Value Function Approximation

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Reinforcement Learning

Introduction: Approximate solution methods

RL Tabular methods are based on the idea to have a **limited number** of **well-defined states** in the MDP.

In such cases, we can:

- Find the **optimal value function** $V(s)$ for each of the states
- Find the **optimal policy** π

Approximate solution methods are the methods we can use in RL when it's impossible to find the optimal value function and the optimal policy, but we can only **find a good approximate solution using limited computational resources**.

This happens for two reasons:

- The **number of states is huge**
- Many states are **similar** but **not identical due to minimal variations in the data featuring them**

The last issue implies that along time **almost every state** encountered **will never have been seen before**.

In other terms, we need to cope with the problem of the size of the state space in two main scenarios:

1. **Discrete but large state space**
    1. Backgammon $\approx 10^{20}$ states, Go $\approx 10^{170}$ states
2. **Continuous state space**
    1. Autonomous driving, robot control, etc, with **infinite** number of states

Large spaces produce two main problems:

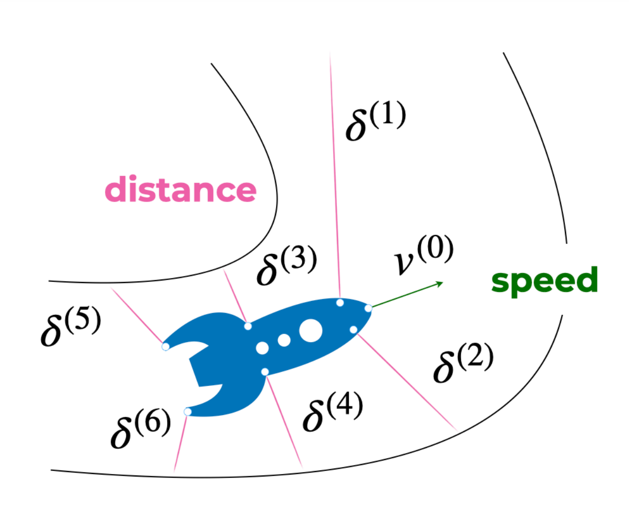
1. It is not possible to store the values for each state individually
2. It is too slow to estimate the value for each state individually

Even if it were possible to manage a large number of different states, there are many situations in which it is not desirable to do so. In fact, in a tabular context each state of the system is by definition independent and different from the other. Therefore there is never an extension of the knowledge acquired about one state to a different state, however similar. In principle therefore, through tabular methods, we could associate very different values with very similar states, especially in the initial stages of the training process. Human experience, however, suggests that it is possible and useful to associate the experience of a state with states that are very similar or even equivalent to states already known. This **generalization** not only makes the process more efficient, but also makes it more effective, allowing us to better reuse the knowledge learned.

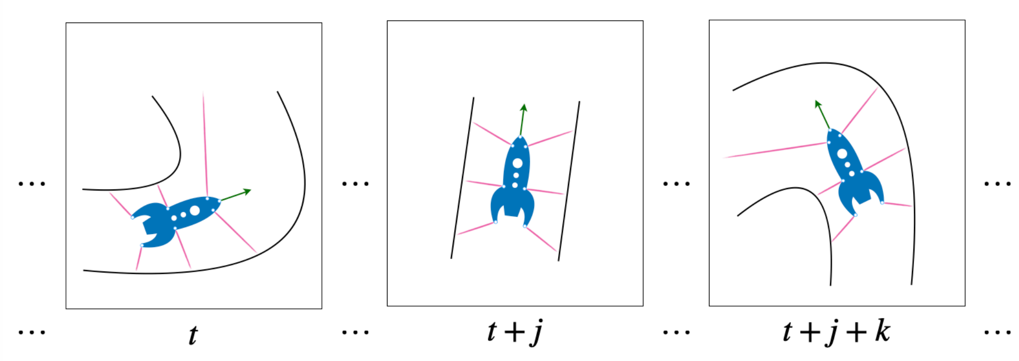
Example

Let's take the example of an **autonomous driving vehicle** moving along a narrow path. The vehicle is equipped with **7 sensors**:

- one for **speed** measured in a **range 0-85 m/s** with precision **0.1**
- 6 for **distance from obstacles**, measured in **meters from 0 to 100** with precision **0.1**



It's easy to see that we may have 850 possible values for seed plus 1000^7 possible combination of distances from obstacles. This means 850×1000^7 states in total. Moreover, we may end up in a high number of states that are **different** but **very similar**.



s_t	s_{t+j}	s_{t+j+k}
22.4	40.1	23.2
100.0	48.2	100.0
60.8	48.4	60.6
26.0	42.1	24.9
34.6	42.8	36.7
28.2	48.2	28.3
26.5	48.4	25.7

State s_t is **very similar** to state s_{t+j+k}, but they are not identical. This implies that most likely when we are at time $t+j+k$ we **have never yet observed** the state s_{t+j+k}. However we could exploit what we have learned from state s_t to evaluate s_{t+j+k}, because they represent almost the same situation.

