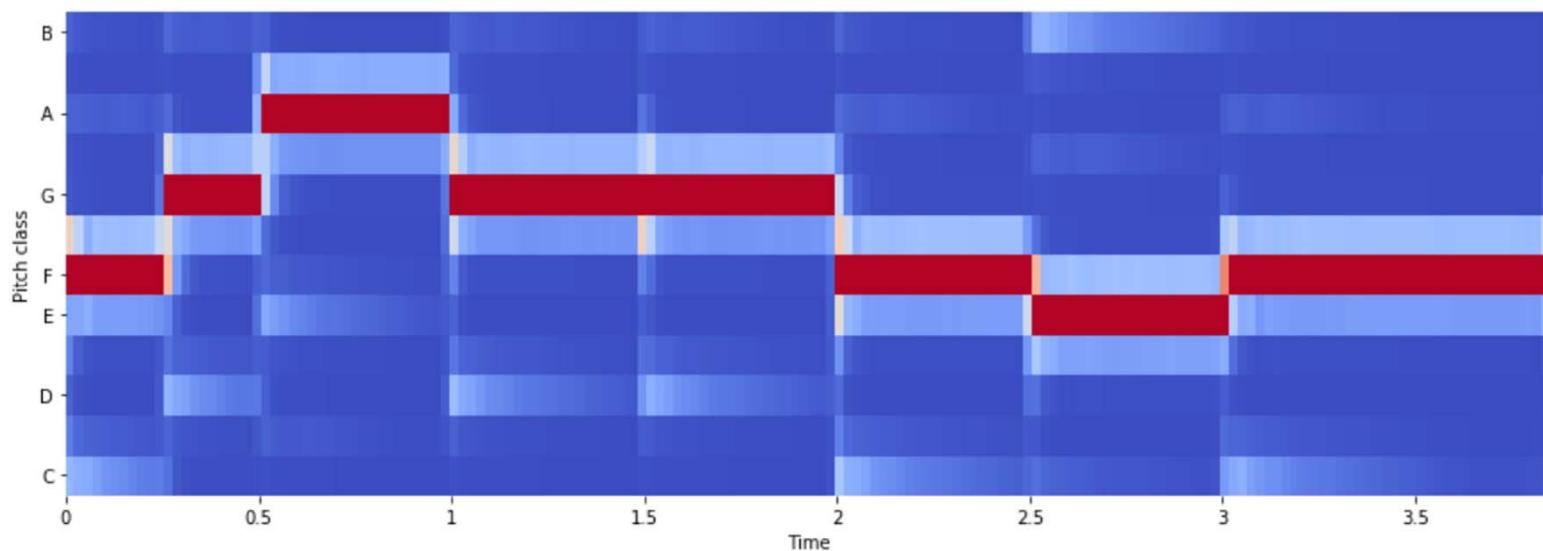
```
from cqt_pytorch import CQT  
  
transform = CQT(  
    num_octaves = 8,  
    num_bins_per_octave = 64,  
    sample_rate = 48000,  
    block_length = 2 ** 18  
)
```

<https://github.com/archinetai/cqt-pytorch>
https://github.com/eloimoliner/CQT_pytorch

Pitch Class Profile / Chromagram

<https://musicinformationretrieval.com/chroma.html>

- “A **chroma vector** ([Wikipedia](#)) is typically a 12-element feature vector indicating how much energy of each pitch class, {C, C#, D, D#, E, ..., B}, is present in the signal”
 - i.e., ignore octaves



Pitch Class Profile / Chromagram

- Good for tasks such as **chord recognition** or cover song identification

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C5/C5S2_ChordRec_Templates.html

Ref: Cho et al, “On the relative importance of individual components of chord recognition systems,” TASLP 2014

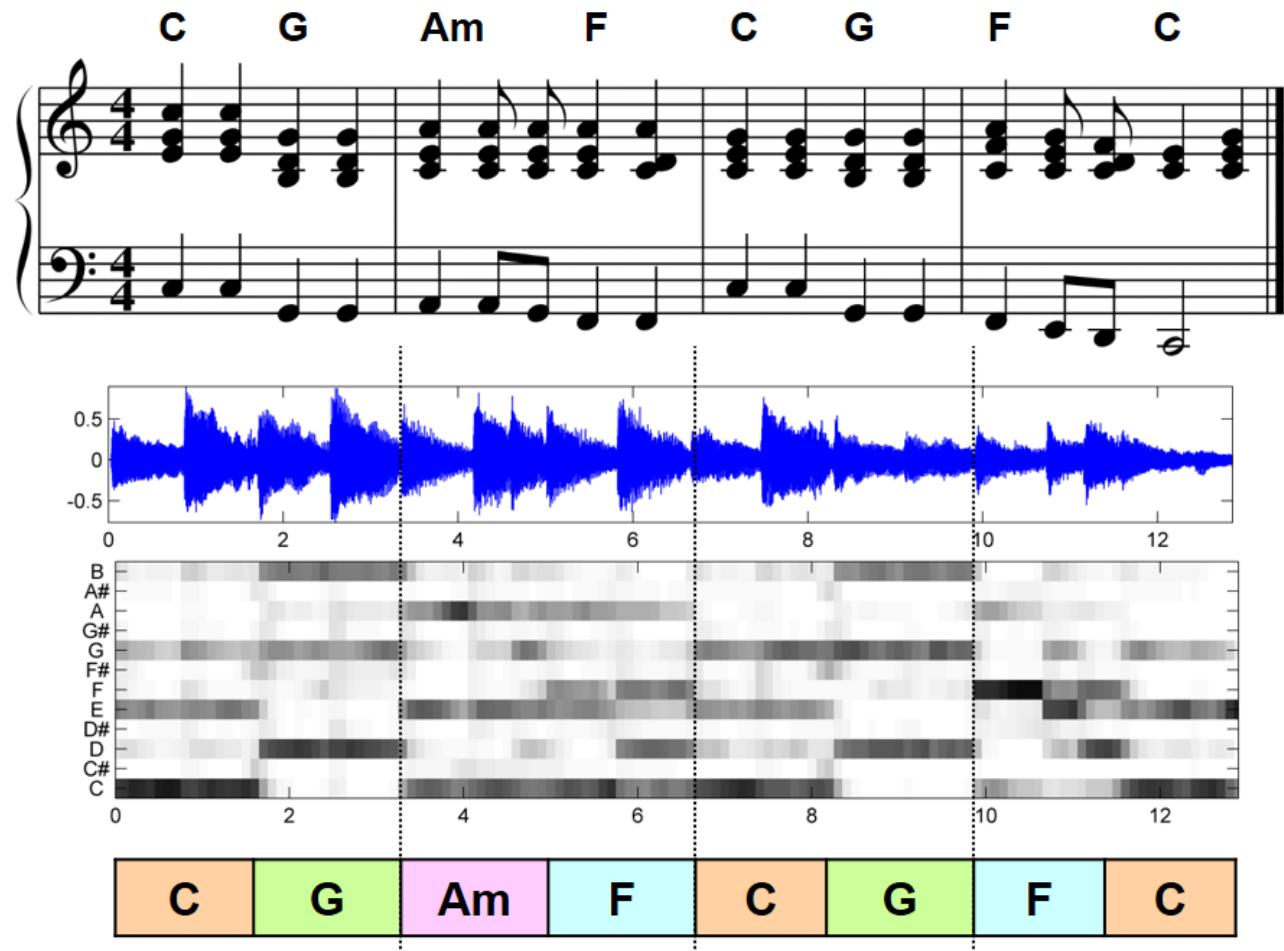
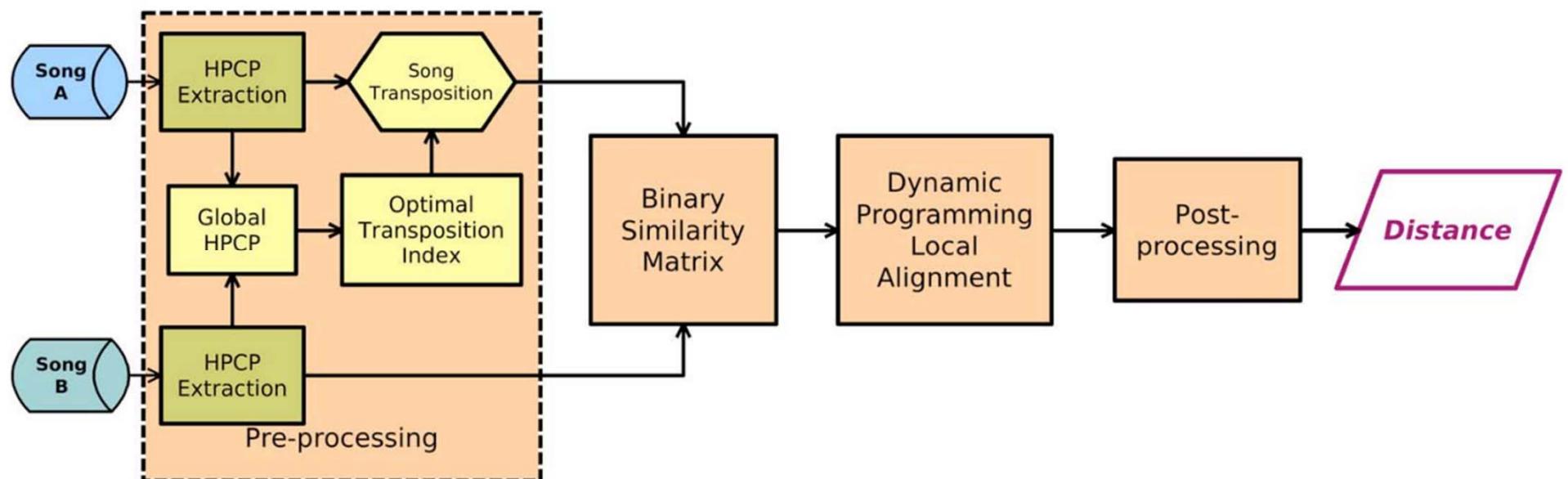


Figure 5.1 from [Müller, FMP, Springer 2015]

Pitch Class Profile / Chromagram

- Good for tasks such as chord recognition or **cover song identification**

https://essentia.upf.edu/tutorial_similarity_cover.html

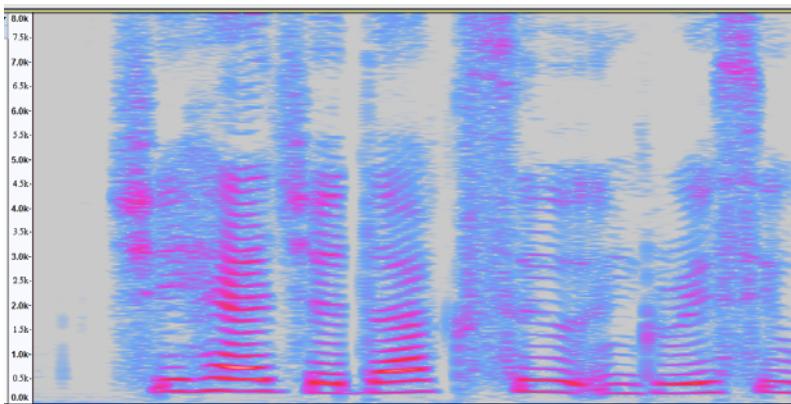


Ref: Serra et al, "Chroma binary similarity and local alignment applied to cover song identification," TASLP 2008

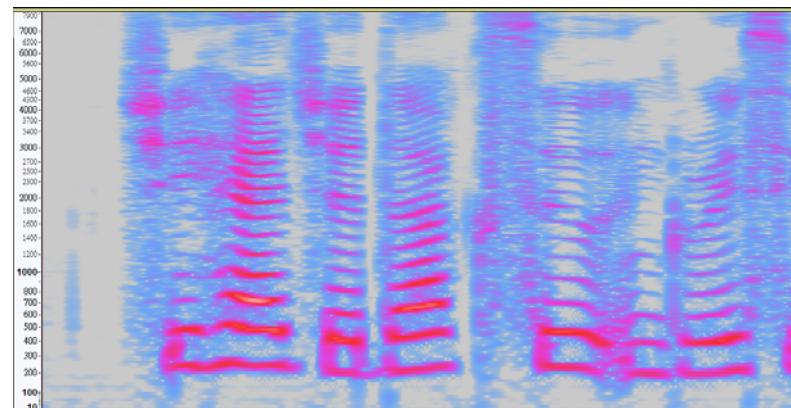
Mel-Spectrogram

- The Mel scale is a perceptual scale of pitches judged by listeners to be equal in distance from one another
- **Finer resolution in the low-frequency range** (NOT exactly logarithmic scale)
- Dimension reduction

linear scale



mel scale



Mel-Spectrogram

https://music-classification.github.io/tutorial/part2_basics/input-representations.html

