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Simultaneous Optimisation of Multiple Quality Characteristics in Manufacturing Processes Using Taguchi's Quality Loss Function

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Taguchi methods have proved to be successful over the last fifteen years or so for the improvement of product quality and process performance. Most Taguchi experiments are concerned with the optimisation of a single quality characteristic. Optimisation of multiple quality characteristics in manufacturing processes is not common and has received very little attention among the Taguchi practitioners. Many engineers using Taguchi methods have employed pure engineering judgement when dealing with multiple quality characteristics in manufacturing process optimisation. This approach is very subjective and therefore always brings an element of uncertainty to the decision-making process. This paper presents an alternative approach for tackling such optimisation problems using Taguchi's quality loss function analysis. The paper also presents a case study to illustrate the potential of the proposed methodology.

Keywords: Multi-response S/N ratio; Optimisation; Quality characteristics: Quality loss fuction; Statistical design of experiments; Taguchi methods

1. Introduction

Taguchi's parameter design (PD) methodology has proved to be an effective approach to producing high-quality products at a relatively low cost. The objective of parameter design (also known as robust design) is to determine the best settings of the process parameters, thereby making the process functional performance insensitive to various sources of variation. In order to accomplish this objective, Taguchi advocates the use of statistical design of experiments (SDOE) [1]. Many successful applications of PD have been reported in US and European manufacturing firms, especially over the last fifteen years [2–5]. The optimisation of multiple quality characteristics using Taguchi experiments has received limited attention among the engineering fraternity. A quality characteristic refers to a performance characteristic that affects the final product quality and is very important to consumers [6]. In optimising multiple quality characteristics, we generally rely on engineering knowledge and experience, which often increases the uncertainty during the decision-making process. This paper presents an alternative approach using Taguchi's quality loss function (TQLF) to determine the optimum conditions during the PD stage. The paper presents an overview of Taguchi's PD with multiple quality characteristics, and a proposed methodology for tackling multiple quality characteristics using Taguchi experiments and finally provides a case study to illustrate the potential of the proposed approach.

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2. Taguchi's PD with Multiple Quality Characteristics – An Overview

Much of the discussion about the methods of Taguchi's PD has been focused on the optimisation of a single quality characteristic. In the optimisation process of multiple quality characteristics, the objective is to determine the best factor settings which will simultaneously optimise all the quality characteristics of interest to the experimenter. Byrne and Taguchi [7] illustrate an example of the optimisation of two quality characteristics: the force required to insert the tube into the connector; and the pull-off force. It was required that the pull-off force be as high as possible, and the insertion force be as low as possible for ease of manufacturing. An informal argument was used to arrive at a compromise solution. Under such circumstances, we often need a powerful methodology to resolve the compromise. It is important to note that engineering judgement, together with past experience, will often bring some uncertainty in the decision-making process. The statistical validity and robustness of the results cannot be assured using the above procedure.

Derringer and Suich [8] made use of modified desirability functions which measure the designer's rquirements over a range of the values of the selected quality characteristics. They have used the approach for the development of a tyre tread compound, which involves four responses (abrasion index, modulus, elongation at break, and hardness) and three independent variables or factors. This method increases the complexity of the computational process and it therefore cannot be easily understood by engineers with limited statistical skills.

In this paper, a methodology is presented for tackling multiple quality characteristics in Taguchi's PD experiments. It will make use of Taguchi's quality loss function (TQLF) for determining the optimum condition of the process. Only static quality characteristics [9] will be considered. The quality loss functions (QLFs) for static quality characteristics is explained in many Taguchi textbooks [10, 11].

3. Proposed Methodology for Optimisation of Multiple Quality Characteristics in Taguchi's PD Experiments

The following issues must be considered when optimising multiple quality characteristics in Taguchi's PD experiments.

The units of each quality characteristic will possibly be different and therefore the loss associated with each characteristic cannot be added directly.

