Formula:

① Multivariate : $M = \begin{array}{|c|c|} \hline x_1 & x_2 \\ \hline \vdots & \vdots \\ \hline \end{array}$ $\rightarrow$ $a = M - \mu$ $\begin{array}{|c|c|} \hline x_1 & x_2 \\ \hline \vdots & \vdots \\ \hline \end{array}$

$\boxed{\text{Step-1}}$

$\mu = \square \quad \square$

$a^T = \begin{bmatrix} x_1 & \cdots \\ x_2 & \cdots \end{bmatrix}$

$a^T . a = \begin{bmatrix} x_1 x_1 & x_1 x_2 \\ x_2 x_1 & x_2 x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(covariance) $\Sigma = \frac{a^T a}{m} = \begin{bmatrix} \frac{x_1 x_1}{m} & \frac{x_1 x_2}{m} \\ \frac{x_2 x_1}{m} & \frac{x_2 x_2}{m} \end{bmatrix} = \begin{bmatrix} \frac{a}{m} & \frac{b}{m} \\ \frac{c}{m} & \frac{d}{m} \end{bmatrix}$

data set no.

$\boxed{\text{Step 2}}$ $|\Sigma| =$ value বের করব

$$\Sigma^{-1} = \frac{\begin{bmatrix} b & -d \\ -c & a \end{bmatrix}}{|\Sigma|}$$

~~Step~~ Test data নিব $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$(X - \mu_1) = \begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{bmatrix}$$

$(X - \mu_1)^T \times \Sigma^{-1} (X - \mu_1)$. বের করব

last $P(x)$ - এ বসাব

$$P(x) = \frac{1}{(2\pi)^{n/2} \, |\Sigma|^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(x-\mu)^T \cdot \Sigma^{-1} (x - \mu_1)\right)}$$

$\boxed{\text{Step 3}}$ check if $P(x) < \varepsilon$. then fault.

② Queueing : $L \rightarrow$ average person in the sys. $\dfrac{\lambda}{\mu-\lambda}$

$L_q \rightarrow$ " " " queue λW_q

$W \rightarrow$ avg time in sys $\dfrac{1}{\mu-\lambda}$

$W_q \rightarrow$ " " " queue $\dfrac{\lambda}{\mu(\mu-\lambda)}$

Shoeshine : potential customer enter : $P_{00}+P_{01}$

" " doesn't " : $1-(P_{00}+P_{01})$

Mean, $L = P_{00} + P_{01} + P_{10} + 2(P_{11}+P_{b1})$ [∵ 21's × 2, 11's × 1]

Avg time, $W = \dfrac{L}{\lambda_a}$ $[\lambda_a = \lambda(P_{00}+P_{01})]$

chair empty $= P_{00}$

ch1 empty $= (P_{00}+P_{01})$

ch2 " $= (P_{10}+P_{00})$

" filled $= (P_{11}+P_{b1}+P_{01})$.

either/both filled $= 1-P_{00}$ or $P_{01}+P_{10}+P_{11}+P_{b1}$

both filled $= P_{11}+P_{b1}$

③ Markov : rains α, not rains β

$$P = \begin{bmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{bmatrix}$$

* random walk – "if monday sunday rained will ~~thursday~~ ~~Tom~~ tuesday rain?" type.

States লিখবে first-এই

sunday (yesterday) monday (today) tuesday

monday (yesterday) tuesday (today).

0 → rained ~~today~~ yesterday, rained today.

1 → doesn't rain " , " "

2 → rained yesterday, not rained today

3 → not " " , " " "

Short-এ

0 → ✓ ✓

1 → ✗ ✓

2 → ✓ ✗

3 → ✗ ✗

এরপর value দেখব (বৃষ্টি হইছে কি হয়নি)

	S	M	M	T	
P_{00}	0 ✓	✓	0 ✓	✓	= দেওয়া থাকে
P_{01}	0 ✓	✓	1 ✗	✓	= Monday conflict 0.
P_{02}	✓	✓	✓	✗	= given
P_{03}	✓	✓	✗	✗	= 0.
P_{10}	✗	✓	✓	✓	= given
P_{11}	✗	✓	✗	✓	= 0
P_{12}	✗	✓	✓	✗	= given
P_{13}	✗	✓	✗	✗	= 0
P_{20}	✓	✗	✓	✓	= 0
P_{21}	✓	✗	✗	✓	= given
P_{22}	✓	✗	✓	✗	= 0
P_{23}	✓	✗	✗	✗	= given
P_{30}	✗	✗	✓	✓	= 0
P_{31}	✗	✗	✗	✓	= given
P_{32}	✗	✗	✓	✗	= 0
P_{33}	✗	✗	✗	✗	= given

এরপর Matrix form করব

$$\begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{bmatrix}$$

Probable ques.