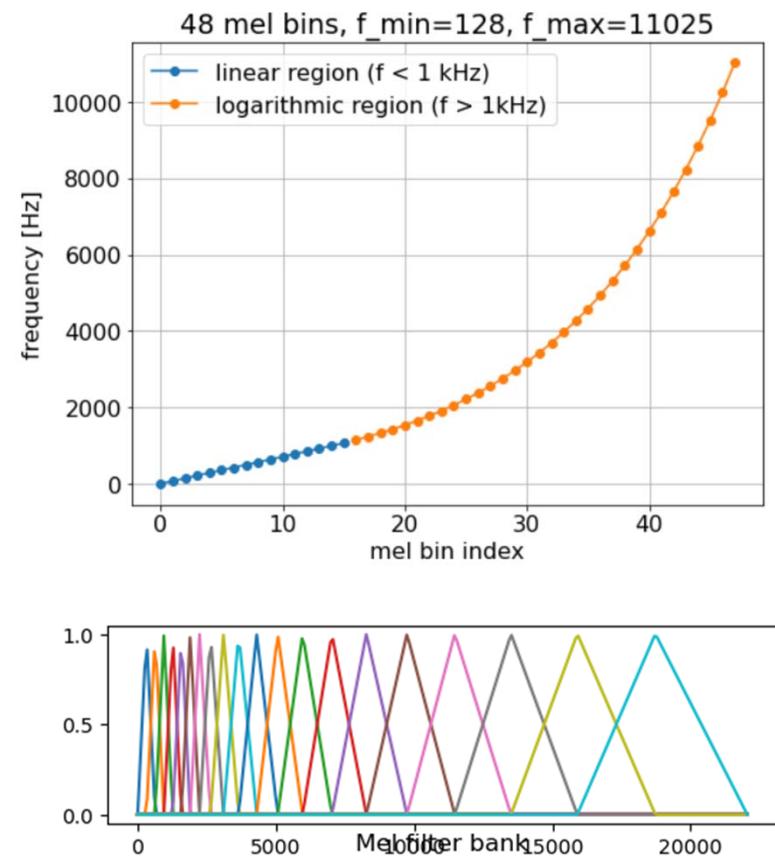
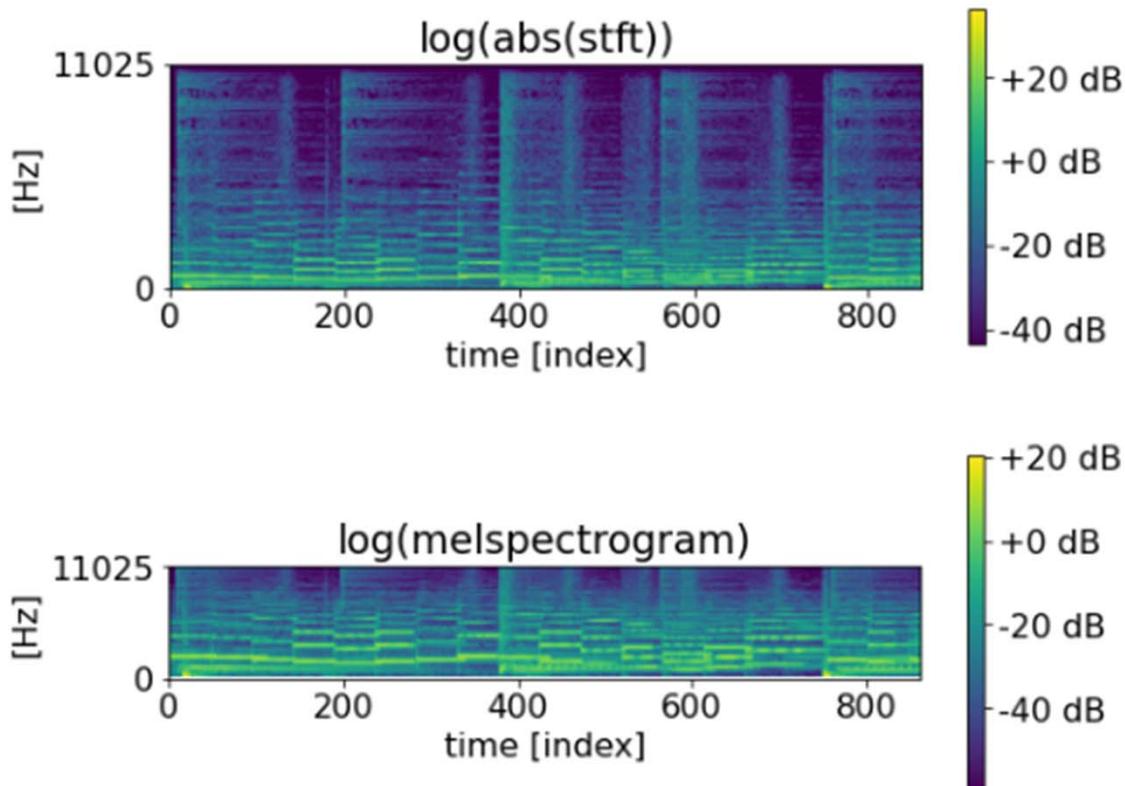


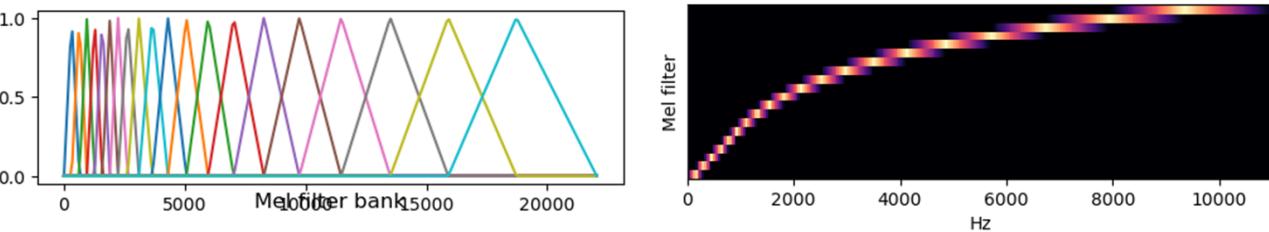
Figure from librosa

Mel-Spectrogram

- Usually treated as the “**default**” feature representation for musical audio, especially in the DL era
 - Easy to compute and understand
 - Reasonably rich information
 - Reasonable size
 - Can be used as input to computer vision (CV) models
 - Possible to go back from mel-spectrograms to waveforms via a “vocoder” (see lecture 5)
- Used in all types of tasks, for both music analysis and generation

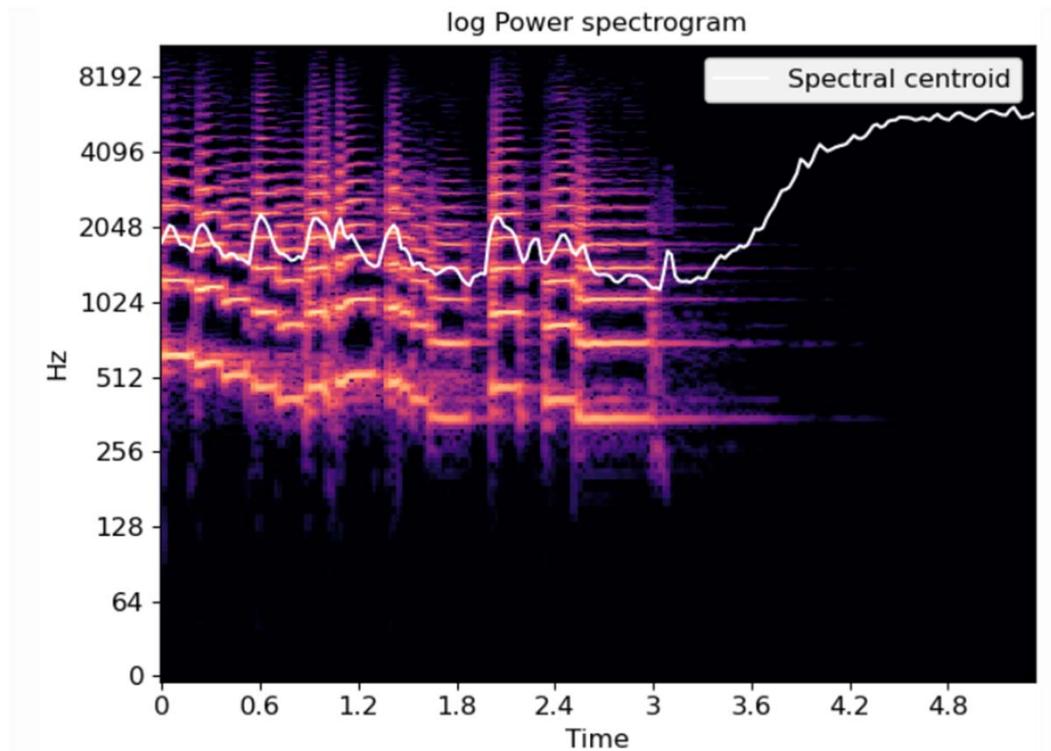
Mel-frequency cepstral coefficients (MFCC)

- Procedure
 1. Compute the spectrogram
 2. Grouping the FFT bins according to the *perceptually motivated* Mel-filter bank
 - Linear till 1,000 Hz and logarithmic above it
 - Usually with triangular overlapping windows
 3. Taking logs and **DCT** for uncorrelating the resulting features
- **Compact representation** of the spectrum (1,024-dim \rightarrow 128 \rightarrow 13 coefficients)
 - Somehow capture the energy distribution in the spectrum
 - Less interpretable



Spectral Centroid

- Each frame of a magnitude spectrogram is normalized and treated as a distribution over frequency bins, from which the mean (centroid) is extracted per frame
- A measure of spectral shape
- Higher centroid values correspond to “**brighter**” textures with more high frequencies
- Other statistics can also be used
 - bandwidth, skewness, kurtosis



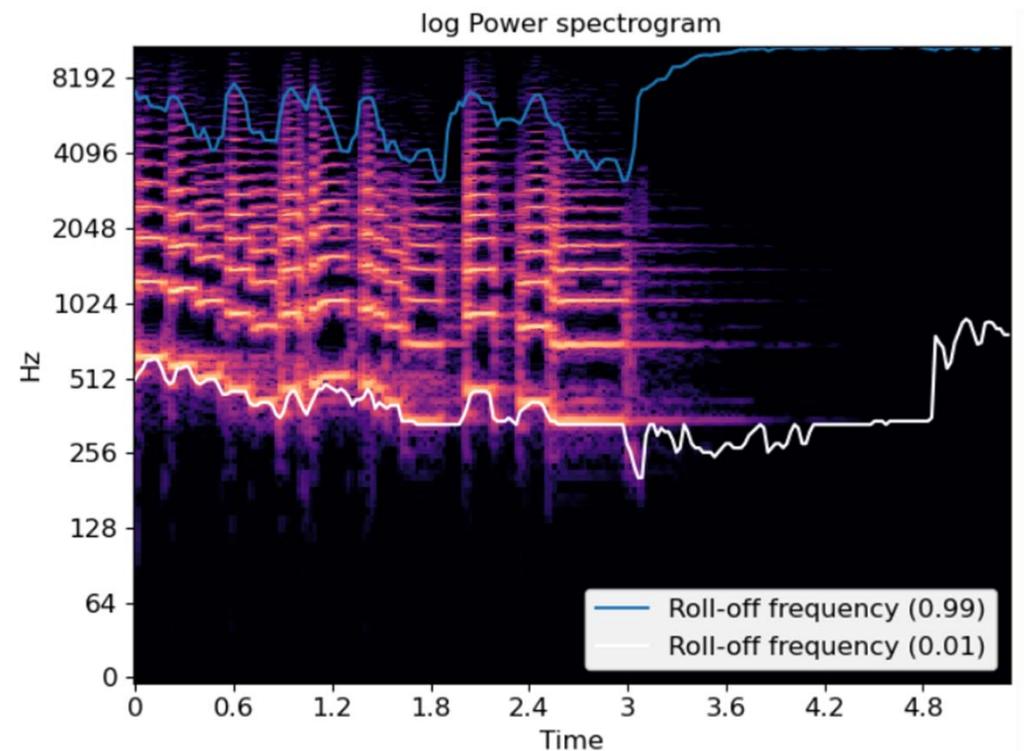
Ref: Tzanetakis and Cook, “Musical genre classification of audio signals,” TASLP 2002

Spectral Contrast

- Each frame of a spectrogram is divided into sub-bands. For each sub-band, the energy contrast is estimated by comparing the mean energy in the top quantile (peak energy) to that of the bottom quantile (valley energy). High contrast values generally correspond to **clear, narrow-band signals**, while low contrast values correspond to **broad-band noise**

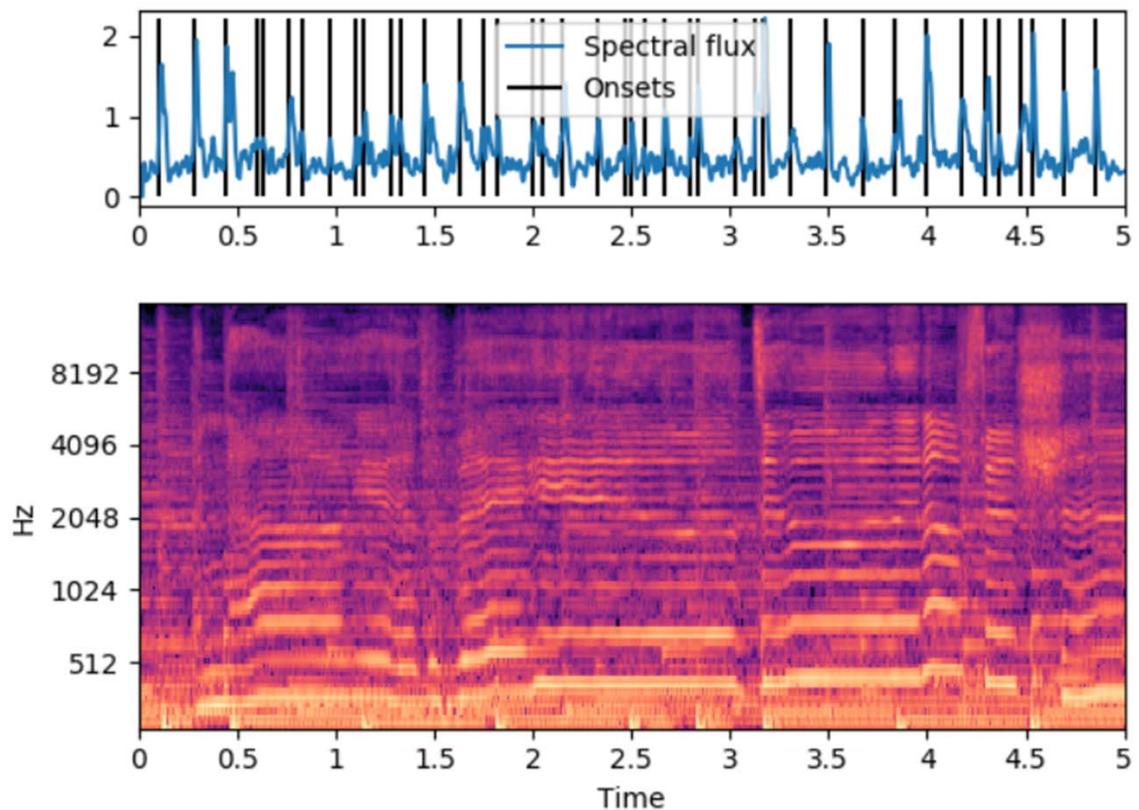
Spectral Rolloff

- The frequency for a spectrogram bin such that at least *roll_percent* (0.85 by default) of the **energy** of the spectrum in a frame is contained **in this bin and the bins below**
- Can be used to approximate the **maximum (or minimum)** frequency by setting *roll_percent* to a value close to 1 (or 0)
- Another measure of spectral shape



