OPTIMIZATION APPROACHES IN RISK MANAGEMENT: APPLICATIONS IN FINANCE AND AGRICULTURE

By

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To my family

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate School
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Along with the fast development of the financial industry in recent decades, novel financial products, such as swaps, derivatives, and structure financial instruments, have been invented and traded in financial markets. Practitioners have faced much more complicated problems in making profit and hedging risks. Financial engineering and risk management has become a new discipline applying optimization approaches to deal with the challenging financial problems.

This dissertation proposes a novel optimization approach using the downside risk measure, conditional value-at-risk (CVaR), in the reward versus risk framework for modeling stochastic optimization problems. The approach is applied to the optimal crop production and risk management problem and two critical problems in the secondary mortgage market: the efficient execution and pipeline risk management problems.

In the optimal crop planting schedule and hedging strategy problem, crop insurance products and commodity futures contracts were considered for hedging against yield and price risks. The impact of the ENSO-based climate forecast on the optimal production and hedge decision was also examined. The Gaussian copula function was applied in simulating the scenarios of correlated non-normal random yields and prices.

Efficient execution is a significant task faced by mortgage bankers attempting to profit from the secondary market. The challenge of efficient execution is to sell or securitize a large number of heterogeneous mortgages in the secondary market in order to maximize expected revenue under a certain risk tolerance. We developed a stochastic optimization model to perform efficient execution that considers secondary marketing functionality including loan-level efficient execution, guarantee fee buy-up or buy-down, servicing retain or release, and excess servicing fee. The efficient execution model balances between the reward and downside risk by maximizing expected return under a CVaR constraint.

The mortgage pipeline risk management problem investigated the optimal mortgage pipeline risk hedging strategy using 10-year Treasury futures and put options on 10-year Treasury futures as hedge instruments. The out-of-sample hedge performances were tested for five deviation measures, Standard Deviation, Mean Absolute Deviation, CVaR Deviation, VaR Deviation, and two-tailed VaR Deviation, as well as two downside risk measures, VaR and CVaR.

CHAPTER 1
INTRODUCTION

Operations Research, which originated from World War II for optimizing the military supply chain, applies mathematic programming techniques in solving and improving system optimization problems in many areas, including engineering, management, transportation, and health care. Along with the fast development of the financial industry in recent decades, novel financial products, such as swaps, derivatives, and structural finance instruments, have been invented and traded in the financial markets. Practitioners have faced much more complicated problems in making profit and hedging risks. Financial engineering and risk management has become a new discipline applying operations research in dealing with the sophisticated financial problems.

This dissertation proposes a novel approach using conditional value-at-risk in the reward versus risk framework for modeling stochastic optimization problems. We apply the approach in the optimal crop planting schedule and risk hedging strategy problem as well as two critical problems in the secondary mortgage market: the efficient execution problem, and the mortgage pipeline risk management problem.

Since Markowitz (1952) proposed the mean-variance framework in portfolio optimization, variance/covariance has become the predominant risk measure in finance. However, this risk measure is suited only to elliptic distributions, such as normal or t-distributions, with finite variances (Szegö 2002). The other drawback of variance risk measure is that it measures both upside and downside risks. In practice, however, finance risk management is concerned mostly with the downside risk. A popular downside risk measure in economics and finance is Value-at-Risk (VaR) (Jorion 2000), which measures α percentile of loss distribution. However, as was shown by Artzner et al. (1999), VaR is ill-behaved and non-convex for general distribution. The disadvantage of VaR is that it only considers risk at α percentile of loss distribution and does not consider the magnitude of the losses in the α-tail (i.e., the worst 1-α percentage of scenarios).

To address this issue, Rockafellar and Uryasev (2000, 2002) proposed Conditional Value-at-Risk, which measures the mean value of the α-tail of loss distribution. It has been shown that CVaR satisfies the axioms of coherent risk measures proposed by Artzner et al. (1999) and has desirable properties. Most importantly, Rockafellar and Uryasev (2000) showed that CVaR constraints in optimization problems can be formulated as a set of linear constraints and incorporated into problems of optimization. This linear property is crucial in formulating the model as a linear programming problem that can be efficiently solved.

This dissertation proposes a mean-CVaR model which, like a mean-variance model, provides an efficient frontier consisting of points that maximize expected return under various risk budgets measured by CVaR. Since CVaR is defined in monetary units, decision makers are able to decide their risk tolerance much more intuitively than with abstract utility functions. It is worth noting that CVaR is defined on a loss distribution, so a negative CVaR value represents a profit.

Although the variance/covariance risk measure has its drawbacks, it has been the proxy of risk measure for modeling the stochastic optimization problem in financial industry. The main reason is that the portfolio variance can be easily calculated given individual variances and a covariance matrix. However, the linear correlation is a simplified model and is limited in capturing the association between random variables. Therefore, a more general approach is needed to model the more complicated relationship between multivariate random variables, e.g., tail dependent. More importantly, the approach should provide a convenient way to create the portfolio loss distribution from marginal ones, which is the most important input data in the mean-CVaR optimization model. This dissertation applies the copula function to model the correlation between random variables. In Chapter 2, the use of copulas to generate scenarios of dependent multivariate random variables is discussed. Furthermore, the simulated scenarios are incorporated into the mean-CVaR model.

Chapter 2 investigates the optimal crop planting schedule and hedging strategy. Crop insurance products and futures contracts are available for hedging against yield and price risks. The impact of the ENSO-based climate forecast on the optimal production and hedging decision is examined. Gaussian copula function is applied in simulating the scenarios of correlated non-normal random yields and prices. Using data of a representative cotton producer in the Southeastern United States, the best production and hedging strategy is evaluated under various risk tolerances for each of three predicted ENSO-based climate phases.

Chapters 3 and 4 are devoted to two optimization problems in the secondary mortgage market. In Chapter 3, the efficient execution problem was investigated. Efficient execution is a significant task faced by mortgage bankers attempting to profit from the secondary market. The challenge of efficient execution is to sell or securitize a large number of heterogeneous mortgages in the secondary market in order to maximize expected revenue under a certain risk tolerance. A stochastic optimization model was developed to perform efficient execution that considers secondary marketing functionality including loan-level efficient execution, guarantee fee buy-up or buy-down, servicing retain or release, and excess servicing fee. Since efficient execution involves random cash flows, lenders must balance between expected revenue and risk. We employ a CVaR risk measure in this efficient execution model that maximizes expected revenue under a CVaR constraint. By solving the efficient execution problem under different risk tolerances specified by a CVaR constraint, an efficient frontier was found, which provides secondary market managers the best execution strategy associated with different risk budgets. The model was formulated as a mixed 0-1 linear programming problem. A case study was conducted and the optimization problem was efficiently solved by the CPLEX optimizer.

Chapter 4 examines the optimal mortgage pipeline risk hedging strategy. Mortgage lenders commit to a mortgage rate while the borrowers enter the loan transaction process. The process is typically for a period of 30-60 days. While the mortgage rate rises before the loans go to closing, the value of the loans declines. Therefore, the lender will sell the loans at a lower price when the loans go to closing. The risk of a fall in value of mortgages still being processed prior to their sale is known as mortgage pipeline risk. Lenders often hedge this exposure by selling forward their expected closing volume or by shorting U.S. Treasury notes or futures contracts. Mortgage pipeline risk is affected by fallout. Fallout refers to the percentage of loan commitments that do not go to closing. As interest rates fall, fallout rises since borrowers who have locked in a mortgage rate are more likely to find better rates with another lender. Conversely, as rates rise, the percentage of loans that go to closing increases. Fallout affects the required size of the hedging instrument because it changes the size of risky pipeline positions. At lower rates, fewer loans will close and a smaller position in the hedging instrument is needed. Lenders often use options on U.S. Treasury futures to hedge against the risk of fallout (Cusatis and Thomas, 2005).

A model was proposed for the optimal mortgage pipeline hedging strategy that minimizes the pipeline risks. A case study considered two hedging instruments for hedging the mortgage pipeline risks: the 10 year Treasury futures and put options on 10-year Treasury futures. To investigate the impact of different risk measurement practices on the optimal hedging strategies, we tested five deviation measures, standard deviation, mean absolute deviation, CVaR deviation, VaR deviation, and two-tailed VaR deviation, as well as two downside risk measures, VaR and

CVaR, in the minimum mortgage pipeline risk model. The out-of-sample performances of the five deviation measures and two downside risk measures were examined.

CHAPTER 2
OPTIMAL CROP PLANTING SCHEDULE AND HEDGING STRATEGY UNDER ENSO-BASED CLIMATE FORECAST

2.1 Introduction

A risk-averse farmer preferring higher profit from growing crops faces uncertainty in the crop yields and harvest price. To manage uncertainty, a farmer may purchase a crop insurance policy and/or trade futures contracts against the yield and price risk. Crop yields depend on planting dates and weather conditions during the growing period. The predictability of seasonal climate variability (i.e., the El Niño Southern Oscillation, ENSO), gives the opportunity to forecast crop yields in different planting dates. With the flexibility in planting timing, the profit can be maximized by selecting the best planting schedule according to climate forecast. Risk – averse is another critical factor when farmers make the decision. Farmers may hedge the yield and price risks by purchasing crop insurance products or financial instruments.

Two major financial instruments for farmers to hedge against crop risks are crop insurances and futures contracts. The Risk Management Agency (RMA) of the United States Department of Agricultural (USDA) offers crop insurance policies for various crops, which could be categorized into three types: the yield-based insurance, revenue-based insurance, and policy endorsement. The most popular yield-based insurance policy, Actual Production History (APH), or Multiple Peril Crop Insurance (MPCI), is available for most crops. The policies insure producers against yield losses due to natural causes. An insured farmer selects to cover a percentage of the average yield together with an election price (a percentage of the crop price established annually by RMA). If the harvest yield is less than the insured yield, an indemnity is paid based on the shortfall at the election price. The most popular revenue-based insurance policy, Crop Revenue Coverage (CRC), provides revenue protection. An insured farmer selects a coverage level of the guarantee revenue. If the realized revenue is below the guarantee revenue, the insured farmer is paid an indemnity to cover the difference between the actual and guaranteed revenue. Catastrophic Coverage (CAT), a policy endorsement, pays 55% of the established price of the commodity on crop yield shortfall in excess of 50%. The cost of crop insurances includes a premium and an administration fee[1]. The premiums on APH and CRC both depend on the crop type, county, practice (i.e., irrigated or non-irrigated), acres, and average yield. In addition, the APH premium depends on price election and yield coverage, and the CRC depends on revenue coverage. The premium on CAT coverage is paid by the Federal Government; however, producers pay the administrative fee for each crop insured in each county regardless of the area planted[2].

In addition to crop insurance coverage, farmers may manage commodity price risk by a traditional hedge instrument such as futures contract. A futures contract is an agreement between two parties to buy or sell a commodity at a certain time in the future, for a specific amount, at a certain price. Futures contracts are highly standardized and are traded by exchange. The cost of futures contract includes commissions and interest foregone on margin deposit. A risk-averse producer may consider using insurance products in conjunction with futures contacts for best possible outcomes.

El Niño Southern Oscillation refers to interrelated atmospheric and oceanic phenomena. The barometric pressure difference between the eastern and western equatorial Pacific is frequently changed. The phenomenon is known as the Southern Oscillation. When the pressure over the western Pacific is above normal and eastern Pacific pressure is below normal, it creates abnormally warm sea surface temperature (SST) known as El Niño. On the other hand, when the

[1] The administration fee is $30 for each APH and CRC contract and $100 for each CAT contract.

[2] Source: http://www2.rma.usda.gov/policies .

east-west barometric pressure gradient is reversed, it creates abnormally cold SST known as La Niña. The term “neutral” is used to indicate SSTs within a normal temperature range. These equatorial Pacific conditions known as ENSO phases refer to different seasonal climatic conditions. Since the Pacific SSTs are predictable, ENSO becomes an index for forecasting climate and consequently crop yields.

A great deal of research has been done on the connection between the ENSO-based climate prediction and crop yields since early 1990s. Cane et al. (1994) found the long-term forecasts of the SSTs could be used to anticipate Zimbabwean maize yield. Hansen et al. (1998) showed that El Niño Southern Oscillation is a strong driver of seasonal climate variability that impact crop yields in the southeastern U.S. Hansen (2002) and Jones et al. (2000) concluded that ENSO-based climate forecasts might help reduce crop risks.

Many studies have focused on the crop risk hedging with crop insurance and other derivative securities. Poitras (1993) studied farmers’ optimal hedging problem when both futures and crop insurance are available to hedge the uncertainty of price and production. Chambers et al. (2002) examined optimal producer behavior in the presence of area-yield insurance. Mahul (2003) investigated the demand of futures and options for hedging against price risk when the crop yield and revenue insurance contracts are available. Coble (2004) investigated the effect of crop insurance and loan programs on demand for futures contract.

Some researchers have studied the impacts of the ENSO-based climate information on the selection of optimal crop insurance policies. Cabrera et al. (2006) examined the impact of ENSO-based climate forecast on reducing farm risk with optimal crop insurance strategy. Lui et al. (2006), following Cabrera et al. (2006), studied the application of Conditional Value-at-Risk (CVaR) in the crop insurance industry under climate variability. Cabrera et al. (2007) included the interference of farm government programs on crop insurance hedge under ENSO climate forecast.

The purpose of this research is two-fold. First, a mean-CVaR optimization model was proposed for investigating the optimal crop planting schedule and hedging strategy. The model maximizes the expected profit with a CVaR constraint for specifying producer's downside risk tolerance. Second, the impact of the ENSO-based climate forecast on the optimal decisions of crop planting schedule and hedging strategy was examined. To this end, we generate the scenarios of correlated random yields and prices by Monte Carlo simulation with the Gaussian copula for each of the three ENSO phases. Using the scenarios associated with a specific ENSO phase as the input data, the mean-CVaR model is solved for the optimal production and hedging strategy for the specified ENSO phase.

The remainder of this article is organized as follows. The proposed model for optimal planting schedule and hedging strategy is introduced in section 2.2. Next, section 2.3 describes a case study using the data of a representative cotton producer in the Southeastern United States. Then, section 2.4 reports the results of the optimal planting schedule, crop insurance policy selection, and hedging position of futures contract. Finally, section 2.5 presents the conclusions.

2.2 Model

2.2.1 Random Yield and Price Simulation

To investigate the impact of ENSO-based climate forecast on the optimal production and risk management decisions, we calibrate the yield and price distributions for an ENSO phase based on the historical yields and prices of the years classified to the ENSO phase based on the Japan Meteorological Agency (JMA) definition (Japan Meteorological Agency, 1991). Then, random yield and price scenarios associated with the ENSO phase are generated by Monte Carlo simulation.

We assume a farmer may plant crops in a number of planting dates across the planting season. The yields of planting dates are positive correlated according to the historical data. In addition, the correlation between the random production and random price is crucial in risk management since negative correlated production and price provide a natural hedge that will affect the optimal hedging strategy (McKinnon, 1967). As a consequence, we consider the correlation between yields of different planting dates and the crop price. Since the distributions of the crop yield and price are not typically normal distributed, a method to simulate correlated multivariate non-normal random yield and price was needed.

Copulas are functions that describe dependencies among variables, and provide a way to create distributions to model correlated multivariate data. Copula function was first proposed by Sklar (1959). The Sklar theorem states that given a joint distribution function F on R^n with marginal distribution F_i, there is a copula function C such that for all $x_1,...,x_n$ in R,

$$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)). \tag{2-1}$$

Furthermore, if F_i are continuous then C is unique. Conversely, if C is a copula and F_i are distribution functions, then F, as defined by the previous expression, is a joint distribution function with margins F_i. We apply the Gaussian copula function to generate the correlated non-normal multivariate distribution. The Gaussian copula is given by:

$$C_\rho(F_1(x_1),...,F_n(x_n)) = \Phi_{n,\rho}(\Phi^{-1}(F_1(x_1)),...,\Phi^{-1}(F_n(x_n))), \tag{2-2}$$

which transfers the observed variable x_i, i.e. yield or price, into a new variable y_i using the transformation

$$y_i = \Phi^{-1}\left[F_i(x_i)\right], \tag{2-3}$$

where $\Phi_{n,\rho}$ is the joint distribution function of a multivariate Gaussian vector with mean zero and correlation matrix ρ. Φ is the distribution function of a standard Gaussian random variable. In moving from x_i to y_i we are mapping observation from the assumed distribution F_i into a standard normal distribution Φ on a percentile to percentile basis.

We use the rank correlation coefficient Spearman's rho ρ_s to calibrate the Gaussian copula to the historical data. For *n* pairs of bivariate random samples $\left(X_i, X_j\right)$, define $R_i = rank(X_i)$ and $R_j = rank(X_j)$. Spearman's sample rho (Cherubini, 2004) is given by

$$\rho_s = 1 - 6\frac{\sum_{k=1}^{n}\left(R_{ik} - R_{jk}\right)}{n(n^2-1)}. \tag{2-4}$$

Spearman's rho measures the association only in terms of ranks. The rank correlation is preserved under the monotonic transformation in equation 3. Furthermore, there is a one-to-one mapping between rank correlation coefficient, Spearman's rho ρ_s, and linear correlation coefficient ρ for the bivariate normal random variables (y_1, y_2) (Kruksal, 1958)

$$\rho_s(y_1, y_2) = \frac{6}{\pi}\arcsin\frac{\rho(y_1, y_2)}{2} \tag{2-5}$$

To generate correlated multivariate non-normal random variables with margins F_i and Spearman's rank correlation ρ_s, we generate the random variables y_i's from the multivariate normal distribution $\Phi_{n,\rho}$ with linear correlation

$$\rho = 2\sin\left(\frac{\pi\rho_s}{6}\right), \tag{2-6}$$

by Monte Carlo simulation. The actual outcomes x_i's can be mapped from y_i's using the transformation

$$x_i = F_i^{-1}\left[\Phi(y_i)\right]. \tag{2-7}$$

2.2.2 Mean-CVaR Model

Since Markowitz (1952) proposed the mean-variance framework in portfolio optimization, variance/covariance has become the predominant risk measure in finance. However, the risk measure is suited only to the case of elliptic distributions, like normal or t-distributions with finite variances (Szegö, 2002). The other drawback of variance risk measure is that it measures both upside and downside risks. In practice, finance risk management is concerned only with the downside risk in most cases. A popular downside risk measure in economics and finance is Value-at-Risk (VaR) (Jorion, 2000), which measures α percentile of loss distribution. However, as was shown by Artzner et al. (1999), VaR is ill-behaved and non-convex for general distribution. The other disadvantage of VaR is that it only considers risk at α percentile of loss distribution and does not consider the magnitude of the losses in the α-tail (the worst 1-α percentage of scenarios). To address this issue, Rockafellar and Uryasev (2000, 2002) proposed Conditional Value-at-Risk, which measures the mean value of α-tail of loss distribution. Figure 2-1 shows the definition of CVaR and the relation between CVaR and VaR. It has been shown that CVaR satisfies the axioms of coherent risk measures proposed by Artzner et al. (1999) and has desirable properties. Most importantly, Rockafellar and Uryasev (2000) showed that CVaR constraints in optimization problems can be formulated as a set of linear constraints and incorporated into the problems of optimization. The linear property is crucial to formulate the model as a mixed 0-1 linear programming problem that could be solved efficiently by the CPLEX solver. This research proposes a mean-CVaR model that inherits advantages of the return versus risk framework from the mean-variance model proposed by Markowitz (1952). More importantly, the model utilizes the CVaR risk measure instead of variance to take the advantages of CVaR. Like mean-variance model, the mean-CVaR model provides an efficient frontier consisting of points that maximize expected return under various tolerances of CVaR losses. Since CVaR is defined in monetary units, farmers are able to decide their risk tolerance much more intuitively compared to abstract utility functions. It is worth noting that CVaR is defined on a loss distribution. Therefore, a negative CVaR value represents a profit. For example, a -$20,000 90% CVaR means the average of the worst 10% scenarios should provide a profit equal to $20,000.