** if it rains on sunday monday. will it rain on thursday?

$\Rightarrow P_{00}^{3}$ (cause thursday 3 days later)

** it rained on moneday, tuesday it won't rain on thursday

$\Rightarrow P_{02}^{2} + P_{03}^{2}$

** Din't ~~do~~ rain monday, tuesday; will rain thursday?

$\Rightarrow P_{30}^{2} + P_{31}^{2}$

* Gambler : *** Patty-Max-এর খেলা type:

winning probability দেওয়া থাকবে কারো → p

① বের করব না জিতারটা → $q = 1-p$.

② কার কত coin বলে দিবে → যোগ করব (N).

③ check করব if $q/p = 1$ or not.

$\neq 1 \rightarrow \left[\dfrac{1-\left(\frac{q}{N}\right)}{1-\left(\frac{q}{N}\right)^{N}}\right]$ (যারটা বের করতেছি তার coin amount).

$= 1 \rightarrow$ (যারটা বের করব তার coin ÷ N.).

*** Barry Garry type খেলা :

probability দেওয়া নাই; condition বলা

কার কত initial Tk বলা আছে

winning prize ও বলা ; dice বা coin ও বলা

① condition wise sum করে probability বের করব

(p) and (q)

② কার কত Tk যোগ ~~করব~~ করব N.

winning prize কে 1coin ধরব

Then N/winning prize করে coin কয়টা বের করব

③ check $q/p = 1$ or not

then আগের মতো formula, $q/p \neq 1 \rightarrow \left[\frac{1-(q/p)^i}{1-(q/p)^N}\right]$

$q/p = 1 \rightarrow i/N$

④ Bayes (Normal টা):

~~ques: এ Given থাকে আর্থাৎ~~

ques-এ "given that blah blah হয়েছে" – মানেই B

যার probability বের করব সে – A.

So, $P(A|B) = \frac{P(A)\, P(B|A)}{P(A)\, P(B|A) + P(A^c)\, P(B|A^c)}$

probability বের করব

Already done এর পর A এর probability জানতে চাচ্ছে

*** $\boxed{P(A|B) = \frac{P(A)\; P(B|A)}{P(B)}}$

$P(B) = P(A)\,P(B|A) + P(A^c)\,P(B|A^c)$

⑤ Random variable: expectation: $E[x] = \sum x\, p(x)$

Discrete → 1. Bernoulli → $P, 1-P$.

2. Geometric → $p(1-p)^x$

3. Bionomial → $p(i) = \binom{n}{i} p^i (1-p)^{n-i}$

4. poisson → $P(i) = e^{-\lambda} \frac{\lambda^i}{i!}$

Continous:

1. uniform → $P\{a \leq x \leq b\} = \int_a^b f(x)dx$

$P\{x \leq a\} = \int_{-\alpha}^{a} f(x)\,dx$

$P\{x \geq a\} = \int_a^{\alpha} f(x)\,dx$

but interval এর আগে PDF বের করব

$(\alpha, \beta) \rightarrow \frac{1}{\beta - \alpha}$

প্রথমে → integration $\int \frac{dx}{(\beta-\alpha)}$; [নিজেদের formula দিয়ে.]

2. Exponential : lifetime থাকলে (দেওয়া থাকে) mean → $\lambda = \frac{1}{mean}$

always math-এ অনুযায়ী formula derive করব.

$$F(x) = P(X \le x)$$
$$= \int_{-\alpha}^{x} \lambda e^{-\lambda y} dy$$
$$= \int_{0}^{x} \lambda e^{-\lambda y} dy.$$
$$= \frac{-\cancel{\lambda}}{\cancel{\lambda}} \left[e^{-\lambda y}\right]_0^x$$
$$= -\left[e^{-\lambda x} - e^{-\lambda 0}\right]$$
$$= -\left[e^{-\lambda x} - 1\right]$$
$$= \left(1 - e^{-\lambda x}\right)$$

So, $P(X > x) = 1 - \left(1 - e^{-\lambda x}\right)$
$$= e^{-\lambda x}$$

প্রথমে এভাবে math করব then.