Spectral Flux

- How quickly the power spectrum of a signal is changing over time
- Usually calculated as the L2-norm between two adjacent normalized spectra
- Usually used for musical onset detection (more related to rhythm than to timbre)



Ref1: https://en.wikipedia.org/wiki/Spectral_flux

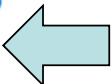
Ref2: https://librosa.org/librosa_gallery/auto_examples/plot_superflux.html

Demonstrations 1

https://colab.research.google.com/github/stevetjoa/musicinformationretrieval/blob/gh-pages/spectral_features.ipynb

Signal Analysis and Feature Extraction

1. Basic Feature Extraction
2. Segmentation
3. Energy and RMSE
4. Zero Crossing Rate
5. Fourier Transform
6. Short-time Fourier Transform and Spectrogram
7. Constant-Q Transform and Chroma
8. Video: Chroma Features
9. Magnitude Scaling
10. Spectral Features
11. Autocorrelation
12. Pitch Transcription Exercise



Demonstrations 2

http://www.ifs.tuwien.ac.at/~schindler/lectures/MIR_Feature_Extraction.html

```
def spectral_centroid(wavedata, window_size, sample_rate):

    magnitude_spectrum = stft(wavedata, window_size)

    timebins, freqbins = np.shape(magnitude_spectrum)

    # when do these blocks begin (time in seconds)?
    timestamps = (np.arange(0,timebins - 1) * (timebins / float(sample_rate)))

    sc = []

    for t in range(timebins-1):

        power_spectrum = np.abs(magnitude_spectrum[t])**2

        sc_t = np.sum(power_spectrum * np.arange(1,freqbins+1)) / np.sum(power_spectrum)

        sc.append(sc_t)

    sc = np.asarray(sc)
    sc = np.nan_to_num(sc)

    return sc, np.asarray(timestamps)
```

Library 1: LibROSA

Spectral features

<code>melspectrogram (*[, y, sr, S, n_fft, ...])</code>	Compute a mel-scaled spectrogram.
<code>mfcc (*[, y, sr, S, n_mfcc, dct_type, norm, ...])</code>	Mel-frequency cepstral coefficients (MFCCs)
<code>rms (*[, y, S, frame_length, hop_length, ...])</code>	Compute root-mean-square (RMS) value for each frame, either from the input signal <code>y</code> or from a spectrogram <code>S</code> .
<code>spectral_centroid (*[, y, sr, S, n_fft, ...])</code>	Compute the spectral centroid.
<code>spectral_bandwidth (*[, y, sr, S, n_fft, ...])</code>	Compute p'th-order spectral bandwidth.
<code>spectral_contrast (*[, y, sr, S, n_fft, ...])</code>	Compute spectral contrast
<code>spectral_flatness (*[, y, S, n_fft, ...])</code>	Compute spectral flatness
<code>spectral_rolloff (*[, y, sr, S, n_fft, ...])</code>	Compute roll-off frequency.

Library 2: Audio Commons Audio Extractor

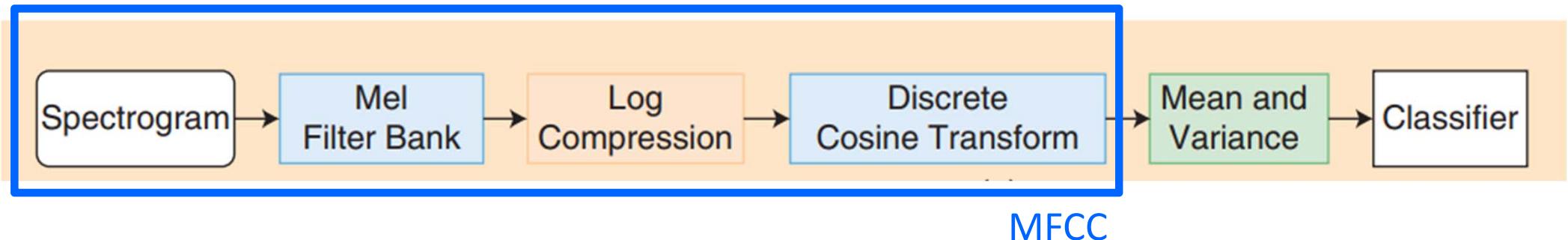
<https://github.com/AudioCommons/ac-audio-extractor>

JSON output example ↗

```
"duration": 6.0,  
"lossless": 1.0,  
"codec": "pcm_s16le",  
"bitrate": 705600.0,  
"samplerate": 44100.0,  
"channels": 1.0,  
"audio_md5": "8da67c9c2acbd13998c9002aa0f60466",  
"loudness": -28.207069396972656,  
"dynamic_range": 0.6650657653808594,  
"temporal_centroid": 0.5078766345977783,  
"log_attack_time": 0.30115795135498047,  
"filesize": 529278,  
"single_event": false,
```

```
"tonality": "G# minor",  
"tonality_confidence": 0.2868785858154297,  
"loop": true,  
"tempo": 120,  
"tempo_confidence": 1.0,  
"note_midi": 74,  
"note_name": "D5",  
"note_frequency": 592.681884765625,  
"note_confidence": 0.0,  
"brightness": 50.56954356039029,  
"depth": 13.000903137777897,  
"metallic": 0.4906048209174263,  
"roughness": 0.7237051954207928,  
"genre": "Genre B",  
"mood": "Mood B"
```

Spectral Features Can be Used to Build a Classifier in ML

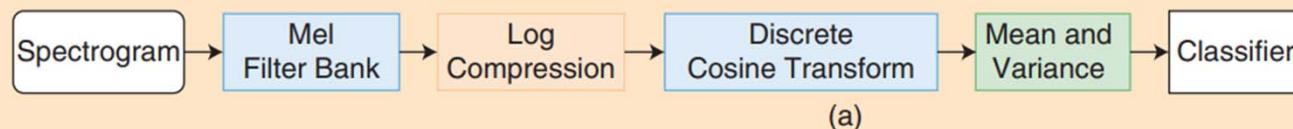


- Procedure
 1. Compute the features per STFT frame (e.g., 13-D frame-level features)
 2. **Temporal pooling** over time for each audio clip (e.g., by taking the **mean and variance**; leading to 26-D clip-level features)
 3. Use that as input to a **classifier** (e.g., random forest, or support vector machine)
- Limits
 - *Clear physical meaning but not sophisticated enough and limited semantic meaning*
 - Only the classifier is *trainable*; the features are hand-crafted

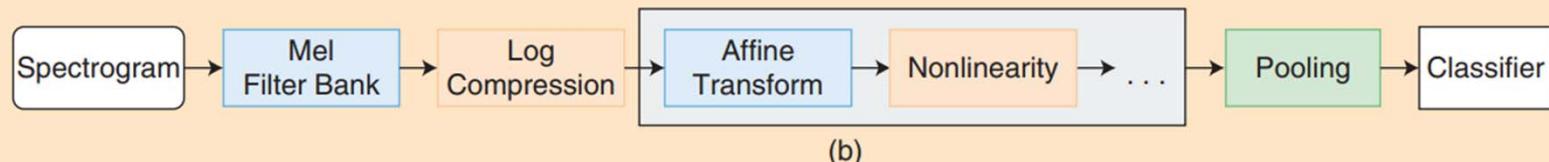
“Feature Learning” in DL

(the blocks inside the black lines are learned)

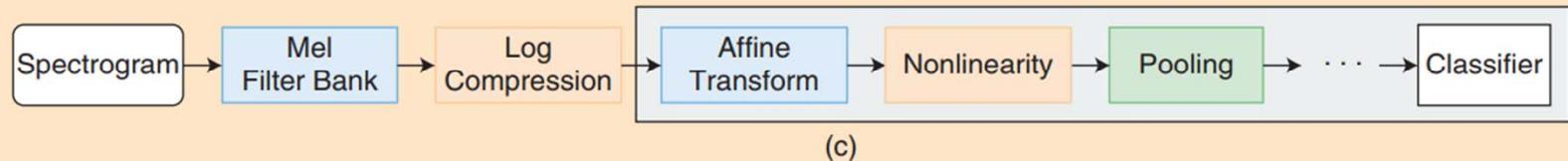
(a) Feature engineering



(b) Low-level feature learning



(c) Convolution neural networks



(d) End-to-end learning

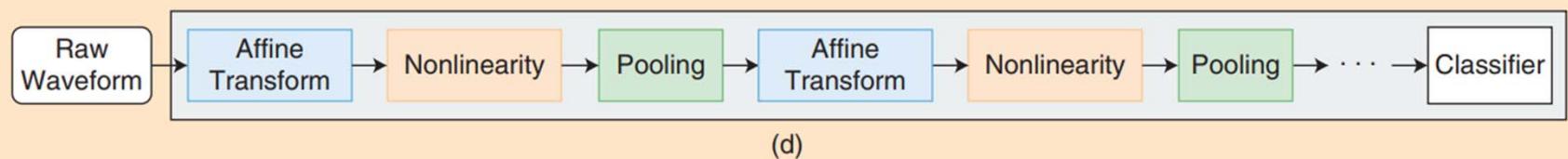
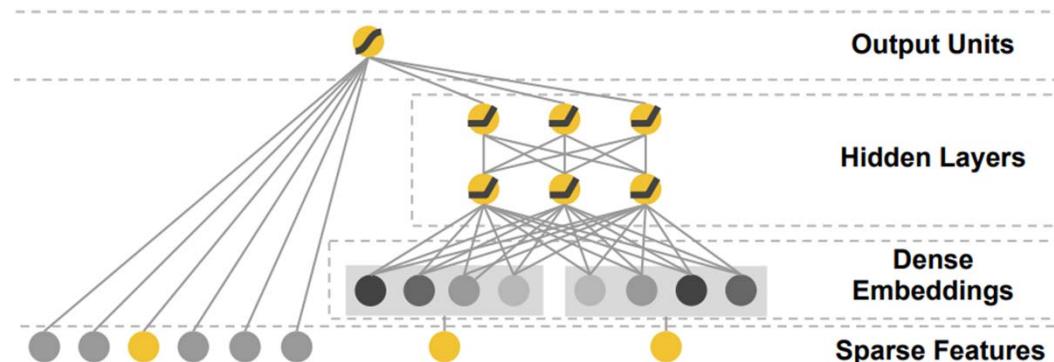


FIGURE 1. The transition of feature representation for music classification: (a) feature engineering [mel-frequency cepstral coefficients (MFCCs)], (b) low-level feature learning, (c) convolutional neural networks, and (d) end-to-end learning. The blocks inside the black lines indicate that they are learned by the algorithms.

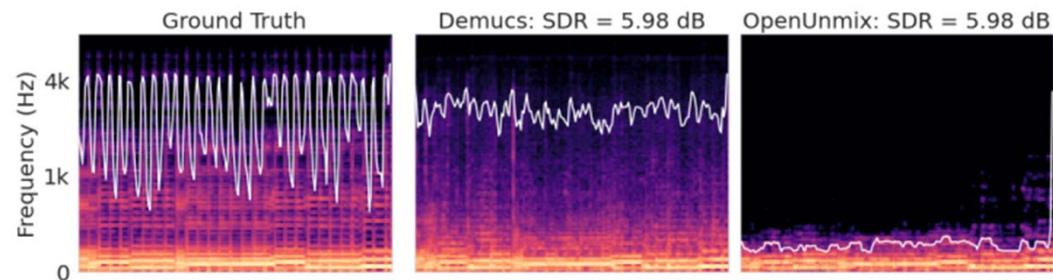
Ref: Nam et al., “Deep learning for audio-based music classification and tagging,” IEEE Signal Processing Magazine, 2019

The Use of Hand-crafted Features in DL

- Hand-crafted features
 - Clear physical meaning
 - Interpretable
- Used as input to music classifiers in the early days
- Can be used alongside learned features (though not popular)
 - The “deep & wide” architecture
- Can be used as objective metrics or loss functions for DL models



Ref: Cheng et al., “Wide & deep learning for recommender systems,” DLRS 2016



Ref: Schaffer et al., “Music separation enhancement with generative modeling,” ISMIR 2022

Different Features are Needed for Different Tasks

- **Timbre representation:** Spectrogram → mel-spectrogram → MFCC
- **Harmonic representation:** Spectrogram → CQT → chroma feature

