

EEE1241

06.01.2019

Course Title: Basic Electrical Engineering

Course Teacher: Ayman Uddin Mahin.

Total Que: 8

Have to answer: 6

Voltage: Potential difference between points at a circuit is known as voltage.

If a total of 1 joule (J) of energy is used, to move a charge of 1 coulomb (C), Here is a difference of 1 volt (V) between the two points.

Current: The rate of charge flow is known as current.

$$I = \frac{dQ}{dt} \text{ or, } I = \frac{Q}{t}$$

Unit: Ampere (A)

Power: Power is the energy per unit of time.

$$\text{power} = \frac{\text{energy}}{\text{time}}$$

$$\text{power} = \text{voltage} \times \text{current}$$

$$\Rightarrow P = V I$$

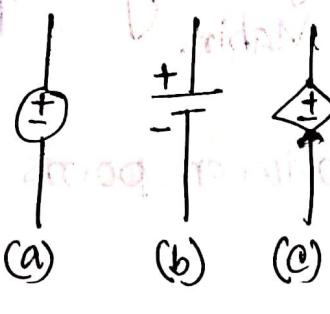
Unit:  $\text{Js}^{-1}$  or Watt (W)

Direct Currents: DC is the unidirectional current of constant magnitude.

$\Delta$  = Dependent source

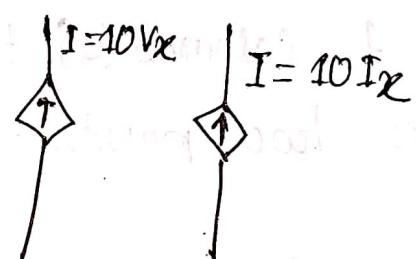
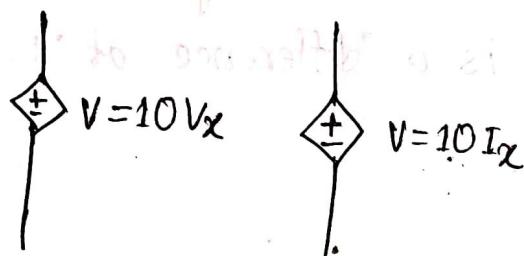
others shape = independent

## Sources:



Voltage source  
(voltage constant)

Current source.  
(current constant)



Voltage controlled	current controlled
Voltage controlled	v. source
a, b, d	independent
c, e	→ dependent

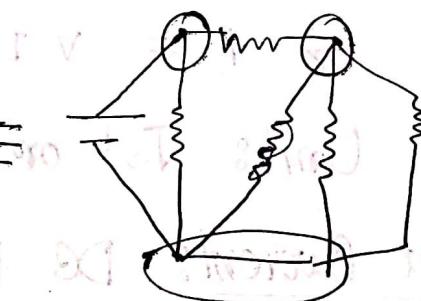
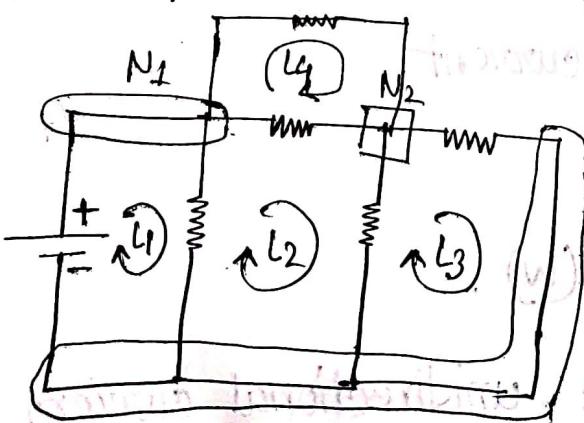
Voltage controlled	current controlled
Controlled	source
current	current
source	source

No: B8

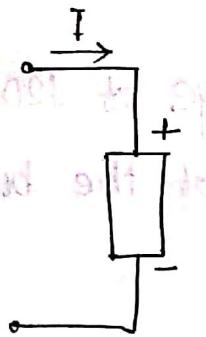
a, b, d → independent  
c, e → dependent

Node: Node is a junction at a circuit, where two or more circuit elements meet together.

Loop: Loop is a closed path

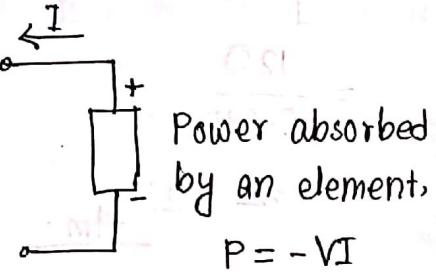


## Power



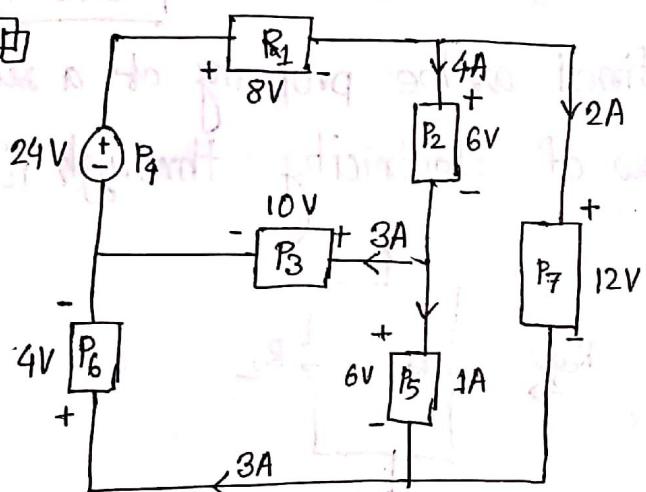
Power absorbed  
by an element,

$$P = VI$$

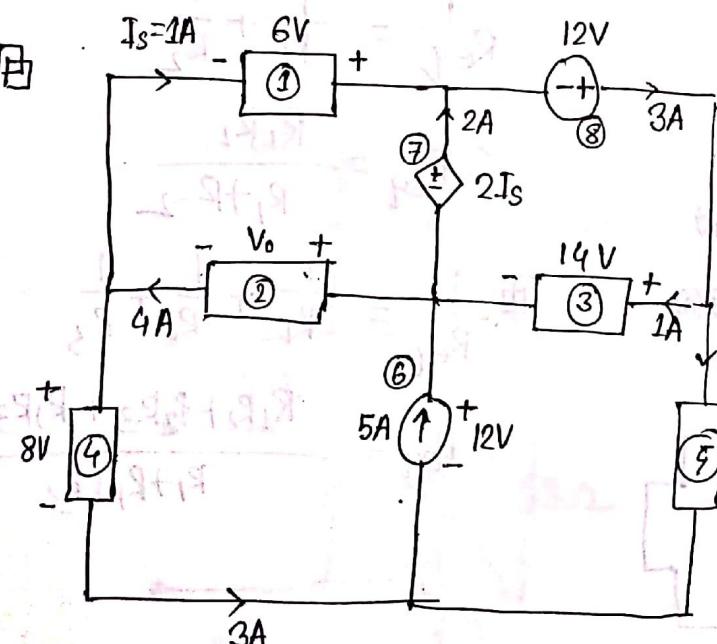


Power absorbed  
by an element,

$$P = -VI$$



\* 1 Que  
Marks 18 = 6



$$\text{Now, } -6 + 4V_o + 14 + 24 + 52 - 60 - 4 - 36 = 0$$

$$\Rightarrow V_o = 4 \text{ Volt.}$$

$$-6+4V_0+14+24+52$$

A bulb consumes 75W at a rated voltage of 120V. Find the current through the bulb and resistance of the bulb.