*** Memoryless : Exponential : $\frac{P\{X > s+t \mid X > t\}}{P\{X > t\}} = P\{X > s\}$

or, $P\{X > s+t\} = P\{X > s\} P\{X > t\}$

or, $e^{-\lambda(s+t)} = e^{-\lambda s} e^{-\lambda t}$

Geometric: $\frac{P\{X \geq s+t \mid X \geq t\}}{P(X \geq t)}$

$$= \frac{P\{X \geq s+t\}}{P\{X \geq t\}} = \frac{p(1-p)^{s+t}}{(1-p)^t} = (1-p)^s$$
$$= P(X \geq s).$$

Date :

Memoryless Property:

*** Exponential Distribution : RV x is memoryless. if
$$P\{x>s+t \mid x>t\} = P\{x>s\} \text{ . for all } s,t \geq 0$$

let x be the lifetime of some instrument.
It has already survived t time and it lives for at least s more time. Both period is same that is the instrument does not know that it has been already used for t time.
The condition for this equation is :
$$\frac{P\{x>s+t, x>t\}}{P\{x>t\}} = P\{x>s\}.$$

$$\Rightarrow P\{x>s+t\} = P\{x>s\} \cdot P\{x>t\}$$

So when x is exponentially distributed for $e^{-\lambda(s+t)} = e^{-\lambda s} \cdot e^{-\lambda t}$, it follows that exponentially distributed random variables are memoryless.

*** Geometric Distribution : Let x be memoryless for $s,t \geq 0$.
Here, PMF for geometric rv is $f(x) = p(1-p)^x$; $x = 0, 1, 2 \ldots$
probability that X is equal or greater than x
is $P(X \geq x) = (1-p)^x$; $x = 0, 1, 2 \ldots$
The conditional probability of interest, $P(x \geq s+t \mid x \geq t\}$
$$= \frac{P(x \geq s+t, x \geq t)}{P(x \geq t)}.$$
$$= \frac{P(x \geq s+t \cancel{, x \geq t})}{P(x \geq t)} = \frac{(1-p)^{s+t}}{(1-p)^t}$$
$$= (1-p)^s = P(x \geq s).$$

(5) TOH $\longrightarrow$ single : $(2^n - 1)$ $\rightarrow$
Double : $2(2^n - 1)$ $\rightarrow$
Triple : $3(2^n - 1)$ $\rightarrow$

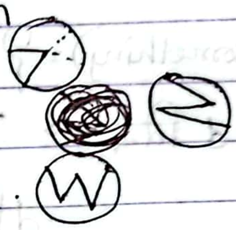
Induction Proof:

Basis	Hypothesis	Induction
T_0	T_n	T_{n+1}
T_0	T_{n-1}	T_n
T_0	T_{n-1}	T_n

(6) Pizza cutting : No intersection formula : (open) $L_n = L_0 + S_n$
(Close) $L_n = 1 + \frac{(n+1)n}{2}$

~~ZigZag $\rightarrow$ $Z_n = L_{3n} - 5n$~~

Zig : $Z_n = L_{2n} - 2n$
Zigzag : $Z_n = L_{3n} - 5n$
W obj : $W_n = L_{4n} - 9n$.



(7) Josephus : $J(1) = 1$
$J(2n) = 2J(n) - 1$ (even)
$J(2n+1) = 2J(n) + 1$ (odd)
$n = 2^m + l$; $l = n - 2^m$.
• so, $J(2^m + l) = 2l + 1$

** Binary $\longrightarrow$ $n = (b_m\ b_{m-1}\ b_{m-2} \ldots\ldots b_2\ b_1\ b_0)_2$
$l = (0\ b_{m-1}\ b_{m-2} \ldots\ldots b_2\ b_1\ b_0)_2$
$2l = (b_{m-1}\ b_{m-2} \ldots\ldots b_2\ b_1\ b_0\ 0)_2$
$J(2^m + l) = 2l + 1 = (b_{m-1}\ b_{m-2} \ldots\ldots b_2\ b_1\ b_0\ 1)_2$