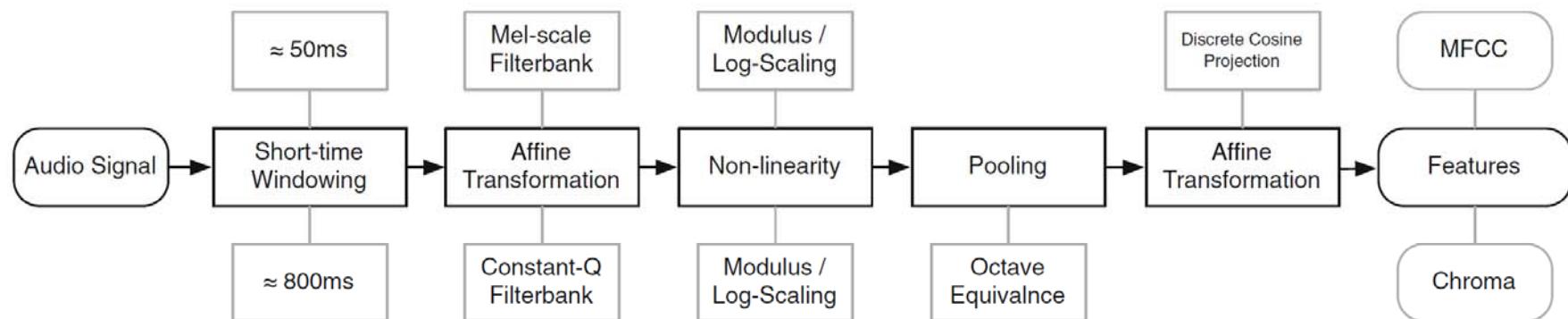
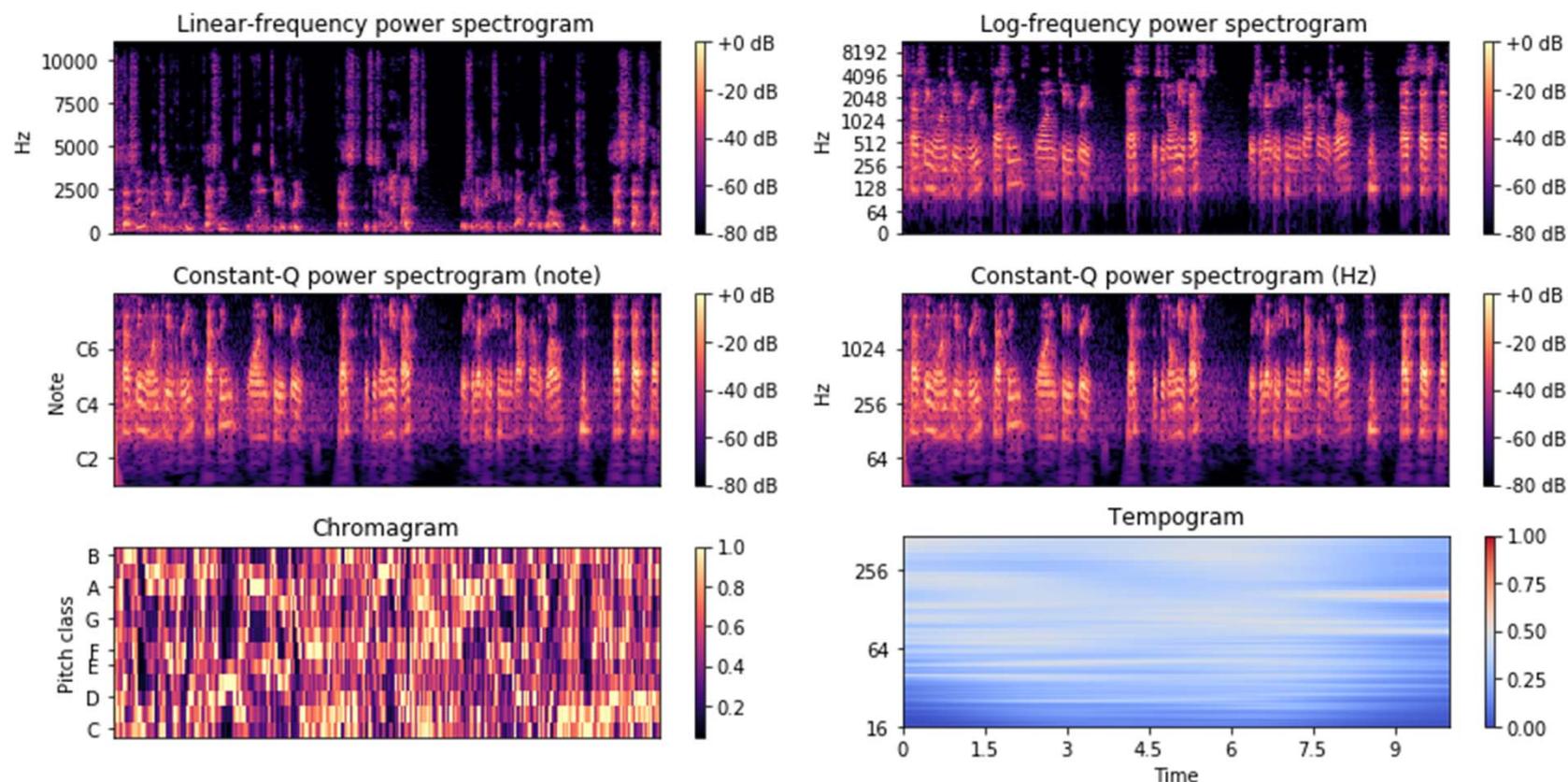


Fig. 3 *State of the art:* standard approaches to feature extraction proceed as the cascaded combination of a few simpler operations; on closer inspection, the main difference between chroma and MFCCs is the parameters used

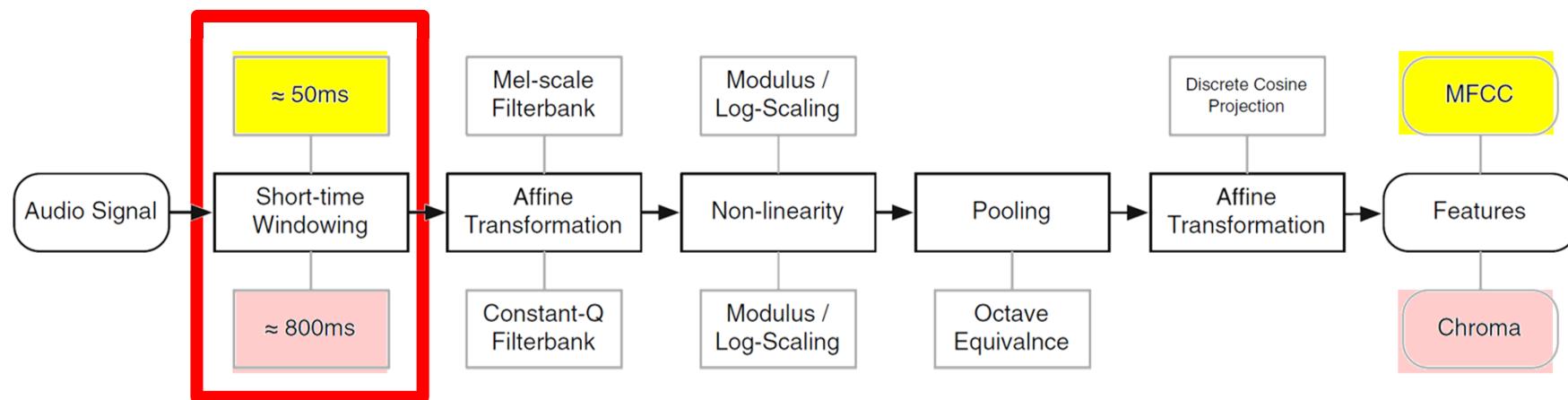
Different Features are Needed for Different Tasks

- We can **combine** different features to train a classifier for tasks such as music *genre* classification or music *emotion* classification



Different Features are Needed for Different Tasks

- **Timbre representation:** Spectrogram → mel-spectrogram → MFCC
- **Harmonic representation:** Spectrogram → CQT → chroma feature



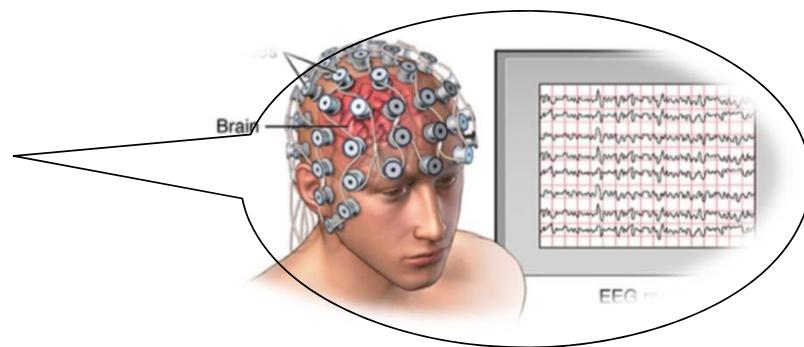
- Wait... why the window size is different?

Outline

- Time and frequency representations for music
- Audio features for music
- **Math in STFT**

Sampling Rate

- Definition: number of samples per second
- Why: analog to digital
- Examples
 - EEG signal: **128 Hz**
 - Telephone audio: **8k Hz**
 - Music audio: **44k Hz**

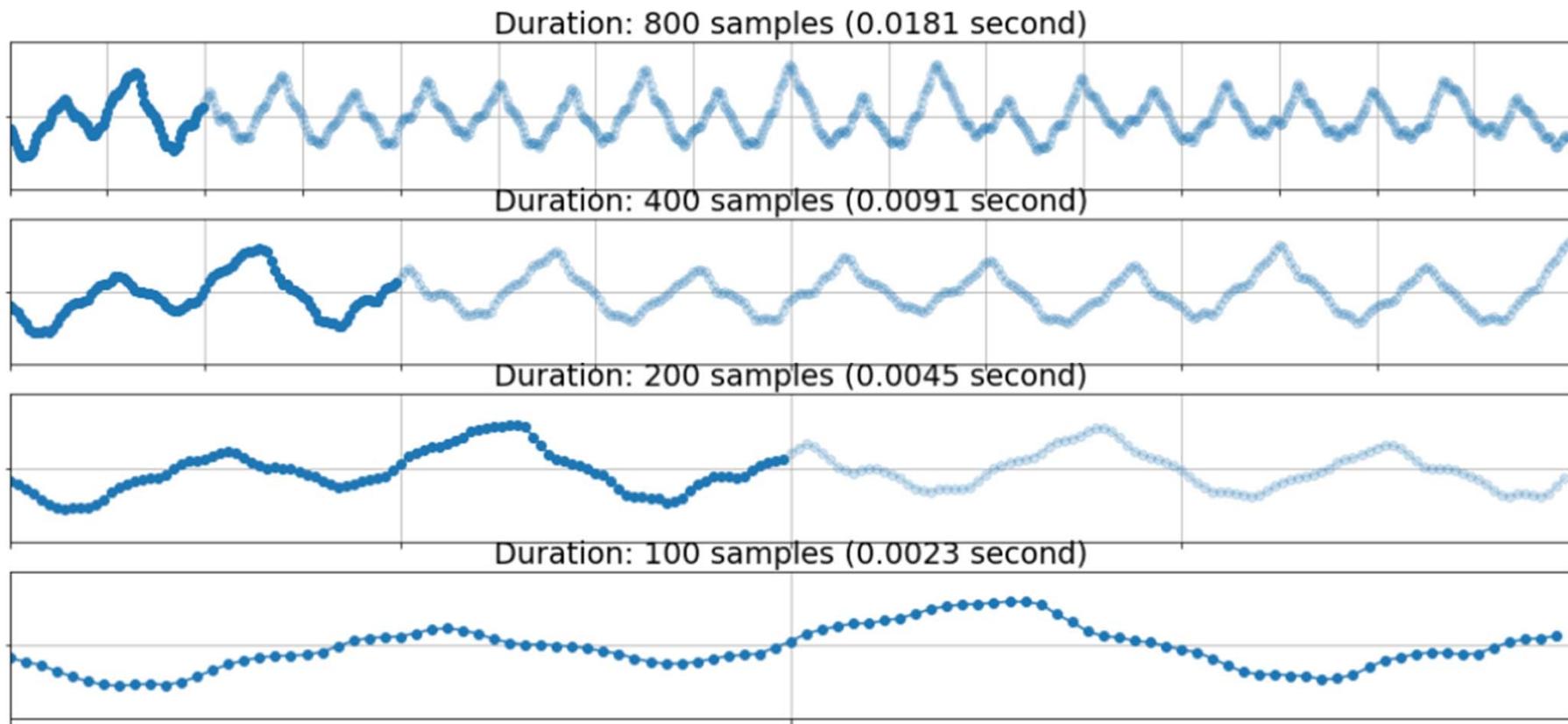


<https://www.brightbraincentre.co.uk/electroencephalogram-eeg-brainwaves/>

```
[ ] import librosa  
x, sr = librosa.load('audio/simple_loop.wav')
```

```
waveform, sample_rate = get_speech_sample()
```

Sampling Rate



https://music-classification.github.io/tutorial/part2_basics/input-representations.html

Sampling Rate

- Usually, a model considered signals with a **certain and fixed** sampling rate
 - The “x”s with different sampling rates are not on the same time basis

```
x, sr = librosa.load('audio/simple_loop.wav')
```

- Make sure your data are under **the same** sampling rate
- Double check the sampling rate assumed by the models you sourced from GitHub
 - Speech: 16k Hz
 - Music: 22k, 44k, or 48k Hz
- If we want to ML/DL model trained on signals with one sampling rate to process a signal under a different sampling rate, “**resampling**” of that signal is needed

Resampling

Library	Time on CPU (ms)
soxr (HQ)	7.2
scipy.signal.resample	13.4
soxr (VHQ)	15.8
torchaudio	19.2

<https://github.com/dofuuz/python-soxr>

```
y = soxr.resample(  
    x,          # 1D(mono) or 2D(frames, channels) array input  
    48000,      # input samplerate  
    16000       # target samplerate  
)
```

Sample Rate independent Models

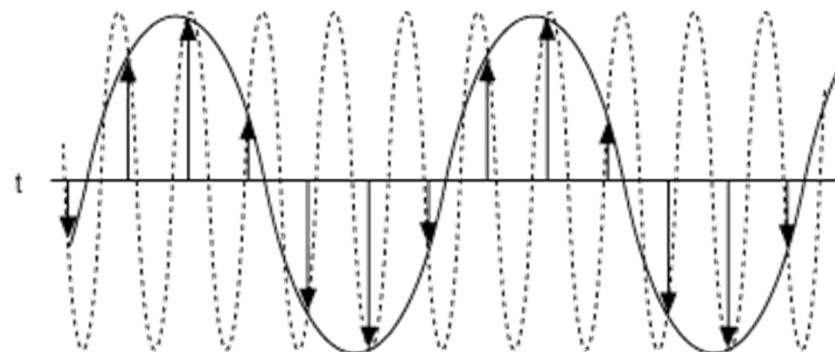
- Usually, a model considered signals with a certain and fixed sampling rate
- Building a **sampling-rate agnostic** model that can take signals sampled at arbitrary sampling rates is an advanced topic

Ref 1: Saito et al, “Sampling-frequency-independent audio source separation using convolution layer based on impulse invariant method,” EUSIPCO 2021

Ref 2: Carson et al, “Sample rate independent recurrent neural networks for audio effects processing,” DAFX 2024

