Magnetic

Spring - 2018

(*) Biot-Savart Law: The magnetic field due to a current carrying conductor at a distance point is inversely proportional to the square of the distance between the conductor and point, and the magnetic field is directly proportional to the length of the conductor and current flowing in the conductor.

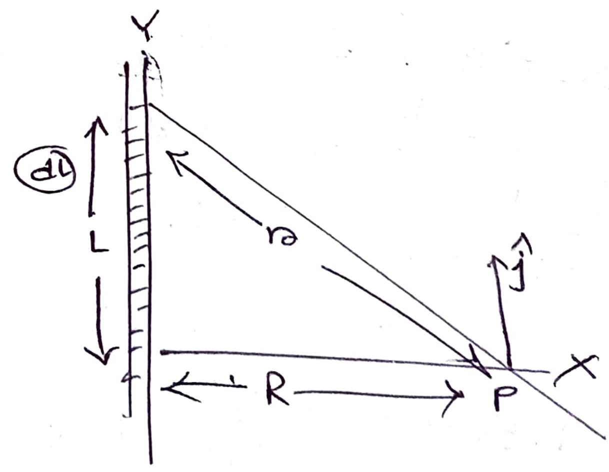
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\bar{l}}{r^2}$$

(*) Ampere's Law: The law states that the line integral of $\vec{B}$ around a close path is equal to μ_0 times the current enclosed by the path. If the path does not enclose the current then,

$$\oint \vec{B} . dl = 0 .$$

(*) Using Biot-Savart's law - magnetic field by a long straight wire carrying current I :-

The magnetic field due to the current I, here will be



$$d\bar{B} = \frac{\mu_0}{4\pi} I \left(\frac{d\bar{l} \times \hat{r}}{r^2} \right)$$

$$= \frac{\mu_0}{4\pi} \frac{I dl}{r^2} (\hat{j} \times \hat{r}) .$$

here, $\hat{j}$ is the vector along Y axis

$\hat{r}$ is " " " r .

or, $$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

The direction of $\vec{dB}$ at point P for all element are the same.

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int_{l=-\infty}^{l=\infty} \frac{\sin\theta \, dl}{r^2}$$

Here, $r = \sqrt{l^2 + R^2}$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{l^2 + R^2}}$$

So, $$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{R\,dl}{(l^2 + R^2)^{3/2}}$$

also, $l = R\tan\phi$, $dl = R\sec^2\phi \, d\phi$

therefore, $$B = \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{+\pi/2} \cos\phi \, d\phi$$

$$= \frac{\mu_0 I}{4\pi R} \left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$$

$$= \frac{\mu_0 I}{4\pi R} \times 2 = \frac{\mu_0 I}{2\pi R}.$$

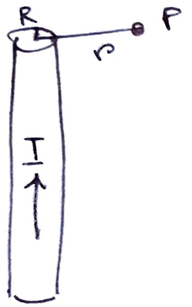
So, $$\vec{B} = \frac{\mu_0 I}{2\pi R} \hat{\theta}_0.$$

Spring - 2019

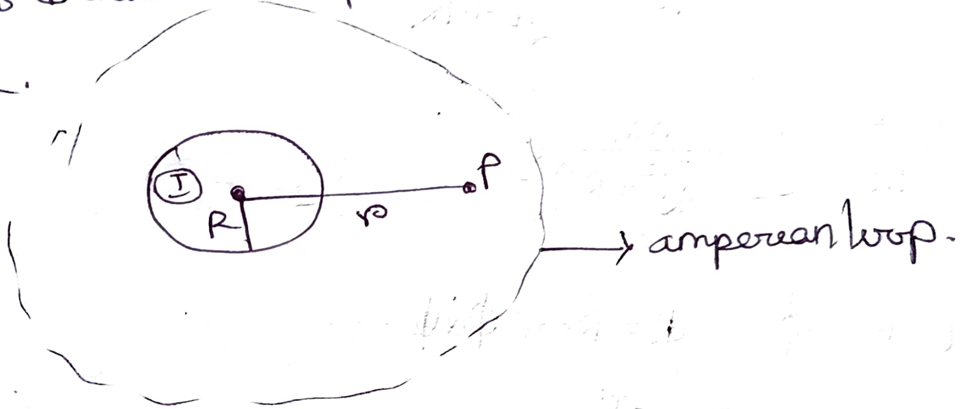
* Magnetic field + Lorentz force.