Here,

$P = 75 \text{ W}$

$V = 120$

$$I = \frac{P}{V} = \frac{75}{120} = 0.625 \text{ A}$$

Ans.

$R = \frac{V}{I}$

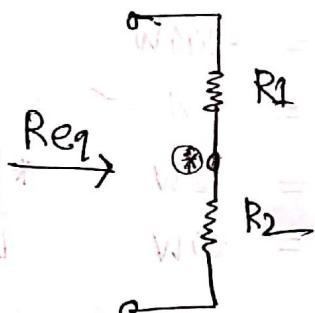
$$= \frac{120}{0.625}$$

$$= 192 \Omega$$

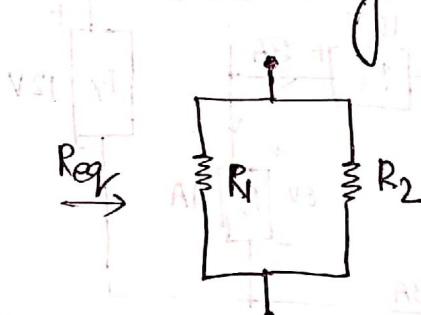
$$\text{Ans.}$$

[13.01.19]

**Resistance:** Resistance is defined as the property of a substance due to which it opposes flow of electricity through it.



$$Req = R_1 + R_2$$

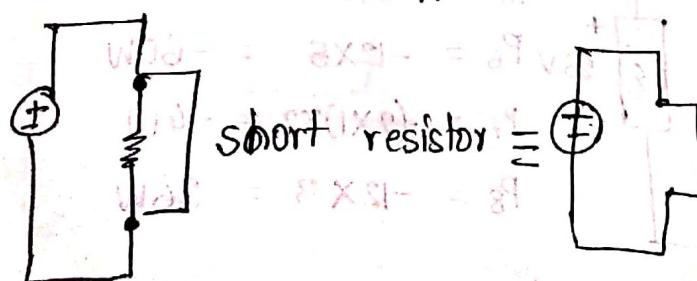


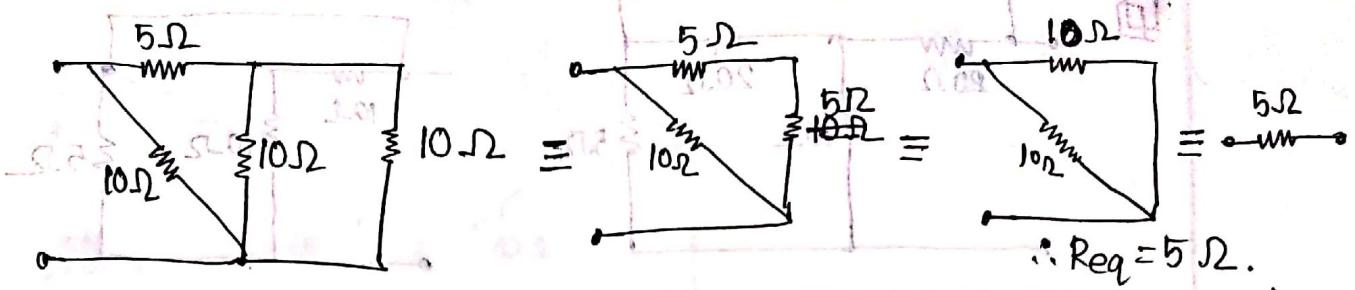
$$\frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore Req = \frac{R_1 R_2}{R_1 + R_2}$$

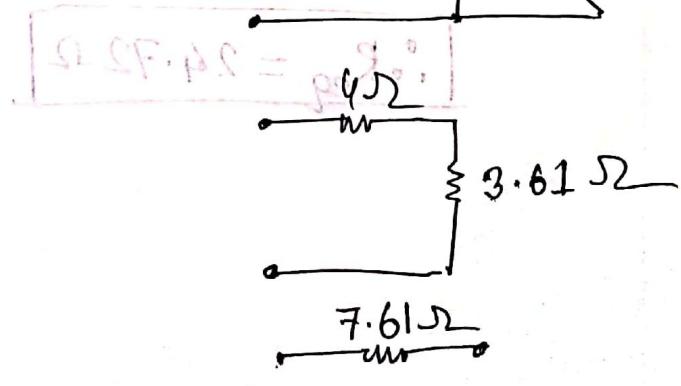
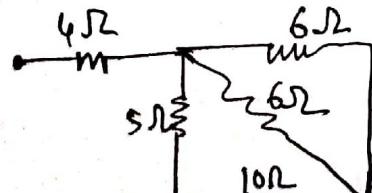
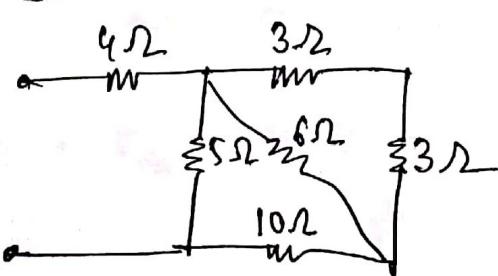
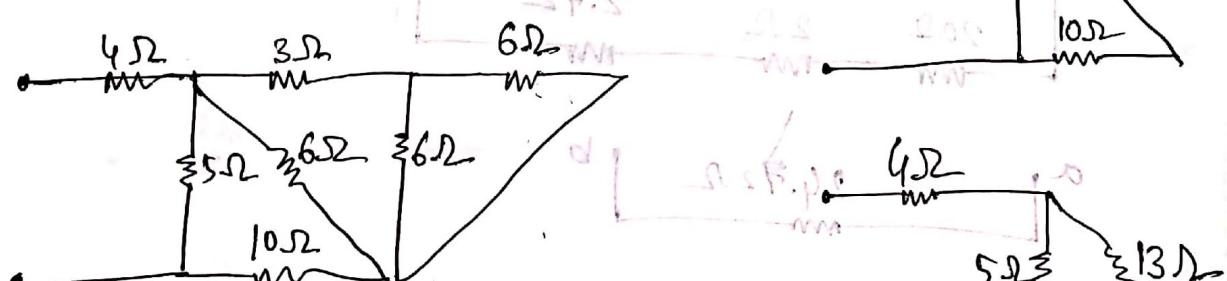
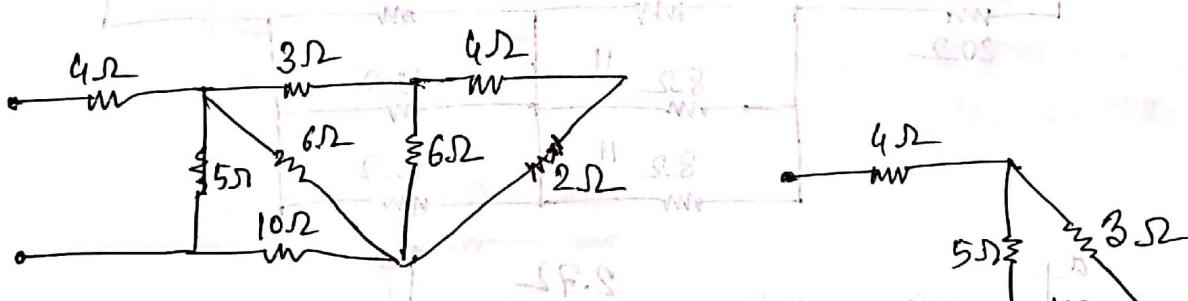
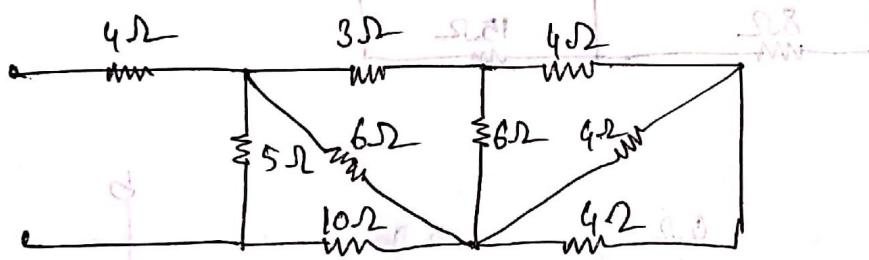
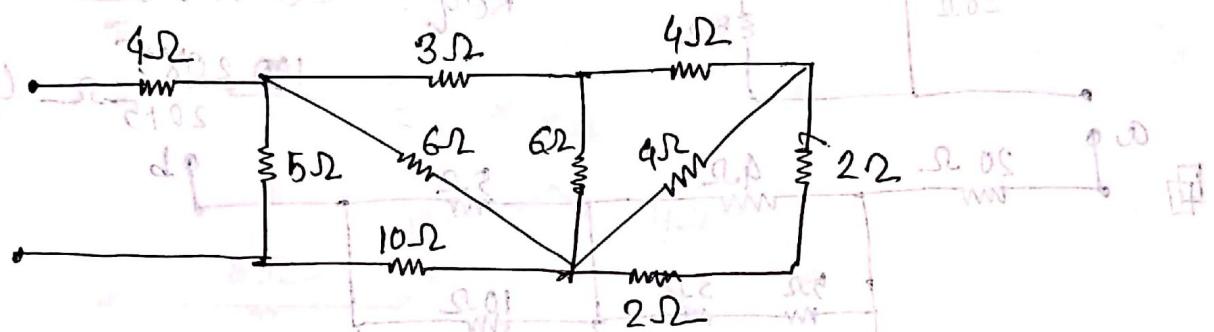
$$\frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