Nyquist–Shannon Sampling Theorem

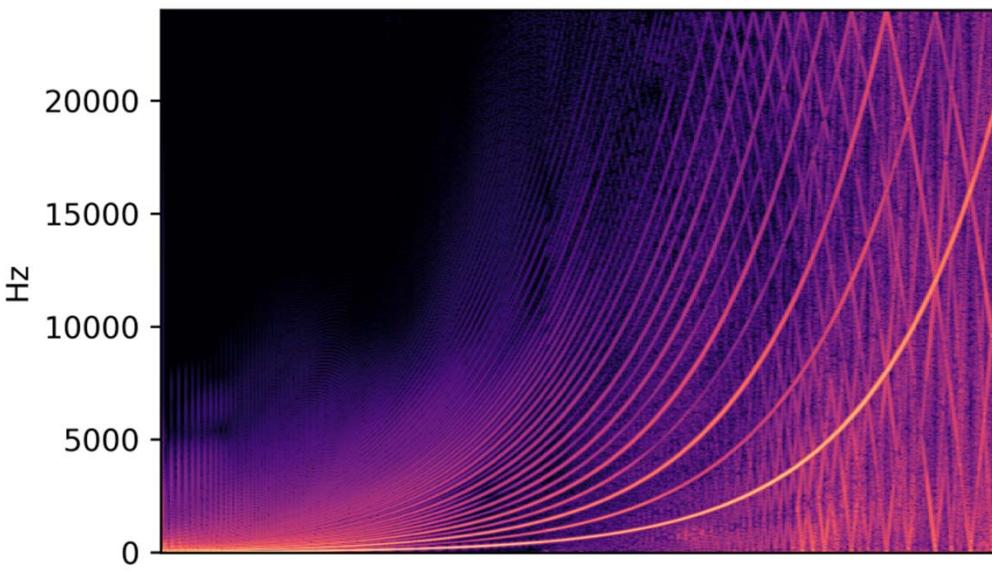
- A signal must be sampled at least **twice** as fast as the bandwidth of the signal to accurately reconstruct the waveform; otherwise, the high-frequency content will alias at a frequency inside the spectrum of interest
- **Sampling freq (Fs) > 2* the highest freq in the signal**



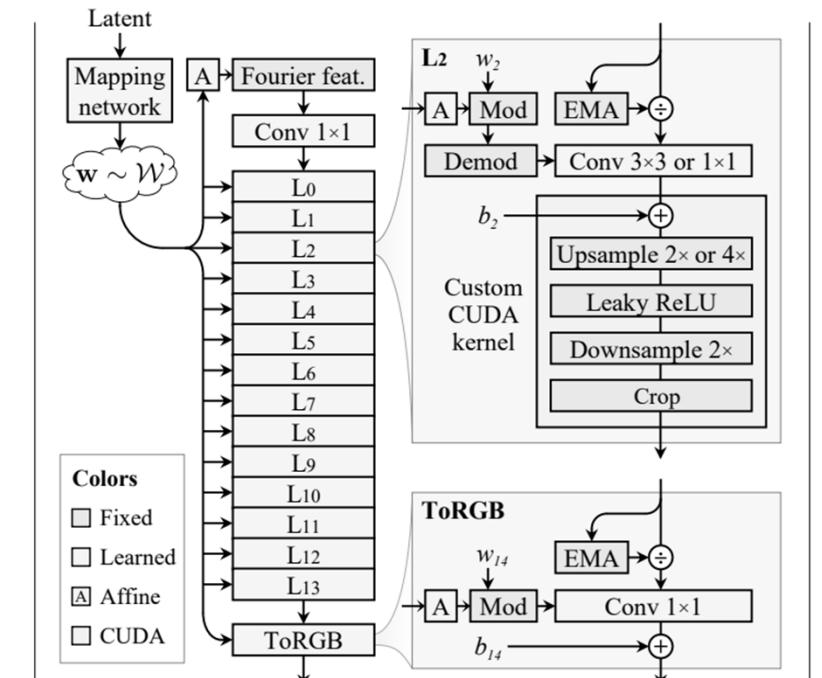
http://zone.ni.com/reference/en-XX/help/370524T-01/siggenhelp/fund_nyquist_and_shannon_theorems/

Aliasing

- High-frequency components fold back



Ref 1: Yeh et al, "PyNeuralFx: A Python package for neural audio effect modeling," ISMIR-LBD 2024



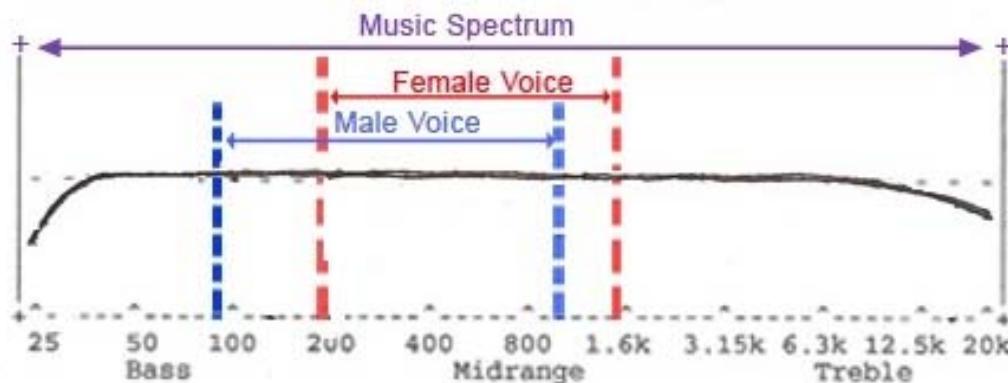
(b) Our alias-free StyleGAN3 generator architecture

<https://nvlabs.github.io/stylegan3/>

Ref 2: Karras et al, "Alias-free generative adversarial networks," NeurIPS 2021

Nyquist–Shannon Sampling Theorem

- Telephone audio: 8k Hz
 - Via phone, we cannot hear frequency higher than 4k Hz



<https://www.quora.com/How-do-HRT-sex-reassignment-and-other-such-procedures-affect-vocal-production-particularly-the-singing-voice>

- **Question:** With $F_s=128$ Hz, we assume that we don't need to care about frequency higher than __ Hz in brain waves

Nyquist–Shannon Sampling Theorem

Brainwaves, Frequencies and Functions

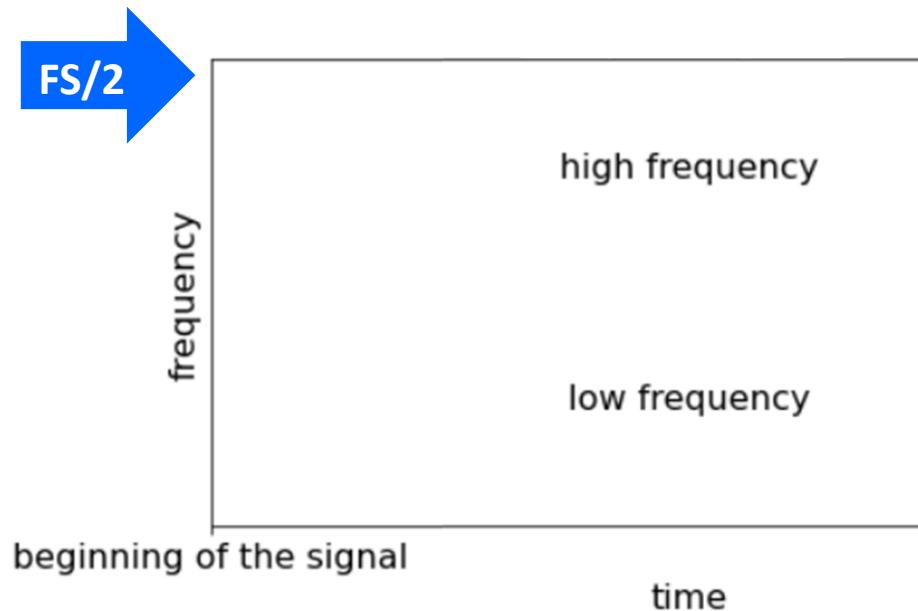
Unconscious		Conscious		
Delta	Theta	Alpha	Beta	Gamma
0,5 – 4 Hz	4 – 8 Hz	8 – 13 Hz	13 – 30 Hz	30-42 Hz
Instinct	Emotion	Consciousness	Thought	Will
Survival Deep sleep Coma	Drives Feelings Trance Dreams	Awareness of the body Integration of feelings	Perception Concentration Mental activity	Extreme focus Energy Ecstasy

<http://altered-states.net/barry/update236/>

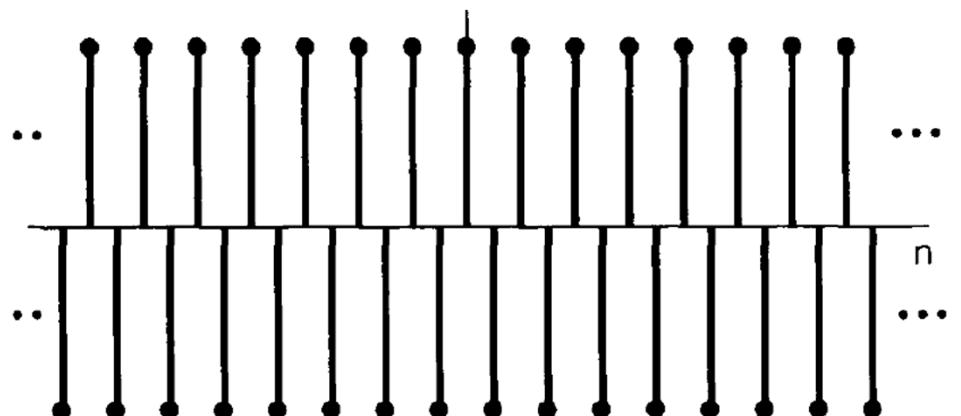
- **Question:** With $F_s=128$ Hz, we assume that we don't need to care about frequency higher than 64 Hz in brain waves

Nyquist–Shannon Sampling Theorem

- Sampling freq (F_s) > 2* the highest freq in the signal
- **Highest freq bin in STFT = $F_s/2$**



Sinusoid with the highest freq given a fixed FS



DTFS: Fourier Series Representation for Discrete-time Periodic Signals

- For **periodic** signals with fundamental **period L** , i.e., $x[n] = x[n + L]$
- The Fourier series coefficients a_k can be interpreted as the inner product of the signal $x[n]$ and a basis function $e^{jk\frac{2\pi}{L}n}$, which is complex-valued
- a_k are **complex-valued**
- a_k are also **periodic with period L** , i.e., $a_k = a_{k+L}$

Synthesis equation: $x[n] = \sum_{k=\langle L \rangle} a_k e^{jk\frac{2\pi}{L}n}$

In discrete-time, $e^{jk\omega n} = e^{j(k+L)\omega n}$; so there are **only L unique basis vectors**

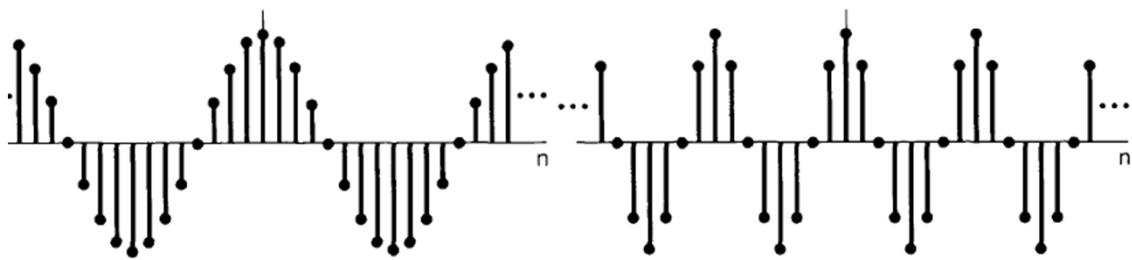
Analysis equation: $a_k = \frac{1}{L} \sum_{n=\langle L \rangle} x[n] e^{-jk\frac{2\pi}{L}n}$

Sum over a set of L successive integers

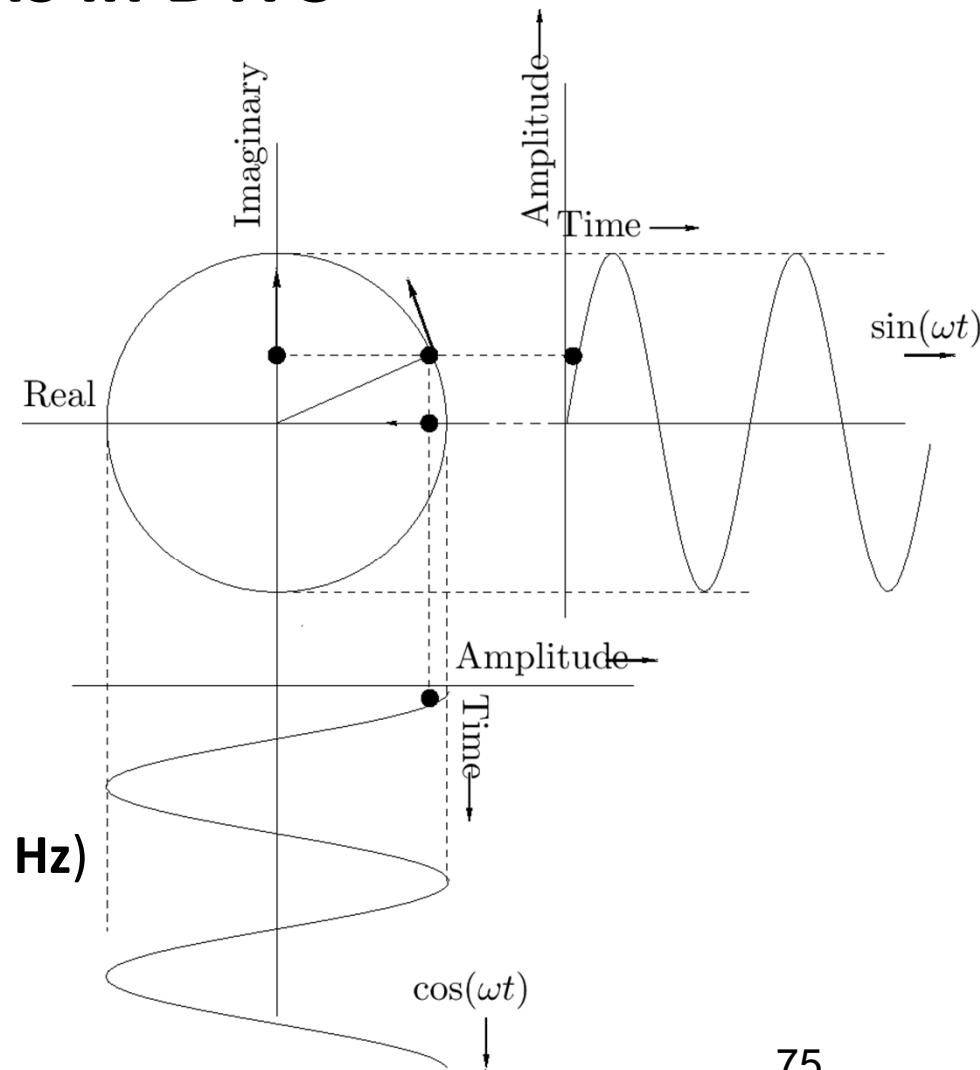
The Basis Functions in DTFS

- $e^{j\omega n} = \cos(\omega n) + j\sin(\omega n)$

$$x[n] = \cos(\pi n/8)$$



$$x[n] = \cos(\pi n/4)$$



- angular or ordinary frequency
 - ω : angular frequency (radians per second)
 - f : ordinary frequency (circles per second; in Hz)
 - $e^{j2\pi fn} = \cos(2\pi fn) + j\sin(2\pi fn)$