Ampere's law around a long wire carrying I:

(Outside the wire).



let us draw an amperean loop around the wire.



According to ampere's law,

$$\oint \bar{B}\,d\bar{l} = \mu_0 i_{enc}$$

$$\Rightarrow \oint B\,dl \sin\theta = \mu_0 i_{enc}$$

$$\cancel{\Rightarrow B\,dl \oint \sin\theta = \mu_0}$$

$$\Rightarrow \bar{B}\,.\,2\pi r = \mu_0 i_{enc} \quad \left[\because \oint dl = 2\pi r\right]$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} \quad \left[\because i_{enc} = I\right]$$

$$\therefore B = \frac{\mu_0 I}{2\pi r} \quad \text{(Ans)}$$

Fall - 2018

* **Magnetic field :** The space around a magnet or current carrying conductors in which a moving charge experiences a sideway deflecting force is called magnetic field.

* **Mag- force :** The force exerted by a magnetic field on a moving charge is called magnetic force.

* **Lorentz force :-** The force exerted on a charge q_0 moving with a velocity v through a region in which both a electric field E and magnetic field B are present.

It is $\rightarrow \bar{F} = \bar{F}_B + \bar{F}_E$

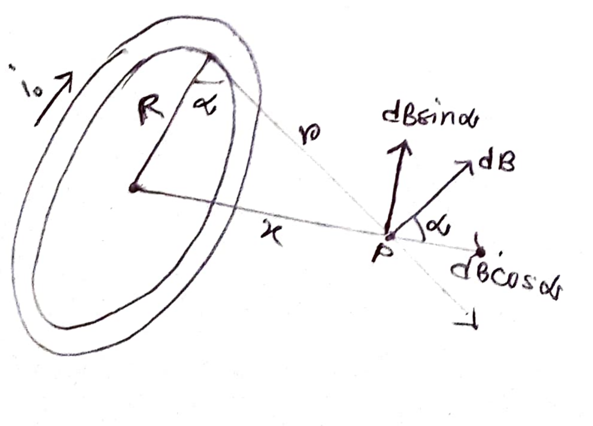
Here, $\bar{F}_B = q(\bar{v} \cdot \times \bar{B})$,

$\bar{F}_E = q\bar{E}$ [Not depend on v].

$$\bar{F} = q(\bar{v} \times \bar{B}) + q\bar{E}$$

$$= q\{\bar{E} + (\bar{v} \times \bar{B})\} \longrightarrow \boxed{\text{Lorentz force}}.$$

* **B at the center of a loop :** (Circular Coil)



If we resolve dB into two components, one $d\bar{B}_{\parallel}$ along axis of loop and another $d\bar{B}_{\perp}$ at right-angles to the axis,
So, we have $d\bar{B}_{\parallel}$ only as $d\bar{B}_{\perp}$ will be opposite to direction.

So, $B = \int dB = \int \frac{\mu_0 I}{4\pi r^2} dl \cos\alpha$ $[\because dB_{\parallel} = dB\cos\alpha]$

We know, $r = \sqrt{R^2 + x^2}$

$$\cos\alpha = R/r = \frac{R}{\sqrt{R^2+x^2}}$$

So, $B = \int \frac{\mu_0 I}{4\pi} \cdot \frac{R\,dl}{\left(\sqrt{R^2+x^2}\right)^2}$

$$= \int \frac{\mu_0 I}{4\pi} \frac{R\,dl}{(R^2+x^2)^{3/2}} = \frac{\mu_0 I R}{4\pi(R^2+x^2)^{3/2}} \int dl$$

$$= \frac{\mu_0 I R}{4\cancel{\pi}(R^2+x^2)^{3/2}} 2\cancel{\pi}R \quad [\because \int dl = 2\pi R]$$