Left shift, add 1

** n^{th} position surviour, কত no. $\rightarrow$ $\boxed{J(n) = n \quad 2l + 1 = 2^m + l \ldots}$

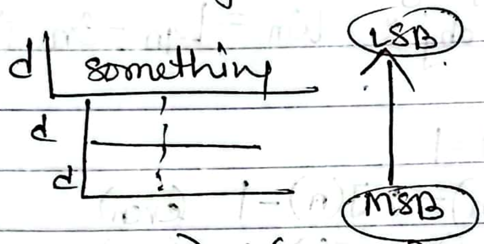
** good person + bad person $\rightarrow$
bad person - দের LCM = m^{th} position

** Radix based Josephus $f(x)$-দেওয়া থাকবে

$$f(1) = \alpha_1$$
$$f(2) = \alpha_2$$
$$f(dn+0) = cf(n) + \beta_0$$
$$f(dn+1) = cf(n) + \beta_1$$
$$f(dn+2) = cf(n) + \beta_2$$

$f(\text{something}) = ?$ $\boxed{d \ c \ \beta, \alpha}$ -এর মান দেয়া থাকবে

d নিব → 'something'-কে d base-এ ভাঙবো.



so $f(\text{something}) = (\ \ \ )_d$

$= (\ \ :\ \ )_c$ -এ নিব

example $= (\alpha_0\ \beta_1\ \beta_2)_c$

$= (\alpha_0 \times c^2 + \beta_1 \times c^1 + \beta_2 \times c^0)$

$= \boxed{\text{Ans.}}$

এটাও binary version

$$f(1) = 1 \leftarrow \alpha$$
$$f(2n+0) = 2f(n) - \beta_0$$
$$f(2n+1) = 2f(n) + \beta_1$$

d

c

④ Perceptron : দুইটা pattern দেয়া থাকবে $w_1 = (\cdot\ \cdot), (\cdot\ \cdot)(\cdot\ \cdot)$

$w_2 = (\ \ ), (\ \ )(\ \ )$.

আবার α ও দেওয়া থাকবে

না থাকলে $\rightarrow \alpha = \frac{1}{\text{no. of data set (এখানে 6)}}$

$$\phi(x) = y = [x_1^2 \quad x_2^2 \quad x_1x_2 \quad x_1 \quad x_2 \quad 1]$$

① w ধরে নিব $\rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

w^T - এতে নিব $\rightarrow [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$

② Table আঁকব

$y = [x_1^2 \quad x_2^2 \quad x_1x_2 \quad x_1 \quad x_2 \quad 1]$

points		y_{ij}
(,)	y_{11}	
(,)	y_{12}	

③ One at a time-এর জন্য.

$w^T y_{ij} \rightarrow [\ \ ] [\ ] \rightarrow$ বের করব if $g(y) > 0$ okk

if $g(y) < 0$

(w change) $\rightarrow w = w + \alpha y_{ij}$

- last-এ table বানাব

y_{ij}	w	$g(y)$

More ~~than~~ at a time-এর জন্য

$w^T y_{ij} \rightarrow g(y) > 0$

$g(y) < 0 \rightarrow$ মনে রাখব পরে যোগ করে w update করব একসাথে

③ Stern brocot tree : $f(LRRL) = ?$ বললে

$L = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $R = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ মনে রাখব

just matrix-গুলা বসায় গুন দিব

ans matrix-এর → $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

যোগ = $\boxed{\dfrac{\text{numerator}}{\text{denominator}}}$ ← যোগ

ans.

** LRRL না দিয়ে fraction দিয়ে sequece চাইলে:

$\dfrac{m}{n}$ → check if $m>n \rightarrow R$

if $m<n \rightarrow L$

প্রতিবারই বড়টা - ছোটটা করব.

	Pass 1	Pass 2	Pass 3	Pass 4	
$m =$	5	5	$(5-2)=3$	$(3-2)=1$	1
$n =$	7	$(7-5)=2$	2	2	$(2-1)1$
	L	R	R	L	সমান হয়ে গেছে

level চাইলে → Pass+1 → 4+1=5th level.