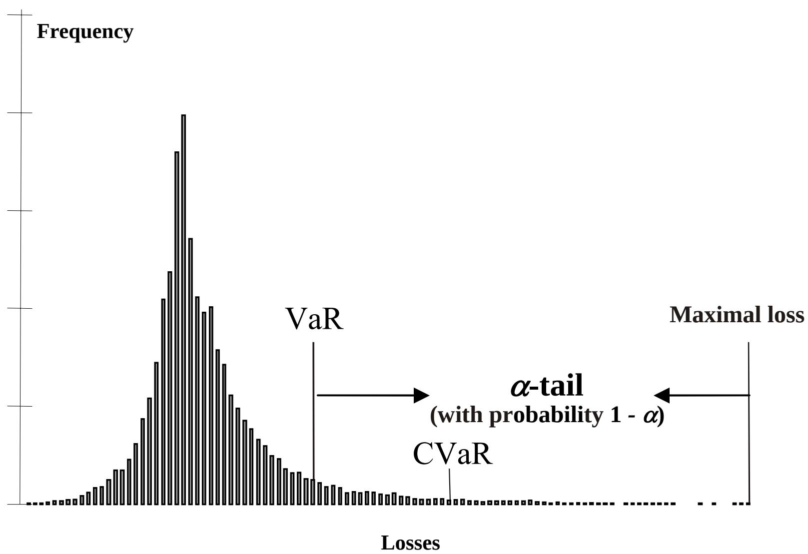

Figure 2-1. Definition of VaR and CVaR associated with a loss distribution

2.2.3 Model Implementation

Assume a farmer who plans to grow crops in a farmland of *Q* acres. There are *K* possible types of crops and more than one crop can be planted. For each crop *k*, there are T_k potential planting dates that give different yield distributions based on the predicted ENSO phase, as well as I_k available insurance policies for the crop. The decision variables x_{kti} and η_k represent the acreages of crop *k* planted in date *t* with insurance policy *i* and the hedge position (in pounds) of crop *k* in futures contract, respectively.

The randomness of crop yield and harvest price in a specific ENSO phase is managed by the joint distribution corresponding to the ENSO phase. We sample *J* scenarios from the joint distribution by Monte Carlo simulation with Gaussian copula, and each scenario has equal probability. Let Y_{ktj} denote the j^{th} realized yield (pound per acre) of crop *k* planted on date *t*, and P_{kj} denote the j^{th} realized cash price (dollar per pound) for crop *k* at the time the crop will be sold.

The objective function of the model, shown in (2-8), is to maximize the expectation of random profit $f(x_{kti}, \eta_k)$ that consists of the random profit from production $f^P(x_{kti})$, from crop insurance $f^I(x_{kti})$, and from futures contract $f^F(\eta_k)$.

$$\max Ef(x_{kti}, \eta_k) = \max\left[Ef^P(x_{kti}) + Ef^I(x_{kti}) + Ef^F(\eta_k)\right], \tag{2-8}$$

The profit from production of crop *k* in scenario *j* is equal to the income from selling the crop, $\sum_{t=1}^{T_k}\left(Y_{ktj}P_{kj}\sum_{i=1}^{I_k}x_{kti}\right)$, minus the production cost, $C_k\sum_{t=1}^{T_k}\sum_{i=1}^{I_k}x_{kti}$, and plus the subsidy, $S_k\sum_{t=1}^{T_k}\sum_{i=1}^{I_k}x_{kti}$., where C_k and S_k are unit production cost and subsidy, respectively. Consequently Equation (2-9) expresses the expected profit from production.

$$Ef^P(x_{kti}) = \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T_k}\left(Y_{ktj}P_{kj} - C_k + S_k\right)\sum_{i=1}^{I_k}x_{kti}. \tag{2-9}$$

Three types of crop insurance policies are considered in the model, including Actual Production History (APH), Crop Revenue Coverage (CRC), and Catastrophic Coverage (CAT). For APH farmers select the insured yield, a percentage α_i from 50 to 75 percent with five percent increments of average yield $\overline{Y}_k$, as well as the election price, a percentage β_i , between

55 and 100 percent, of the of the established price P_k established annually by RMA. If the harvest is less than the yield insured, the farmer is paid an indemnity based on the difference $\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}$ at price $\beta_i P_k$. The indemnity of APH insurance policy $i \in I_{APH}$ for crop *k* in the j^{th} scenario is given by

$$D_{kij} = \max\left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}, 0\right] \times \beta_i P_k \qquad \forall i \in I_{APH} \tag{2-10}$$

For CRC, producers elect a percentage of coverage level γ_i between 50 and 75 percent. The guaranteed revenue is equal to the coverage level γ_i times the product of $\sum_{t=1}^{T_k}\overline{Y}_k x_{kti}$ and the higher of the base price (early-season price) P_k^b and the realized harvest price in the j^{th} scenario of crop *k*, P_{kj}^h. The base price and harvest price of crop *k* are generally defined based on the crop's futures price in planting season and harvest season, respectively. If the calculated revenue $\sum_{t=1}^{T_k} Y_{ktj} x_{kti} P_{kj}$ is less than the guaranteed one, the insured will be paid the difference. Equation (2-11) shows the indemnity of CRC insurance policy $i \in I_{CRC}$ for crop *k* in the j^{th} scenario.

$$D_{kij} = \max\left[\gamma_i \sum_{t=1}^{T_k}\overline{Y}_k x_{kti} \times \max\left[P_k^b, P_{kj}^h\right] - \sum_{t=1}^{T_k} Y_{ktj} x_{kti} P_{kj}, 0\right] \qquad \forall i \in I_{CRC}, \tag{2-11}$$

The CAT insurance pays 55 % of the established price of the commodity on crop losses in excess of 50 %. The indemnity of CAT insurance policy $i \in I_{CAT}$ for crop *k* in the j^{th} scenario is given by

$$D_{kij} = \max\left[\sum_{t=1}^{T_k}\left(0.5\overline{Y}_k - Y_{ktj}\right)x_{kti}, 0\right] \times 0.55 P_k \ . \tag{2-12}$$

The cost of insurance policy *i* for crop *k* is denoted by R_{ki}, which includes a premium and an administration fee. For the case of CAT, the premium is paid by the Federal Government. Therefore, the cost of CAT only contains a \$ 100 administrative fee for each crop insured in each county.

The expected total profit from insurance is equal to the indemnity from the insurance coverage minus the cost of the insurance is given by

$$Ef^{I}\left(x_{kti}\right)=\left\{\frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{i=1}^{I_k}D_{kij}-R_{ki}\sum_{t=1}^{T_k}x_{kti}\right. . \tag{2-13}$$

The payoff of a futures contract for crop *k* in scenario *j* for a seller is given by

$$\pi_{kj}^{F}=(F_k-f_{kj})\eta_k , \tag{2-14}$$

where F_k is the futures price of crop *k* in the planting time, f_{kj} is the j^{th} realized futures price of crop *k* in the harvest time, and η_k is the hedge position (in pounds) of crop *k* in futures contract. It is worth noting that the futures price f_{kj} is not exactly the same as the local cash price P_{kj} at harvest time. Basis, defined in (2-15), refers to the difference that induces the uncertainty of futures hedging known as the basis risk. The random basis can be estimated from comparing the historical cash prices and futures prices.

$$\text{Basis} = \text{Cash Price} - \text{Futures Price}. \tag{2-15}$$

The cost of a futures contract, C_k^F, includes commissions and interest foregone on margin deposit. Equation (2-16) expresses the expected profit from futures contract.

$$Ef^{F}\left(\eta_k\right)=\frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\pi_{kj}^{F}-C_k^{F}\right) \tag{2-16}$$

We introduce binary variables z_{ki} in constraint (2-17) and (2-18) to ensure only one insurance policy can be selected for each crop *k*.

$$\sum_{t=1}^{T_k} x_{kti} \leq Q \cdot z_{ki} \qquad \forall i, k \tag{2-17}$$

$$\sum_{i=1}^{I_k} z_{ki} = 1 \qquad \forall k \tag{2-18}$$

where

$$z_{ki} = \begin{cases} 1 & \text{if crop } k \text{ is insured by policy } i, \\ 0 & \text{otherwise.} \end{cases}$$

Constraint (2-19) restricts the total planting area to a given planting acreage *Q*. The equality in this constraint can be replaced by an inequality $(\leq)$ to represent farmers choosing not to grow the crops when the production is not profitable.

$$\sum_{k=1}^{K} \sum_{t=1}^{T_k} \sum_{i=1}^{I_k} x_{kti} = Q \tag{2-19}$$

To model producer's risk tolerance, we impose the CVaR constraint

$$\alpha CVaR(L(x_{kti}, \eta_k)) \leq U \,. \tag{2-20}$$

where $L(x_{kti}, \eta_k)$ is a random loss equal to the negative random profit $f(x_{kti}, \eta_k)$ defined in (8). The definition of $\alpha CVaR(L(x_{kti}, \eta_k))$ is given by

$$\alpha CVaR(L(x_{kti}, \eta_k)) = E\left[L(x_{kti}, \eta_k) \middle| L(x_{kti}, \eta_k) \geq \zeta_\alpha\left(L(x_{kti}, \eta_k)\right)\right], \tag{2-21}$$

where $\zeta_\alpha\left(L(x_{kti}, \eta_k)\right)$ is the α-quintile of the distribution of $L(x_{kti}, \eta_k)$. Therefore, constraint (2-20) enforces the conditional expectation of the random loss $L(x_{kti}, \eta_k)$ given that the random loss exceeds α-quintile to be less than or equal to U. In other words, the expected loss of α-tail, i.e. (1- α)100% worst scenarios, is upper limited by an acceptable CVaR upper bound U. Rockafellar

& Uryasev (2000) showed that CVaR constraint (2-20) in optimization problems can be expressed by linear constraints (2-22), (2-23), and (2-24)

$$\zeta_{\alpha}\left(L(x_{kti},\eta_k)\right)+\frac{1}{J(1-\alpha)}\sum_{j=1}^{J} z_j \le U\,, \tag{2-22}$$

$$z_j \ge \sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} L(x_{kti},\eta_k)-\zeta_{\alpha}\left(L(x_{kti},\eta_k)\right) \qquad \forall j\,, \tag{2-23}$$

$$z_j \ge 0 \qquad \forall j\,, \tag{2-24}$$

where z_j are artificial variables introduced for the linear formulation of CVaR constraint.

Note that the maximum objective function contains indemnities D_{kij} that include a *max* term shown in equation (2-10), (2-11), and (2-12). To implement the model as a mix 0-1 linear problem, we transform the equations to an equivalent linear formulation by disjunctive constraints (Nemhauser and Wolsey, 1999). For example, equation (2-10), $D_{kij}=\max\left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k-Y_{ktj}\right)x_{kti},0\right]\times\beta_i P_k$, can be represented by a set of mix 0-1 linear constraints

$$
\begin{gathered}
D_{kij} \ge 0\,,\\
D_{kij} \ge \left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k-Y_{ktj}\right)x_{kti}\right]\times\beta_i P_k\,,\\
D_{kij} \le \left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k-Y_{ktj}\right)x_{kti}\right]\times\beta_i P_k + MZ_{kij}\,,\\
D_{kij} \le M(1\text{-}Z_{kij})\,,\\
\left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k-Y_{ktj}\right)x_{kti}\right]\times\beta_i P_k \le M(1\text{-}Z_{kij})\,,\\
\left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k-Y_{ktj}\right)x_{kti}\right]\times\beta_i P_k \ge -MZ_{kij}\,.
\end{gathered}
\tag{2-25}
$$

where M is a big number and Z_{kij} is a 0-1 variable. Similarly, equation (2-11) and (2-12) can be transformed into a set of mix 0-1 linear constraints in the same way. Consequently, the optimal crop production and hedging problem has been formulated as a mix 0-1 linear programming problem.

2.2.4 Problem Solving and Decomposition

Although the mix 0-1 linear programming problem can be solved with optimization software, the solving time increases exponentially when the problem becomes large. To improve the solving efficiency, we may decompose the original problem into sub-problems that could be solved more efficiently than the original problem. Since only one insurance policy could be selected for each crop, we decomposed the original problem into sub-problems in which each crop is insured by a specific insurance policy. The original problem contains *K* types of crops, and for the k^{th} type of crop there are I_k eligible insurance policies. Therefore, the number of the sub-problems is equal to the number of all possible insurance combinations of the *K* crops,

$$\prod_{k=1}^{K} I_k \ .$$

The formulation of the sub-problem is the same as the original problem except that the index *i's* are fixed and the equation (2-17) and (2-18) are removed. Solving sub-problems gives the optimal production strategy and futures hedge amount under a specific combination of insurance policies for *K* crops. The solution of the sub-problem with the highest optimal expected profit among all sub-problems gives the optimal solution of the original problem in which the optimal production strategy and futures hedge position are provided from the sub-problem solution and the optimal insurance coverage is the specific insurance combination of the sub-problem.

2.3 Case Study

Following the case study in Cabrera et al. (2006), we consider a representative farmer who grows cotton on a non-irrigated farm of 100 acres in Jackson County, Florida. Dothan Loamy Sand, a dominant soil type in the region, is assumed. The farmer may trade futures contracts from the New York Board of Trade and/or purchase crop insurance to hedge the crop yield and price risk. Three types of crop insurances, including Actual Production History (APH), Crop Revenue Coverage (CRC), and Catastrophic Coverage (CAT), are eligible for cotton and the farmer may select only one eligible insurance policy to hedge against the risk or opt for none. For APH, the eligible coverage levels of yield are from 65% to 75% with 5% increments, and the election price is assumed to be 100% of the established price. In addition, the available coverage levels of revenue for CRC are from 65% to 85% with 5% increments.

To investigate the impact of ENSO-based climate forecast in the optimal decisions of production and hedging strategy, we select historical climate data from 1960 to 2003 for the numerical implementation. ENSO phases during this period included 11 years of El Niño, 9 years of La Niña, and the remaining 25 years of Neutral, according to the Japan Meteorological Index (Table 2-1).

Table 2-1. Historical years associated with ENSO phases from 1960 to 2003

EL Niño		Neutral				La Niña	
1964	1987	1960	1975	1984	1994	1965	1989
1966	1988	1961	1978	1985	1995	1968	1999
1970	1992	1962	1979	1986	1996	1971	2000
1973	1998	1963	1980	1990	1997	1972	
1977	2003	1967	1981	1991	2001	1974	
1983		1969	1982	1993	2002	1976	

The cotton yields during the period of 1960-2003 were simulated using the CROPGRO-Cotton model (Messina et al., 2005) in the Decision Support System for Agrotechnology Transfer (DSSAT) v4.0 (Jones et al., 2003) based on the historical climate data collected at Chipley weather station. The input for the simulation model followed the current management practices of variety, fertilization and planting dates in the region. More specifically, a medium to full season Delta & Pine Land® variety (DP55), 110 kg/ha Nitrogen fertilization in two applications, and four planting dates, 16 Apr, 23 Apr, 1 May, and 8 May, were included in the yield simulation, which was further stochastically resampled to produce series of synthetically generated yields following the historical distributions (for more details see Cabrera et al., 2006).

Assume cotton would be harvested and sold in December. The December cotton futures contact was used to hedge the price risk. In addition, assume the farmer will settle the futures contract on the last trading date, i.e. seventeen days from the end of December. The historical settlement prices of the December futures contract on the last trading date from 1960 to 2003 were collected from the New York Board of Trade.

The statistics and the rank correlation coefficient Spearman's rho matrix of yields and futures price are summarized in Table 2-2, which shows that crop yields for different planting dates are highly correlated and the correlation of yields is decreasing when the corresponding two planting dates are getting farther. In addition, the negative correlation between yields and futures price is found in the El Niño and Neutral phases, but not in La Niña. We assumed the random yields and futures price follow the empirical distributions of yields and futures price.

Table 2-2. Marginal distributions and rank correlation coefficient matrix of yields of four planting dates and futures price for the three ENSO phases

ENSO	Variable	Statistics of Marginal Distribution		Rank Correlation Coefficient Matrix Spearman's rho				
		Mean	Standard Deviation	Yield on 4/16	Yield on 4/23	Yield on 5/1	Yield on 5/8	Futures Price
El Niño	Yield on 4/16 (lb)	815.0	71.7	1.00	0.93	0.75	0.74	-0.36
	Yield on 4/23 (lb)	804.6	79.4	0.93	1.00	0.63	0.57	-0.23
	Yield on 5/1 (lb)	795.4	99.8	0.75	0.63	1.00	0.75	-0.22
	Yield on 5/8 (lb)	793.7	79.1	0.74	0.57	0.75	1.00	-0.42
	Futures Price ($/lb)	0.5433	0.1984	-0.36	-0.23	-0.22	-0.42	1.00
Neutral	Yield on 4/16 (lb)	808.9	108.8	1.00	0.84	0.77	0.62	-0.16
	Yield on 4/23 (lb)	818.4	100.6	0.84	1.00	0.75	0.64	-0.28
	Yield on 5/1 (lb)	825.8	86.2	0.77	0.75	1.00	0.75	-0.01
	Yield on 5/8 (lb)	824.5	68.0	0.62	0.64	0.75	1.00	-0.19
	Futures Price ($/lb)	0.5699	0.1872	-0.16	-0.28	-0.01	-0.19	1.00
La Niña	Yield on 4/16 (lb)	799.1	99.8	1.00	0.97	0.67	0.60	0.13
	Yield on 4/23 (lb)	790.7	85.3	0.97	1.00	0.73	0.68	0.20
	Yield on 5/1 (lb)	793.9	90.6	0.67	0.73	1.00	0.97	-0.13
	Yield on 5/8 (lb)	809.3	94.1	0.60	0.68	0.97	1.00	-0.08
	Futures Price ($/lb)	0.4669	0.1851	0.13	0.20	-0.13	-0.08	1.00

We further estimated the local basis defined in Equation (2-15). The monthly historical data on average cotton prices received by Florida farmers from the USDA National Agricultural Statistical Service were collected (1979 to 2003) as the cotton local cash prices. By subtracting the futures price from the local cash price, we estimated the historical local basis. Using the Input Analyzer in the simulation software Arena, the best fitted distribution based on minimum square error method was a beta distribution with probability density function

$$-0.13+0.15\times \text{BETA}\,(2.76, 2.38).$$

We calibrated the Gaussian copula based on the sample rank correlation coefficient Spearman's rho matrix for the three ENSO phases. For each ENSO phase, we sampled 2,000 scenarios of correlated random yields and futures price based on the Gaussian copula and the empirical distributions of yields and futures price by Monte Carlo simulation. Furthermore, we simulated the basis and calculated the local cash price from the futures price and basis.