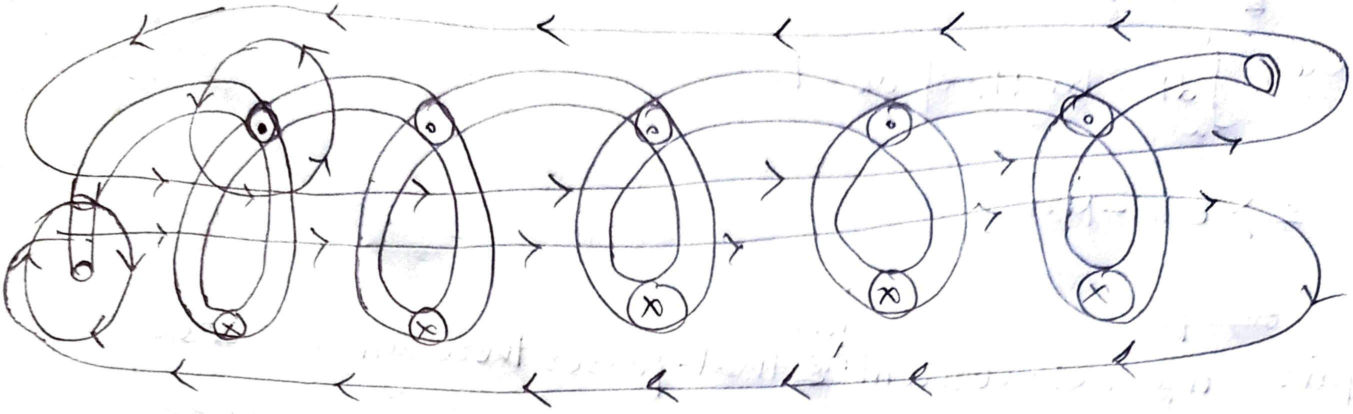
$$= \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

At the center $x=0$,

So, $B = \frac{\mu_0 I R^2}{2 R^{\cancel{2}\cdot 3/\cancel{2}}} = \frac{\mu_0 I R^2}{2R^3} = \frac{\mu_0 I}{2R}$

Fall-2014

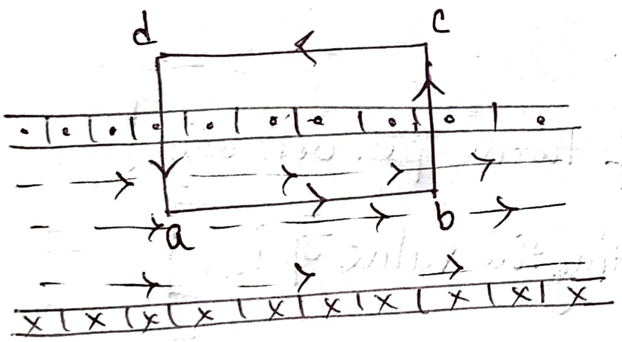
⊛ B for Solenoid using Ampere's law:



Outside the solenoid:

Magnitude of magnetic field reaches zero.

Inside the solenoid:



We know, $\oint \bar{B} d\bar{l} = \mu_0 i_{enc}$.

$\Rightarrow \int_a^b \bar{B} \cdot d\bar{l} + \int_b^c \bar{B} \cdot d\bar{l} + \int_c^d \bar{B} \cdot d\bar{l} + \int_d^a \bar{B} \cdot d\bar{l} = \mu_0 i_{enc}$.

$\Rightarrow \int_a^b B dl \cos\theta + \int_b^c B dl \cos\theta + \int_c^d B dl \cos\theta + \int_d^a B dl \cos\theta = \mu_0 i_{enc}$.

$\Rightarrow \int_a^b B dl \cos 0^\circ + \int_b^c B dl \cos 90^\circ + \int_c^d B dl \cos 180^\circ + \int_d^a B dl \cos 90^\circ = \mu_0 i_{enc}$

$\Rightarrow \int_a^b B dl + 0 + 0 + 0 = \mu_0 i_{enc}$.

$$\Rightarrow B\int_a^b dl = \mu_0 i_{enc}$$

~~$\Rightarrow B \cdot ab = \mu_0 i_{enc}$~~

Here, $\int_a^b dl$ = Length of ab = h.

So, $\Rightarrow Bh = \mu_0 i_{enc}$

~~or, B~~

Again, the net current i_{enc} that passes through the area bounded by the path of integration is not equal to I in the solenoid because it is the integrated one that is enclosed more than one turn.

~~So, $I = i_{enc}(nh)$.~~

~~Now, $Bh = \mu_0 ($~~

So, $i_{enc} = I(nh)$ [n be the no. of turns per unit length]

Now, $Bh = \mu_0 I(nh)$. [Putting the value of i_{enc}]

$$\Rightarrow B = \mu_0 In.$$

or, $B = \mu_0 I \frac{N}{l}$ [N = total number of turns]