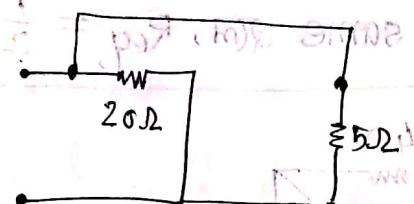
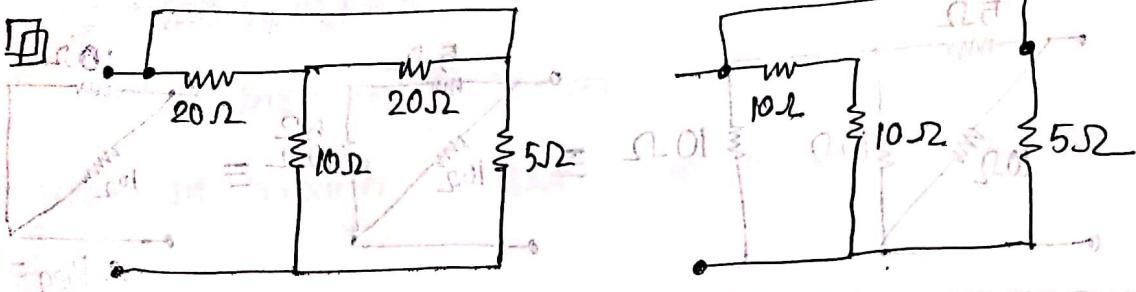
$$\therefore Req = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1 + R_2 + R_3}$$





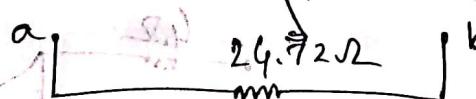
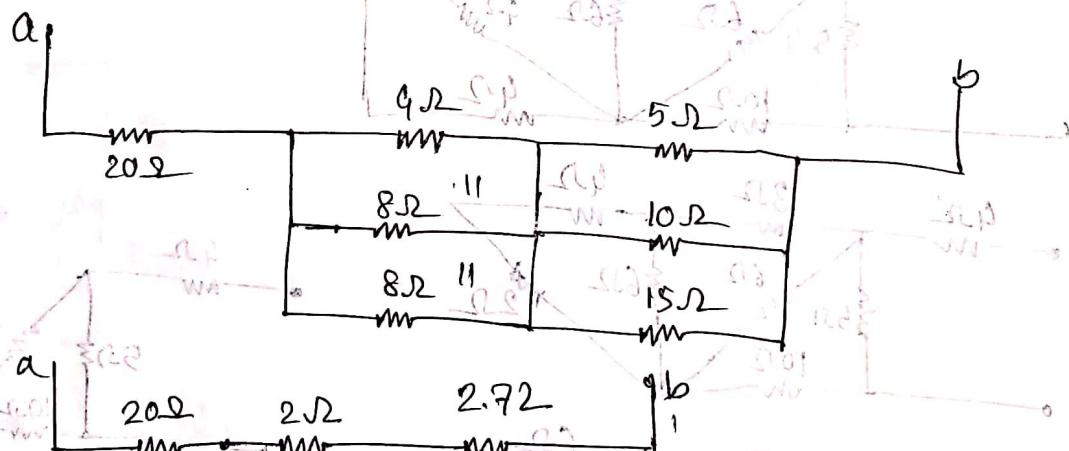
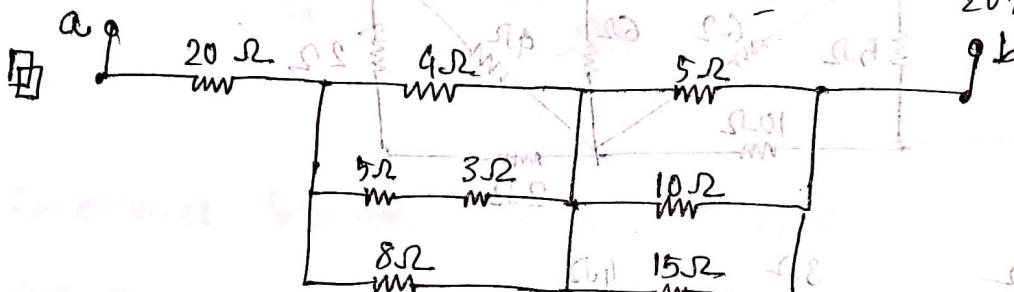
# If parallel resistor has value same & (m),  $\text{Req} = \frac{1}{2}$  (one resistor).



