NP Complete Problem - Clique

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1 Introduction

Complete Graph: A graph of n vertices with all possible edges.

Subgraph: A subgraph H of graph G has $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ such that edges in $E(H)$ connect only vertices in H.

Clique: A complete subgraph of a graph G.

2 What are Cliques?

Cliques are complete subgraphs of a graph G - for example, a clique of size 5 is a subset of 5 completely interconnected nodes in G. Using adjacency matrices, it is easy to find cliques of up to 3 nodes (see homework 1). The clique problem asks the question: in a graph G, is there a clique of size k?

3 Why are Cliques Relevant?

In a social network, we can look at cliques to see what groups of users are completely interconnected.

4 Proof of NP

Given a subgraph of k vertices, how can we determine if it is a clique? If it has $\frac{k(k-1)}{2}$ edges, then it is complete and is a clique of size k.

5 Proof of NP-Hard

To show the clique problem is in NP-Hard, we will reduce 3-SAT to the clique problem. Given a boolean expression in 3-SAT, take each clause and create a cluster of 3 nodes per clause in a graph G, assigning each node to a variable in that clause. In G, connect two nodes if (1) both nodes are in a different cluster, and (2) both nodes can simultaneously be true (i.e. don't connect x and $\neg x$). Thus the nodes that are connected can simultaneously be true in the 3-SAT statement. If we had a solver for a k clause statement, we would have k variables, one fram each clause, that are assigned to be true - this corresponds with k nodes in our gadget which would all share an edge. This would be a clique of size k. If we had such a solver, then we'd have have a polynomial time solver for the clique problem. We can see that the clique problem is as hard as 3-SAT, so clique is in NP-Hard.