We assumed the futures commission and opportunity cost of margin to be $0.003 per pound, the production cost of cotton was $464 per acre, and the subsidy for cotton in Florida was $349 per acre. Finally, the parameters of crop insurance are listed in Table 2-3.

Table 2-3. Parameters of crop insurance (2004) used in the farm model analysis

Crop Insurance Parameters	Values
APH premium 65%~75%	$19.5/acre ~$38/acre
CRC premium 65%~85%	$24.8/acre~$116.9/acre
Established Price for APH	$0.61/lb
Average yield	814 lb/acre

Source: www.rma.usda.gov

2.4 Results and Discussion

This section reports the results of optimal planting schedule and hedging strategy with crop insurance and futures contract for the three predicted ENSO phases. In section 2.4.1 we assumed crop insurances were the only risk management tool for crop yield and price risk together with an unbiased futures market[3]. In section 2.4.2 we considered both insurance and futures contracts were available and assumed the future market being unbiased. In section 2.4.3 we investigated the optimal decision under biased futures markets.

2.4.1 Optimal Production with Crop Insurance Coverage

This section considers crop insurance as the only crop risk management tool. Since the indemnity of CRC depends on the futures price, we assume the futures market is unbiased, i.e., $F = Ef$ where F is the futures price in planting time and f is the random futures price in harvest time. Table 2-4 shows that the optimal insurance and production strategies for each ENSO phase with various 90%CVaR upper bounds ranged from -\$20,000 to -\$2,000 with increments of \$2000.

Remarks in Table 2-4 are summarized as follows. First, the ENSO phases affected the expected profit and the feasible region of the downside risk. The Neutral year has highest expected profit and lowest downside loss. In contrast, the La Niña year has lowest expected profit and highest downside loss. Second, the 65%CRC and 70%CRC crop insurance policies are desirable to the optimal hedging strategy in all ENSO phases when 90%CVaR constraint is lower than a specific value depending on the ENSO phase. In contrast, the APH insurance policies are not desirable for all ENSO phases and 90%CVaR upper bounds. Third, risk management can be conducted through changing the planting schedule. The last two rows associated with the Neutral phase shows that planting 100 acres in date 3 provides a 90% CVaR of -\$6,000 that can be reduced to -\$8,000 by changing the planting schedule to 85 acres in date 3 and 15 acres in date 4. Last, changing the insurance coverage together with the planting schedule may reduce the downside risk. In the La Niña phase, planting 100 acres in date 4 provides a 90%CVaR of -

[3] Although only crop insurance contracts were considered, the unbiased futures market assumption is need since the indemnity of CRC depends on the futures price.

$4,000 that can be reduced to -$10,000 by purchasing a 65%CRC insurance policy and shifting the planting date from date 4 to date 1.

Table 2-4. Optimal insurance and production strategies for each climate scenario under the 90% CVaR tolerance ranged from -$20,000 to -$2,000 with increment of $2000

ENSO Phases	90%CVaR Upper Bound	Optimal Expected Profit	Optimal Insurance Strategy	Optimal Planting Schedule			
				Date1	Date2	Date3	Date4
El Niño	<-18000	infeasible					
	-16000	28364	CRC70%	100	0	0	0
	-14000 to -4000	28577	CRC65%	100	0	0	0
	>-2000	28691	No	100	0	0	0
Neutral	<-20000	infeasible					
	-18000	31149	CRC70%	0	0	100	0
	-16000 to -10000	31240	CRC65%	0	0	100	0
	-8000	31779	No	0	0	85	15
	>-6000	31793	No	0	0	100	0
La Niña	<-12000	infeasible					
	-10000 to -6000	20813	CRC65%	100	0	0	0
	>-4000	21572	No	0	0	0	100

No = no insurance. Planting dates: Date1 = April 16, Date2 = April 23, Date3 = May 1, Date4 = May 8. Negative CVaR upper bounds represent profits.

2.4.2 Hedging with Crop Insurance and Unbiased Futures

In this section, we consider managing the yield and price risk with crop insurance policies and futures contracts when the futures market is unbiased. Since the crop yield is random, we define the hedge ratio of the futures contract as the hedge position in the futures contract divided by the expected production. The optimal solutions of the planting schedule, crop insurance coverage, and futures hedge ratio with various 90%CVaR upper bounds ranged from -$24,000 to $0 with increment of $4,000 for the three ENSO phases (Table 2-5).

From Table 2-5, when the futures market is unbiased, the futures contract dominating all crop insurance policies is the only desirable risk management tool. The optimal hedge ratio increases when the upper bound of 90%CVaR decreases. This means that to achieve lower downside risk, higher hedge ratio is needed. Next, we compare the hedge ratio in different ENSO phases with the same CVaR upper bound, the La Niña phase has the highest optimal hedge ratio

Table 2-5. Optimal solutions of planting schedule, crop insurance coverage, and futures hedge ratio with various 90% CVaR upper bounds ranged from -$24,000 to $0 with increment of $4,000 for the three ENSO phases

ENSO Phases	90%CVaR Upper Bound	Optimal Expected Profit	Optimal Insurance Strategy	Expected Production	Optimal Hedge Amount	Optimal Hedge Ratio	Optimal Planting Schedule			
							Date1	Date2	Date3	Date4
El Niño	-24000	infeasible								
	-20000	28520	No	81422	57092	0.70	100	0	0	0
	-16000	28563	No	81422	4482	0.52	100	0	0	0
	-12000	28605	No	81422	28895	0.35	100	0	0	0
	-8000	28646	No	81422	15238	0.19	100	0	0	0
	-4000	28686	No	81422	1726	0.02	100	0	0	0
	<0	28691	No	81422	0	0	100	0	0	0
Neutral	-24000	infeasible								
	-20000	31645	No	82482	49598	0.60	0	0	100	0
	-16000	31695	No	82482	32721	0.40	0	0	100	0
	-12000	31744	No	82482	16547	0.20	0	0	100	0
	-8000	31792	No	82482	529	0.01	0	0	100	0
	-4000	31793	No	82482	0	0	0	0	100	0
La Niña	-16000	infeasible								
	-12000	21438	No	80911	44812	0.55	0	0	0	100
	-8000	21516	No	80911	18705	0.23	0	0	0	100
	-4000	21572	No	80911	0	0	0	0	0	100

No = no insurance. Planting dates: Date1= April 16, Date2= April 23, Date3= May 1, Date4= May 8. Negative CVaR upper bounds represent profits.

and the Neutral phase has the lowest one. Similar to the result in Table 2-4, the Neutral phase has the highest expected profit and the lowest feasible downside loss. In contrast, the La Niña phase has the lowest expected profit and the highest feasible downside loss. Furthermore, we compare two risk management tools: insurance (in Table 2-4) and futures (in Table 2-5). The futures contract provides higher expected profit under the same CVaR upper bound, as well as a larger feasible region associated with the CVaR constraint. Finally, the optimal production strategy with futures hedge is to plant 100 acres in date 1 for the El Niño phase, in date 3 for the Neutral phase, and in date 4 for the La Niña phase.

2.4.3 Biased Futures Market

In the section 2.4.2 we assumed the futures market is unbiased. However, the futures prices observed from futures market in the planting time may be higher or lower than the expected futures price in the harvest time. This section examines the impact of biased futures prices on the optimal insurance and futures hedging decisions in the three ENSO phases. We first illustrate the optimal hedge strategies and the optimal planting schedule. Then, the performance of the optimal hedge and planting strategies is introduced by the efficient frontiers on the expected profit versus CVaR risk diagram.

Table 2-6. Optimal insurance policy and futures hedge ratio under biased futures prices

Bias		-10%		-5%		0%		5%		10%	
ENSO Phases	90% CVaR Upper Bound	Insurance	Hedge Ratio	Insurance	Hedge Ratio	Insurance	Hedge Ratio	Insurance	Hedge Ratio	Insurance	Hedge Ratio
	-28000									x	
	-26000							x		No	1.03
	-24000					x		No	1.00	No	1.24
	-22000			x		No	0.83	No	1.18	No	1.41
	-20000	x		65%CRC	0.48	No	0.70	No	1.33	No	1.57
	-18000	65%CRC	0.45	65%CRC	0.29	No	0.61	No	1.47	No	1.72
El Niño	-16000	70%CRC	0.20	65%CRC	0.15	No	0.52	No	1.60	No	1.87
	-14000	70%CRC	0.04	65%CRC	0.03	No	0.44	No	1.73	No	2.02
	-12000	65%CRC	-0.01	65%CRC	-0.07	No	0.35	No	1.86	No	2.16
	-10000	65%CRC	-0.10	65%CRC	-0.15	No	0.27	No	1.98	No	2.30
	-8000	65%CRC	-0.20	65%CRC	-0.24	No	0.19	No	2.11	No	2.44
	-6000	65%CRC	-0.29	65%CRC	-0.33	No	0.10	No	2.23	No	2.58
	-4000	65%CRC	-0.39	65%CRC	-0.41	No	0.02	No	2.35	No	2.73
	-2000	65%CRC	-0.48	65%CRC	-0.50	No	0.00	No	2.48	No	2.87
	0	65%CRC	-0.58	65%CRC	-0.58	No	0.00	No	2.60	No	3.01
	-30000									x	
	-28000							x		No	1.06
	-26000							No	0.99	No	1.27
	-24000					x		No	1.19	No	1.42
	-22000			x		No	0.73	No	1.33	No	1.56
	-20000	x		No	0.66	No	0.60	No	1.46	No	1.68
Neutral	-18000	70%CRC	0.28	70%CRC	0.12	No	0.50	No	1.57	No	1.80
	-16000	70%CRC	0.10	70%CRC	-0.02	No	0.40	No	1.68	No	1.92
	-14000	70%CRC	-0.04	70%CRC	-0.12	No	0.30	No	1.79	No	2.04
	-12000	70%CRC	-0.16	No	0.20	No	0.20	No	1.89	No	2.15
	-10000	70%CRC	-0.28	No	0.09	No	0.10	No	2.00	No	2.27
	-8000	70%CRC	-0.39	No	0.00	No	0.01	No	2.10	No	2.38
	-6000	70%CRC	-0.51	No	-0.11	No	0.00	No	2.20	No	2.49
	-4000	70%CRC	-0.62	No	-0.21	No	0.00	No	2.30	No	2.59
	-2000	70%CRC	-0.74	No	-0.31	No	0.00	No	2.39	No	2.70
	0	70%CRC	-0.86	No	-0.42	No	0.00	No	2.49	No	2.81
	-20000									x	
	-18000							x		No	1.06
	-16000					x		No	1.04	No	1.23
	-14000			x		No	0.75	No	1.20	No	1.37
	-12000	x		No	0.67	No	0.55	No	1.33	No	1.51
La Niña	-10000	No	0.56	No	0.46	No	0.39	No	1.45	No	1.64
	-8000	No	0.33	No	0.27	No	0.23	No	1.57	No	1.77
	-6000	No	0.11	No	0.09	No	0.08	No	1.68	No	1.90
	-4000	No	-0.10	No	-0.08	No	0.00	No	1.80	No	2.02
	-2000	No	-0.30	No	-0.25	No	0.00	No	1.91	No	2.15
	0	No	-0.49	No	-0.41	No	0.00	No	2.02	No	2.27

x = infeasible. No = No insurance. Negative CVaR upper bounds represent profits.

Table 2-6 shows the optimal insurance policy and futures hedge ratio associated with different 90% CVaR upper bounds under biased futures prices for the three ENSO phases. When the futures price is unbiased or positive biased, the futures contract is the only desirable instrument for crop risk management and all insurance policies are not needed in the optimal hedging strategy. On the other hand, when the futures price is negative biased, the optimal hedging strategy includes 65%CRC (or 70% CRC in some cases) insurance policies and futures contract in the El Niño phase for all feasible 90%CVaR upper bounds. In the Neutral phase, the optimal hedging strategy consists of the 70%CRC insurance policy and futures contract for all CVaR upper bounds with the deep negative biased (-10%) futures price and for the CVaR upper bounds between -18000 and -14000 with the negative biased (-5%) futures price. In addition, no insurance policy is desirable under the La Niña phase.

Mahul (2003) showed that the hedge ratio contains two parts: a pure hedge component and a speculative component. The pure hedge component refers to the hedge ratio associated with unbiased futures price. A positive biased futures price induces the farmer to select a long speculative position and a negative biased futures price implies a short speculative position. Therefore, the optimal futures hedge ratio under positive (negative) biased futures prices should be higher (lower) than that under the unbiased futures price. However, the optimal hedge ratios in Table 2-6 do not agree with the conclusion when the futures price is negative biased. We use the optimal hedge ratios in the La Niña phase to illustrate how optimal futures hedge ratios change corresponding to the bias of the futures price[4]. Figure 2-2 shows the bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase.

[4] Since the optimal hedging strategy in the La Niña phase contains only futures contracts, the optimal hedge ratios are compatible.

When the CVaR constraint is not strict (i.e. the upper bound of 90%CVaR equals zero) the optimal hedge ratio curve follows the pattern claimed in Mahul (2003). The hedge ratio increases (decreases) with the positive (negative) bias of futures price in a decreasing rate. However, when the CVaR constraint becomes stricter (i.e., the CVaR upper bound equals -$8,000), the optimal hedge ratio increases not only with the positive bias but also with the negative one. It is because the higher negative bias of futures price implies a heavier cost (loss) is involved in futures hedge. It makes the CVaR constraint become stricter such that a higher pure hedge component is required to satisfy the constraint. The net change of the optimal hedge ratio, including an increment in the pure hedge component and a decrement in the speculative component, depends on the loss distribution, the CVaR upper bound, and the bias of futures price.

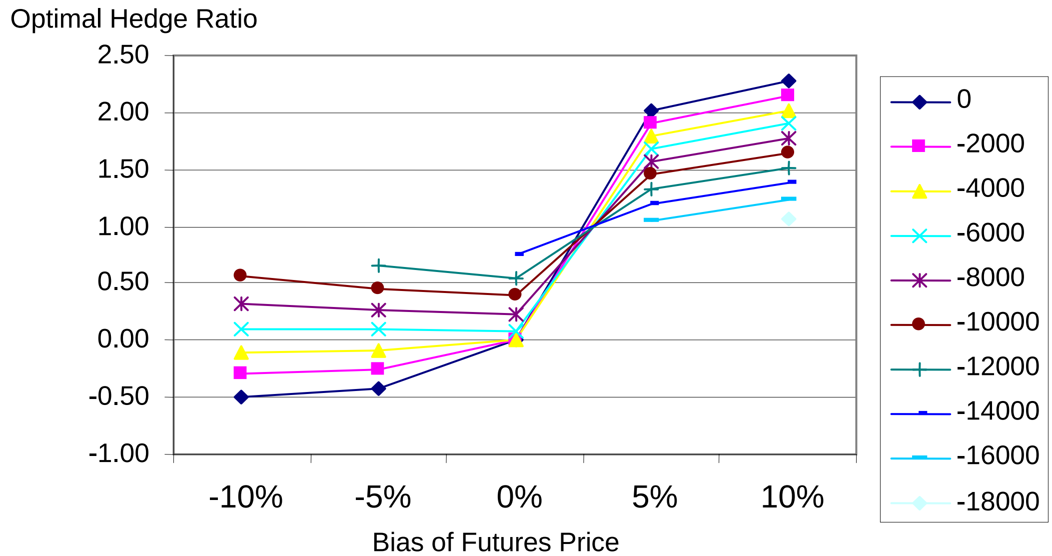

Figure 2-2. Bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase

Table 2-7 reported the optimal planting schedules for different biases of futures price in the three ENSO phases. For the El Niño phase, the optimal planting schedule (i.e., planting 100 acres in date 1) was not affected by the biases of futures price and 90%CVaR upper bounds.

Table 2-7. Optimal planting schedule for different biases of futures price in ENSO phases

	90%CVaR	El Niño				Neutral				La Niña			
Bias	upper bound	Date 1	Date 2	Date 3	Date 4	Date 1	Date 2	Date 3	Date 4	Date 1	Date 2	Date 3	Date 4
	-20000		x				x						
	-18000	100	0	0	0	0	0	97	3				
	-16000	100	0	0	0	0	0	100	0				
	-14000	100	0	0	0	0	0	100	0				
	-12000	100	0	0	0	0	0	100	0		x		
-10%	-10000	100	0	0	0	0	0	100	0	0	0	0	100
	-8000	100	0	0	0	0	0	100	0	0	0	0	100
	-6000	100	0	0	0	0	0	100	0	0	0	0	100
	-4000	100	0	0	0	0	0	100	0	0	0	0	100
	-2000	100	0	0	0	0	0	100	0	0	0	0	100
	0	100	0	0	0	0	0	100	0	0	0	0	100
	-22000		x				x						
	-20000	100	0	0	0	0	0	39	61				
	-18000	100	0	0	0	0	0	100	0				
	-16000	100	0	0	0	0	0	100	0				
	-14000	100	0	0	0	0	0	100	0		x		
	-12000	100	0	0	0	0	0	44	56	0	0	0	100
-5%	-10000	100	0	0	0	0	0	46	54	0	0	0	100
	-8000	100	0	0	0	0	0	85	15	0	0	0	100
	-6000	100	0	0	0	0	0	85	15	0	0	0	100
	-4000	100	0	0	0	0	0	92	8	0	0	0	100
	-2000	100	0	0	0	0	0	100	0	0	0	0	100
	0	100	0	0	0	0	0	93	7	0	0	0	100
	-24000		x				x						
	-22000	100	0	0	0	0	0	100	0				
	-20000	100	0	0	0	0	0	100	0				
	-18000	100	0	0	0	0	0	100	0				
	-16000	100	0	0	0	0	0	100	0		x		
	-14000	100	0	0	0	0	0	100	0	0	0	0	100
0%	-12000	100	0	0	0	0	0	100	0	0	0	0	100
	-10000	100	0	0	0	0	0	100	0	0	0	0	100
	-8000	100	0	0	0	0	0	100	0	0	0	0	100
	-6000	100	0	0	0	0	0	100	0	0	0	0	100
	-4000	100	0	0	0	0	0	100	0	0	0	0	100
	-2000	100	0	0	0	0	0	100	0	0	0	0	100
	0	100	0	0	0	0	0	100	0	0	0	0	100

x = infeasible. Date 1 = 16 Apr, Date 2 = 23 Apr, Date 3 = 1 May, Date 4 = 8 May. Negative CVaR upper bounds represent profits.