Different quality characteristics have different relative weights and therefore a certain relative weight may be assigned to each characteristic prior to optimisation.

If the experiment involves target-the-best quality characteristics (e.g. length, thickness, diameter, time, force, pressure, and viscosity), the adjustment factor(s) should be identified. The adjustment factor(s) will then be used for tuning the mean performance onto the target value.

Having considered the above issues, the following steps can be taken into account. It is important to note that these steps will serve only as a guide to engineers with limited skills in multiple response process optimisation using Taguchi's PD experiments.

Step 1. Identification of Factors for the Experiment

The selection of factors or process variables are crucial to the success of any optimisation problem. In a Taguchi's PD experiment, the factors can be classified into control, noise, and signal factors. Some of the possible ways to identify the factors include the use of brainstorming and historical data. A typical brainstorming session includes people from manufacturing, design, quality, and the shop-floor. It is also good practice to use brainstorming in conjunction with cause-and-effect analysis. The cause-and-effect analysis will provide a better picture of the problem and the possible causes which influence the problem. Having identified the factors for the experiment, it is important to define the level of each factor.

Step 2. Selection of Quality Characteristic(s) or Responses for the Experiment

It is important that there must be a correlation between the quality characteristic(s) chosen for a certain experiment and the factors selected for experimentation. The selection of appropriate quality characteristic(s) requires a sound engineering knowledge of the process under investigation. To select a good response, it is advisable to start with the engineering or economic goal. Having determined the goal, identify the basic mechanism and physical laws affecting it, then choose the appropriate quality characteristic to increase the understanding of these mechanisms and laws.

Quality characteristics for Taguchi's PD experiments can be divided into two main categories:

1. Static quality characteristics.
2. Dynamic quality characteristics.

Static quality characteristics are further classified into smaller-the-better (STB), larger-the-better (LTB), nominal-the-best (NTB) and classified attributes (CA). More information on this topic can be obtained from [1, 9, 10]. A process is said to exhibit dynamic quality characteristic when the state of a particular factor has a direct impact on the quality characteristic. Such a factor is called a tuning or adjustment factor. The advantage of using such characteristics is that the experimenters may gain a better understanding of the process. However, the identification of dynamic characteristics for many processes has proved to be a complex procedure [12].

Step 3. Computation of Normalised Quality Loss for each Quality Characteristic

Quality loss is the loss associated with a product owing to the deviation in the functional performance of the product from its target. In this paper, only three static quality characteristics (STB, LTB, and NTB) are considered as these are the most commonly used in industry. The Eq. for the quality loss functions of these quality characteristics are available in most Taguchi textbooks. Let L_{ij} be the quality loss for the ith quality characteristic at the jth trial condition or run in the experimental design matrix. As each quality characteristic has different units of measurement, it is important to normalise the quality loss [13]. The normalised quality loss can be computed using:

$$y_{ij} = \frac{L_{ij}}{L_{i*}} \tag{1}$$

where y_{ij} = normalised quality loss
L_{i*} = maximum quality loss for the ith quality characteristic among all the trial conditions.

It is important to note that y_{ij} varies from a minimum of zero to a maximum of one.

Step 4. Computation of Total Normalised Quality Loss (Y_j)

For computing the total normalised quality loss (Y_j) corresponding to each trial condition, we must assign a weighting factor for each quality characteristic considered in the optimisation process. If w_i represents the weighting factor for the ith quality characteristic, k is the number of quality characteristics and y_{ij}

is the loss function associated with the ith quality characteristic at the jth trial condition, then Y_j can be computed using:

$$Y_j = \sum_{i}^{k} w_i \, y_{ij} \tag{2}$$

Step 5. Computation of Multiple Signal-to-Noise ratio (η_j)

After the total normalised quality loss (Y_j) corresponding to each trial condition has been calculated, the next step is to compute the multiple signal-to-noise ratio (η_j) at each design point. This is given by:

$$\eta_j = -10 \log(Y_j) \tag{3}$$

Step 6. Determination of Significant Factor/Interaction Effects and Optimal Settings