Proof 3টা → ① $\dfrac{m}{n}$, $\dfrac{m'}{n'}$ cosecutive fraction. (Induction করব) | $m'n - mn' = 1$ − এর উপরে

② $\dfrac{m}{n} < \dfrac{m'}{n'}$ so is $\dfrac{m}{n} < \dfrac{m+m'}{n+n'} < \dfrac{m'}{n'}$

$\dfrac{m+m'}{n+n'} - \dfrac{m}{n}$ → করব

$\dfrac{m'}{n'} - \dfrac{m+m'}{n+n'}$ → করব.

③ Is there any positive a/b with $a \perp b$ in the tree?

if $\frac{a}{b} \neq \frac{m+m'}{n+n'}$ ~~and~~

and if $\frac{m+m'}{n+n'} < a/b$ $\quad$ $m \leftarrow m+m'$, $n \leftarrow n+n'$ | move to right

else move to left.

এভাবে inequality equality-র দিকে যাবে

must find a/b after $(a+b)$ steps

MLE & MAP Classifier.

Date :

Proofs:

① $\boxed{\frac{m}{n} \text{ and } \frac{m'}{n'} \text{ are consecutive fractions}}$

$\frac{m}{n} \times \frac{m'}{n'}$
$m'n - mn'$

we have $m'n - mn' = 1$

Basis: $\Rightarrow \frac{m}{n} = \frac{0}{1}$ and $\frac{m'}{n'} = \frac{1}{0}$; $m'n - mn' = 1 \cdot 1 - 0 \cdot 0 = 1$

Hypothesis: $m'n - mn' = 1$ is true for any stage in Stern Brocot tree.

Induction: median of $\frac{m}{n}$ and $\frac{m'}{n'}$ is $\frac{m+m'}{n+n'}$

So, $(m+m')n - m(n+n') = \cancel{mn} + nm' - \cancel{mn} - mn' = m'n - mn'$
$= 1$

$m'(n+n') - n'(m+m') = m'n + \cancel{m'n'} - n'm - \cancel{n'm'} = m'n - n'm$
$= 1$ (Proved)

② $\boxed{\text{If } \frac{m}{n} < \frac{m'}{n'} \text{ and all values are nonnegative then } \frac{m}{n} < \frac{m+m'}{n+n'} < \frac{m'}{n'}}$

So, $\frac{m+m'}{n+n'} - \frac{m}{n} = \frac{\cancel{mn} + m'n - \cancel{mn} - mn'}{n(n+n')} = \frac{m'n - mn'}{n(n+n')} = \frac{1}{n(n+n')}$

$\frac{m'}{n'} - \frac{m+m'}{n+n'} = \frac{m'n + \cancel{m'n'} - mn' - \cancel{m'n'}}{n'(n+n')} = \frac{m'n - mn'}{n'(n+n')} = \frac{1}{n'(n+n')}$

as $n>0$ and $n'>0$ and so is $\frac{1}{n(n+n')}, \frac{1}{n'(n+n')} > 0$

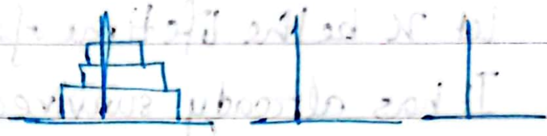
Thus, $\frac{m}{n} < \frac{m+m'}{n+n'} < \frac{m'}{n'}$.

RECURRENT PROBLEMS

Date:

3 types of recurrent problems: ① Tower of Hanoi
② Pizza Cutting
③ Josephus.

Tower of Hanoi (Derivations):



Single TOH: $T_n = T_{n-1} + 1 + T_{n-1}$

$$= 2T_{n-1} + 1$$
$$= 2(2T_{n-2} + 1) + 1$$
$$= 2^2 T_{n-2} + 2 + 1$$
$$= 2^2 (2T_{n-3} + 1) + 2 + 1$$
$$= 2^3 T_{n-3} + 2^2 + 2 + 1$$
$$\vdots$$
$$= 2^n T_{n-n} + 2^{n-1} + 2^{n-2} \ldots\ldots + 2^n.$$
$$= \frac{(2^n - 1)}{(2-1)} = \boxed{2^n - 1}$$ **

Basis: $T_0 = 2^n - 1 = 2^0 - 1 = 0.$

Hypothesis: $T_n = 2^n - 1$

Induction: $\cancel{T_{n+1} = 2^{n+1} - 1}$

$$\cancel{= 2^n . 2 - 1}$$
$$\cancel{= 2(T_n + 1) - 1}$$
$$\cancel{= 2T_n + 2 - 1 = 2T_n}$$

$$T_{n+1} = 2T_n + 1$$
$$= 2(2^n - 1) + 1$$
$$= 2^{n+1} - 2 + 1$$
$$= 2^{n+1} - 1.$$