Table 2-7. Optimal planting schedule for different biases of futures price in ENSO phases (cont'd)

Bias	90%CVaR upper bound	El Niño Date 1	El Niño Date 2	El Niño Date 3	El Niño Date 4	Neutral Date 1	Neutral Date 2	Neutral Date 3	Neutral Date 4	La Niña Date 1	La Niña Date 2	La Niña Date 3	La Niña Date 4
	-28000						x						
	-26000		x			0	0	30	70				
	-24000	100	0	0	0	0	0	47	53				
	-22000	100	0	0	0	0	0	59	41				
	-20000	100	0	0	0	0	0	68	32				
	-18000	100	0	0	0	0	0	82	18		x		
5%	-16000	100	0	0	0	0	0	92	8	38	0	0	62
	-14000	100	0	0	0	0	0	99	1	33	0	0	67
	-12000	100	0	0	0	0	0	100	0	35	0	0	65
	-10000	100	0	0	0	0	0	100	0	35	0	0	65
	-8000	100	0	0	0	0	0	100	0	33	0	0	67
	-6000	100	0	0	0	0	0	100	0	34	0	0	66
	-4000	100	0	0	0	0	0	100	0	38	0	0	62
	-2000	100	0	0	0	0	0	100	0	39	0	0	61
	0	100	0	0	0	0	0	100	0	37	0	0	63
	-30000						x						
	-28000		x			0	0	31	69				
	-26000	100	0	0	0	0	0	45	55				
	-24000	100	0	0	0	0	0	54	46				
	-22000	100	0	0	0	0	0	67	33				
	-20000	100	0	0	0	0	0	75	25		x		
	-18000	100	0	0	0	0	0	87	13	33	17	0	50
10%	-16000	100	0	0	0	0	0	96	4	45	0	0	55
	-14000	100	0	0	0	0	0	100	0	47	0	0	53
	-12000	100	0	0	0	0	0	100	0	47	0	0	53
	-10000	100	0	0	0	0	0	100	0	50	0	0	50
	-8000	100	0	0	0	0	0	100	0	54	0	0	46
	-6000	100	0	0	0	0	0	100	0	56	0	0	44
	-4000	100	0	0	0	0	0	100	0	59	0	0	41
	-2000	100	0	0	0	0	0	100	0	61	0	0	39
	0	100	0	0	0	0	0	100	0	63	0	0	37

x = infeasible. Date 1 = 16 Apr, Date 2 = 23 Apr, Date 3 = 1 May, Date 4 = 8 May. Negative CVaR upper bounds represent profits.

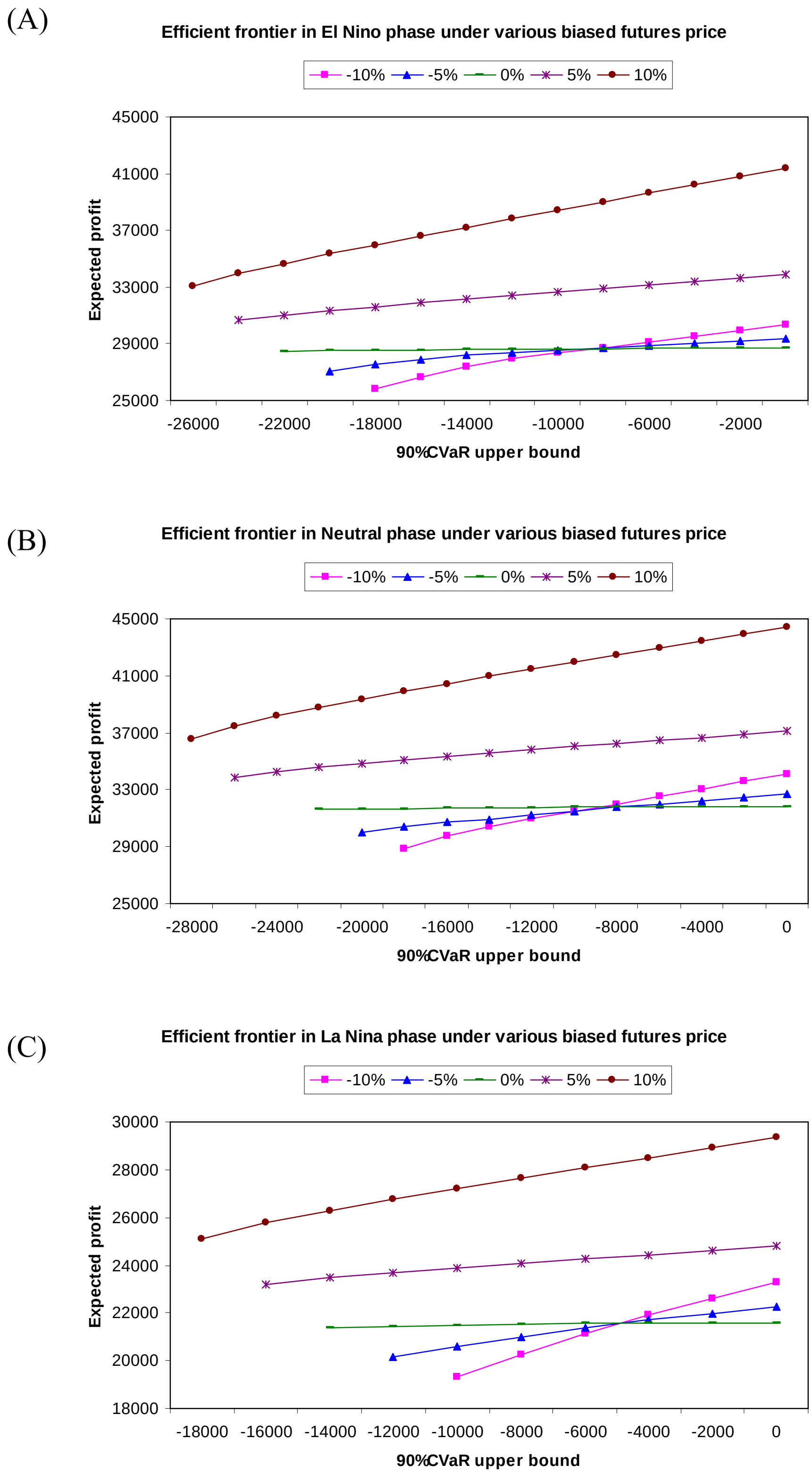

Figure 2-3. The efficient frontiers under various biased futures price. (A) El Niño year. (B) Neutral year. (C) La Niña year.

For the Neutral phase, however, the optimal planting strategy was to plant on date 3 and date 4 depending on the 90%CVaR upper bounds. More exactly, the date 3 is the optimal planting date for all risk tolerances under unbiased futures market. When futures prices are positive biased, the lower the 90%CVaR upper bounds (i.e., the stricter the CVaR constraint) was, the more planting acreages moved to date 4 from date 3. This result was based on the fact that there was no insurance coverage involved in the optimal hedging strategy. When the futures prices are negative biased, the optimal planting schedule had the same pattern as positive biased futures markets but was affected by the existence of insurance coverage in the optimal hedging strategy. For example, when the 90%CVaR upper bounds were within the range of -$8,000 and -$14,000 under a -5% biased futures price, the optimal planting acreage in date 4 went down to zero due to a 75%CRC in the optimal hedging strategy. For the La Niña phase, the optimal planting schedule was to plant 100 acres in date 4 when future prices were unbiased or negative biased. When future price was negative biased, the stricter CVaR constraint was, the more planting acreage shifted from date 4 to date 1. For deep negative biased futures price together with strict CVaR constraint (i.e., -10% biased futures price and -$18,000 90%CVaR upper bound), the optimal planting schedule included date 1, date 2, and date 4.

Figure 2-3 shows mean-CVaR efficient frontiers associated with various biased futures prices for three ENSO phases. With the efficient frontiers, the farmer may make the optimal decision based on his/her downside risk tolerance and trade off between expected profit and downside risk. The three graphs show that the Neutral phase has highest expected profit and lowest feasible CVaR upper bound. In contrast, the La Niña phase has the lowest expected profit and highest feasible CVaR upper bound. The pattern of the efficient frontiers in the three graphs is the same. The higher positive bias of futures price is, the higher expected profit would be provided. However, the higher negative bias of futures price provides a higher expected profit under a looser CVaR constraint and a lower expected profit under a stricter CVaR constraint.

2.5 Conclusion

This research proposed a mean-CVaR model for investigating the optimal crop planting schedule and hedging strategy when the crop insurance and futures contracts are available for hedging the yield and price risk. Due to the linear property of CVaR, the optimal planting and hedging problem could be formulated as a mixed 0-1 linear programming problem that could be efficiently solved by many commercial solvers such as CPLEX. The mean-CVaR model is powerful in the sense that the model inherits the advantage of the return versus risk framework (Markowitz, 1952) and further utilizes CVaR as a (downside) risk measure that can cope with general loss distributions. Compared to using utility functions for modeling risk aversion, the mean-CVaR model provides an intuitive way to define risk. In addition, a problem without nonlinear side constraints could be formulated linearly under the mean-CVaR framework, which could be solved more efficiently comparing to the nonlinear formulation from the utility function framework.

A case study was conducted using the data of a representative cotton producer in Jackson County in Florida to examine the optimal crop planting schedule and risk hedging strategy under the three ENSO phases. The eligible hedging instruments for cotton include futures contracts and three types of crop insurance policies: APH, CRC, and CAT. We first analyzed the best production and risk hedging problem with three types of insurance policies. The result showed that 65%CRC or 70%CRC would be the optimal insurance coverage when the CVaR constraint reaches a strict level. Furthermore, we examined the optimal hedging strategy when crop insurance policies and futures contracts are available. When futures price are unbiased or positive biased, the optimal hedging strategy only contains futures contracts and all crop insurance policies are not desirable. However, when the futures price is negative biased, the optimal hedging strategy depends on the ENSO phases. In the El Niño phase, the optimal hedging strategy consists of the 65%CRC (or 70%CRC for some CVaR upper bounds) and futures contracts for all CVaR upper bound values. In the Neutral phase, when futures price is deep negative biased (-10%), the optimal hedging strategy consists of the 70%CRC and futures contracts for all CVaR upper bound values. Under a -5% negative biased futures price, optimal hedging strategy contains the 70%CRC and futures contracts when the CVaR upper bound is within the range of -$18,000 and -$14,000. Otherwise, the optimal hedging strategy contains only futures contract. In the La Niña phase, the optimal hedging strategy contains only futures contract for all CVaR upper bound values and all biases of futures prices between -10% and 10%. The optimal futures hedge ratio increases with the increasing CVaR upper bound when the insurance strategy is unchanged. For a fixed CVaR upper bound, the optimal hedge ratio increases when the positive bias of futures price increases. However, when the futures price is negative biased, the optimal hedge ratio depends on the value of CVaR upper bound.

The case study provides some insight into how planting schedule incorporated with insurance and futures hedging may manipulate the downside risk of a loss distribution. In our model, we used a static futures hedging strategy that trades the hedge position in the planting time and keeps the position until the harvest time. A dynamic futures hedging strategy may be considered in the future research. The small sample size for the El Niño and La Niña phases may limit the case study results. In addition, we assumed the cost of futures contract as the commission plus an average interest foregone for margin deposit and the risk of daily settlement that may require a large amount of cash for margin account was not considered. It may reduce the value of futures hedging for risk-averse farmers.

CHAPTER 3
EFFCIENT EXECUTION IN THE SECONDARY MORTGAGE MARKET

3.1 Introduction

Mortgage banks (or lenders) originate mortgages in the primary market. Besides keeping the mortgages as a part of the portfolio, a lender may sell the mortgages to mortgage buyers (or conduits) or securitize the mortgages as mortgage-backed securities (MBSs) through MBS swap programs in the secondary market. In the United States, three government-sponsored enterprises (GSEs) (Fannie Mae, Freddie Mac, and Ginnie Mae) provide MBS swap programs in which mortgage bankers can deliver their mortgages into appropriate MBS pools in exchange for MBSs.

In practice, most mortgage bankers prefer to participate in the secondary market based on the following reasons. First, mortgage banks would get funds from secondary marketing and then use the funds to originate more mortgages in the primary market and earn more origination fees. Second, the value of a mortgage is risky and depends on several sources of uncertainties, i.e., default risk, interest rate risk, and prepayment risk. Mortgage bankers could reduce risks by selling or securitizing mortgages in the secondary market. More exactly, when mortgages are sold as a whole loan, all risks would be transferred to mortgage buyers. On the other hand, when mortgages are securitized as MBSs, the risky cash flows of mortgages are split into guarantee fees, servicing fees, and MBS coupon payments, which belong to MBS issuers, mortgage servicers, and MBS investors, respectively. In this case, mortgage bankers are exposed only to risk from retaining the servicing fee and other risky cash flows are transferred to different parties.

A significant task faced by mortgage bankers attempting to profit from the secondary market is efficient execution. The challenge of efficient execution is to sell or securitize a large number of heterogeneous mortgages in the secondary market in order to maximize expected revenue through complex secondary marketing functionality. In addition, to deal with the uncertain cash flows from the retained servicing fee, the balance between mean revenue and risk is also an important concern for mortgage bankers.

In this chapter, we develop a stochastic optimization model to perform an efficient execution that considers secondary marketing functionality, including loan-level efficient execution, guarantee fee buy-up or buy-down, servicing retain or release, and excess servicing fee. Further, we employ Conditional Value-at-Risk (CVaR), proposed by Rockafellar and Uryasev (2000), as a risk measure in the efficient execution model that maximizes expected revenue under a CVaR constraint. By solving the efficient execution problem under different risk tolerances specified by a CVaR constraint, an efficient frontier could be found.

A great deal of research has focused on mortgage valuation (Kau, Keenan, Muller, and Epperson (1992); Kau (1995); Hilliard, Kau and Slawson (1998); and Downing, Stanton, and Wallace (2005)), MBS valuation (Schwartz and Torous (1989); Stanton (1995); Sugimura (2004)), and mortgage servicing right valuation (Aldrich, Greenberg, and Payner (2001); Lin, Chu, and Prather (2006)). However, academic literature addressing topics of mortgage secondary marketing is scant. Hakim, Rashidian, and Rosenblatt (1999) addressed the issue of fallout risk, which is an upstream secondary marketing problem. To the best of our knowledge, we have not seen any literature focusing on efficient execution.

The organization of this chapter is as follows: Section 2 discusses mortgage securitization. We describe the relationship between MBS market participants and introduce the Fannie Mae MBS swap program. Section 3 presents our model development. Section 4 reports our results, and the final section presents our conclusions.

3.2 Mortgage Securitization

Mortgage bankers may sell mortgages to conduits at a price higher than the par value5 to earn revenue from the whole loan sales. However, for lenders who possess efficient execution knowledge, mortgage securitization through MBS swap programs of GSEs may bring them higher revenue than the whole loan sale strategy. In this research, we consider pass-through MBS swap programs provided by Fannie Mae (FNMA). To impose considerations of MBS swap programs of other GSEs is straightforward.

In this section, we describe the relationship between participants in the pass-through MBS market and detail the procedure of mortgage securitization through a MBS swap program. Participants in the MBS market can be categorized into five groups: borrowers, mortgage bankers, mortgage servicers, MBS issuers, and mortgage investors. The relationship between these five participants in the pass-through MBS market is shown in Figure 3-1.

In Figure 3-1, solid lines show cash flows between participants and dashed lines represent mortgage contracts and MBS instruments between them. Mortgage bankers originate mortgage loans by signing mortgage contracts with borrowers who commit to making monthly payments in a fixed interest rate known as the mortgage note rate. To securitize those mortgages, mortgage bankers deliver the mortgages into an MBS swap in exchange for MBSs. Further, mortgage bankers sell the MBSs to MBS investors and receive MBS prices in return.

MBS issuers provide MBS insurance to protect the MBS investors against default losses and charge a base guarantee fee. The base guarantee fee is a fixed percentage, known as the

[5] Mortgage bankers underwrite mortgages at a certain mortgage note rate. The par value is the value of the mortgage when the discount interest rate equals the mortgage note rate. In other words, the par value of a mortgage is its initial loan balance.

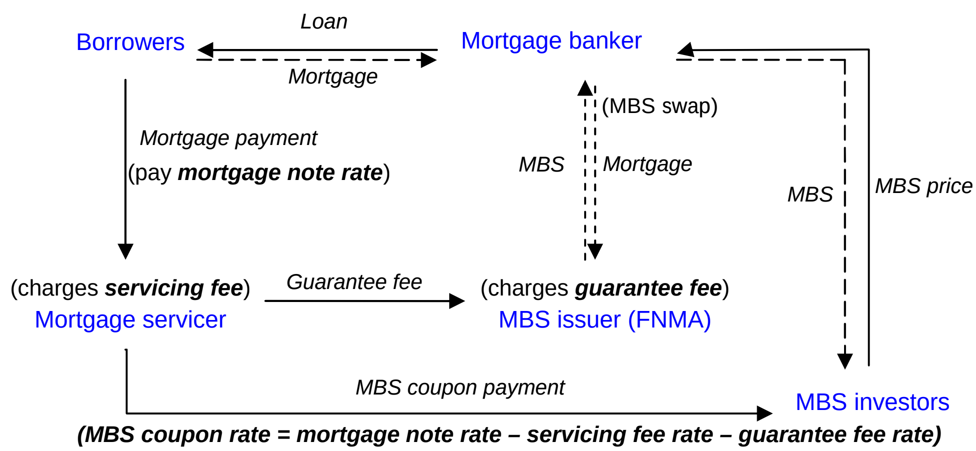

Figure 3-1. The relationship between participants in the pass-through MBS market. Mortgage bankers originate mortgage loans by signing mortgage contracts with borrowers who commit to making monthly payments with a fixed interest rate known as the mortgage note rate. To securitize those mortgages, mortgage bankers deliver the mortgages into an MBS swap in exchange for MBSs. Further, mortgage bankers sell the MBSs to MBS investors and receive MBS prices in return. The MBS issuer provides MBS insurance and charges a base guarantee fee. Mortgage servicers provide mortgage servicing and a base servicing fee is disbursed for the servicing. Both fees are a fixed percentage (servicing fee rate or guarantee fee rate) of the outstanding mortgage balance and decline over time as the mortgage balance amortizes. Deducting guarantee fees and servicing fees from mortgage payments, the remaining cash flows that pass-through to the MBS investors are known as MBS coupon payments with a rate of return equal to the mortgage note rate minus the servicing fee rate minus the guarantee fee rate.

guarantee fee rate, of the outstanding mortgage balance, and which declines over time as the mortgage balance amortizes.