In Taguchi's PD experiment with multiple quality characteristics, the optimal condition for processes with STB and LTB quality characteristics is obtained by selecting the factor levels with the highest multiple signal-to-noise ratio (η). However, for NTB quality characteristics, we must identify the factor(s) which influences only the mean quality characteristic but has no effect on the η (also called adjustment factor). In other words, for NTB quality characteristics, a multiple signal-to-noise ratio analysis may have to be performed first, followed by an analysis of the mean quality characteristic. The idea is to reduce variability in the functional performance of product and then bring the mean characteristic onto its target value [14]. In order to identify the significant factor or interaction effects, the use of analysis of variance (ANOVA) is recommended. ANOVA is a powerful tool to subdivide the total variability into useful components of variability [15]. In the case of a multiple quality characteristic optimisation problem, we must separate the total variability of the multiple signal-to-noise ratios into the contributions made by each of the factors (or process parameters) and the error term.

Step 7. Perform the Confirmation Trial or Experiment

The purpose of a confirmation trial or experiment is to verify that the optimal factor settings actually yield an improvement. It is important to note that the multiple signal-to-noise ratio value for the confirmation experiment cannot be estimated by Eq. (3). It is advisable to compare the signal-to-noise ratio values (predicted and observed) separately. If the predicted signal-to-noise ratio for each quality characteristic is close enough to the observed signal-to-noise ratio, we can conclude that the interactions among the factors were not important for the study. On the other hand, if the predicted and observed signal-to-noise ratios do not match, it is then an indication of the presence of interactions and therefore further experiments may be required to verify this.

4. Case Study

Peace [16] considered a case study of the optimisation of three quality characteristics for a double-sided surface mount technology electronic assembly operation. Six controllable factors (A, B, C, D, E, and F) and one interaction effect ($A \times B$) were studied using an L_8 orthogonal array experiment advocated by Taguchi. Factors A, B, C, D, E, and F were assigned to columns 1, 2, 4, 5, 6, and 7 of the L_8 array. The interaction between A and B was assigned to column 3 of the array. The quality characteristics of interest were solder paste mass, solder paste height, and glue torque. Here, the first and second quality characteristics belong to the nominal-the-best category. The third quality characteristic belongs to the larger-the-better category. Four observations were taken corresponding to each trial condition. The results of the analysis from Peace's study are:

1. For reducing the variation in the solder paste mass, the optimal settings for the significant effects are: A_2 B_1 F_1.
2. Similarly, for reducing the variation in solder paste height, the optimal settings for the significant effects are: B_2 D_2 F_1.
3. For glue torque, the optimal settings for the significant effects are: A_2 D_2 F_1.

The optimal settings for the process can be obtained by combining the above three cases. The optimal settings are therefore determined as: A_2 $B_{1 \text{ or } 2}$ D_2 F_1. Factors E and C were determined as adjustment factors for solder paste mass and solder paste height, respectively. Here, adjustment factors are those which affect only the mean response and not the signal-to-noise ratio. For factor B, level 1 is better for reducing the variation in the solder paste mass and level 2 is better for reducing the variation in the solder paste height. Based on engineering knowledge, it was concluded that for factor B, level 2 is better than level 1. However, research has shown that engineering judgement for multiresponse optimisation problems will usually bring some uncertainty to the decision-making process [17,18]. Moreover, engineering judgement is subjective in nature. Therefore, the above case study was analysed by strictly following the proposed methodology described in the paper. The first step was to compute the normalised quality loss associated with each characteristic. Table 1 illustrates the normalised quality loss values for each characteristic.

After the normalised quality loss values for each characteristic have been determined, the next step is to compute the total normalised quality loss (Y_j) corresponding to each trial

Table 1. Normalised quality loss values of selected quality characteristics.