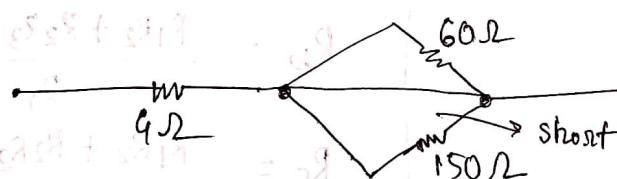
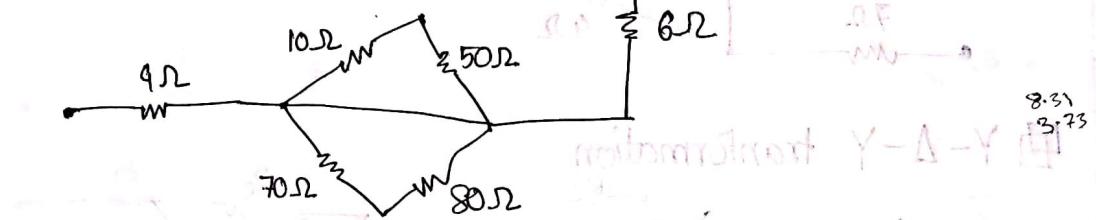
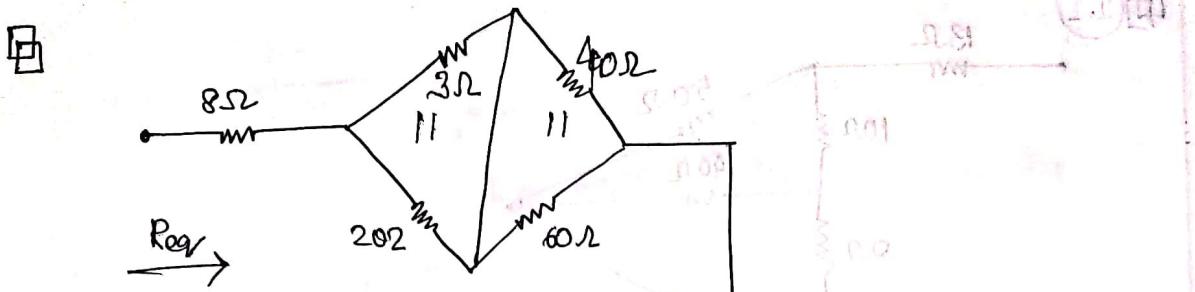


$$Req = 20 \text{ } \Omega$$

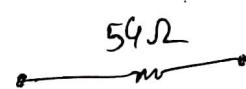
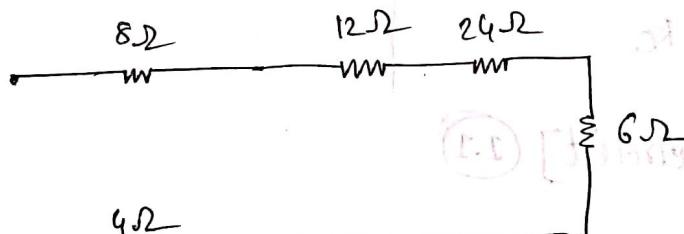
$$\frac{100}{20+5} \Omega = 4 \Omega$$



$$Req = 24.72 \Omega$$

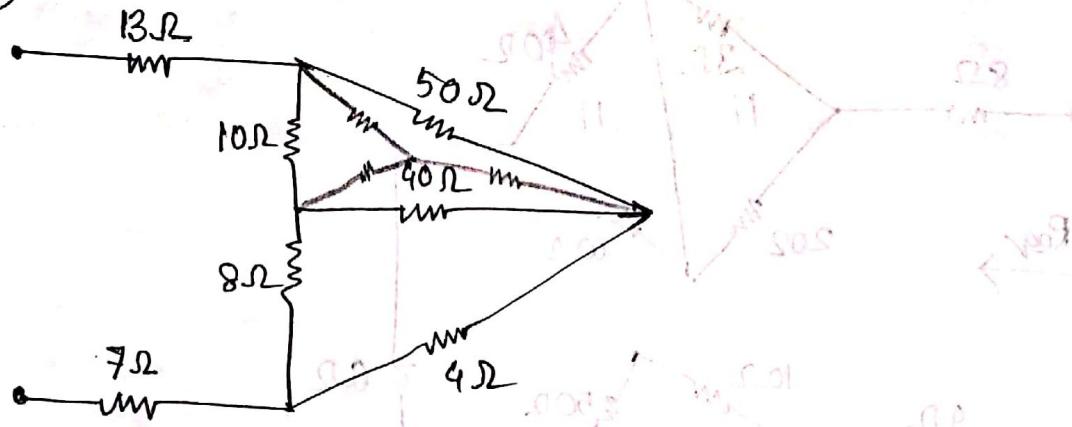


Short: If resistor at  
কোনো wire connected এমন  
short হয়, তাহলে resistance 0.

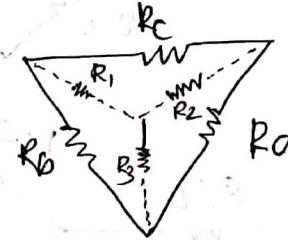
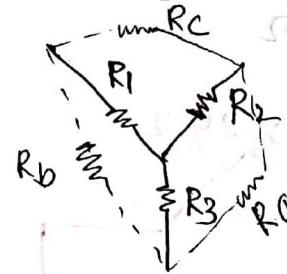
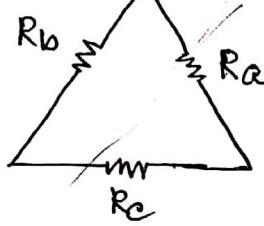


$$\therefore R_{eq} = 54\Omega$$

1.1



### Y-Δ-Y transformation



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

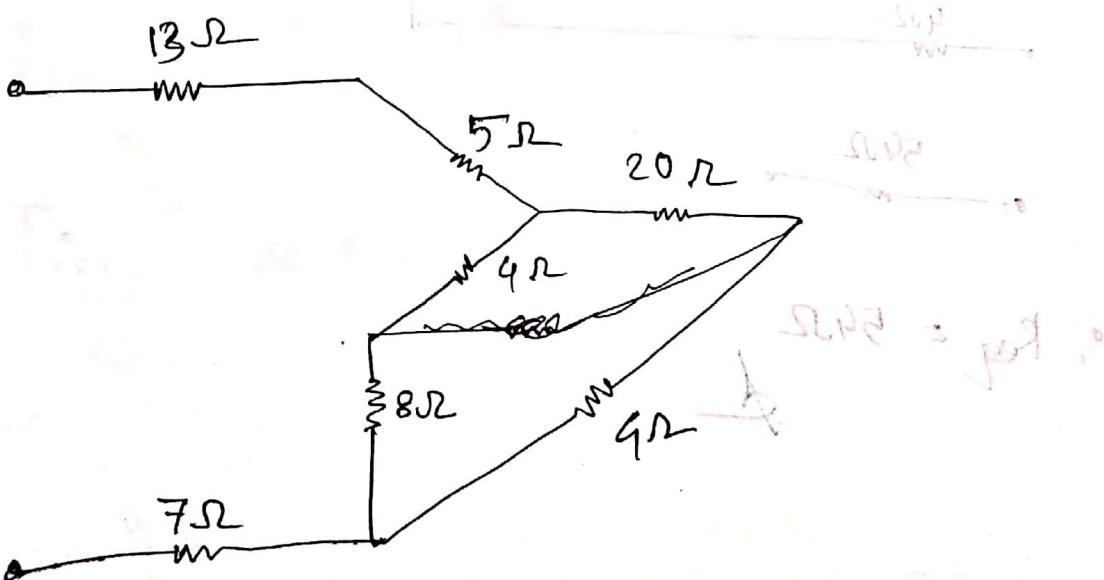
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