Mortgage bankers negotiate the base guarantee fee rate with Fannie Mae and have the opportunity to “buy-down” or “buy-up” the guarantee fee. When lenders buy-down the guarantee fee, the customized guarantee fee rate is equal to the base guarantee fee rate minus the guarantee fee buy-down spread. Further, lenders have to make an upfront payment to Fannie Mae. On the other hand, the buy-up guarantee fee allows lenders to increase the guarantee fee rate from the base guarantee fee rate and receive an upfront payment from Fannie Mae. For example, if a lender wants to include a 7.875% mortgage with a 0.25% base guarantee fee and a 0.25% base servicing fee in a 7.5% pass-through MBS (Figure 3-2), the lender can buy-down the guarantee fee rate to 0.125% from 0.25% by paying Fannie Mae an upfront amount equal to the present value of the cash flows of the 0.125% difference and maintaining the 0.25% base servicing fee.

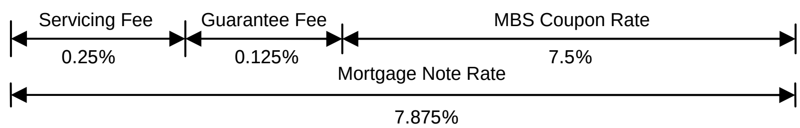

Figure 3-2. Guarantee fee buy-down. A lender may include a 7.875% mortgage in a 7.5% pass-through MBS by buying-down the guarantee fee to 0.125% from 0.25% and maintaining the 0.25% base servicing fee.

If a lender chooses to include an 8.125% mortgage in the 7.5% pass-through MBS (Figure 3-3), the lender can buy-up the guarantee fee by 0.125% in return for a present value of the cash flows of the 0.125% difference. The buy-down and buy-up guarantee fee features allow lenders to maximize the present worth of revenue.

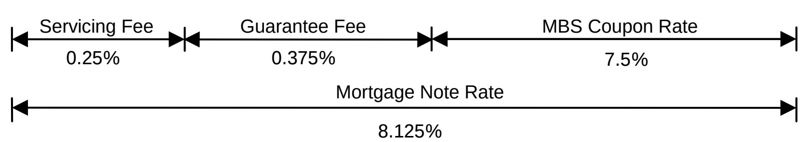

Figure 3-3. Guarantee fee buy-up. A lender may include an 8.125% mortgage in a 7.5% pass-through MBS by buying-up the guarantee fee to 0.375% from 0.25% and maintaining the 0.25% base servicing fee.

Mortgage servicers provide mortgage services, including collecting monthly payments from borrowers, sending payments and overdue notices, and maintaining the principal balance report, etc. A base servicing fee is disbursed for the servicing, which is a fixed percentage, known as the base servicing fee rate, of the outstanding mortgage balance, and which declines over time as the mortgage balance amortizes. Mortgage bankers have the servicing option to sell the mortgage servicing (bundled with the base servicing fee) to a mortgage servicer and receive an upfront payment from the servicer or retain the base servicing fee and provide the mortgage servicing.

Deducting guarantee fees and servicing fees from mortgage payments, the remaining cash flows that pass-through to the MBS investors are known as MBS coupon payments, which contain a rate of return known as the MBS coupon rate (or pass-through rate), equal to the mortgage note rate minus the servicing fee rate minus the guarantee fee rate.

Fannie Mae purchases and swaps more than 50 types of mortgages on the basis of standard terms. This research focuses on pass-through MBS swaps of 10-, 15-, 20-, and 30-year fixed-rate mortgages. Mortgages must be pooled separately by the time to maturity. For instance, 30-year fixed-rate mortgages are separated from 15-year fixed-rate ones. For each maturity, Fannie Mae provides different MBS pools characterized by MBS coupon rates that generally trade on the half percent (4.5%, 5.0%, 5.5%, etc.). Mortgage lenders have the option to deliver individual mortgages into one of these eligible MBS pools, which allows lenders to maximize revenue.

Further, the mortgage note rate must support the MBS coupon rate plus the servicing fee rate plus the guarantee fee rate. Therefore, when securitizing a mortgage as an MBS, mortgage bankers have to manipulate the servicing fee rate and guarantee fee rate so that Equation (3-1) is satisfied.

$$\begin{aligned} \textit{Mortgage Note Rate} = \textit{Servicing Fee Rate} + \textit{Guarantee Fee Rate} \\ + \textit{MBS Coupon Rate.} \end{aligned} \tag{3-1}$$

Mortgage bankers could retain an excess servicing fee from the mortgage payment, which, like the base servicing fee, is a fixed percentage, known as the excess servicing fee rate, of the outstanding mortgage balance and which declines over time as the mortgage balance amortizes. In Equation (3-2), the excess servicing fee rate is equal to the excess of the mortgage note rate over the sum of the MBS coupon rate, customized guarantee fee rate, and base servicing fee rate. In other words, the servicing fee rate in Equation (3-1) consists of the base servicing fee rate and the excess servicing fee rate.

$$\begin{aligned} \textit{Excess Servicing Fee Rate} = \textit{Mortgage Note Rate} - \textit{MBS Coupon Rate} \\ - \textit{Guarantee Fee Rate} - \textit{Base Servicing Fee Rate.} \end{aligned} \tag{3-2}$$

In the example shown in Figure 3-3, the excess servicing fee may be sold to Fannie Mae by buying-up the guarantee fee. Another option for mortgage bankers is to retain the excess servicing fee in their portfolio and to receive cash flows of the excess servicing fee during the life of the mortgage. The value of the excess servicing fee is equal to the present value of its cash flows. This value is stochastic since borrowers have the option to terminate mortgages before maturity and the interest rate used to discount the future cash flows is volatile. Therefore, efficient execution becomes a stochastic optimization problem. Similar to the guarantee fee buy-up and buy-down features, the excess servicing fee allows lenders to maximize the expected revenue.

3.3 Model

Efficient execution is a central problem of mortgage secondary marketing. Mortgage bankers originate mortgages in the primary market and execute the mortgages in the secondary market to maximize their revenue through different secondary marketing strategies. In the secondary market, each mortgage can be executed in two ways, either sold as a whole loan, or pooled into an MBS with a specific coupon rate. When mortgages are allocated into an MBS pool, we consider further the guarantee fee buy-up/buy-down option, the mortgage servicing retain/release option, and excess servicing fee to maximize the total revenue.

Based on secondary marketing strategies, mortgage bankers may retain the base servicing fee and the excess servicing fee when mortgages are securitized. The value of the retained servicing fee is random and affected by the uncertainty of interest rate term structure and prepayment. Therefore, a stochastic optimization model is developed to maximize expected revenue under a risk tolerance and an efficient frontier can be found by optimizing expected revenue under different risk tolerances specified by a risk measure.

3.3.1 Risk Measure

Since Markowitz (1952), variance (and covariance) has become the predominant risk measure in finance. However, the risk measure is suited only to the case of elliptic distributions, like normal or t-distributions with finite variances (Szegö (2002)). The other drawback of variance risk measure is that it measures both upside and downside risks. In practice, finance risk managers are concerned only with the downside risk in most cases.

A popular downside risk measure in economics and finance is Value-at-Risk (VaR) (Duffie and Pan (1997)), which measures α percentile of loss distribution. However, as was shown by Artzner et al. (1999), VaR is ill-behaved and non-convex for general distribution. The other disadvantage of VaR is that it only considers risk at α percentile of loss distribution and does not consider how much worse the α-tail (the worst 1-α percentage of scenarios) could be.

To address this issue, Rockafellar and Uryasev ((2000) and (2002)) proposed Conditional Value-at-Risk (CVaR), which is the mean value of α-tail of loss distribution. It has been shown that CVaR satisfies the axioms of coherent risk measures proposed by Artzner et al. (1999) and has desirable properties. Most importantly, Rockafellar and Uryasev (2000) showed that CVaR constraints in optimization problems can be expressed by a set of linear constraints and incorporated into problems of optimization.

This research uses CVaR as the measure of risk in developing the efficient execution model that maximizes expected revenue under a CVaR constraint. Thanks to CVaR, an efficient execution model could be formulated as a mixed 0-1 linear programming problem.

It is worth mentioning that the prices of MBSs, prices of whole loan sale mortgages, upfront payment of released servicing, and upfront payment of guarantee fee buy-up or buy-down are deterministic numbers that can be observed from the secondary market. However, the revenue from retained base servicing fees and excess servicing fees are equal to the present value of cash flows of the fees. Because of the randomness of interest rate term structure and prepayment, the revenue from the fees is varied with different scenarios. Lenders could simulate the scenarios based on their own interest rate model and prepayment model. This research assumes the scenarios are given input date.

3.3.2 Model Development

In this subsection, we present the stochastic optimization model. The objective of the model is to maximize the revenue from secondary marketing. Four sources of revenue are included in the model: revenue from MBSs or whole loan sale, revenue from the base servicing fee, revenue from the excess servicing fee, and revenue from the guarantee fee buy-up/buy-down.

(1) Revenue from MBSs or whole loan sale:

$$f_1(z_c^m, z_w^m) = \sum_{m=1}^{M} \left[L^m \times \sum_{c=1}^{C^m} \left(P_c^{t^m} \times z_c^m \right) + P_w^m \times z_w^m \right], \tag{3-3}$$

where

M = total number of mortgages,
m = index of mortgages (m = 1, 2, …, M),
L^m = loan amount of mortgage m,
C^m = number of possible MBS coupon rates of mortgage m,
c = index of MBS coupon rate,

$P_c^{t^m}$ = price of MBS with coupon rate index c and maturity of t^m,
t^m = maturity of mortgage m,
P_w^m = whole loan sale price of mortgage m,
$z_c^m = \begin{cases} 1, \text{if mortgage m is pooled into MBS with coupon rate index } c, \\ 0, \text{otherwise,} \end{cases}$
$z_w^m = \begin{cases} 1, \text{if mortgage } m \text{ is sold as a whole loan,} \\ 0, \text{otherwise.} \end{cases}$

Equation (3-4) enforces that each mortgage could be either sold as a whole loan or delivered into a specific MBS pool.

$$\sum_{c=1}^{C^m} z_c^m + z_w^m = 1 . \tag{3-4}$$

If mortgage m is securitized as an MBS with coupon rate index $\hat{c}$, then $z_{\hat{c}}^m = 1$, $z_c^m = 0$ for all $c \neq \hat{c}$, and $z_w^m = 0$, and the revenue from mortgage m equals $L^m \times P_{\hat{c}}^{t^m} \times z_{\hat{c}}^m$. On the other hand, if mortgage m is sold as a whole loan, then $z_c^m = 0$ for all c, and $z_w^m = 1$, and the revenue from mortgage m equals $P_w^m \times z_w^m$. The total revenue from the M mortgages is shown in Equation (3-3).

(2) Revenue from the base servicing fee of securitized mortgages:

$$f_2(z_{sbo}^m, z_{sbr}^m) = \sum_{m=1}^{M} \left(L^m \times B^m \times z_{sbo}^m \right) + \sum_{m=1}^{M} \left(\sum_{k=1}^{K} p_k \left(L^m \times R_{sb}^m \times K_{sr}^{mk} \right) - c_s^m \right) z_{sbr}^m , \tag{3-5}$$

where

$z_{sbr}^m = \begin{cases} 1, \text{if the servicing of mortgage } m \text{ is retained,} \\ 0, \text{otherwise,} \end{cases}$
$z_{sbo}^m = \begin{cases} 1, \text{if the servicing of mortgage } m \text{ is sold,} \\ 0, \text{otherwise,} \end{cases}$
B^m = base servicing value of mortgage m,
p_k = the probability of scenario k,
K = number of scenarios,
c_s^m = servicing cost of mortgage m,

K_{sr}^{mk} = retained servicing fee multiplier of mortgage m under scenario *k*.

The retained servicing multipliers K_{sr}^{mk} for scenario k could be generated by simulation. Mortgage bankers first simulate the random discounted cash flows of the retained servicing fee by using their own interest rate model and prepayment model. Then, a retained servicing multiplier K_{sr}^{mk} could be found associated with each scenario. In this research, we treat the retained servicing multiplier K_{sr}^{mk} as input data. Details of how to get the retained servicing multiplier K_{sr}^{mk} is beyond the scope of this research.

Equation (3-6) enforces that when a mortgage is securitized, the servicing of the mortgage can be either released or retained, and the revenue from mortgage servicing exists only if the mortgage is securitized as an MBS instead of being sold as a whole loan.

$$z_w^m + z_{sbo}^m + z_{sbr}^m = 1 . \tag{3-6}$$

More exactly, if mortgage m is sold as a whole loan, then $z_w^m = 1$, $z_{sbo}^m = z_{sbr}^m = 0$, and the revenue from the base servicing fee equals zero. On the other hand, if mortgage m is securitized as an MBS and the servicing of mortgage m is sold, then $z_{sbo}^m = 1, z_{sbr}^m = 0$, $z_w^m = 0$, and the upfront payment from the mortgage servicer equals $L^m \times B^m \times z_{sbo}^m$; otherwise, $z_{sbr}^m = 1$, $z_{sbo}^m = 0$, $z_w^m = 0$, and the expected revenue from the released servicing equals the expected revenue from base servicing fee $\sum_{k=1}^{K} p_k \left(L^m \times R_{sb}^m \times K_{sr}^{mk} \times z_{sbr}^m \right)$ minus servicing cost $c_s^m \times z_{sbr}^m$. The total revenue from the M mortgages is expressed in Equation (3-5).

(3) Revenue from the excess servicing fee of securitized mortgages:

$$f_3(r_{ser}^m) = \sum_{m=1}^{M} \sum_{k=1}^{K} p_k \left(L^m \times K_{sr}^{mk} \times r_{ser}^m \right), \tag{3-7}$$

where r_{ser}^m = retained excess servicing fee of mortgage *m*.

If mortgage m is securitized as an MBS, the mortgage generates expected revenue $\sum_{k=1}^{K} p_k \left(L^m \times K_{sr}^{mk} \times r_{ser}^m \right)$ from retaining the excess servicing fee. The total revenue from the M mortgages is shown in Equation (3-7).

(4) Revenue from the guarantee fee buy-up/buy-down of securitized mortgages:

$$f_4(r_{gu}^m, r_{gd}^m) = \sum_{m=1}^{M} \left(K_u^m \times L^m \times r_{gu}^m \right) - \sum_{m=1}^{M} \left(K_d^m \times L^m \times r_{gd}^m \right), \tag{3-8}$$

where

r_{gu}^m = guarantee fee buy-up spread of mortgage *m*,
r_{gd}^m = guarantee fee buy-down spread of mortgage *m*,
K_u^m = guarantee fee buy-up multiplier of mortgage *m*,
K_d^m = guarantee fee buy-down multiplier of mortgage *m*.

Guarantee fee buy-up and buy-down multipliers (or ratios) K_u^m and K_d^m, announced by Fannie Mae, are used to calculate the upfront payment of a guarantee fee buy-up and buy-down. Lenders buy-up the guarantee fee of mortgage m to receive an upfront $K_u^m \times L^m \times r_{gu}^m$ from Fannie Mae. On the other hand, they can buy-down the guarantee fee of mortgage m and make an upfront payment $K_d^m \times L^m \times r_{gd}^m$ to Fannie Mae. The total revenue from the *M* mortgages is shown in Equation (3-8).

Guarantee fee buy-up/buy-down and retaining the excess servicing fee are considered only when mortgage m is securitized. Equation (3-9) enforces r_{ser}^m, r_{gu}^m, and r_{gd}^m to be zero when mortgage m is sold as a whole loan.

$$z_w^m + r_{gu}^m + r_{gd}^m + r_{ser}^m \leq 1 \tag{3-9}$$

From equation (3-1), when mortgages are securitized as MBSs, the mortgage note rate has to support the MBS coupon rate, servicing fee rate, and guarantee fee rate. Equation (3-10) places a mathematic expression in the restriction.

$$\sum_{c=1}^{C^m} R_c z_c^m + r_{gu}^m - r_{gd}^m + r_{ser}^m \le R_n^m - R_{sb}^m - R_{gb}^m \quad (3\text{-}10)$$

where

R_c = MBS coupon rate related to index *c*,
R_n^m = note rate of mortgage *m*,
R_{sb}^m = base servicing fee of mortgage *m*,
R_{gb}^m = base guarantee fee of mortgage *m*,

Next, we introduce the CVaR constraint

$$CVaR_\alpha(L) \le U \quad (3\text{-}11)$$

where L is the loss function, α is the percentile of CVaR, and U is the upper bound of CVaR losses. Equation (3-11) restricts the average of α-tail of loss distribution to be less than or equal to U. In other words, the average losses of the worst 1-α percentage of scenarios should not exceed U. It is worth mentioning that CVaR is defined on a loss distribution. Therefore, we should treat revenue as negative losses when we use CVaR constraint in maximum revenue problem.

Rockafellar & Uryasev (2000) proposed that CVaR constraints in optimization problems can be expressed by a set of linear constraints.

$$\zeta + (1-\alpha)^{-1} \sum_{k=1}^{K} p_k z_k \le U \quad (3\text{-}12)$$

$$z_k \ge L_k - \zeta \quad \forall k = 1,\dots,K \quad (3\text{-}13)$$

$$z_k \ge 0 \qquad \forall k = 1,\dots,K \quad (3\text{-}14)$$

$$\begin{aligned} L_k = &-\sum_{m=1}^{M}\left[L^m \times \sum_{c=1}^{C^m}\left(P_c^m \times z_c^m\right) + P_{whole}^m \times z_w^m\right] - \sum_{m=1}^{M}\left(L^m \times B^m \times z_{sbo}^m\right) - \sum_{m=1}^{M}\left(L^m \times R_{sb}^m \times K_{sr}^{mk} \times z_{sbr}^m\right) \\ &-\sum_{m=1}^{M}\left(L^m \times K_{sr}^{mk} \times r_{ser}^m\right) - \sum_{m=1}^{M}\left(K_u^m \times L^m \times r_{gu}^m\right) + \sum_{m=1}^{M}\left(K_d^m \times L^m \times r_{gd}^m\right). \end{aligned} \quad (3\text{-}15)$$

where L_k is loss value in scenario k, and ζ and z_k are real variables. Users of the model could specify their risk preference by selecting the value of α and *U*.

More constraints could be considered in the model based on the mortgage banker's preference. For instance, mortgage bankers may want to control the retained excess servicing fee based on the future capital demand or risk consideration. The excess servicing fee could be limited by an upper bound in three levels, including aggregate level, group level, and loan level. Constraint (3-16) limits average excess servicing fee across all mortgages by an upper bound U_{se}^{a}.

$$\sum_{m=1}^{M} L^m \times r_{se}^m \le U_{se}^a \left(\sum_{m=1}^{M} L^m (1 - z_w^m) \right). \tag{3-16}$$

We further categorized mortgages into groups according to the year to maturity. Constraint (3-17) limits the group-level average excess servicing fee by an upper bound U_{se}^{j} for each group *j*.

$$\sum_{m \in j} L^m \times r_{se}^m \le U_{se}^j \left(\sum_{m \in j} L^m (1 - z_w^m) \right). \tag{3-17}$$

In addition, constraint (3-18) restrict the loan level excess servicing fee to an upper bound U_{se}^{m}.

$$0 \le r_{ser}^m \le U_{se}^m. \tag{3-18}$$

Furthermore, constraints (3-19) and (3-20) impose an upper bound U_{gu}^{m} and U_{gd}^{m} in the guarantee fee buy-up and buy-down spreads, respectively.

$$0 \le r_{gu}^m \le U_{gu}^m \tag{3-19}$$

$$0 \le r_{gd}^m \le U_{gd}^m \tag{3-20}$$

Those upper bounds are determined by the restrictions of an MBS swap program or sometimes by the decision of mortgage bankers. For example, the maximum guarantee fee buy-down spread accepted by Fannie Mae is the base guarantee fee rate.