Trial number	y_{1j} (solder paste mass)	y_{2j} (solder paste height)	y_{3j} (glue torque)
1	0.179	0.292	0.813
2	0.397	0.250	0.850
3	1.000	0.378	1.000
4	0.324	0.007	0.665
5	0.177	0094	0.380
6	0.126	0.492	0.404
7	0.165	0.012	0.297
8	0.394	1.000	0.934

Table 2. Total normalised quality loss values.

Trial number	Y_j
1	1.285
2	1.497
3	2.379
4	0.997
5	0.651
6	1.022
7	0.474
8	2.327

condition. Here, a weighting method is used to integrate the three normalised loss functions into a total loss function using Eq. (2). The weighting ratio for the three responses are set as 1:1:1 (i.e. each characteristic has equal importance or relative weighting) as assumed in the original case study (see [16]). The total normalised quality loss values for each trial are presented in Table 2.

The total normalised quality loss function is further transformed into a multiple signal-to-noise ratio (η). For multiple response optimisation problems, the optimal condition is the one that yields the highest multiple signal-to-noise ratio. The multiple signal-to-noise ratio values (based on Eq. (3) and the design matrix are shown in Table 3.

5. Determination of Significant Factor/Interaction Effects and Optimal Settings

Having computed the η values, it is essential to determine which of the above factor/interaction effects has an impact on the process performance. In order to investigate which factor/interaction effects significantly affect the multiple signal-to-noise ratio (η), it was decided to use the analysis of variance (ANOVA). ANOVA is used to subdivide the total variability of the multiple signal-to-noise ratios, which is measured by the sum of the squared deviations from the overall mean of the multiple signal-to-noise ratio, into contributions by each factor/interaction effect and by the error. The total sum of squares (SS_t) of the multiple signal-to-noise ratio can be computed using the following equation:

$$SS_t = \sum_{j=1}^{n} \eta_j^2 - n\eta_m^2 \tag{4}$$

where η_j = multiple signal-to-noise ratio of each trial condition
η_m = overall mean of the multiple signal-to-noise ratio
n = number of trials in the experiment

From Table 3, we obtain $SS_t = 42.48$

The total sum of squares of the multiple signal-to-noise ratio can then be decomposed into three main sources: the sum of squares due to the main effects (A, B, C, D, E, and F); the sum of squares due to the interaction effect (A $\times$ B); and the sum of squares due to the error.

The sum of squares due to a factor (say, factor A) can be calculated using:

$$SS_A = \frac{A_1^2}{n_1} + \frac{A_2^2}{n_2} - n\eta_m^2 \tag{5}$$

where A_1 = sum of the multiple signal-to-noise ratio values at level 1 of factor A
A_2 = sum of the multiple signal-to-noise ratio values at level 2 of factor A
n_1 = number of observations at level 1 of factor A
n_2 = number of observations at level 1 of factor B

For the interaction effect, we calculate the sum of the multiple signal-to-noise ratio values at level 1 and level 2 (using the AB interaction column in Table 3) and substitute them into Eq. (5).

As each factor was studied at two levels, the degrees of freedom associated with each factor/interaction effect are equal to one. The total degree of freedom for the experiment is 7 (i.e. 8–1). Moreover, the total degree of freedom for the factor/interaction effect is also 7. Therefore, the error degree of freedom is zero and as a result the ANOVA cannot be performed. In order to pursue the calculations, the inactive and smaller effects are added together to obtain a non-zero estimate of the error variance [19]. This process is called pooling, which can be used to combine the factor or interaction effects with low magnitudes of sum of squares. The results of pooled ANOVA performed on a multiple signal-to-noise ratio (η) are shown in Table 4. The results have shown that only factors A, D, and F have a significant impact on the multiple signal-to-noise ratio. Peace [16] also obtained similar results, apart from factor B. In fact, factor B was the major concern about the engineer's judgement in terms of determining the optimal level

Table 3. Multiple signal-to-noise ratio values.