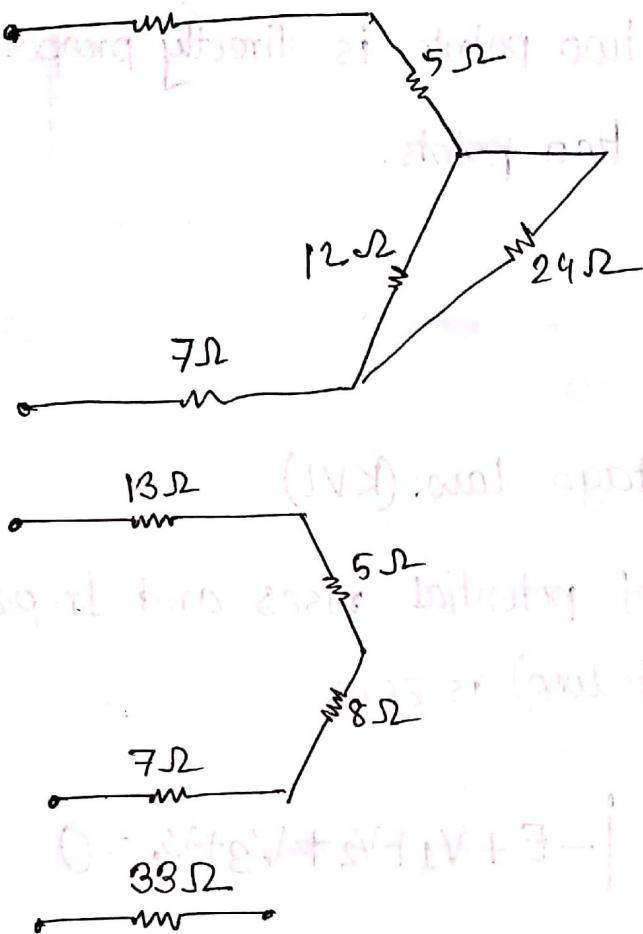
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

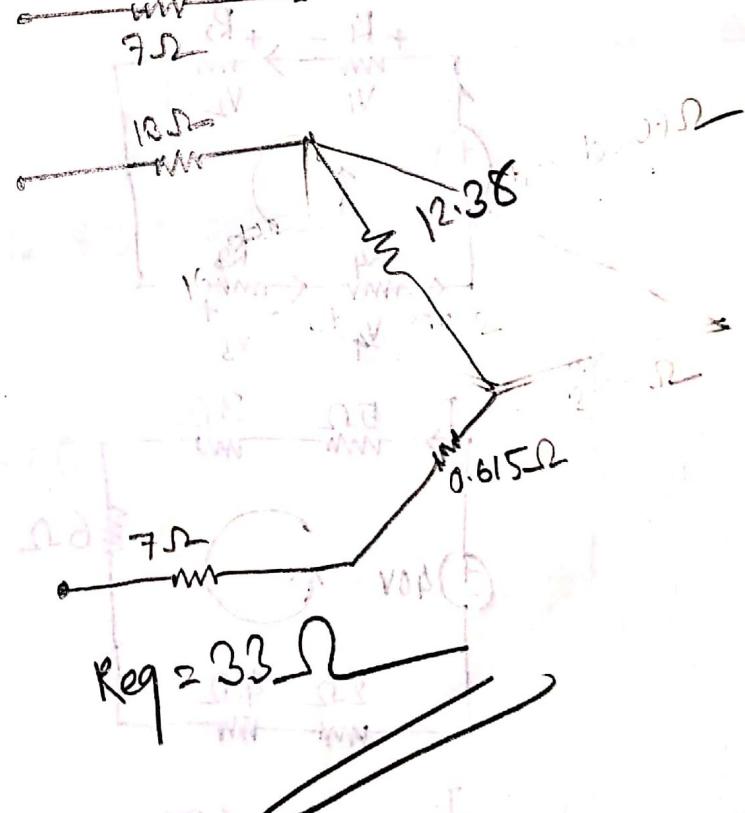
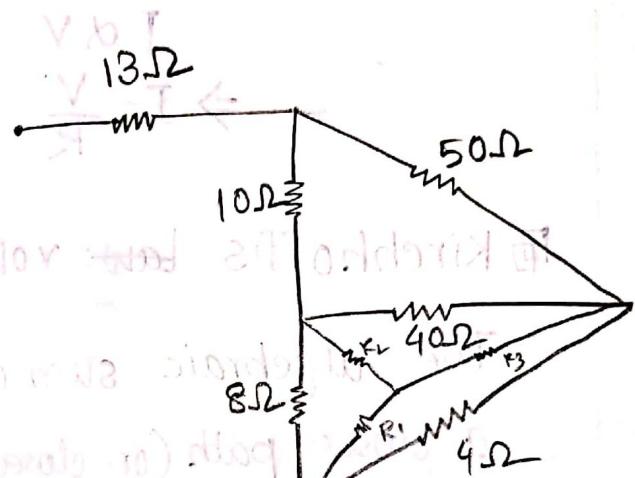
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

### \* Solve [previous circuit] 1.1





$$\therefore \text{Req} = 33\Omega$$



$$\text{Req} = 33\Omega$$

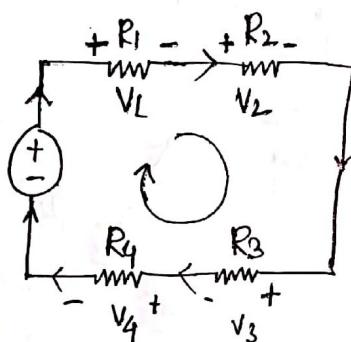


■ Ohm's Law: Ohm's Law state that the current through a conductor between two points is directly proportional to the voltage across the two points.

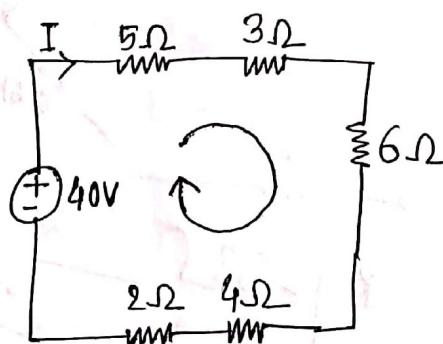
$$\begin{aligned} I &\propto V \\ \Rightarrow I &= \frac{V}{R} \end{aligned}$$

■ Kirchhoff's law: voltage law: (KVL)

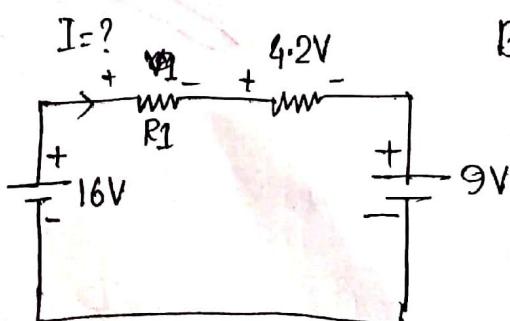
The algebraic sum of potential rises and drops around a closed path (or closed loop) is zero.