Discrete-time Fourier Transform (DFT): Extending DTFS to Deal with Aperiodic, Finite-Duration DT Signals

- For **periodic DT** signal $x[n] \xrightarrow{\mathcal{FS}} a_k$

Synthesis: $x[n] = \sum_{k=\langle L \rangle} a_k e^{jk\omega n}$

Analysis: $a_k = \frac{1}{N} \sum_{n=\langle L \rangle} x[n] e^{-jk\omega n}$

- For **aperiodic, finite-duration DT** signal $x[n] \xrightarrow{\mathcal{DFT}} X_k$

Synthesis: $x[n] = \sum_{k=0}^{L-1} X_k e^{jk\omega n}$

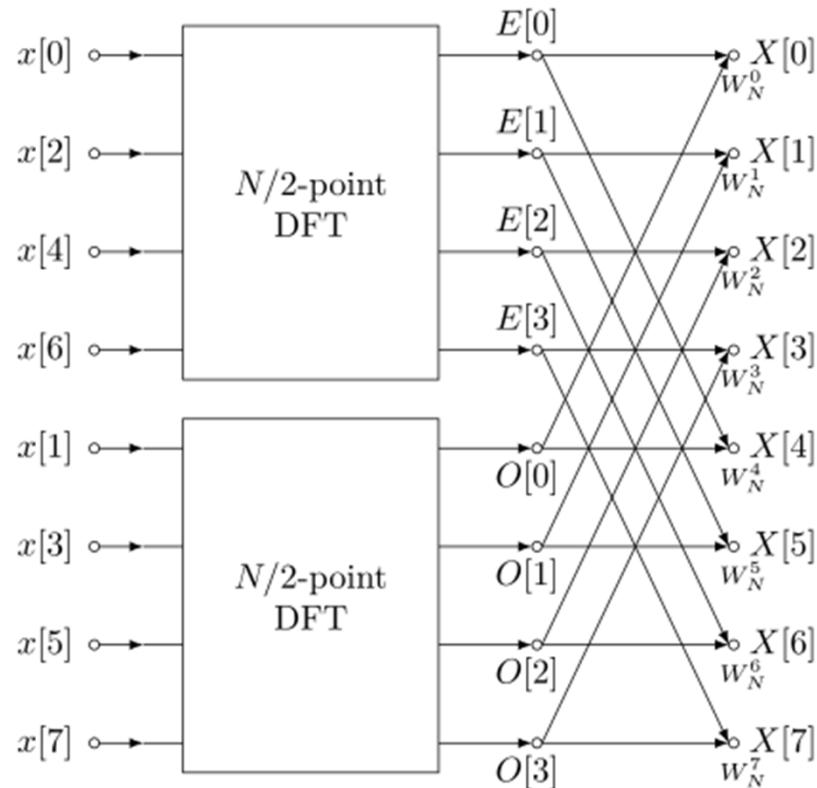
Analysis: $X_k = \frac{1}{L} \sum_{n=0}^{L-1} x[n] e^{-jk\omega n}$

NOTE: While the length of $x[n]$ is N , the length of X_k is also L

DFT and FFT (Fast Fourier Transform)

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_DFT-FFT.html

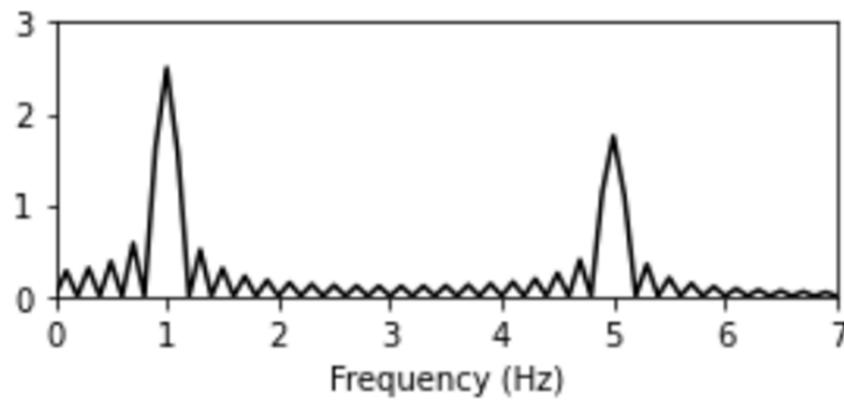
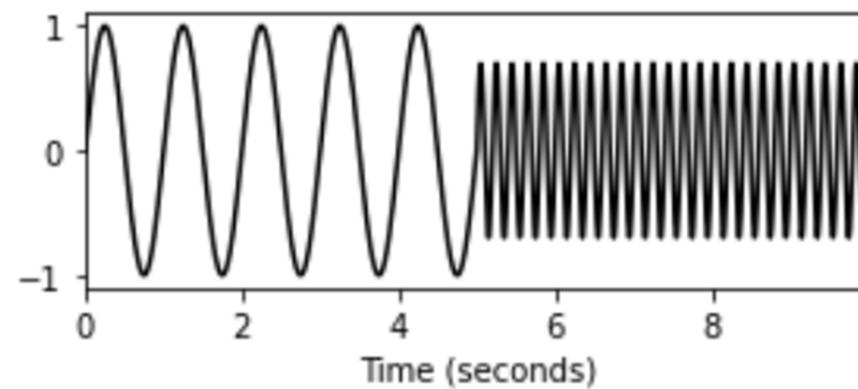
- FFT is a fast algorithm to compute the DFT



[https://en.wikipedia.org/wiki/
Fast_Fourier_transform](https://en.wikipedia.org/wiki/Fast_Fourier_transform)

FFT cannot Localize Temporal Events

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html



STFT: Short-time Fourier Transform

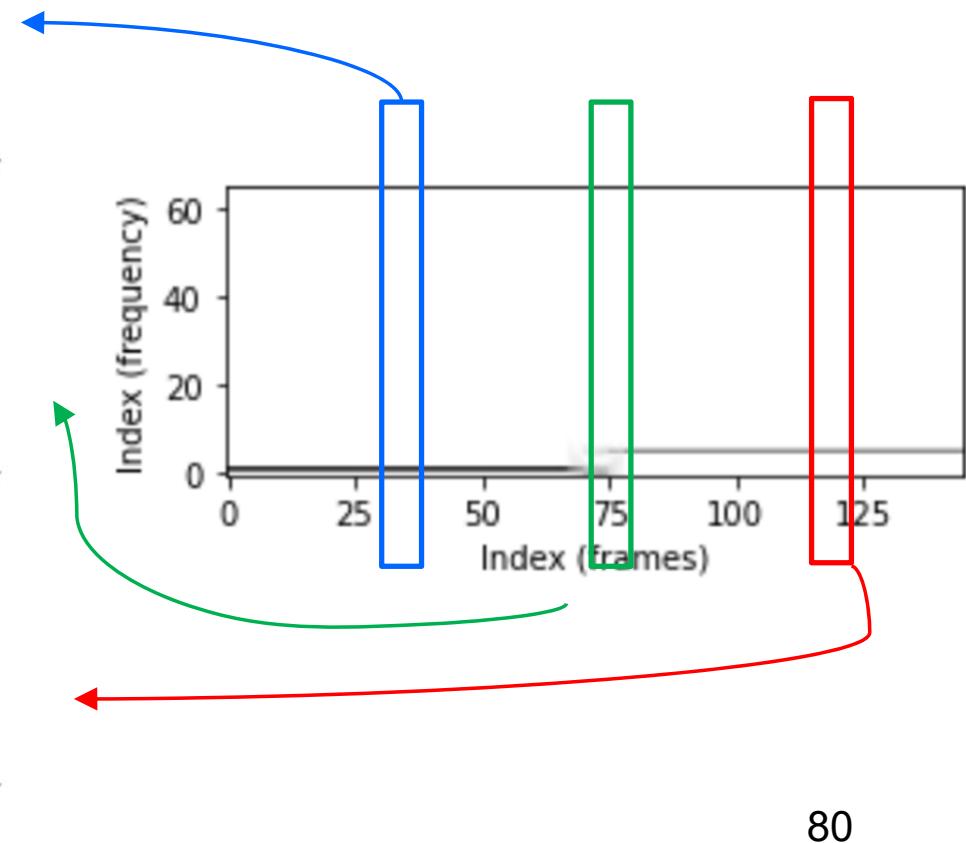
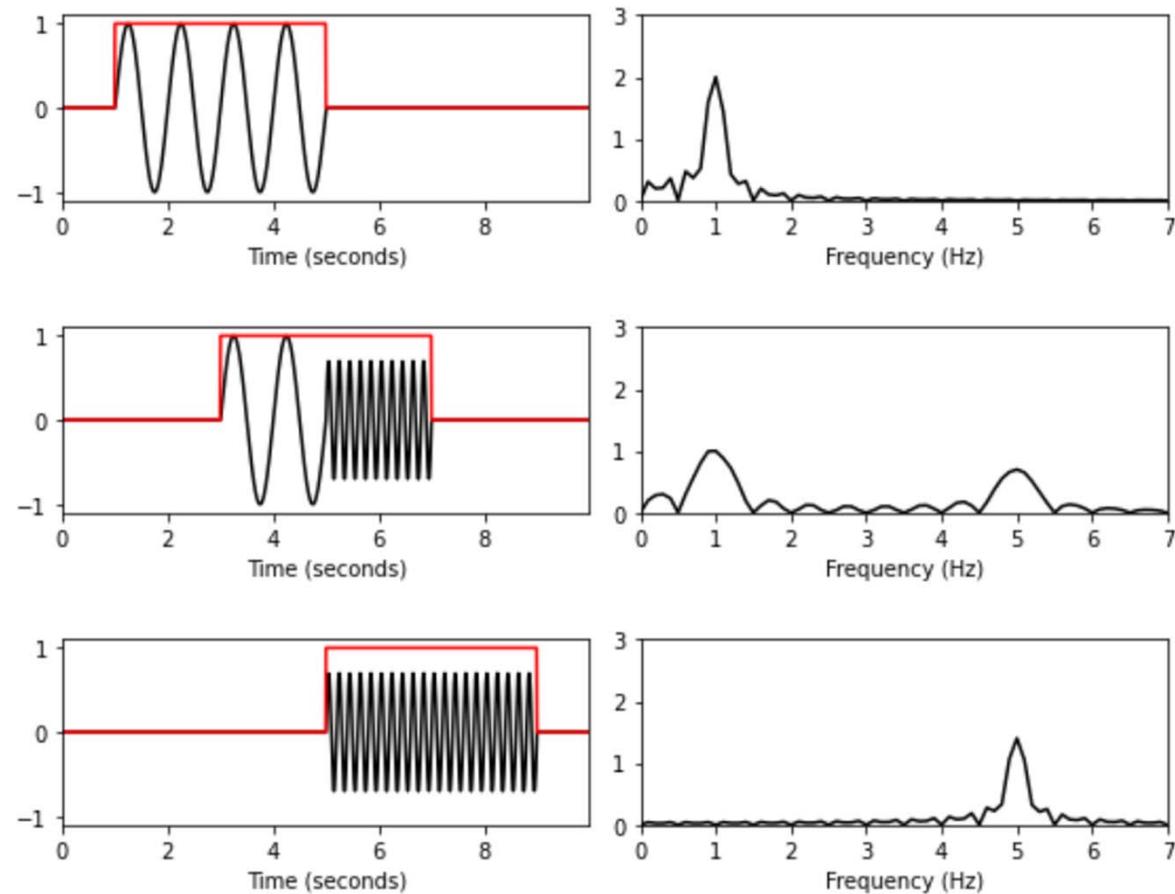
https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html

$$\mathcal{X}(m, k) := \sum_{n=0}^{N-1} x(n + mH) w(n) \exp(-2\pi i kn/N)$$

- $x[n]$: real-valued discrete-time signal of length L obtained by equidistant sampling with respect to a fixed sampling rate
- $w[n]$: window function of length $N < L$
- H : “hop size”