Date :

Double TOH :

$T_n = T_{n-1} + 2 + T_{n-1}$

$= 2T_{n-1} + 2$

$= 2^2(T_{n-1} + 2) + 2$

$= 2^3 T_{n-3} + 2^3 + 2^2 + 2$

$\vdots$

$= 2^n T_{n-n} + 2^n + 2^{n-1} \dots + 2$

$= 2[2^0 + 2^1 + \dots + 2^{n-1}]$

$= 2 \frac{(2^{n-1+1} - 1)}{(2-1)} = 2(2^n - 1)$

Basis: $T_0 = 2(2^0 - 1) = 0$.

Hypothesis: $T_{n-1} = 2(2^{n-1} - 1)$.

Induction: $T_n = 2T_{n-1} + 2$.

$= 2[2(2^{n-1} - 1)] + 2$

$= 2(2^{n-2} - 2) + 2$

$= 2^{n-1} - 4 + 2$

$= 2(2^n - 1)$

Triple TOH :

$T_n = T_{n-1} + 3 + T_{n-1}$

$= 2T_{n-1} + 3$

$= 2(2T_{n-2} + 3) + 3$

$= 2^2 T_{n-2} + 2.3 + 3$.

$= 2^2(2T_{n-3} + 3) + 2.3 + 3$

$= 2^3 T_{n-3} + 2^2.3 + 2.3 + 3$.

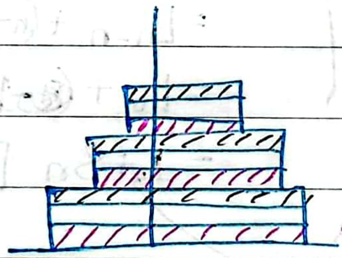
$\vdots$

$= 2^n T_{n-n} + 2^{n-1}.3 + \dots + 2^0.3$.

$= 3[2^0 + 2^1 + \dots + 2^{n+1}]$

$= 3 \frac{(2^{n-1+1} - 1)}{(2-1)}$

$= 3(2^n - 1)$



Triple TOH = 1.5 × Double TOH

Base: $T_0 = 3(2^0 - 1) = 0$

Hypothesis: $T_{n-1} = 3.(2^{n-1} - 1)$

Induction: $T_n = 2T_{n-1} + 3$

$= 2.3(2^{n-1} - 1) + 3$

$= 2.3.2^{n-1} - 2.3 + 3$

$= 2^n.3 - 2.3 + 3$.

$= 2^n.3 - 3$. [∵ -6+3]

$= 3(2^n - 1)$.

(10) MLE + MAP/Bayes:

Class + Data ques-এ থাকবে.

** $\boxed{\mu \text{ (mean)}, \ \sigma^2 \text{ (variance)}}$; Data দিয়ে দিলে [example-length]

class-এর info.

in MLE → $P(\text{Data} \mid \text{Class}) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(l-\mu)^2}{2\sigma^2}}$

in MAP → $P(\text{Class} \mid \text{Data}) = \dfrac{P(\text{Class})\, P(\text{Data} \mid \text{Class})}{P(\text{Data})}$.

** Data না থাকলে — μ, σ^2 থাকলে + Prior probability বলে দিলে (as in ~~P(Salmon~~ Salmon is twice Sea Bass)

$$P_1(\text{Class} \mid \text{Data}) = P_2(\text{Class} \mid \text{Data})$$

→ এভাবে L-এর মান বের করব (log use করা লাগে solve-এ)

using MAP cause MLE is not affected by prior probability

** Suppose subject numbers + data set points দেওয়া

μ, σ^2 নাই।

তাইলে বের করব,

$$\text{mean, } \mu = \frac{\sum \text{data}}{(\text{total class number})}$$

$$\text{variance } \sigma^2 = \frac{\sum (\mu - \text{data})^2}{(\text{total class number})}$$

prior probability : বের করব

$$P(\text{Class1}) = \frac{\text{no. of class1}}{\text{total in both class}}$$

$$P(\text{class2}) = \frac{\text{no of class2}}{\text{total in both class.}}$$

Rules:

$$\binom{r}{k} \rightarrow \begin{cases} 0 & \text{if } k<0 \ (-ve) \\ 1 & \text{if } k=0 \\ 0 & \text{if } k>r \\ 1 & \text{if } k=r \end{cases}$$