Finally, we impose non-negativity constraints

$$r_{ser}^{m}, r_{gu}^{m}, r_{gd}^{m} \geq 0, \tag{3-21}$$

and binary constraints

$$z_{c}^{m}, z_{whole}^{m}, z_{sbr}^{m}, z_{sbo}^{m} \in \{0,1\} \qquad \forall m = 1, 2, ..., C^{m}. \tag{3-22}$$

The notations and model formulation are summarized in the Appendix.

3.4 Case Study

In this section, a case study is conducted. First, we present the data set of mortgages, MBSs, base servicing fee and guarantee fee multipliers, and scenarios of retained servicing fee multiplier. Next, we introduce the solver that is used to solve the mixed 0-1 linear programming problem. Then, we show the results including efficient frontier and sensitivity analysis.

3.4.1 Input Data

In the case study, we consider executing 1,000 fixed-rate mortgages in the secondary market. For each mortgage, the data includes years to maturity (YTM), loan amount, note rate, and guarantee fee. Table 3-1 summarizes the data on mortgages. Mortgages are categorized into four groups according to YTM. For each group, the table shows number of mortgages, and the minimum, mean, and maximum value of loan amount, note rate, and guarantee fee.

MBS pools are characterized by the MBS coupon rate and YTM. In this case study, we consider 13 possible MBS coupon rates from 3.5% to 9.5%, increasing in increments of 0.5%, and four different YTM: 10, 15, 20, and 30 years. There are a total of 50 MBS pools since the MBS pool with a 3.5% coupon rate is not available for 20-year and 30-year mortgages. Table 3-2 shows the prices of MBSs for different MBS pools.

This case study assumes the base servicing rate is 25 bp (1bp = 0.01%) for all mortgages. The base servicing value is 1.09 for mortgages with maturity of 10, 15, and 20 years and 1.29 for mortgages with maturity of 30 years. In addition, the servicing cost are assumed to be zero. The guarantee fee buy-up and buy-down multipliers are summarized in Table 3-3, which shows that guarantee fee buy-up and buy-down multipliers depend on mortgage note rate and maturity. A mortgage with a higher note rate and longer maturity has larger multipliers. In addition, buy-down multipliers are larger than the buy-up multipliers under the same note rate and maturity.

In the case study, we consider 20 scenarios that are uniformly distributed across the range between 0 and $2K_{sr}^{m}$, where K_{sr}^{m} is the expected retained servicing fee multiplier for mortgage *m* summarized in Table 3-3. Equation (3-23) defines the probability mass function for the variable of retained servicing fee multiplier.

$$f_X(x) = \begin{cases} 0.05, x \in \{S : S = (.05 + .1z)K_{sr}^{m}, 0 \le z \le 19, z \in I\}, \\ 0, \text{ otherwise.} \end{cases} \tag{3-23}$$

From Table 3-3, the expected retained servicing fee multiplier for mortgage *m* depends on note rate and maturity. A mortgage with a larger note rate and longer maturity has a higher expected retained servicing fee multiplier.

The large-scale mixed 0-1 linear programming problem was solved by CPLEX-90 on an Intel Pentium 4, 2.8GHz PC. The running times for solving the instances of the efficient execution problem are approximately one minute with a solution gap[6] of less than 0.01%.

[6] Solution gap defines a relative tolerance on the gap between the best integer objective and the object of the best node remaining. When the value *|best node-best integer |/(1e-10 +|best integer|)* falls below this value, the mixed integer programming (MIP) optimization is stopped.

3.4.2 Result

The efficient execution model solved the optimal execution solution under different upper bounds of CVaR losses across the range from -\$193,550,000 to -\$193,200,000, increasing in increments of \$50,000, under a fixed α value to get an efficient frontier in the expected revenue versus CVaR risk diagram.

The procedure was repeated for α values of 0.5, 0.75, 0.9, and 0.95 to get efficient frontiers under different risk preferences associated with α values. The efficient frontiers are shown in Figure 3-4, and the solutions of efficient execution under different α and *U* values are listed in Table 3-4.

Table 3-1. Summary of data on mortgages

		Loan Amount (\$)				Note Rate (%)			Guarantee Fee (%)		
YTM	# of Mortgages	Min	Mean	Max	Sum	Min	Mean	Max	Min	Mean	Max
10	13	\$539,323	\$164,693	\$330,090	\$2,141,009	4.38	5.00	5.75	0.125	0.146	0.40
15	148	\$34,130	\$172,501	\$459,000	\$25,530,131	4.00	5.29	8.00	0.125	0.187	0.80
20	11	\$38,920	\$137,024	\$291,320	\$1,507,260	5.13	5.97	7.25	0.125	0.212	0.80
30	828	\$24,000	\$194,131	\$499,300	\$160,740,083	4.75	5.92	7.875	0.125	0.212	1.05

Table 3-2. Summary of data on MBS prices of MBS pools

	MBS coupon rate (%)												
YTM	9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5
10	107.69	107.69	107.69	107.66	107.31	106.94	105.84	104.94	103.69	102.45	101.13	99.703	93.381
15	107.69	107.69	107.69	107.66	107.31	106.97	105.84	104.63	103.19	101.5	99.688	97.531	93.381
20	107.73	107.73	107.72	107.63	106.69	105.78	104.69	103.64	101.78	99.719	97.047	90.263	N/A
30	107.73	107.73	107.72	107.63	106.69	105.78	104.5	103.13	100.91	98.469	95.313	90.263	N/A

Table 3-3. Guarantee fee buy-up and buy-down and expected retained servicing multipliers

Note	30-YR		20-YR		15-YR		10-YR		30-YR	10-, 15-, 20- YR
Rate	Buy-up	Buy-down	Buy-up	Buy-down	Buy-up	Buy-down	Buy-up	Buy-down	Expected Retained Servicing	Expected Retained Servicing
4	5.65	7.6	4.80	6.52	3.60	5.55	3.42	5.27	5.75	4.00
5	4.95	6.9	4.20	5.93	2.95	4.90	2.80	4.65	5.74	3.60
6	3.15	5.0	2.67	4.41	1.50	3.29	1.38	3.18	4.19	2.76
7	1.65	3.3	1.40	3.00	0.95	2.75	0.90	2.70	2.22	2.00
8	0.95	2.5	0.75	2.35	0.55	2.35	0.50	2.30	1.60	0.62

Figure 3-4 shows the trade-off between CVaR (α-tail risk) and expected revenue. For a fixed α value, when the upper bound of CVaR increases, the optimal expected revenue increases in a decreasing rate. On the other hand, for a fixed upper bound of CVaR, a high α value implies high risk aversion. Therefore, the associated optimal expected revenue becomes lower.

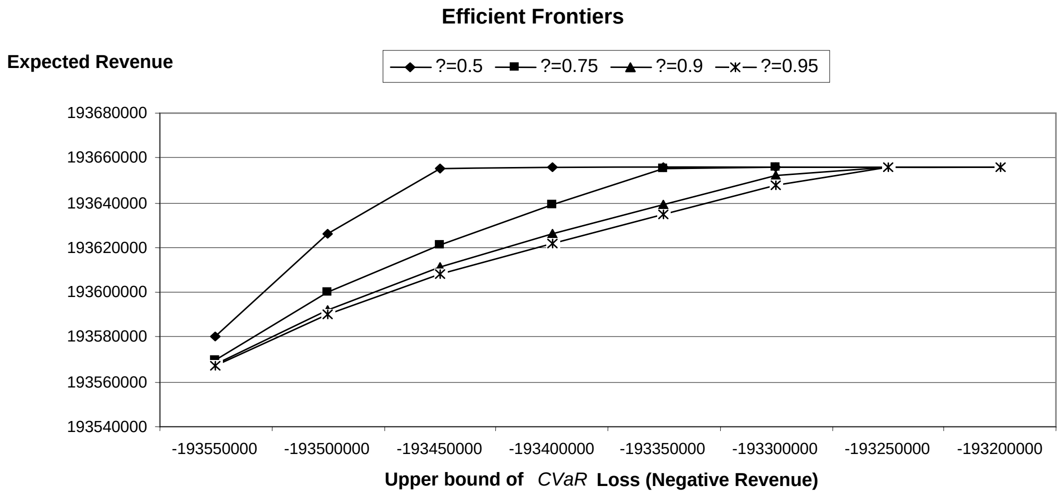

Figure 3-4: Efficient Frontiers. Plot of maximum expected revenue associated with different upper bound of CVaR losses across the range from -$193,550,000 to -$193,200,000 in increments of $50,000 under a fixed α value in the expected revenue versus CVaR risk diagram to get an efficient frontier. Repeat the procedure for different α values of 0.5, 0.75, 0.9, and 0.95 to get efficient frontiers under different risk preferences associated with α values.

Table 3-4 summaries the solution of efficient execution under different risk preferences specified by α and *U*, which includes the number of mortgages sold as a whole loan, number of mortgages securitized as MBSs with a specific coupon rate ranging from 3.5% to 9.5% that increases in 0.5% increments, the number of retained mortgage servicing and released mortgage servicing, the sum of guarantee fee buy-up and buy-down amount, and the sum of excess servicing fee amount.

3.4.4 Sensitivity Analysis

A sensitivity analysis was conducted in servicing fee multipliers, mortgage prices, and MBS prices. The sensitivity analysis is performed under a CVaR constraint with α =75% and U = -$193,400,000.

Table 3-5 shows that when all servicing fee multipliers increase by a fixed percentage, the number of mortgages sold as a whole loan decreases since lenders could get higher revenue from securitization due to the increasing servicing fee. In addition, the number of retained servicing and the amount of excess servicing fee increase due to the increasing retained servicing fee multipliers, and the number of released servicing decreases because the base servicing values, i.e. the upfront payments of released servicing, do not increase associated with retained servicing multipliers. An interesting result is that the increasing servicing fee multipliers increase (decrease) the guarantee fee buy-down (buy-up) amount, and the number of mortgages pooled into low coupon rate MBSs (4.5% and 4%) increases and the number of mortgages pooled into high coupon rate MBSs (7%, 6.5%, and 6%) decreases.

Table 3-6 shows that when all mortgage prices increase by a fixed percentage, the number of mortgages sold as a whole loan increases so that lenders can take advantage of high mortgage price. Since the number of whole loan sale mortgages increases which implies that the number of securitized mortgages decreases, the sum of buy-up and buy-down guarantee fee and excess servicing fee slightly decrease. Furthermore, the number of released servicing decreases since the number of securitized mortgage decreases.

Table 3-7 shows that when MBS price increases, the number of whole loan sale decreases, the number of securitized mortgages increases in the lower MBS coupon rate pools (4%, 4.5%, and 5%), and the number of retained servicing slightly increases.

3.5 Conclusion

This research proposed a stochastic optimization model to perform the efficient execution analysis. The model considers secondary marketing functionalities, including the loan-level execution for an MBS/whole loan, guarantee fee buy-up/buy-down, servicing retain/release, and excess servicing fee. Since secondary marketing involves random cash flows, lenders must balance between expected revenue and risk. We presented the advantages of CVaR risk measure and employed it in our model that maximizes expected revenue under a CVaR constraint. By solving the efficient execution problem under different risk tolerances, efficient frontiers could be found. We conducted a sensitivity analysis in parameters of expected retained servicing fee multipliers, mortgage prices, and MBS prices. The model is formulated as a mixed 0-1 linear programming problem. The case study shows that realistic instances of the efficient execution problem can be solved in an acceptable time (approximately one minute) by using CPLEX-90 solver on a PC.

Table 3-4. Summary of efficient execution solution under different risk preferences

α	Upper Bound of CVaR Losses U	Total Revenue	# of Whole Loan Sale	# of mortgages pooled into MBS (with coupon rate %)													# of Released Servicing	# of Retained Servicing	Sum of buy-up amount (%)	Sum of buy-down amount (%)	Sum of Excess servicing fee amount (%)
				9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5					
0.5	-193550000	195380000	155	0	0	0	0	0	5	30	183	429	134	59	5	0	845	0	40.41	63.76	11.29
	-193500000	193626000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	16.72	63.64	36.85
	-193450000	193655000	155	0	0	0	0	0	5	21	192	428	134	54	11	0	845	0	0.077	60.87	57.22
	-193400000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193350000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193300000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193250000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193200000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
0.75	-193550000	193570000	155	0	0	0	0	0	5	30	183	429	134	59	5	0	845	0	45.10	63.76	6.60
	-193500000	193600000	155	0	0	0	0	0	5	30	183	428	135	59	5	0	845	0	28.94	63.64	23.13
	-193450000	193621000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	20.04	63.64	33.53
	-193400000	193639000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	7.67	63.64	45.89
	-193350000	193655000	155	0	0	0	0	0	5	21	192	428	134	55	10	0	845	0	0.69	60.87	56.11
	-193300000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193250000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
	-193200000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73

Table 3-4. Summary of efficient execution solution under different risk preferences (cont'd.)

α	Upper Bound of CVaR Losses *U*	Total Revenue	# of Whole Loan Sale	# of mortgages pooled into MBS (with coupon rate %) 9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5	# of Released Servicing	# of Retained Servicing	Sum of buy-up amount (%)	Sum of buy-down amount (%)	Sum of Excess servicing fee amount (%)
0.9	-193550000	193568000	155	0	0	0	0	0	5	30	183	429	134	59	5	0	845	0	46.12	63.76	5.57
	-193500000	193592000	155	0	0	0	0	0	5	30	183	428	135	59	5	0	845	0	33.06	63.64	19.01
	-193450000	193611000	155	0	0	0	0	0	5	30	183	428	135	59	5	0	845	0	24.66	63.64	27.41
	-193400000	193626000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	16.99	63.64	36.58
	-193350000	193639000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	9.40	63.64	44.17
	-193300000	193652000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	1.04	63.64	52.53
	-193250000	193656000	155	0	0	0	0	0	5	20	193	428	133	54	12	0	845	0	0	60.57	58.50
	-193200000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73
0.95	-193550000	193567000	155	0	0	0	0	0	5	30	183	429	134	59	5	0	845	0	46.46	63.76	5.23
	-193500000	193590000	155	0	0	0	0	0	5	30	183	428	135	59	5	0	845	0	34.57	63.64	17.50
	-193450000	193608000	155	0	0	0	0	0	5	30	183	428	135	59	5	0	845	0	25.96	63.64	26.11
	-193400000	193622000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	20.22	63.64	33.35
	-193350000	193635000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	12.18	63.64	41.39
	-193300000	193648000	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	5.16	63.64	48.41
	-193250000	193656000	155	0	0	0	0	0	5	20	193	428	133	54	12	0	845	0	0	60.57	58.50
	-193200000	193656000	155	0	0	0	0	0	5	14	199	428	116	70	13	0	845	0	0	56.80	66.73

Table 3-5 Sensitivity analysis in servicing fee multiplier

Servicing Fee Multipliers Increment	# of Whole Loan Sale	# of mortgages pooled into MBS (with coupon rate %)													# of Released Servicing	# of Retained Servicing	Sum of buy-up amount (%)	Sum of buy-down amount (%)	Sum of Excess servicing fee amount (%)
		9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5					
0%	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	7.6741	63.6374	45.8949
10%	155	0	0	0	0	0	4	15	197	321	221	71	16	0	768	77	0	38.4944	107.926
20%	114	0	0	0	0	0	4	8	157	64	496	114	43	0	661	225	0	74.0746	371.006
30%	78	0	0	0	0	0	3	3	126	48	505	149	88	0	260	662	0	137.755	537.436
40%	74	0	0	0	0	0	3	2	127	7	495	174	118	0	207	719	0	165.085	626.538
50%	73	0	0	0	0	0	3	0	84	45	235	430	130	0	156	771	0	175.951	804.253

Table 3-6. Sensitivity analysis in mortgage price

Mortgage Price Increment	# of Whole Loan Sale	# of mortgages pooled into MBS (with coupon rate %)													# of Released Servicing	# of Retained Servicing	Sum of buy-up amount (%)	Sum of buy-down amount (%)	Sum of Excess servicing fee amount (%)
		9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5					
0%	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	7.6741	63.6374	45.8948
0.5%	196	0	0	0	0	0	5	14	199	426	82	65	13	0	804	0	0	55.144	62.6578
1%	277	0	0	0	0	0	5	14	198	406	36	52	12	0	723	0	0	48.773	51.4078
1.5%	313	0	0	0	0	0	5	14	197	388	36	35	12	0	687	0	0	45.1942	49.2382
2%	429	0	0	0	0	0	5	14	192	301	32	17	10	0	571	0	0	30.587	44.2382

Table 3-7. Sensitivity analysis in MBS price

MBS Price Increment	# of Whole Loan Sale	# of mortgages pooled into MBS (with coupon rate %)													# of Released Servicing	# of Retained Servicing	Sum of buy-up amount (%)	Sum of buy-down amount (%)	Sum of Excess servicing fee amount (%)
		9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.5					
0%	155	0	0	0	0	0	5	30	183	428	135	56	8	0	845	0	7.6746	63.6374	45.8949
1%	70	0	0	0	0	0	5	17	196	428	206	54	24	0	858	72	0.25	63.7476	62.0756
2%	43	0	0	0	0	0	5	20	193	428	210	74	27	0	864	93	0.25	65.3748	66.4206
3%	29	0	0	0	0	0	5	20	193	427	212	86	28	0	866	105	0.25	65.523	68.1438
4%	10	0	0	0	0	0	5	20	193	428	212	104	28	0	866	124	0.25	66.3766	67.9206
5%	1	0	0	0	0	0	5	21	192	428	212	113	27	1	867	132	0.25	67.6766	67.4706

CHAPTER 4
MORTGAGE PIPELINE RISK MANAGEMENT

4.1 Introduction

Mortgage lenders commit to a mortgage interest rate while the loan is in process, typically for a period of 30-60 days. If the interest rate rises before the loan goes to closing, the value of the loans will decline and the lender will sell the loans at a lower price. The risk that mortgages in process will fall in value prior to their sale is known as interest rate risk. In addition, the profit of mortgage origination is affected by fallout. The loan borrowers have the right to walk away at any time before closing. This right is actually a put option for the borrowers of the loan commitments. As interest rates fall, fallout rises because borrowers who have locked in a mortgage rate are more likely to find better rates with another lender. Conversely, as rates rise the percentage of loans that close increases. Fallout affects the required size of the hedging instrument because it affects the size of the pipeline position to be hedged. At lower rates, fewer rate loans will close and a smaller position in the hedging instrument is needed. Those risks in the mortgage underwriting process are known as the mortgage pipeline risks. Lenders often hedge this exposure by selling forward their expected closing volume or by shorting U.S. Treasury notes or futures contracts. In addition, options on U.S. Treasury note futures are used to hedge against the risk of fallout (Cusatis and Thomas, 2005).