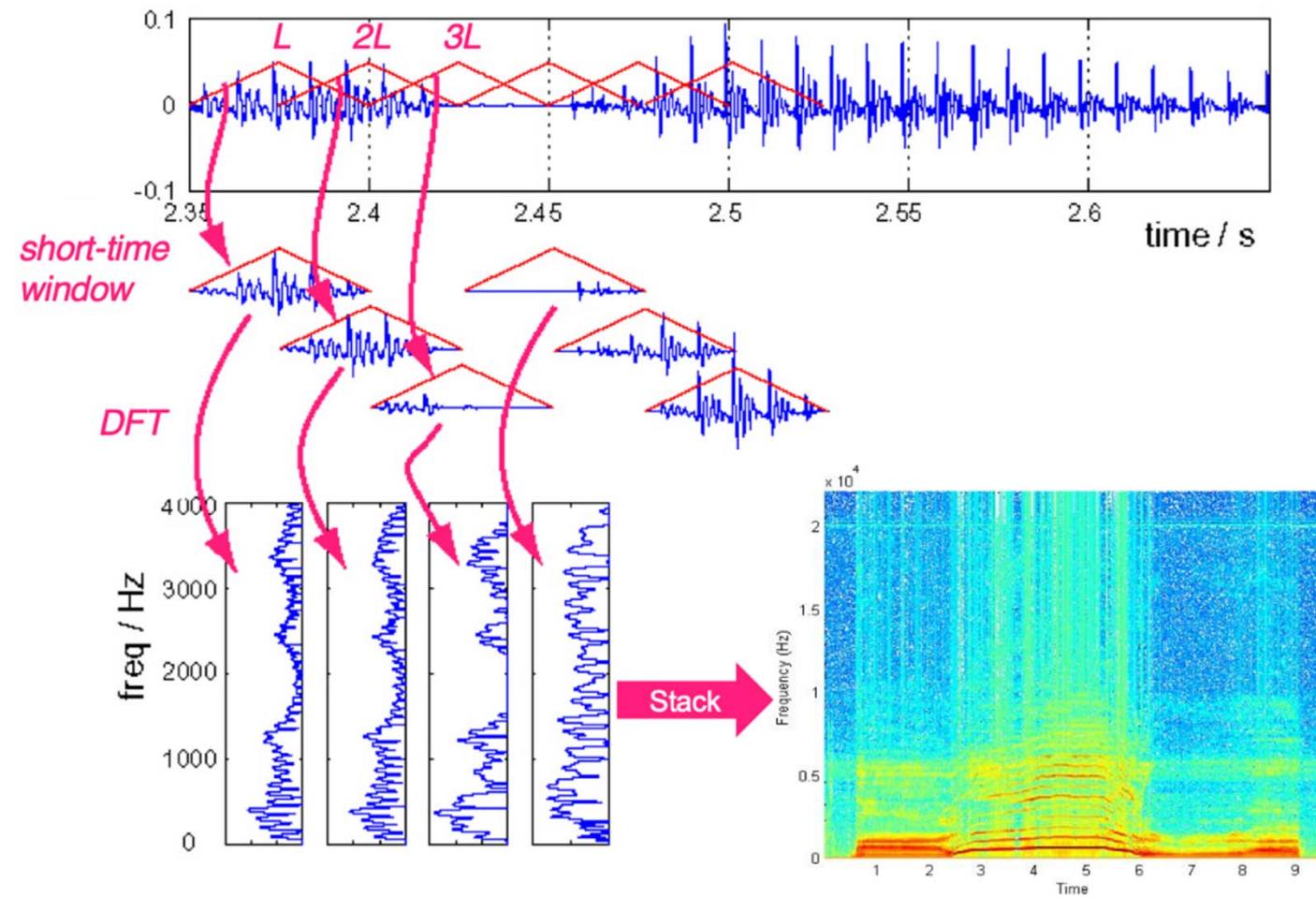
Windowed FFT

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html



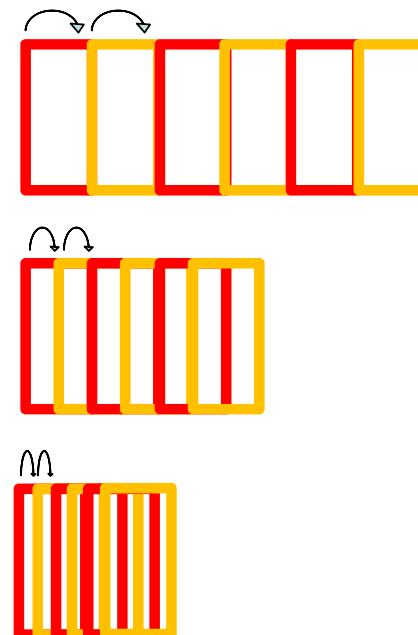
Windowed FFT

<https://source-separation.github.io/tutorial/basics/representations.html>



Hop Size in STFT

- Hop size
 - Can be proportional to the window size
 - $\text{hop_size} = \text{win_size}$
 - $\text{hop_size} = 0.5 * \text{win_size}$
 - $\text{hop_size} = 0.1 * \text{win_size}$
 - Can also be not proportional to the window size

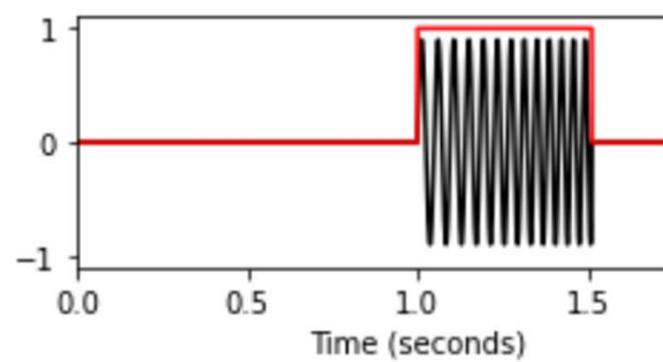


Window Functions in STFT

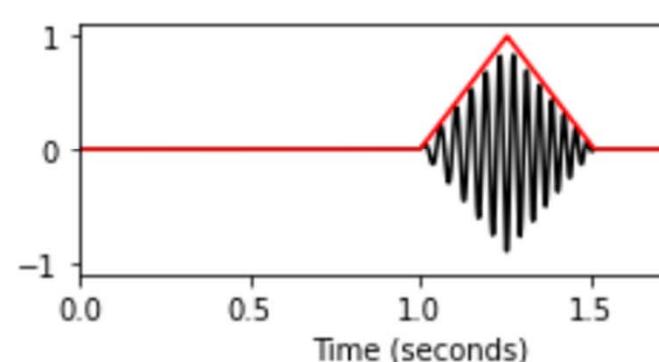
https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Window.html

- Hann window is often used

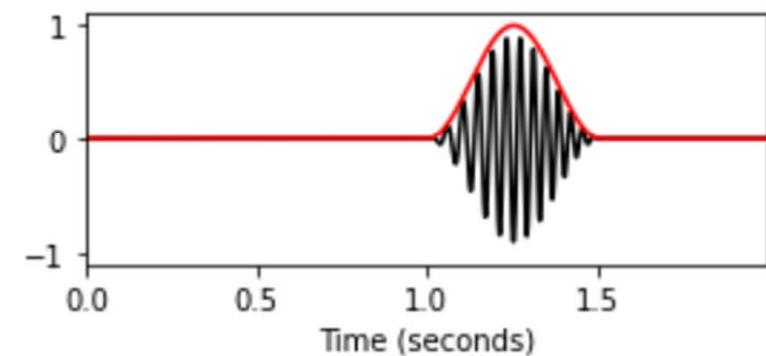
Rectangular window:



Triangular window:



Hann window:

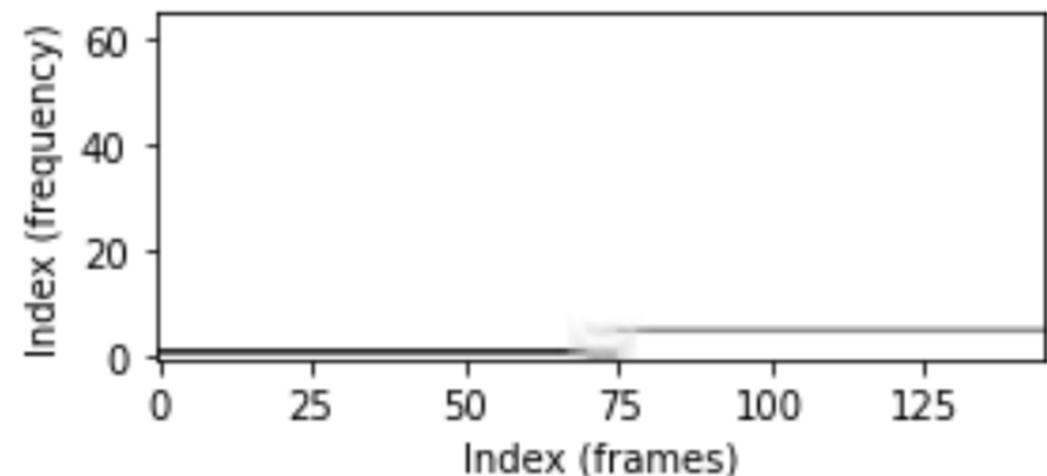


Interpretation of Time and Frequency Indices in STFT

https://www.audiolabs-erlangen.de/resources/MIR/FMP/C2/C2_STFT-Basic.html

$$F_{\text{coef}}(k) := \frac{k \cdot F_s}{N}$$

- **Frequency resolution:** $\frac{F_s}{N}$
- **Temporal resolution:** $\frac{H}{F_s}$



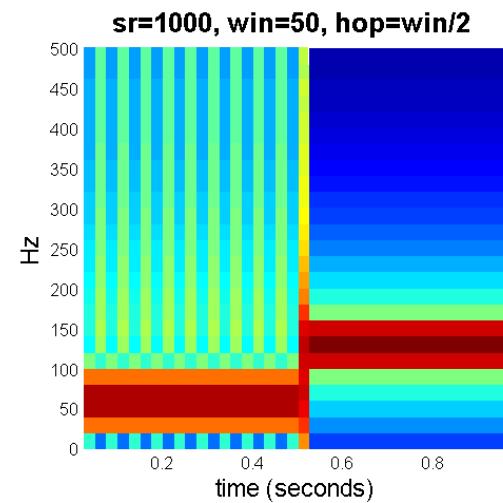
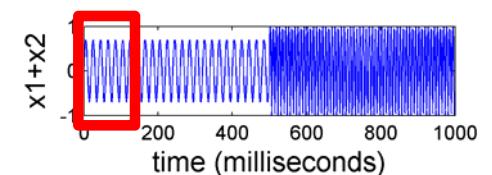
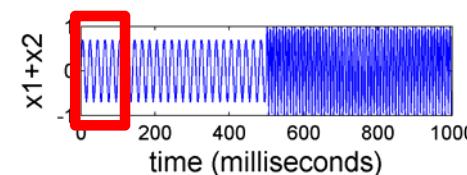
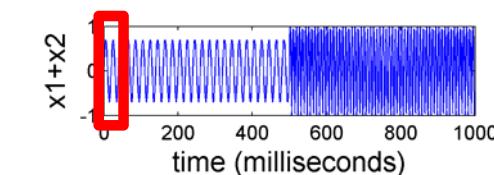
$$T_{\text{coef}}(m) := \frac{m \cdot H}{F_s}$$

Trade-off between Frequency/Temporal Resolution in STFT

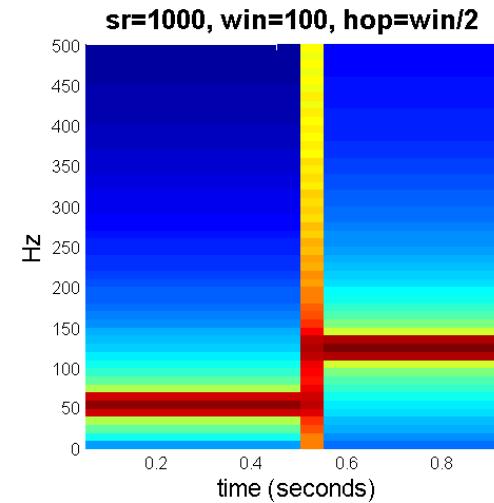
- **Frequency resolution:** $\frac{F_s}{N}$
 - Longer window size (N) → finer frequency resolution (but larger resulting STFT)
→ can localize events along the frequency axis
- **Temporal resolution:** $\frac{H}{F_s}$
 - Smaller hop size (H) → finer temporal resolution (but larger resulting STFT)
→ can localize events along the time axis
- If $H = N/R$ (e.g., $R = 4$)
 - Temporal resolution: $\frac{N}{RF_s}$ (no good freq/time resolution at the same time)

Trade-off between Frequency/Temporal Resolution in STFT

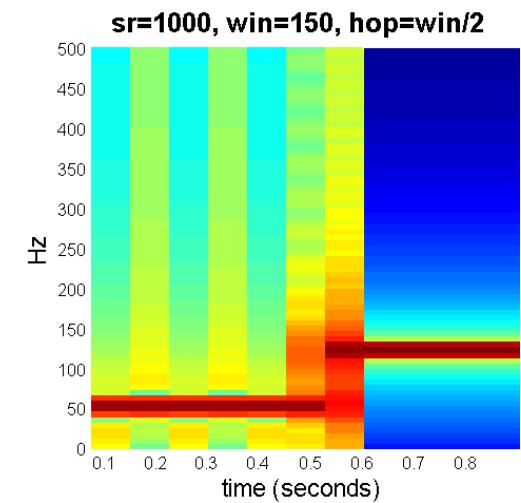
- Different window size (`win_size = 50, 100, 150`)
- `hop_size = win_size/2`