$$-E + V_1 + V_2 + V_3 + V_4 = 0$$



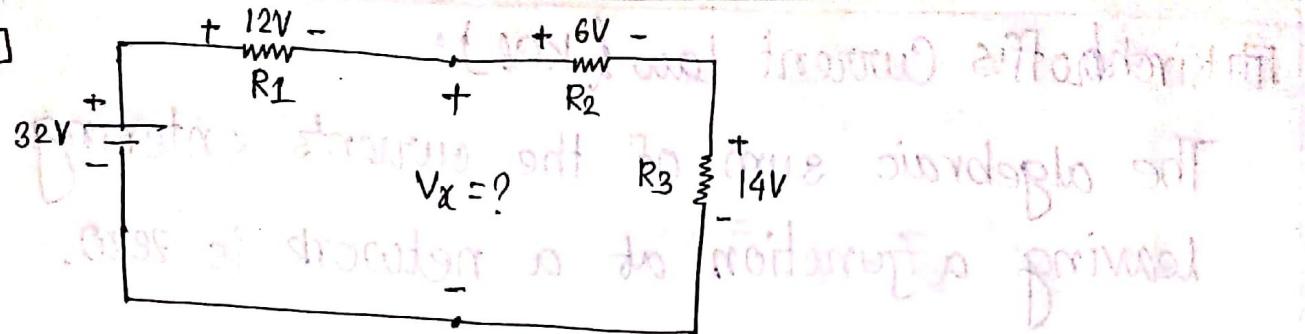
$$\begin{aligned} -40 + (5 \times I) + (3 \times I) + (6 \times I) + (4 \times I) &= 0 \\ \Rightarrow -40 + I \times 20 &= 0 \\ \Rightarrow I &= 2 \text{ A} \end{aligned}$$



By applying (KVL),

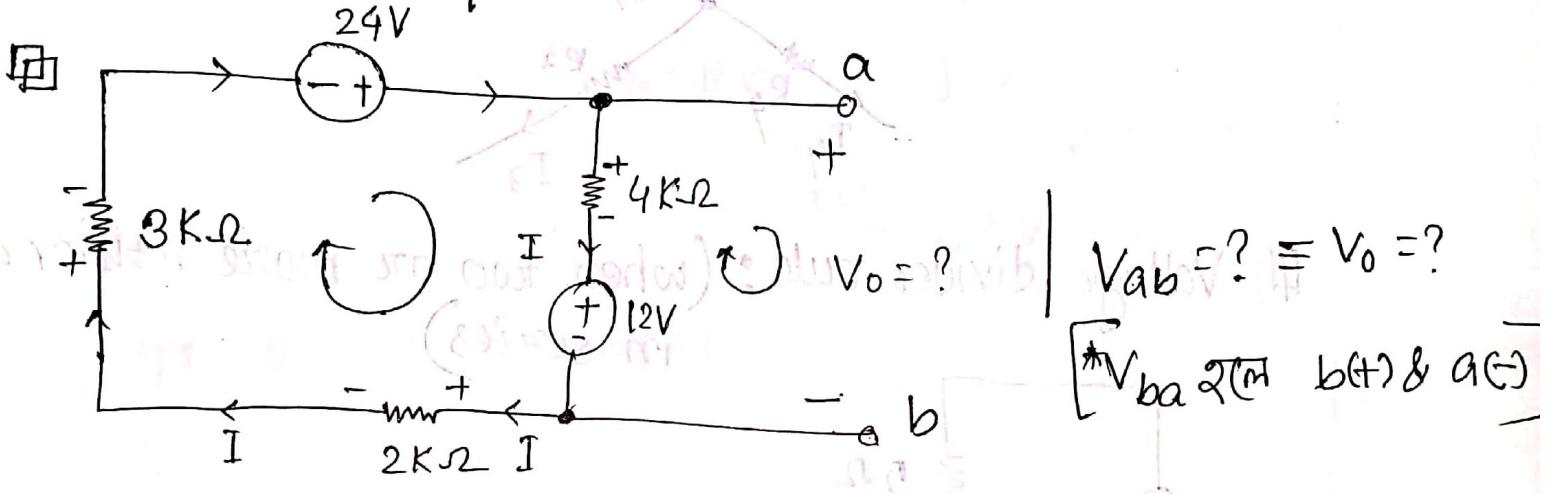
$$\begin{aligned} -16 + V_1 + 4.2 + 9V &= 0 \\ \Rightarrow V_1 &= 2.8 \text{ V} \end{aligned}$$

A



$$\begin{aligned} V_x - 32 + 12 &= 0 \\ \Rightarrow V_x &= 20 \text{ V} \end{aligned}$$

$$\begin{aligned} -V_x + 6 + 14 &= 0 \\ \therefore V_x &= 20 \text{ V} \end{aligned}$$

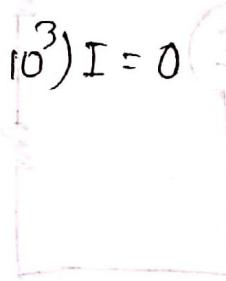


$$(3 \times 10^3) \times I - 24 + (4 \times 10^3) I - 12 + (2 \times 10^3) I = 0$$

$$\Rightarrow I = 4 \times 10^3 \text{ A}$$

$$12 - (4 \times 10^3) \times (4 \times 10^3) + V_0 = 0$$

$$\Rightarrow V_0 = 4 \text{ V.}$$



$$G = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$zL = L$$

$$\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = zL$$

$$V(R_1 + R_2 + R_3) = zL$$



$$14$$

$$R_3$$

$$zL$$

$$V$$

$$14$$

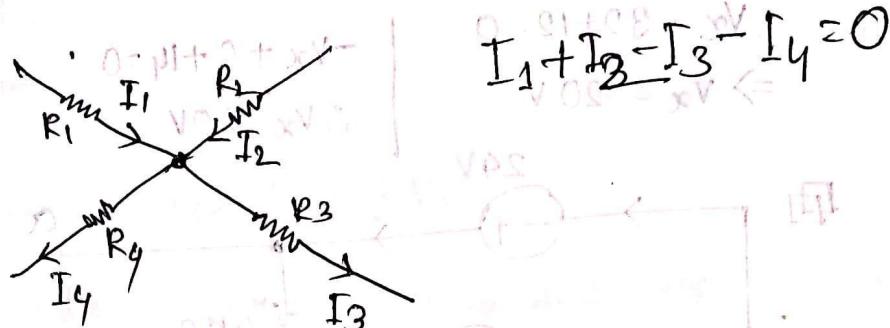
$$R_2$$

$$zL$$

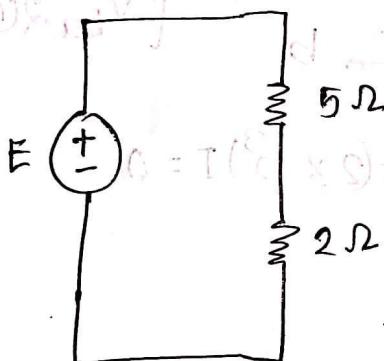
$$V$$

## Kirchhoff's Current Law (KCL):

The algebraic sum of the currents entering and leaving a junction at a network is zero.