Mortgage lenders are concerned with two types of risk: volatility and extreme loss. The volatility measures the uncertainty of the value of loan commitments. High volatility of the exposure means the mortgage institution has high uncertainty in the value of locked loans. The extreme loss measures the worse case scenario. The downside risk measure is used to decide the reservation of economic capital prepared for the extreme loss. To reduce the volatility and downside risk of locked loan values, mortgage pipeline managers hedge the exposure by purchasing financial instruments whose values are sensitive to mortgage rates. A direct way is to sell the MBS forward contracts. However, MBS forward contracts are traded over the counter. In other words, the contract is not standardized and the lack of liquidity may induce high transaction costs. An alternative approach is to sell an equivalent amount of Treasury futures. As rates go up, the value of the short position will rise, offsetting the losses on the loan commitments in the pipeline. The fallout is also a critical source of risk in pipeline management. Mortgage loans provide a put option to borrowers allowing them to not close the loan commitment. This embedded put option reduces the number of loans that go to closing as rates go down and vise versa. In other words, the fallout effect varies the loan amount that need to be hedged. This phenomenon is called "negative convexity" in the fixed income market. Figure 4.1 shows the negative convexity in the price-rate diagram. Curve A shows the convexity of fixed income security value with respect to interest rate. For locked mortgages in pipeline, when rates drop below the locked rate, the value of loans in pipeline is increasing in a decreasing rate because of fallout. Therefore, the value of locked mortgages with respect to rates has a negative convexity property shown in Curve B.

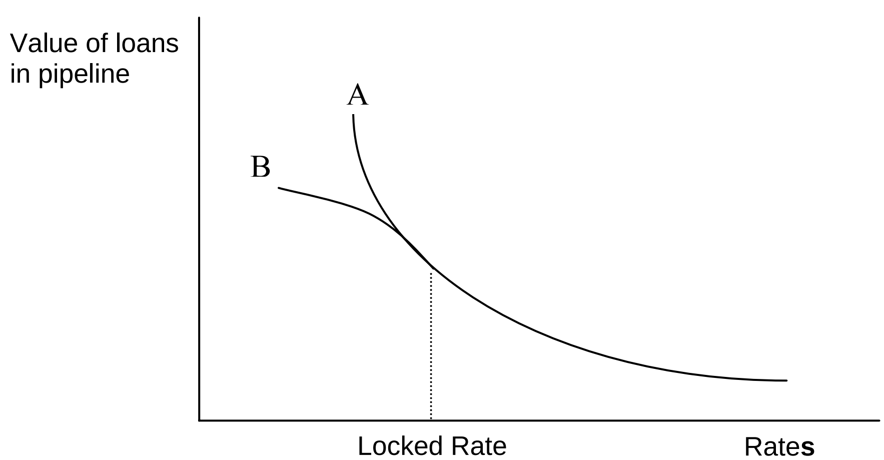

Figure 4-1. Negative convexity. When rates drop below the locked rate, the value of loans in pipeline is increasing in a decreasing rate because of fallout.

Put options on Treasury futures is a preferred instrument for hedging the fallout risk. It is well known that the 10-year Treasury notes are the preferred choice for hedging pipeline risk, mainly because the duration more closely matches the loan portfolio (Taflia, 2003). This research considers using the put options on 10-year Treasury futures and the 10-year Treasury futures for hedging the mortgage pipeline risk. The other advantage in selecting these two hedge instruments is that those instruments are traded in exchange with high liquidity. So the hedge portfolio could be rebalanced without unexpected transaction costs. It is worth mentioning that the spread between mortgage rates and Treasury rates creates basis risk. So an optimal cross hedge model for the pipeline risk management is developed in next section.

This research investigates the best risk measures for mortgage pipeline risk hedging strategy. Five deviation risk measures and two downside risk measures are selected for the optimal pipeline risk hedge model. We tested the hedging performance by optimizing the hedging strategy associated with each risk measure based on an (in-sample) historical dataset. Then, the out-of-sample performance of the optimal hedging strategy is examined.

4.2 Model

4.2.1 Locked Loan Amount Evaluation

To hedge the pipeline risk, we first model the value of locked loan amount with respected to the change in mortgage rate. The fallout of mortgage loans varies for different regions. A general way to estimate the fallout is using a regression model based on the regional historical data. This research defined the fallout as a function of the price change in the first generic 10-year Treasury futures given by

$$Fallout = a\exp(b \cdot \Delta TY1), \qquad (4\text{-}1)$$

where a and b are coefficients, and $\Delta TY1$ is the price change on the mortgage backed security. Although the fallout effect increases when rates drop, there are always some loans that do not go to closing even the rates go up. To model this effect, the fallout is assumed to be greater than a minimum fallout level

$$Fallout \geq \underline{Fallout}, \tag{4-2}$$

where $\underline{Fallout}$ is the lower bound of the fallout, which is also regional dependent.

The value change of the locked loan amount can be formulated as the value change of the fallout adjusted loan amount. We assume that the value change of mortgage loans can be approximately equivalent to the value change of 10-year Treasury futures.

$$\Delta LV = (1 - Fallout) \cdot \Delta TY1, \tag{4-3}$$

where ΔLV is the value change of locked loan amount.

4.2.2 Pipeline Risk Hedge Agenda

Assume a pipeline risk manager has 100 million dollars loan amount flowing into pipeline every two weeks. The lock period of the mortgage loans is four weeks. The risk manager hedges the loan exposure twice during the lock period, one at the time point when the loans are originated, i.e., in the start date of the first week of the lock period, and the other in the middle of the lock period, i.e., on the beginning of the third week. In this hedge agenda, the risk manager hedges the pipeline risk every two weeks with two different locked loan amounts. The first part of the loans was locked in the current mortgage rate with a loan amount of 100 million dollars. The second part of the loans was locked in the rate two weeks earlier. The loan amount is fallout adjusted. Therefore, the total value change is given by

$$\Delta LV_t = \Delta LV1_t + \Delta LV2_t, \tag{4-4}$$

where $\Delta LV1_t$ is the value change of the loans originated at time t-1 during [t-1,t], and $\Delta LV2_t$ is the value change of the loans originated at time t-2 during [t-1,t]. $\Delta LV1_t$ is given by

$$\begin{aligned}\Delta LV1_t &= LV1_t - LV1_{t-1} \\ &= (1 - Fallout_t) \cdot (TY1_t - TY1_{t-1})/100 \cdot LV1_{t-1} \\ &= \left(1 - \max\left[a \cdot \exp(b(TY1_t - TY1_{t-1})), \underline{Fallout}\right]\right) \cdot (TY1_t - TY1_{t-1})/100 \cdot LV1_{t-1},\end{aligned} \tag{4-5}$$

where $Fallout_t$ is the fallout during [t-1,t], and $LV1_{t-1}$ is the loan amount originated at time t-1.

$\Delta LV2_t$ is given by

$$\begin{aligned}\Delta LV2_t &= LV2_t - LV2_{t-1} \\ &= (1 - Fallout_{t,t-1}) \cdot (TY1_t - TY1_{t-1})/100 \cdot LV2_{t-1}\end{aligned} \tag{4-6}$$

The value change of the loans originated at time t-2 during [t-1,t] considered fallouts in two consecutive periods. The second fallout, $Fallout_{t,t-1}$, is calculated based on the value change of 10-year Treasury futures during [t-2,t] and adjusted by the first one, $Fallout_{t-1}$,

$$Fallout_{t,t-1} = \max\left[a \cdot \exp(b(TY1_t - TY1_{t-2})) - Fallout_{t-1}, \underline{Fallout}\right], \tag{4-7}$$

and the time t-1 value of the locked loans originated at time t-2, $LV2_{t-1}$, is given by

$$LV2_{t-1} = (1 - Fallout_{t-1}) \cdot LV2_t. \tag{4-8}$$

4.2.3 Model Development

An optimization model for the mortgage pipeline risk hedge was developed. The model considers shorting the first generic 10-year Treasury futures (TY1) and buying the put options on 10-year Treasury futures, such as TYZ7P 118, for pipeline risk hedge. As mortgage rates rise, the locked loan values as well as the value of 10-year Treasury futures drop. The increasing value in the short position of 10-year Treasury futures offsets the losses in the locked loan value. On the other hand, the decreasing value of the 10-year Treasury futures increases the value of long position of the put option on 10-year Treasury futures. To investigate the performance of different risk measures on the pipeline hedging strategies, five deviation measures: standard deviation, mean absolute deviation, CVaR deviation, VaR deviation, and two-tailed VaR deviation, and two downside risk measures: CVaR and VaR, were tested in the pipeline hedge model . The notations used in the model formulation are listed as follows.

x = vector $[x_1, x_1]$ of decision variables (hedge positions),
x_1 = hedge position of 10-year Treasury futures,
x_2 = hedge position of put options on 10-year Treasury futures,
θ = random vector $[\theta_0, \theta_1, \theta_2]$ of parameters (prices),
θ_j = the *j*th scenario of random vector $[\theta_0, \theta_1, \theta_2]$,
θ_{0j} = the *j*th scenario of value change in the locked loan amount in one analysis period,
θ_{1j} = the *j*th scenario of price change in 10-year Treasury futures in one analysis period,
θ_{2j} = the *j*th scenario of price change in put options on 10-year Treasury futures in one analysis period,
θ_0 = value change in the locked loan amount in one analysis period,
θ_1 = price change in 10-year Treasury futures in one analysis period,
θ_2 = price change in put options on 10-year Treasury futures in one analysis period,
J = number of scenarios,
$L(x, \theta_j)$ = the *j*th scenario of loss based on hedge position *x* and *j*th scenario of prices θ_j,

The model formulation associated with each risk measure is described as follows. The objective function (4-9) of the minimum standard deviation model is equivalent to the standard deviation since the value of the sum of the square is larger than one. A drawback of the standard deviation measure is that the squared term amplifies the effect of outliers.

The formulation of minimum standard deviation model is given by

$$Min \sum_{j=1}^{J} \left(L(x, \theta_j) - \frac{\sum_{j=1}^{J} L(x, \theta_j)}{J} \right)^2 \tag{4-9}$$

$$\text{s.t. } L(x, \theta_j) = -\theta_{0j} - (\theta_{1j} x_1 + \theta_{2j} x_2) \qquad \forall j .$$

To overcome the drawback of standard deviation mentioned above, the mean absolute deviation takes the absolute value instead of the squared value in calculating dispersion.

The formulation of minimum mean absolute standard deviation model is given by

$$Min \sum_{j=1}^{J} \left| L(x,\theta_j) - \frac{\sum_{j=1}^{J} L(x,\theta_j)}{J} \right|$$

$$\text{s.t.} \qquad L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \qquad \forall j \, .$$

The nonlinear programming problem was formulated as an equivalent linear programming problem

$$
\begin{aligned}
& Min \;\; z_j \\
\text{s.t.} \qquad & z_j \geq L(x,\theta_j) - \frac{\sum_{j=1}^{J} L(x,\theta_j)}{J} \\
& z_j \geq -L(x,\theta_j) + \frac{\sum_{j=1}^{J} L(x,\theta_j)}{J} \\
& L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \qquad \forall j.
\end{aligned}
$$

Conditional Value-at-Risk ($CVaR_\alpha$) deviation measures the distance between the mean value and $CVaR_\alpha$ of a distribution. In other words, the minimum $CVaR_\alpha$ deviation model minimizes the difference between 50% percentile and mean of α tail of a distribution. The minimum $CVaR_\alpha$ deviation model can be formulated as a linear programming problem

$$Min \qquad \zeta_\alpha + \frac{1}{J(1-\alpha)} \sum_{j=1}^{J} z_j - \frac{\sum_{j=1}^{J} L(x,\theta_j)}{J}$$

$$
\begin{aligned}
\text{s.t.} \quad & z_j \geq L(x,\theta_j) - \zeta_\alpha \qquad \forall j \\
& L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \qquad \forall j \\
& z_j \geq 0 \qquad \forall j
\end{aligned}
$$

where ζ_α is a real number variable representing the α percentile of a distribution.

Value-at-Risk (VaR_α) deviation measures the distance between the mean value and VaR_α of a distribution. The minimum $CVaR_\alpha$ deviation model minimizes the difference between 50% percentile and α percentile of a distribution. The minimum VaR_α deviation model can be formulated as a 0-1 mixed linear programming problem

$$Min \quad \zeta_\alpha - \frac{\sum_{j=1}^{J} L(x,\theta_j)}{J}$$

$$\begin{aligned}
\text{s.t. } & L(x,\theta_j) - \zeta_\alpha \le My_j \qquad \forall j \\
& \sum_{j=1}^{J} y_j \le (1-\alpha)J \\
& L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \qquad \forall j \\
& y_j \in \{0,1\} \qquad \forall j
\end{aligned}$$

where M is a big number used to enforce $y_j = 1$ when $L(x,\theta_j) - \zeta_\alpha > 0$.

The two-tailed VaR_α deviation measures the distance between two VaR_α values in two tails of a distribution. This measure ignores the outliers beyond α and 1- α tails of a distribution. The property can be used in forecast and robust regression.

$$\begin{aligned}
& Min\ \zeta_\alpha + \zeta_{1-\alpha} \\
& \text{s.t. } L(x,\theta_j) - \zeta_\alpha \le My_j \qquad \forall j \\
& \sum_{j=1}^{J} y_j \le (1-\alpha)J \\
& -L(x,\theta_j) - \zeta_{1-\alpha} \le Mz_j \qquad \forall j \\
& \sum_{j=1}^{J} z_j \le (1-\alpha)J \\
& L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \qquad \forall j \\
& y_j, z_j \in \{0,1\} \qquad \forall j
\end{aligned}$$

In addition to the deviation measures, two downside risk measures are considered in the mortgage pipeline risk models.

Conditional Value-at-Risk ($CVaR_\alpha$) measures the mean of the α tail of a distribution. The minimum $CVaR_\alpha$ model minimizes the mean of the worst (1- α) *100 % loss scenarios. The minimum $CVaR_\alpha$ model can be formulated as a linear programming problem.

$$Min \quad \zeta_\alpha + \frac{1}{J(1-\alpha)}\sum_{j=1}^{J} z_j$$

$$\begin{aligned} \text{s.t.} \quad & z_j \geq L(x,\theta_j) - \zeta_\alpha \quad \forall j \\ & L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \quad \forall j \\ & z_j \geq 0 \quad \forall j \end{aligned}$$

Value-at-Risk (VaR_α) measures the α percentile of a distribution. The minimum VaR_α model minimizes the loss value at α percentile of a distribution. The minimum VaR_α model can be formulated as a 0-1 mixed linear programming problem

$$Min \quad \zeta_\alpha$$

$$\begin{aligned} \text{s.t.}\ & L(x,\theta_j) - \zeta_\alpha \leq My_j \quad \forall j \\ & \sum_{j=1}^{J} y_j \leq (1-\alpha)J \\ & L(x,\theta_j) = -\theta_{0j} - (\theta_{1j}x_1 + \theta_{2j}x_2) \quad \forall j \\ & y_j \in \{0,1\} \quad \forall j. \end{aligned}$$

4.3 Case Study

4.3.1 Dataset and Experiment Design

The dataset for the case study includes biweekly data of the price of the generic first 10-year Treasury furthers (TY1)[7], the price of put options on the 10-year Treasury futures[8], and the

[7] The TY1 is the near contract generic TY future. A genetic is constructed by pasting together successive "Nth" contract prices from the primary months of March, June, September, and December.

[8] The strike prices of the put options are selected based on the price of TY1.

mortgage rate index MTGEFNCL[9]. All prices and index are the close price on Wednesday from 8/13/2003 to 10/31/2007.

The dataset includes 110 scenarios. To test the performance of six deviation measures and two downside risk measures in the pipeline risk management, we used the first 60 time series data as known historical data and solve the optimal hedge positions associated with each risk measure. Next, the optimal hedge positions associated with different risk measures are applied in hedging the next (the 61st) scenario, and a hedge error can be calculated for each model. Then, the window is rolled one step forward. The dataset from the second to the 61st scenarios are used to compute the optimal hedging positions, and the one step forward out-of-sample hedge error can be get from the 62nd scenario. This procedure was repeated for the remaining 50 scenarios, and 50 out-of-sample hedge errors were used to evaluate the performances of eight risk measures in pipeline risk hedge.

To investigate whether more historical data provides better solution in out-of-sample performance, an alternative approach is to fix the window start date. The growing window approach keeps all the known historical data for solving the optimal hedge positions. The results from two approaches are compared.

4.3.2 Analyses and Results

The experiment was conducted in a UNIX workstation with 2 Pentium4 3.2 GHz processor and 6GB of memory. The optimization models were solved using CPLEX optimizer.

Figure 4-2 shows the values of naked (unhedged) and hedged pipeline positions. It shows that no matter which risk measure is selected, the pipeline hedge reduces the volatility of the value of locked loans dramatically. Figure 4-3 and Figure 4-4 show the out-of-sample hedge

[9] The MTGEFNCL index represents the 30 Year FNMA current coupon, which has been used as an index for the 30 year mortgage rate.

performances associated with risk measures using rolling window approach and growing window approach, respectively. The risk measures selected for the optimal hedging model includes: standard deviation (STD), mean absolute deviation (MAD), CVaR deviation with 90% confident interval ($CVaR_{90}$ Dev), VaR deviation with 90% confident interval (VaR_{90} Dev), two-tailed VaR deviation with 90% confident interval ($2TVaR_{90}$), two-tailed VaR deviation with 99% confident interval ($2TVaR_{90}$), $CVaR_{90}$, and VaR_{90}. The Figures show that the standard deviation has the worst out-of-sample hedge performance among all risk measures.