4টা proof →

1. Symmetry Identity $\binom{r}{k} = \binom{r}{r-k}$
2. Absorption Identity $\binom{r}{k} = \frac{r}{k} \times \binom{r-1}{k-1}$
3. Addition Identity $\binom{r}{k} = \binom{r-1}{k} + \binom{r-1}{k-1}$
4. Summation Identity. $\sum_{k \le n} \binom{r+k}{k} = \binom{r+n+1}{n}$

Date :

Symmetry Identity: $\binom{n}{k} = \binom{n}{n-k}$

$\binom{4}{3} = \binom{4}{4-3} = \binom{4}{1}$

$\binom{4}{3} \to$ নিব ; $\binom{4}{1} \to$ নিব না ; same.

$\boxed{\text{proof:}}$ $LHS = \binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$

$$= \frac{n!}{(n-(n-k))!\;(n-k)!}$$

$$= \binom{n}{n-k}.$$

$\boxed{\text{Symmetry holds or not?}}$

** if $n<0$; n negative.

let $n=-1$

whether $\binom{-1}{k} = \binom{-1}{-1-k}$ holds or not?

** when $k=0$;

$LHS = \binom{-1}{0} = 1$

$RHS = \binom{-1}{-1-0} = \binom{-1}{-1} = 0 \; [\because k<0]$

$LHS \neq RHS$.

doesn't hold

** when $k>0$; (+ve).

$$L.H.S = \binom{-1}{k} = \frac{-1 \cdot -2 \cdot -3 \cdots\cdots (-1-k+1)}{k!}$$

$$= \frac{(-1)^k \cdot -2 \cdot -3 \cdots\cdots -k}{1 \cdot 2 \cdot 3 \cdots\cdots k}$$

$$= (-1)^k = \pm 1$$

Date :

$$R.H.S = \binom{-1}{-1-K} = 0$$

$\Rightarrow$ then k totally becomes negative

Thus L.H.S $\neq$ RHS. (doesn't hold)

** if $K<0$; (-ve)

$$LHS = \binom{-1}{-k} = 0$$

$$RHS = \binom{-1}{-1-(-K)} = \binom{-1}{-1+k}$$

$$= \frac{-1 . -2 . -3 \ldots\ldots (-1+1-k-1)}{(k-1)!}$$

$$= \frac{-1 \; -2 \; -3 \ldots\ldots -(k-1)}{1 . 2 . 3 \ldots\ldots (k-1)}$$

$$= (-1)^{k-1} = \pm 1.$$

Again LHS $\neq$ RHS. (doesn't hold)

CamScanner

FLOOR: $\lfloor x \rfloor$ greatest integer less than or equal to x

is n such that

$$\boxed{\begin{array}{l} n \leq x \\ n+1 > x \end{array}}$$

CEIL: $\lceil x \rceil$ The least integer greater than or equal to x is n such that $n \geq x$,

$n-1 < x$.

$\lfloor 2.6 \rfloor = 2$

$\lfloor 3 \rfloor = 3$.

$\lfloor -1.5 \rfloor = -2$

$\lceil 2.6 \rceil = 3$.

$\lceil 3 \rceil = 3$.

$\lceil -1.5 \rceil = -1$

Rules:

1. $\lfloor x \rfloor = x$ or $\lceil x \rceil = x \iff x$ is integer
2. $\lfloor -x \rfloor = -\lceil x \rceil$

   $\lceil -x \rceil = -\lfloor x \rfloor$
3. $\lfloor x \rfloor = n \iff n \leq x < n+1$
4. $\lfloor x \rfloor = n \iff x-1 < n \leq x$
5. $\lceil x \rceil = n \iff n-1 < x \leq n$.
6. $\lceil x \rceil = n \iff x \leq n < x+1$

5. $\lfloor x+n \rfloor = \lfloor x \rfloor + n$, integer n.
6. $n \leq x \iff n \leq \lfloor x \rfloor$
7. $x \leq n \iff \lceil x \rceil \leq n$.

