Date of Examination :24/11/2019

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
Department : Arts and Sciences
Program : Bachelor of Science in Computer Science and Engineering
Semester Final Examination: Spring 2019
Year: 1st **Semester: 2nd**
Course Number: MATH 1219
Course Name: Mathematics II

Time: 3 (Three) hours **Full Marks: 70**

Instruction: There are 7 (seven) questions in group A and B. Answer 5 (five) questions, taking 3 (three) from Group A and 2 (two) from Group B.
Marks allotted are indicated in the right margin.

Group-A

1. a. Evaluate the following indefinite integrals: [14]

(i) $\int \frac{\sqrt{a+x}}{\sqrt{a-x}}dx$, (ii) $\int \tan^{-1}\frac{2x}{1-x^2}dx$, (iii) $\int \frac{a\sin^3 x - b\cos^3 x}{\sin^2 x\cos^2 x}dx$,

(iv) $\int \frac{x^2}{(x+1)(x+2)^2}dx$.

2. a. Evaluate the following definite integrals: [8]

(i) $\int_8^{15} \frac{1}{(x-3)\sqrt{x+1}}dx$, (ii) $\int_0^{\pi/2} \frac{1}{3+2\cos x}dx$.

b. Show that $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}dx = \frac{\pi}{2}\log\frac{1}{2}$. [6]

3. a. Obtain Walli's formula for $I_n = \int_0^{\pi/2} \sin^n x\, dx$. [6]

b. Define Beta and Gamma function. Hence evaluate (i) $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$, (ii) $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}} dx$. [8]

4 a. Find the arc length of the curve $ay^2 = x^3$ from the origin to the point whose abscissa is b. [5]

b. Find the area bounded by the parabolas $y^2 = 5x+6$ and $x^2 = y$. [5]

c. Find the volume of the solid generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about the initial line. [4]

Group-B

5. a. Define differential equation, and its order and degree. Solve $x(y^2+1)dx + y(x^2+1)dy = 0$. [5]

b. When a differential equation is said to be homogeneous? Is $(x^2+y^2)dy = xydx$ homogeneous? Solve it. [5]

c. Is the differential equation $(2x-y+1)dx+(2y-x-1)dy=0$ exact? Solve it. [4]

6. a. Reduce the differential equation $\frac{dy}{dx}+x\sin 2y = x^3\cos^2 y$ in the form of linear differential equation and solve it. [5]

b. What is Clairaut's equation? Solve: $y = 2px + p^2$, where $p = \frac{dy}{dx}$. [5]

c. Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y = e^{-x}$. [4]

7. a. Solve $\frac{d^2y}{dx^2}+4y=\cos 2x$. [4]

b. Solve $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=\sin(\ln x)$. [5]

c. By direct integration, solve $\frac{\partial^2 z}{\partial x\partial y}=x^2y$ subject to the condition $z(x,0)=x^2$ and $z(1,y)=\cos y$. [5]