Trial number	A	B	AB	C	D	E	F	η_i
1	1	1	1	1	1	1	1	−1.0880
2	1	1	1	2	2	2	2	−1.7530
3	1	2	2	1	1	2	2	−3.7630
4	1	2	2	2	2	1	1	0.0140
5	2	1	2	1	2	1	2	1.8670
6	2	1	2	2	1	2	1	−0.0946
7	2	2	1	1	2	2	1	3.2470
8	2	2	1	2	1	1	2	−3.6680

Table 4. ANOVA on the multiple signal-to-noise ratio.

Source	Degrees of freedom	Sum of squares	Mean square	F-ratio	Per cent contribution
A	1	7.88	7.88	5.63*	15.25
D	1	17.96	17.96	12.83**	39.00
F	1	11.04	11.04	7.89**	22.70
Pooled error	4	5.589	1.40	–	23.05
Total	7	42.47	–	–	100.00

From F-tables: $F_{0.05,1,4} = 7.71$ and $F_{0.10,1,4} = 4.54$
*factor is significant at 10% significance level.
**factor is significant at both 5% and 10% significance levels.

due to a trade-off in its levels. This study has shown that the simultaneous optimisation of multiple quality characteristics does not necessarily provide similar results, compared to optimising multiple quality characteristics, taken one at a time. In other words, in multiple response optimisation problems, it is unreasonable to optimise one characteristic at a time. On many occasions, the optimum conditions obtained for one quality characteristic (or response) are not completely compatible with those for other quality characteristics. Moreover, a factor may have a significant influence on the response when optimising each quality characteristic separately. However, the same factor may have very little influence when optimising all responses simultaneously. A simple and powerful methodology has been presented in this paper to optimise multiple quality characteristics simultaneously with the use of Taguchi's quality loss function.

6. Determination of Optimal Settings

After the ANOVA has been performed, the next step is to determine the optimal condition which provides the best settings of each factor or process variable involved in the experiment. In multiple response problems like those above, the optimal setting of each factor is the one which yields the highest multiple signal-to-noise ratio. In this case study, as two quality characteristics (solder paste mass and height) belong to target-the-best category, it was important to identify the adjustment factor/signal factor which affects only the mean performance and not the signal-to-noise ratio. The adjustment factors for solder paste mass and solder paste height were E and C, respectively. The results of ANOVA (refer to Table 4) have clearly shown that both factors C and E have no impact on the multiple signal-to-noise ratio. This finding suggests that the proposed methodology is powerful for identifying the factors which influence the mean response and not the multiple signal-to-noise ratio. The best settings for factors C and E were levels 1 and 2, respectively. The following factor settings have yielded the highest multiple signal-to-noise ratio:

Factor A – Level 2	Factor B – Level 1	Factor C – Level 1
Factor D – Level 2	Factor E – Level 2	Factor F – Level 1

In future work, the same case will be studied with different relative weights assigned to quality characteristics and then its impact on the selection of important factor effects and optimal condition will be determined.

7. Conclusions

Optimisation of multiple quality characteristics in Taguchi parameter design (PD) experiments is not common and has received little attention. Engineering judgement has been primarily used to optimise multiple quality characteristics in Taguchi PD experiments. This approach is subjective in nature and therefore always brings an element of uncertainty to the decision-making process. Moreover, a factor that is statistically significant in a single quality characteristic case may not be significant when considered in a multiresponse case. Therefore, it was essential to develop a simple and practical step-by-step approach for tackling multiple response or quality characteristic problems in Taguchi PD experiments. The methodology uses the Taguchi's quality loss function (TQLF) for identifying the significant factor/interaction effects and also for determining the optimal condition of the process. In order to demonstrate the potential of the proposed methodology, the paper presents a simple case study carried out for optimising three quality characteristics for a double-sided surface mounting technology electronic assembly operation. Future work will look into the impact of different relative weightings of quality characteristics on the multiple signal-to-noise ratio.

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