Date :

☐ If n is an m-bit integer, then prove that

$$m = \lfloor \log n \rfloor + 1$$

n is an m-bit integer

n is like $\underbrace{1xxxx\cdots x}_{m\text{ bits.}}$; which means.

n is at least $\underbrace{1000\cdots\cdot 0}_{m\text{ bits}} = 2^{m-1}$

n is at most $\underbrace{1111\cdots 1}_{m\text{ bits}} = \underbrace{1000\cdots 0}_{m+1\text{ bits}} - 1 = 2^m - 1$

So, $2^{m-1} \leq n < 2^m$

$\Rightarrow \log 2^{m-1} \leq \log n < \log 2^m$

$\Rightarrow (m-1)\cancel{\log 2} \leq \log n < m\cancel{\log 2}$

$\Rightarrow (m-1) \leq \log n < m$

$\Rightarrow \lfloor \log n \rfloor = (m-1)$ [$\because n \leq x < n+1 \Leftrightarrow \lfloor x \rfloor = n$]

$\Rightarrow m = \lfloor \log n \rfloor + 1$

Date :

Example : $n = 35$; $m = ?$

$m = \lfloor \log_2 35 \rfloor + 1$

$= 5 + 1$

$= 6$ (Ans.)

6 bits is needs to write 32. and 5 for 31.

□ Prove/disprove → $\lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$; for $x \geq 0$.

If x is integer, $\lfloor x \rfloor = x$.

$L.H.S = \lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor = RHS$.

again if x is not integer,

let, $m = \lfloor \sqrt{\lfloor x \rfloor} \rfloor$

$\Rightarrow m \leq \sqrt{\lfloor x \rfloor} < m+1$ [$\because n = \lfloor x \rfloor \Leftrightarrow n \leq x < n+1$]

$\Rightarrow m^2 \leq \lfloor x \rfloor < (m+1)^2$

$\Rightarrow m^2 \leq x < (m+1)^2$ [$\because n \leq x \Leftrightarrow n \leq \lfloor x \rfloor$]

$\Rightarrow m^2 \leq \sqrt{x} < (m+1)^2$

$\Rightarrow m \leq \sqrt{x} < (m+1)$

$\Rightarrow m = \lfloor \sqrt{x} \rfloor$ [$\because n \leq x < n+1 \Leftrightarrow n = \lfloor x \rfloor$]

$LH.S = RHS$. (Proved)

□ MOD: $n \% m$.

$n = \lfloor \frac{n}{m} \rfloor \times m + n \bmod m$.

$\Rightarrow n \bmod m = n - m \lfloor \frac{n}{m} \rfloor$; $y \neq 0$.

□ Properties:

$$c(x \bmod y) = cx \bmod cy$$

$$\begin{aligned}
LHS &= c(x \bmod y) \\
&= c\left(x - y\left\lfloor \frac{x}{y} \right\rfloor\right). \\
&= cx - cy\left\lfloor \frac{x}{y} \right\rfloor \\
&= cx - cy\left\lfloor \frac{cx}{cy} \right\rfloor \\
&= cx \bmod cy. = RHS.
\end{aligned}$$

Date :

prove or disprove $(x \bmod ny) \bmod y = x \bmod y$.

if y is 0. $\therefore \rightarrow ny = 0$.

$$L.H.S = (x \bmod 0) \bmod y$$
$$= x \bmod y = RHS.$$

if $y \neq 0$,

$$LHS = (x \bmod ny) \bmod y$$
$$= \left(x - ny\left\lfloor \frac{x}{ny} \right\rfloor\right) \bmod y$$
$$= \left(x - ny\left\lfloor \frac{x}{ny} \right\rfloor\right) - y\left\lfloor \frac{x - ny\lfloor \frac{x}{ny} \rfloor}{y} \right\rfloor$$
$$= x - ny\left\lfloor \frac{x}{ny} \right\rfloor - y\left\lfloor \frac{x}{y} - \frac{ny\lfloor \frac{x}{ny} \rfloor}{y} \right\rfloor$$
$$= x - \cancel{ny\left\lfloor \frac{x}{ny} \right\rfloor} - y\left\lfloor \frac{x}{y} \right\rfloor + \cancel{ny\left\lfloor \frac{x}{ny} \right\rfloor}$$
$$= x - y\left\lfloor \frac{x}{y} \right\rfloor$$
$$= x \bmod y = R.H.S. \text{ (proved)}.$$