size: 26 x 39



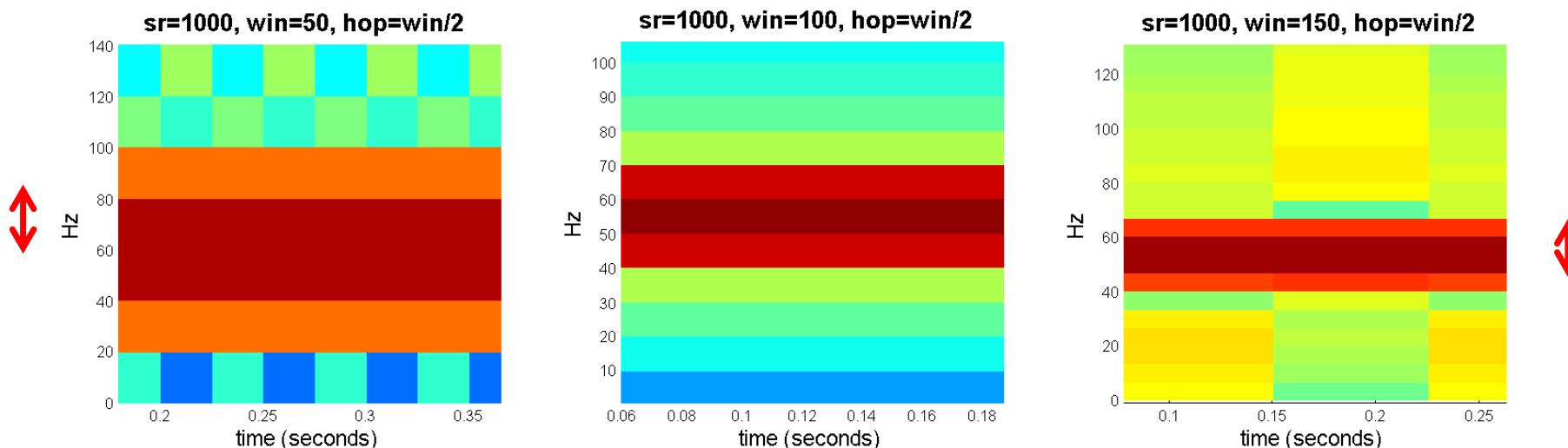
size: 51 x 19



size: 76 x 12

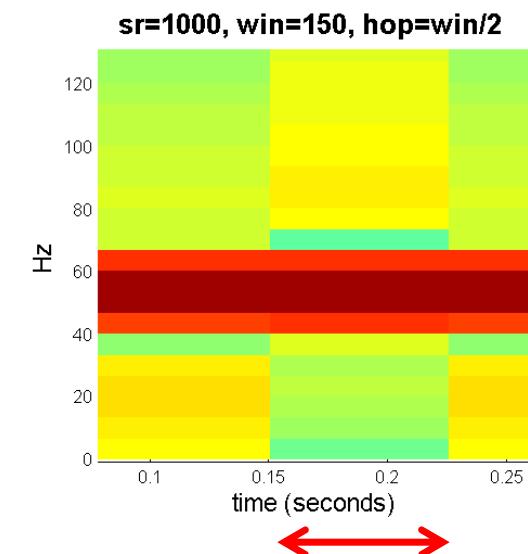
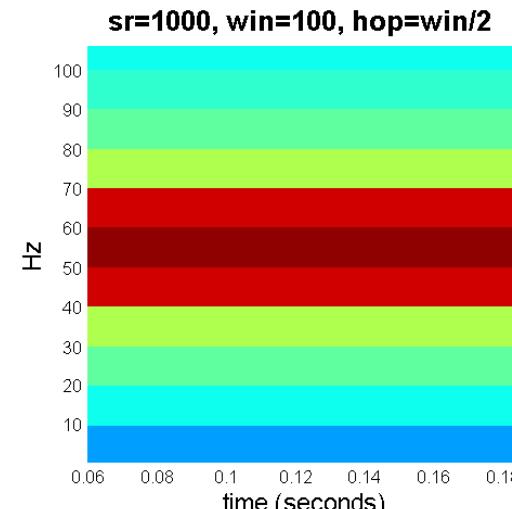
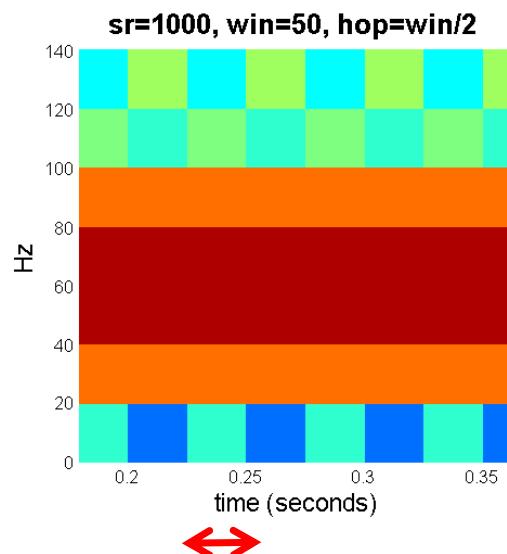
Frequency Resolution in STFT

- **freq_resolution = Fs/win_size**
 - longer window → better frequency resolution
 - freq_resolution = 20, 10, 6.6667 (Hz), respectively



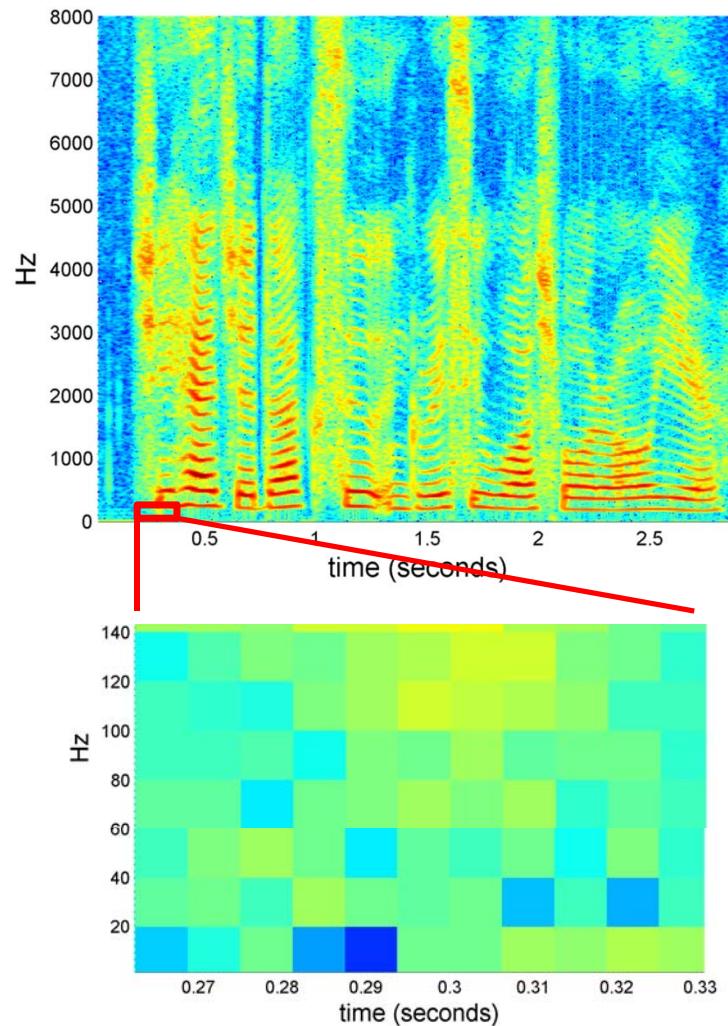
Understanding STFT

- **temporal_resolution: hop_size/Fs**
 - longer hop → worse temporal resolution
 - temp_resolution = 25, 50, 75 (ms), respectively



Quiz

1. The figure on the top-right is the spectrogram of a signal. What is the underlying sampling rate?
2. The figure on the bottom-right is a zoom-in of the above figure. We can see that the frequency resolution is 20 Hz. What is the window size (in samples)?
3. The temporal resolution is close to 6.6 ms. What's the hop size (in samples), approximately?



Quiz

4. Given an EEG headset that samples signals at 128 Hz, if we want to be able to discriminate frequency components that differ by 0.5 Hz in frequency, what is the minimal window size (in samples) we need to use? What is the length of such a window in seconds then?
5. Following the previous question, if we further want to discriminate events that differ in time by 0.5 second, what is the maximal hop size (in samples) we need to use?

Brainwaves, Frequencies and Functions

Unconscious		Conscious		
Delta	Theta	Alpha	Beta	Gamma
0,5 – 4 Hz	4 – 8 Hz	8 – 13 Hz	13 – 30 Hz	30-42 Hz
Instinct	Emotion	Consciousness	Thought	Will

Quiz

6. Given a music signal with $sr = 44,100$ Hz, when we use a window size of 1,024 samples, what would be the frequency resolution?
7. According to the figure on the right, we know that the fundamental frequency (f_0) of A1 is 55 Hz, that of A#1 is 58.27 Hz, etc. Following the previous question, which notes does the first frequency bin of the STFT would cover?

Note name	Keyboard	Frequency Hz
A0		27.500
B0		30.868
C1		32.703
D1		36.708
E1		41.203
F1		43.654
G1		48.999
A1		55.000
B1		61.735
C2		65.406
D2		73.416
E2		82.407
F2		87.307
G2		97.999
A2		110.00
B2		123.47
C3		130.81
D3		146.83
E3		164.81
F3		174.61
G3		196.00
A3		220.00
B3		246.94
C4		261.63
D4		293.67
		311.13

Quiz

6. Given a music signal with $sr = 44,100$ Hz, when we use a window size of 1,024 samples, what would be the frequency resolution?

Sol: 43.1 Hz

7. According to the figure on the right, we know that the fundamental frequency (f_0) of A1 is 55 Hz, that of A#1 is 58.27 Hz, etc. Following the previous question, which notes does the first frequency bin of the STFT would cover?

Note name	Keyboard	Frequency Hz
[0, 43.1)		27.500
B0		30.868
C1		32.703
D1		36.708
E1		38.891
[43.1, 86.2)		43.654
G1		48.999
A1		55.000
B1		61.735
C2		65.406
D2		73.416
E2		82.407
[86.2, 129.3)		87.307
G2		97.999
A2		110.00
B2		123.47
[129.3, 172.4)		130.81
C3		146.83
E3		164.81
[172.4, 215.5)		174.61
G3		196.00
A3		220.00
B3		246.94
C4		261.63
D4		293.67
		211.12

Quiz

8. Following the '6' and '7';
how if we use a window size of 4,096 samples?

[0, 10.8)
[10.8, 21.5)
[21.5, 32.3)
[32.3, 43.1)
[43.1, 53.8)
...

(Note: Musical notes after F#3 would be covered by only one frequency bin now)

Note name	Keyboard	Frequency Hz
[21.5, 32.3)		27.500 30.868
[32.3, 43.1)		32.703 36.708
D1		34.648
E1		41.203
[43.1, 53.8)		43.654 48.999
G1		46.249 51.913
A1		55.000
B1		61.735
C2		65.406
D2		73.416
E2		82.407
F2		87.307
G2		97.999
A2		110.00
B2		123.47
C3		130.81
D3		146.83
E3		164.81
F3		174.61
G3		196.00
A3		220.00
B3		246.94
C4	261.63
D4		293.67
		311.13

Real Example 1: Piano Transcription

- Curtis Hawthorne et al., “**Onsets and Frames: Dual-objective piano transcription**,” ISMIR 2018

<https://archives.ismir.net/ismir2018/paper/000019.pdf>

Q: What is the frequency resolution?
And the temporal resolution?

Our onset and frame detectors are built upon the convolution layer acoustic model architecture presented in [13], with some modifications. We use *librosa* [15] to compute the same input data representation of mel-scaled spectrograms with log amplitude of the input raw audio with 229 logarithmically-spaced frequency bins, a hop length of 512, an FFT window of 2048, and a sample rate of 16kHz. We present the network with the entire input sequence, which allows us to feed the output of the convolutional frontend into a recurrent neural network (described below).

Real Example 2: Beat Tracking

- Sebastian Böck, Florian Krebs, and Gerhard Widmer, “**Joint beat and downbeat tracking with recurrent neural networks**,” ISMIR 2016

http://www.cp.jku.at/research/papers/Boeck_etal_ISMIR_2016.pdf

120 BPM = 120 beats per minute
= 2 beats per second

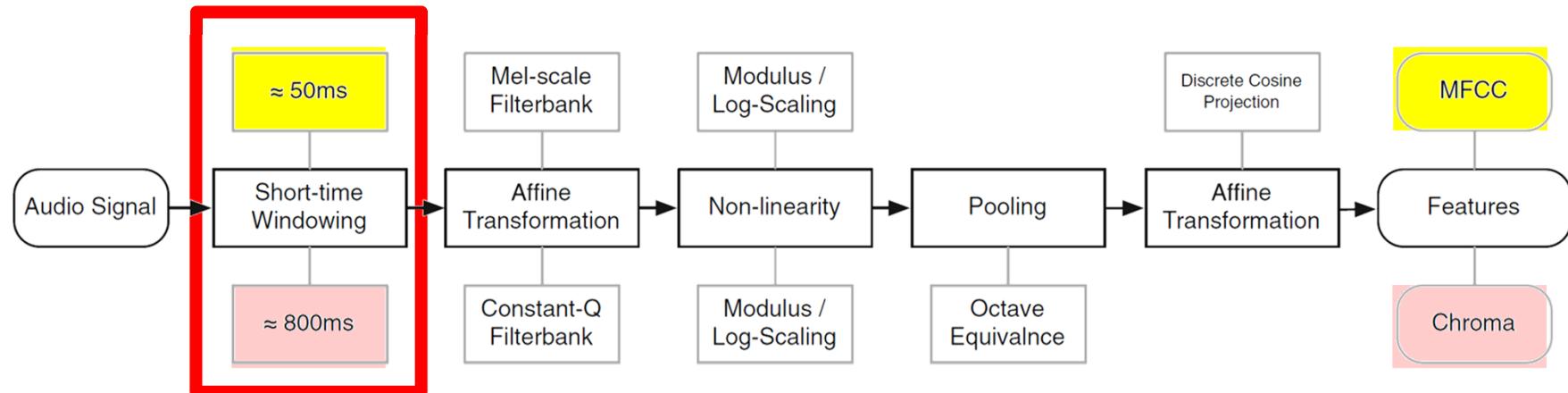
(16th note in 4/4 meter: 125 ms)

2.1 Signal Pre-Processing

The audio signal is split into overlapping frames and weighted with a Hann window of same length before being transferred to a time-frequency representation with the Short-time Fourier Transform (STFT). Two adjacent frames are located 10 ms apart, which corresponds to a rate of 100 fps (frames per second). We omit the phase portion of the complex spectrogram and use only the magnitudes for further processing. To enable the network to capture features which are precise both in time and frequency, we use three different magnitude spectrograms with STFT lengths of 1024, 2048, and 4096 samples (at a signal sample rate of 44.1 kHz). To reduce the dimensionality of the features, we limit the frequencies range to [30, 17000] Hz and process the spectrograms with logarithmically spaced

Different STFT Parameters are Needed for Different Tasks

- **Timbre/rhythm** related tasks tend to use **smaller** windows
- **Pitch/harmony** related tasks tend to use **longer** windows



- Make sure you use the right **Fs**, **window size** and **hop size!**
 - Especially when using models from open source projects
 - STFTs of the same matrix size may have different physical meanings

Other Time-Frequency Representations

- STFT of multiple window sizes

we use three different magnitude spectrograms with STFT lengths of 1024, 2048, and 4096 samples (at a signal sample rate of 44.1 kHz). To reduce the dimensionality of the

- CQT
- Wavelets
- Scattering transform
- In DL, STFT is a common choice