Table 4-1 and Table 4-2 show the mean value, standard deviation, and maximum loss of the 50 out-of-sample losses of hedged position based on rolling window and growing window approach, respectively. Comparing these two tables we can see that the two approaches are not significantly different. For the rolling window approach, the two tailed 99% VaR has the best hedge performance among all risk measures. In addition, the mean standard deviation has the

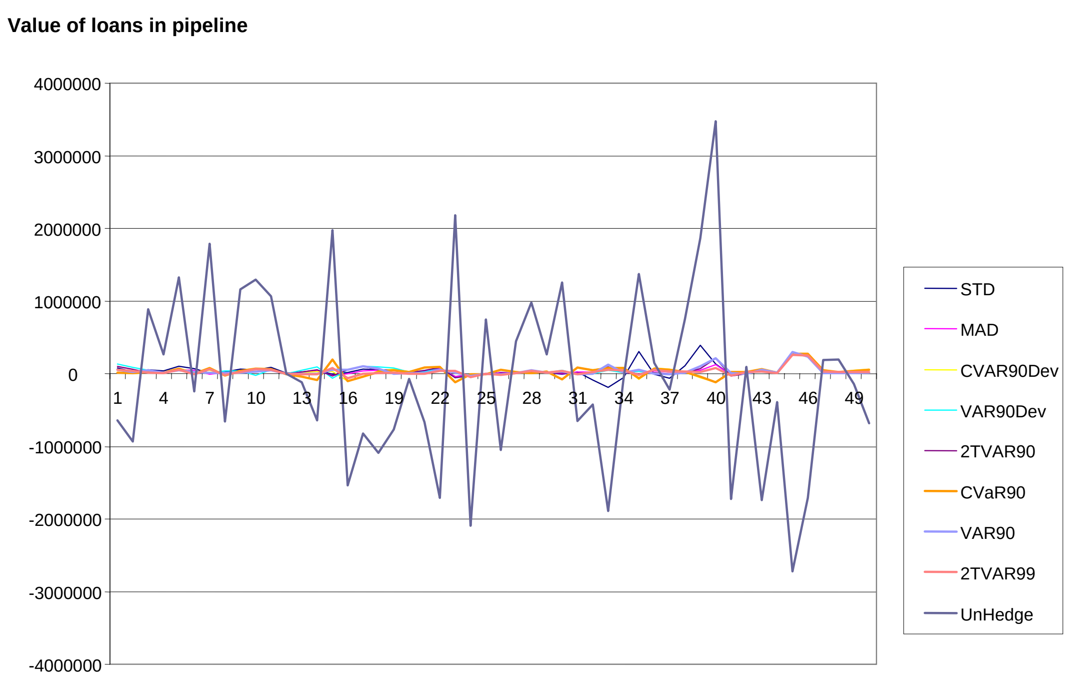

Figure 4-2 Value of naked pipeline position and hedged pipeline positions associated with different risk measures.

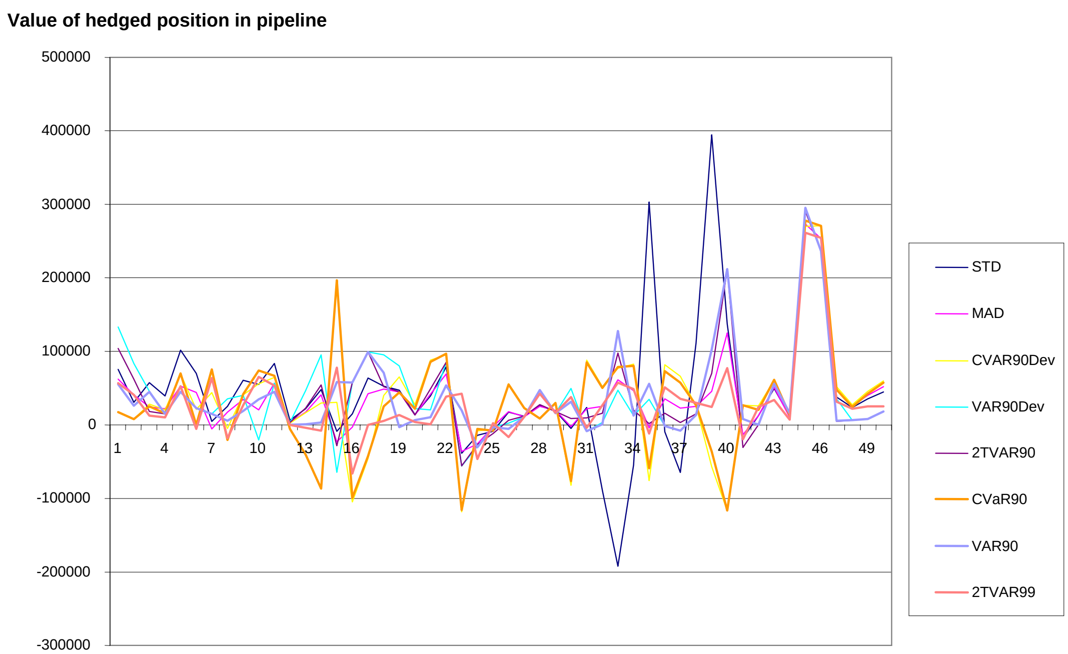

Figure 4-3 Out-of-sample hedge errors associated with eight risk measures using rolling window approach

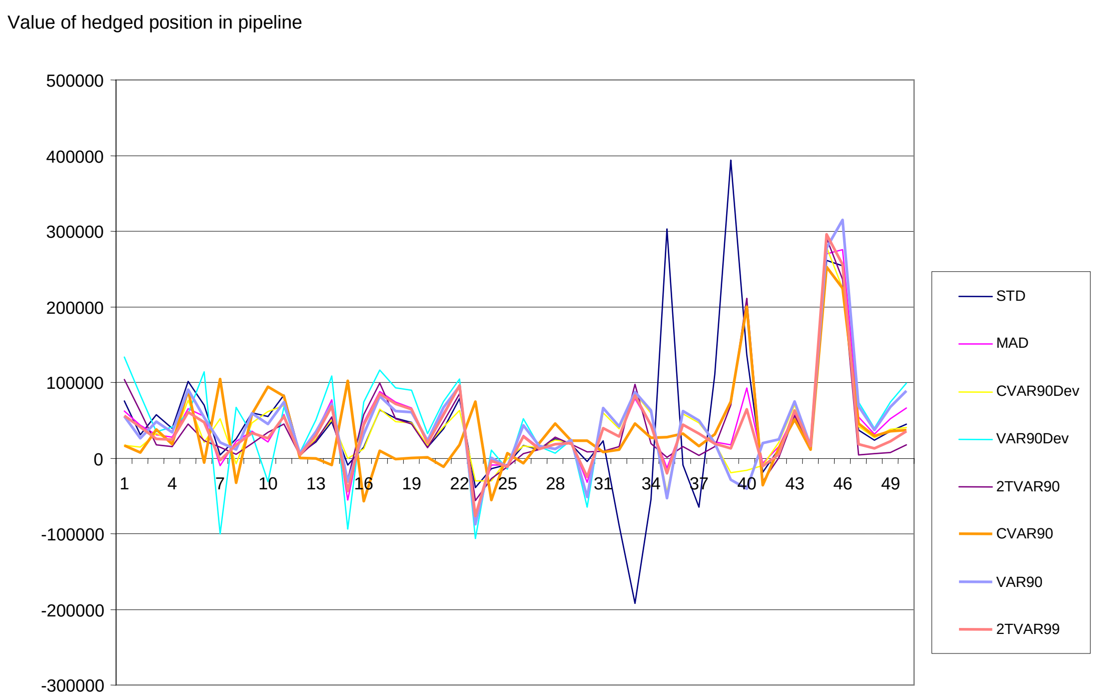

Figure 4-4. Out-of-sample hedge errors associated with eight risk measures using growing window approach

best performance in reducing the volatility. For the growing window approach, CVaR deviation has best performance in mean value and standard deviation, and the CVaR has the best performance in maximum loss.

Table 4-1 Mean value, standard deviation, and maximum loss of the 50 out-of-sample losses of hedged position based on rolling window approach

	STD	MAD	$CVaR_{90}$ Dev	VaR_{90} Dev	$2TVaR_{90}$	$2TVaR_{99}$	$CVaR_{90}$	VaR_{90}
Mean	45260	36648	31863	42667	37571	30178	31317	38491
StDev	94182	54407	71278	66014	62836	55409	77217	62209
Max Loss	394484	273282	270711	295226	290365	261490	278274	295226

Table 4-2 Mean value, standard deviation, and maximum loss of the 50 out-of-sample losses of hedged position based on growing window approach

	STD	MAD	$CVaR_{90}$ Dev	VaR_{90} Dev	$2TVaR_{90}$	$2TVAR_{99}$	$CVaR_{90}$	VaR_{90}
Mean	45260	42028	35546	47363	37571	38296	35944	44566
StDev	94182	59247	54723	76232	62836	59040	60523	65864
Max Loss	394484	276070	278462	314817	290365	296030	252958	314817

4.4 Conclusion

This chapter studies the optimal mortgage pipeline risk management strategy. We developed an optimization model to minimize the pipeline risk under five different deviation risk measures and two downside risk measures. A case study using 10-year Treasury futures and options on the 10-year Treasury futures as the hedging instruments shows that under the growing window approach, the CVaR deviation has a better performance for hedging the volatility and CVaR is better for hedging the downside risk. On the other hand, when the rolling window approach is used, the two tailed 99% VaR has a better performance in general and the mean absolute deviation performs well in hedging volatility. It is worth mentioning that the result of the case study shows the standard deviation is the worst risk measure for the out-of-sample pipeline risk hedge.

CHAPTER 5
CONCLUSION

This dissertation has shown a novel stochastic optimization model using conditional value-at-risk as the risk measure in the reward versus risk framework. This optimization approach was applied in modeling the optimal crop production and risk hedging strategy. To get a better performance in capturing the correlation between marginal distributions of multivariate random variables, the copula function was used to generate the portfolio loss distribution. Incorporating the copula based loss distribution within the CVaR optimization model produces a powerful model for solving the optimal planting schedule and hedging strategy problem. In addition, the ENSO-based climate forecast information was used to get a better prediction of the crop yield in the coming season. A case study showed that 65%CRC or 70%CRC would be the optimal insurance coverage when the CVaR constraint reaches a strict level. Furthermore, the optimal hedging strategy with crop insurance products and futures contracts was examined. When futures price are unbiased or positive biased, the optimal hedging strategy only contains futures contracts and no crop insurance policies are desirable. However, when the futures price is negative biased, the optimal hedging strategy depends on the ENSO phases. The optimal futures hedge ratio increases with the increasing CVaR upper bound when the insurance strategy is unchanged. For a fixed CVaR upper bound, the optimal hedge ratio increases when the positive bias of futures price increases. However, when the futures price is negative biased, the optimal hedge ratio depends on the value of CVaR upper bound.

For the secondary mortgage market efficient execution problem, a stochastic optimization model is proposed to perform the efficient execution analysis. The model considers secondary marketing functionalities, including the loan-level execution for an MBS/whole loan, guarantee fee buy-up/buy-down, servicing retain/release, and excess servicing fee. Since secondary marketing involves random cash flows, lenders must balance between expected revenue and risk. The advantage of the CVaR risk measure was introduced and employed in the model that maximizes expected revenue under a CVaR constraint. By solving the efficient execution problem under different risk tolerances, efficient frontiers could be found, which provides optimal execution strategies associated with different risk budgets. A sensitivity analysis was conducted in parameters of expected retained servicing fee multipliers, mortgage prices, and MBS prices.

In the optimal mortgage pipeline risk management strategy problem, we developed an optimization model to minimize the mortgage pipeline risks under five different deviation risk measures and two downside risk measures. A case study using 10-year Treasury futures and out options on the 10-year Treasury futures as the hedging instruments was conducted. The result shows that the CVaR Deviation and CVaR have better out-of-sample performances in controlling the volatility and maximum loss, respectively, under the growing window approach. In addition, the two-tailed 99% VaR deviation performs better on mean loss and maximum loss for the rolling window approach. In contrast, standard deviation performed the worst.

APPENDIX A
EFFICIENT EXECUTION MODEL FORMULATION

Notations:

Indices:

m = index of mortgages *(1,2,…,M)*,
M = total number of mortgages,
j = index of mortgage groups *(1,2,…,J)*,
J = total number of groups,
k = index of scenarios *(1,2,…,K)*,
K = total number of scenarios,
c = index of MBS coupon rate *(1,2,…,C^m)*,
C^m = number of possible MBS coupon rates of mortgage m.

Decision Variables:

$$z_c^m = \begin{cases} 1, \text{ if mortgage } m \text{ is pooled into MBS with coupon rate index } c, \\ 0, \text{ otherwise}, \end{cases}$$

$$z_w^m = \begin{cases} 1, \text{ if mortgage } m \text{ is sold as a whole loan}, \\ 0, \text{ otherwise}, \end{cases}$$

$$z_{sbo}^m = \begin{cases} 1, \text{ if the servicing of mortgage } m \text{ is retained}, \\ 0, \text{ otherwise}, \end{cases}$$

$$z_{sbr}^m = \begin{cases} 1, \text{ if the servicing of mortgage } m \text{ is sold}, \\ 0, \text{ otherwise}, \end{cases}$$

r_{gu}^m = guarantee fee buy-up spread of mortgage m,
r_{gd}^m = guarantee fee buy-down spread of mortgage m,
r_{ser}^m = retained excess servicing fee spread of mortgage m,
z_k = real variables used in CVaR constraint formulation,
ζ = real variables used in CVaR constraint formulation.

Input Data:

L^m = loan amount of mortgage m,
$P_c^{t^m}$ = price of MBS with coupon rate index c and maturity t^m,
t^m = maturity of mortgage m,
P_w^m = whole loan sale price of mortgage m,
R_c = MBS coupon rate related to index c,
K_u^m = guarantee fee buy-up multiplier of mortgage m,
K_d^m = guarantee fee buy-down multiplier of mortgage m,

R_n^m = note rate of mortgage *m*,
R_{sb}^m = base servicing fee of mortgage *m*,
R_{gb}^m = base guarantee fee of mortgage *m*,
B^m = base servicing value of mortgage *m*,
c_s^m = servicing cost of mortgage *m*,
p_k = the probability of scenario *k*,
K_{sr}^{mk} = retained servicing fee multiplier of mortgage *m* under scenario *k*,
U_{gu}^m = upper bound of guarantee fee buy-up spread of mortgage *m*,
U_{gd}^m = upper bound of guarantee fee buy-down spread of mortgage *m*,
U_{se}^m = upper bound of retained excess servicing fee of mortgage *m*,
U_{se}^a = upper bound of average retained excess servicing fee of all mortgages,
U_{se}^j = upper bound of average retained excess servicing fee of mortgages in group *j*,
U = upper bound of CVaR losses,
α = percentile of CVaR.

Model Formulation:

Max

$$\sum_{m=1}^{M}\left[L^m \times \sum_{c=1}^{C^m}\left(P_c'^{m} \times z_c^m\right) + P_w^m \times z_w^m\right]$$
[revenue from MBSs or whole loan sale]

$$\sum_{m=1}^{M}\left(L^m \times B^m \times z_{sbo}^m\right) + \sum_{m=1}^{M}\left(\sum_{k=1}^{K} p_k\left(L^m \times R_{sb}^m \times K_{sr}^{mk}\right) - c_s^m\right) z_{sbr}^m$$
[revenue from the base servicing fee of securitized mortgages]

$$+\sum_{m=1}^{M}\sum_{k=1}^{K} p_k\left(L^m \times K_{sr}^{ms} \times r_{ser}^m\right)$$
[revenue from the excess servicing fee of securitized mortgages]

$$+\sum_{m=1}^{M}\left(K_u^m \times L^m \times r_{gu}^m\right) - \sum_{m=1}^{M}\left(K_d^m \times L^m \times r_{gd}^m\right)$$
[revenue from the guarantee fee buy-up/buy-down of securitized mortgages]

s.t.

$$\sum_{c=1}^{C^m} z_c^m + z_w^m = 1 \qquad \forall m = 1,2,\ldots,M$$
[mortgage m must be either securitized as an MBS or sold as a whole loan]

$$z_w^m + z_{sbo}^m + z_{sbr}^m = 1 \qquad \forall m = 1,2,\ldots,M$$
[if mortgage m is sold, there is no servicing; otherwise, servicing either be sold or be retained]

$$z_w^m + r_{gu}^m + r_{gd}^m + r_{ser}^m \le 1 \qquad \forall m = 1,2,\ldots,M$$
[if mortgage m is sold, there is no guarantee fee and excess servicing fee]

$$\sum_{c=1}^{C^m} R_c z_c^m + r_{gu}^m - r_{gd}^m + r_{ser}^m \le R_n^m - R_{sb}^m - R_{gb}^m \qquad \forall m = 1,2,\ldots,M$$
[if mortgage m is securitized, note rate = MBS coupon rate + servicing fee rate + guarantee fee rate]

$$\zeta + (1-\alpha)^{-1} \sum_{k=1}^{K} p_k z_k \le U$$
$$z_k \ge L_k - \zeta \quad \forall k = 1,\ldots,K$$
$$z_k \ge 0, \qquad \forall k = 1,\ldots,K$$
[$CVaR_\alpha(loss) \le U$: the expected loss (negative revenue) of the worst 1-α percent of senarios should be less than or equal to U]

$$
\begin{aligned}
L_k = & -\sum_{m=1}^{M}\left[L^m \times \sum_{c=1}^{C^m}\left(P_c^m \times z_c^m\right) + P_{whole}^m \times z_w^m\right] - \sum_{m=1}^{M}\left(L^m \times B^m \times z_{sbo}^m\right) \\
& -\sum_{m=1}^{M}\left(L^m \times R_{sb}^m \times K_{sr}^{mk} \times z_{sbr}^m\right) - \sum_{m=1}^{M}\left(L^m \times K_{sr}^{mk} \times r_{ser}^m\right) \\
& -\sum_{m=1}^{M}\left(K_u^m \times L^m \times r_{gu}^m\right) + \sum_{m=1}^{M}\left(K_d^m \times L^m \times r_{gd}^m\right)
\end{aligned}
$$

$$\sum_{m=1}^{M} L^m \times r_{se}^m \le U_{se}^a \left(\sum_{m=1}^{M} L^m (1 - z_w^m)\right)$$ [the average excess servicing fee of all securitized mortgages is restricted to an upper bound.]

$$\sum_{m \in j} L^m \times r_{se}^m \le U_{se}^j \left(\sum_{m \in j} L^m (1 - z_w^m)\right) \quad \forall j = 1, 2, ..., J$$ [the average excess servicing fee of securitized mortgages of group j is restricted to an upper bound]

$$0 \le r_{ser}^m \le U_{se}^m \qquad \forall m = 1, 2, ..., M$$ [for each securitized mortgage m, the excess servicing fee is restricted to upper bound]

$$0 \le r_{gu}^m \le U_{gu}^m \qquad \forall m = 1, 2, ..., M$$ [for each securitized mortgage m, the guarantee fee buy-up spread is restricted to an upper bound]

$$0 \le r_{gd}^m \le U_{gd}^m \qquad \forall m = 1, 2, ..., M$$ [for each securitized mortgage m, the guarantee fee buy-down spread is restricted to an upper bound]

$$z_c^m, z_{whole}^m, z_{sbr}^m, z_{sbo}^m \in \{0,1\} \qquad \forall m = 1, 2, ..., C^m$$ [binary constraints]

$$r_{ser}^m, r_{gu}^m, r_{gd}^m \ge 0$$ [non-negative constraints]

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BIOGRAPHICAL SKETCH

Chung-Jui Wang was born in Taipei, Taiwan. He earned his bachelor's (1994) and master's (1996) degrees in civil engineering from National Central University. After 2 years military service, he worked for Evergreen Construction Co. for two years. In 2002, he began his master's and doctoral studies in the Department of Industrial and Systems Engineering at the University of Florida. He finished his Ph.D. in operation research with concentration in quantitative finance in December 2007.