Section 1.1

We motivate the following discussion by noting that the syntax of a program in a language is usually a nested or tree like structure. Recursion is then an important technique in constructing and manipulating such structures.

Inductive specification is a method for specifying a set of values. For example, let's consider a set $S \subseteq \mathbb{N}$. We can define S as follows:

Definition 1. *A natural number $n \in S$ if and only if*

1. $n = 0$, *or*
2. $n - 3 \in S$.

We call this definition the ***top down definition*** *of S*

Definition 2. *Define the set S to be the smallest set contained in $\mathbb{N}$ and satisfying the following two properties:*

1. $0 \in S$, *and*
2. *if* $n \in S$, *then* $(n + 3) \in S$.

We call this definition the ***bottom up definition*** *of S*

Definition 3.

$$\frac{}{0 \in S}$$

$$\frac{n \in S}{(n+3) \in S}$$

We call this definition the ***rules of inference*** *of S*

Exercise 1.3 [*] Find a set T of natural numbers such that $0 \in T$, and whenever $n \in T$, then $n + 3 \in T$, but $T \neq S$, where S is the set defined in definition 1.1.2.

Solution 1.3 Let T be the natural numbers. Then $0 \in T$ by definition of the natural numbers. We must show that $n + 3 \in T$ whenever $n \in T$.

Let A be the Peano system (A_i, i, S_i) where $i \in \mathbb{N}$ is the distinguished element of A, and S_i is the successor function of A defined by the rule

$$S(n) = n + 3, R(S) = \{n | n \geq i\}$$