Feature vectors

The first step towards the generalization of the notion of state is to represent a state as a feature vector $\mathbf{x}(s)$ such that

$$\mathbf{x}(s) = \begin{bmatrix} x_1(s) \\ x_2(s) \\ \dots \\ x_n(s) \end{bmatrix} \tag{1}$$

Featurs may include three different types of knowledge:

1. **Algorithmic state**: data cached by the agent not to represent the current agent situation nor the environment, but just information useful to take algorithmic decisions
2. **Situational state**: the summary of the current agent situation (e.g., its position in space, the internal state of agent's devices such as batteries)
3. **Epistemic state**: the agent current knowledge about the environment (e.g., sensors, environment features)

Linear value function approximation

Now we can set up a set of tools to formalize the goal of approximating the value function $V(s)$ as a linear combination of state features.

Value function for a policy as a weighted combination of features

$$\hat{V}(s; \mathbf{w}) = \sum_{i=1}^{n} x_i(s) w_i = \mathbf{x}(s)^T \mathbf{w} \tag{2}$$

Objective function (we want to learn w)

This is the mean squared error between the true value of state s under policy π and our approximation $\hat{V}(s; \mathbf{w})$.

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[\left(V^\pi(s) - \hat{V}(s; \mathbf{w}) \right)^2 \right] \tag{3}$$

Weight update

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \tag{4}$$

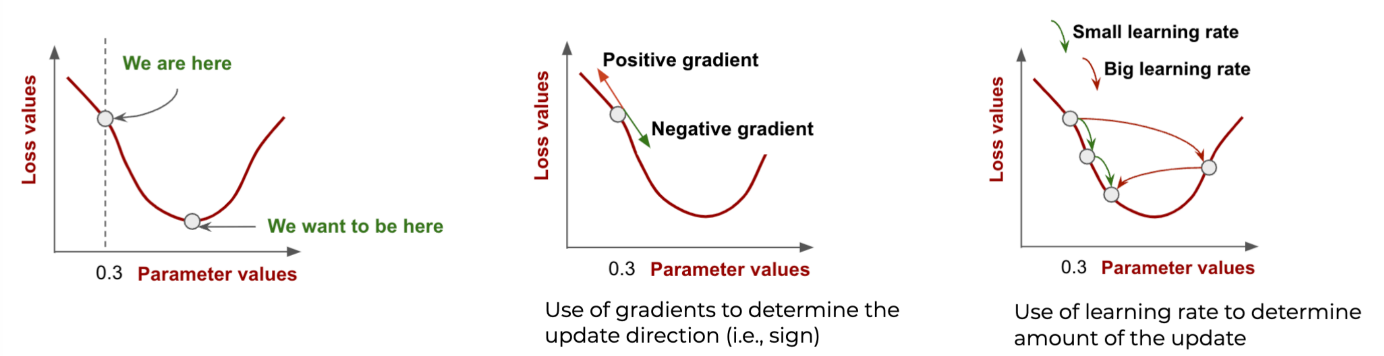
Review of gradient descent as updating method

Since our goal is to minimize $J(\mathbf{w})$ and $J(\mathbf{w})$ is differentiable we can learn how to minimize it by updating the parameters using **gradient descent**. In general, given a differentiable function $f(\mathbf{w})$ we do

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \nabla f(\mathbf{w}_t) \tag{5}$$

where α is the learning rate that specifies how large the update step should be and $\nabla f((w))$ is

$$\nabla f(\mathbf{w}) = \left(\frac{\partial f(\mathbf{w})}{\partial w_1}, \frac{\partial f(\mathbf{w})}{\partial w_2}, \dots, \frac{\partial f(\mathbf{w})}{\partial w_d} \right)^T \tag{6}$$



In particular, we are interested in **Stochastic Gradient Descent (SGD)** because it does not compute the gradient from the whole dataset but **estimates the gradient from a randomly selected subset of data**, making is feasible to perform the update on-line as RL requires.

With SGD we do not compute an average of several points in the function but just the update for a single point repeated multiple times:

$$\Delta \mathbf{w} = \alpha \left(V^{\pi}(s) - \hat{V}(s; \mathbf{w}) \right) \nabla_{\mathbf{w}} V(s) \tag{7}$$

Model Free VFA for Policy Evaluation

The reason why we cannot simply solve this by a supervised approach is that we do not have the true value $V_\pi(s)$ for any state s. Thus, we want to perform policy evaluation without a model.

Review of Model Free Policy Evaluation

- We assume to have a policy π
- We want to estimate V_π and/or Q_π

The general approach we used was to maintain a look up table of the estimates V_π and/or Q_π by updating them:

- **Monte Carlo**: update estimates after each episode
- **TD methods**: update estimates after each step

VFA variants

What we need to change is the estimate update step including fitting the parameters of VFA.