## Voltage divider rule: (when two or more resistor are in series)



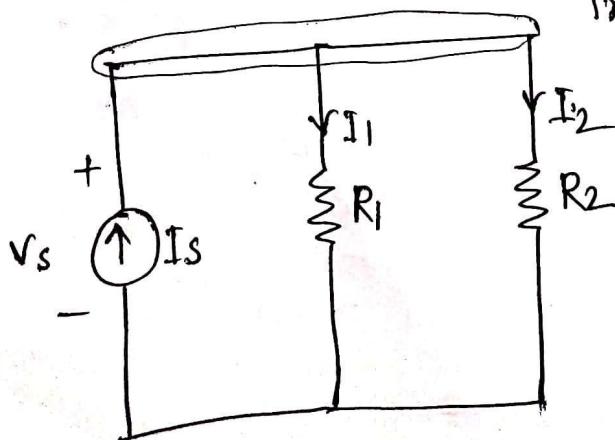
$$R_T = R_1 + R_2$$

$$I = \frac{E}{R_T}$$

$$V_1 = IR_1 = \frac{E}{R_T} R_1 = \frac{E}{R_1 + R_2} R_1 E$$

$$V_2 = IR_2 = \frac{R_2}{R_1 + R_2} E$$

## Current Divider Rule: (when two or more resistor are in parallel)



$$I_S - I_1 - I_2 = 0$$

$$\Rightarrow I_S = I_1 + I_2$$

$$\Rightarrow I_S = \frac{V}{R_1} + \frac{V}{R_L}$$

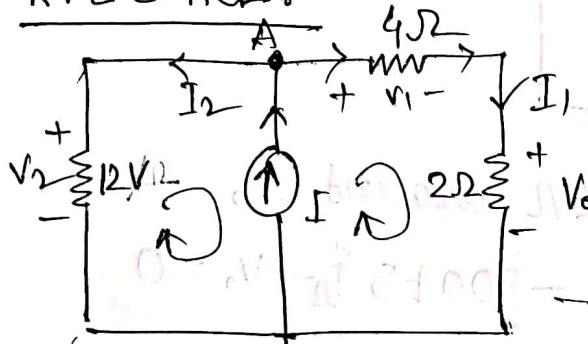
$$\Rightarrow I_S = V \left( \frac{R_1 + R_L}{R_1 R_L} \right)$$

$$\therefore V = \frac{R_1 R_L}{R_1 + R_L} V_S$$

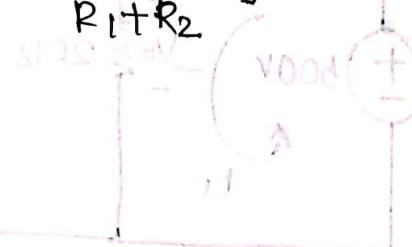
$$I_1 = \frac{V}{R_1} \left( \frac{R_1 R_2}{R_1 + R_2} \right) I_s \quad I_2 = \frac{V}{R_2} \left( \frac{1}{R_2} + \frac{R_1 R_2}{R_1 + R_2} \right) I_s$$

$$\therefore I_1 = \frac{R_2}{R_1 + R_2} I_s$$

KVL & KCL:



$$\therefore I_2 = \frac{R_1}{R_1 + R_2} I_s$$



$$I = ?$$

$$0 = I - i_1 - i_2$$

$$RV - V + V_1 + V_2 = 0$$

$$12 + 4I_1 = 4I_1 + 2I_2$$

$$V = V_1 + V_2$$

$$= 24 + 12 = 36V$$

$$I_1 = \frac{12}{4} = 3A$$

$$V_1 = 4 \times I_1 = 4 \times 3 = 12V$$

$$V_2 = V = 36V$$

$$\therefore I_2 = \frac{V_2}{R_2} = \frac{36}{12} = 3A$$

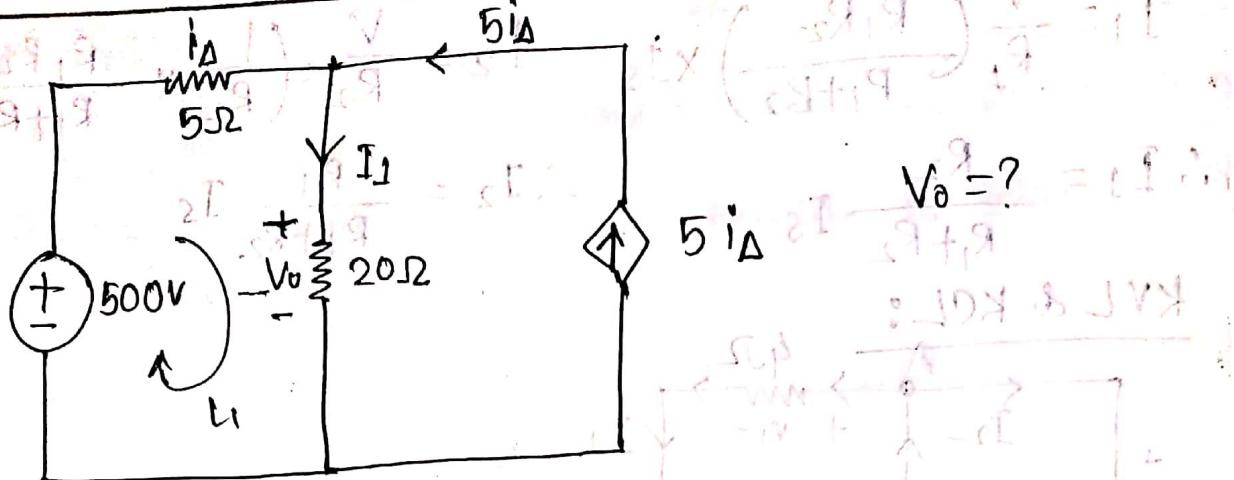
KCL at node A:

$$I = I_1 + I_2 = 0$$

$$\Rightarrow I = I_1 + I_2$$

$$= 3(3+3)A$$

$$= 9A.$$



$$i_\Delta + 5i_\Delta - I_0 = 0$$

$$\Rightarrow I_0 = 6i_\Delta$$

$$\therefore I_0 = (6 \times 5) A$$

~~2 20A 24A~~

$$V_0 = 20 \times I_0$$

$$= (20 \times 24) V$$

$$2 480 V.$$

$$V_0 = ?$$

KCL at node 1:

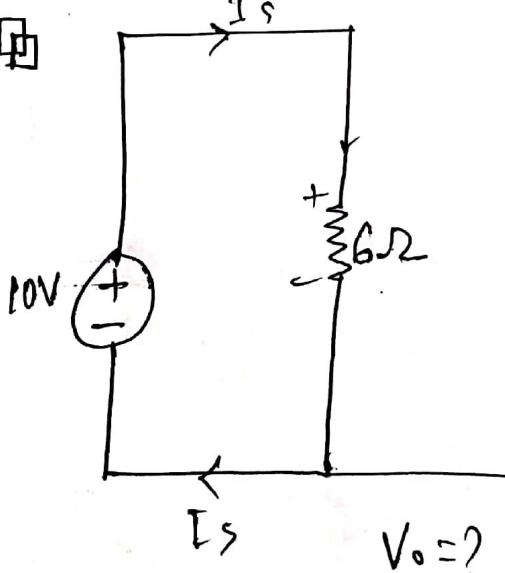
$$i_\Delta + 5i_\Delta - I_0 = 0$$

$$-500 + 5i_\Delta + V_0 = 0$$

$$\Rightarrow -500 + 5i_\Delta + 20I_0 = 0$$

$$\Rightarrow -500 + 5i_\Delta + 120i_\Delta = 0 \quad \text{[since } I_0 = 6i_\Delta]$$

$$\Rightarrow i_\Delta = 5A \quad 4A$$



10V

$i_s$

$V_0 = ?$

At node 1:  $i_s + 3i_\Delta = 0$

At node 2:  $i_s + 3i_\Delta = 0$

$$-10 + 6I_s = 0$$

$$\Rightarrow I_s = \frac{10}{6} = 1.67 A$$

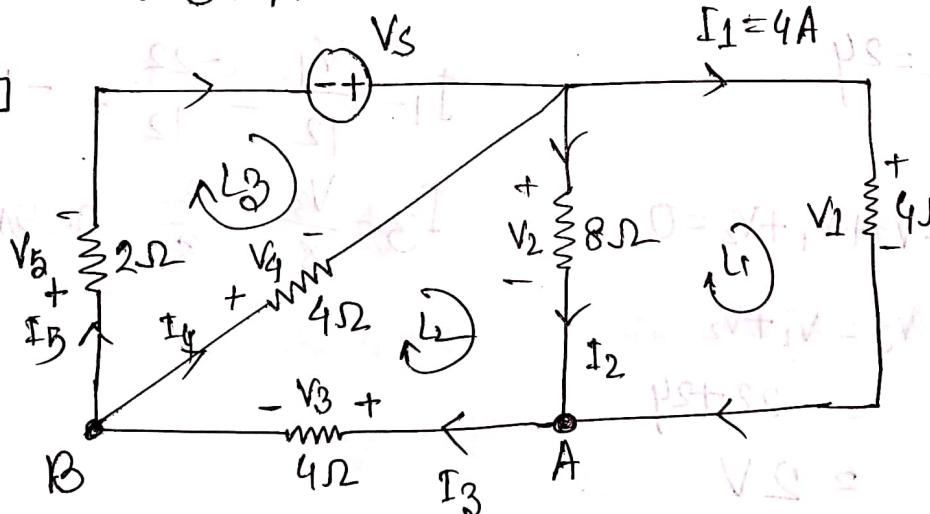
$$-(3 \times 1.67) + 2I + 3I = 0$$

$$\Rightarrow I = \frac{3 \times 1.67}{5} = 1 A$$

$$V_o = I \times 3$$

$$= 3 \times 1$$

$$= 3 V$$



$$[ V_1 = 4 \times 4 = 16 V ]$$

$$[-V_2 + V_1 = 0]$$

$$\Rightarrow V_2 = 16 V$$

$$I_2 = \frac{V_2}{8} = \frac{16}{8} = 2 A$$

KCL at node A,

$$I_1 + I_2 - I_3 = 0$$

$$\Rightarrow I_3 = 4 + 2 = 6 A$$

$$I_1 = 4 A$$

$$V_S = ?$$

KVL at Loop 3,

$$32 - V_S + 40 = 0$$

$$V_S = 72 V$$

$$V_5 = I_5 \times 2 \\ = 16 \times 2 \\ = 32 V$$

$$V_3 = I_3 \times 4 \\ = 6 \times 4 \\ = 24 V$$

KVL at L2,

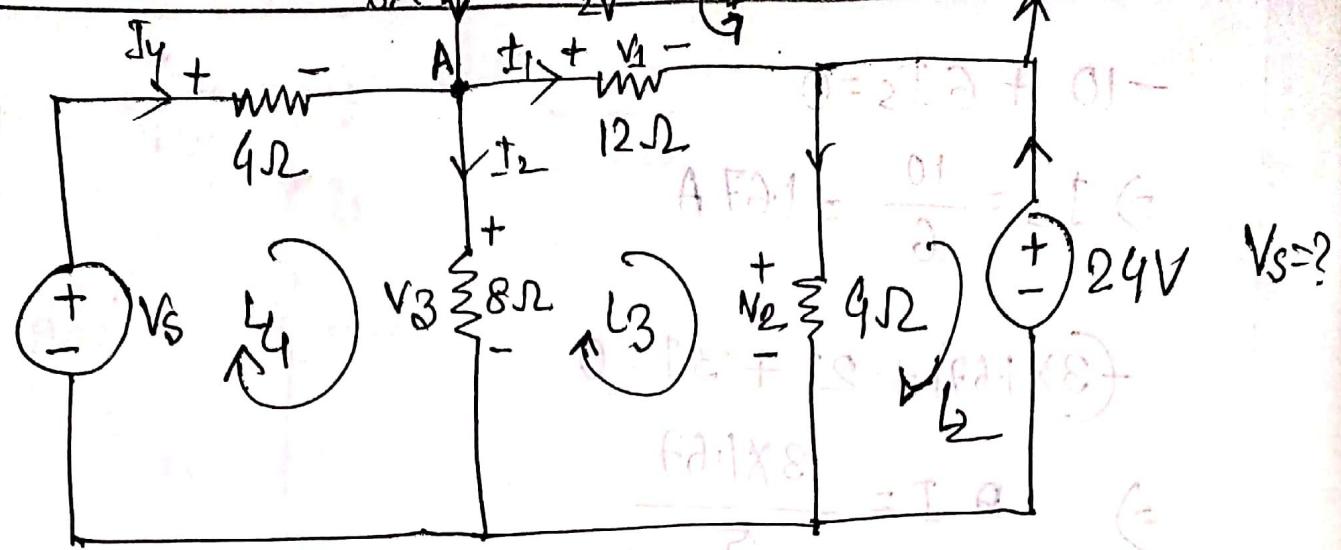
$$V_3 + V_4 + V_2 = 0$$

$$\Rightarrow V_4 = -(V_3 + V_2)$$

$$= -40 V$$

KCL at node B,

$$I_3 = I_4 + I_5 \quad I_5 = I_3 - I_4 \\ \Rightarrow I_5 = 16 A$$



In L<sub>1</sub>,  $V_1 + (4 \times 6) - 2 = 0$

$$\Rightarrow V_1 = -22V$$

In L<sub>2</sub>,  $-V_2 + 24 = 0$

$$\Rightarrow V_2 = 24$$

In L<sub>3</sub>,  $-V_3 + V_1 + V_2 = 0$

$$\Rightarrow V_3 = V_1 + V_2$$

$$= -22 + 24$$

$$= 2V$$

at Node A,

$$I_1 + I_3 = 6 + I_4$$

$$\therefore I_4 = (-1.83 + 0.25) - 6$$

$$\therefore I_4 = -7.58A$$

In L<sub>4</sub>,  $V_S + V_4 + V_3 = 0$

$$\Rightarrow V_S = V_4 + V_3 = (4 \times I_4) + 2 = -32.32V.$$

$$I_1 = \frac{V_1}{12} = \frac{-22}{12} = -1.83A$$

$$I_3 = \frac{V_3}{8} = \frac{2}{8} = 0.25A$$

$$V_{AB} = 12V$$

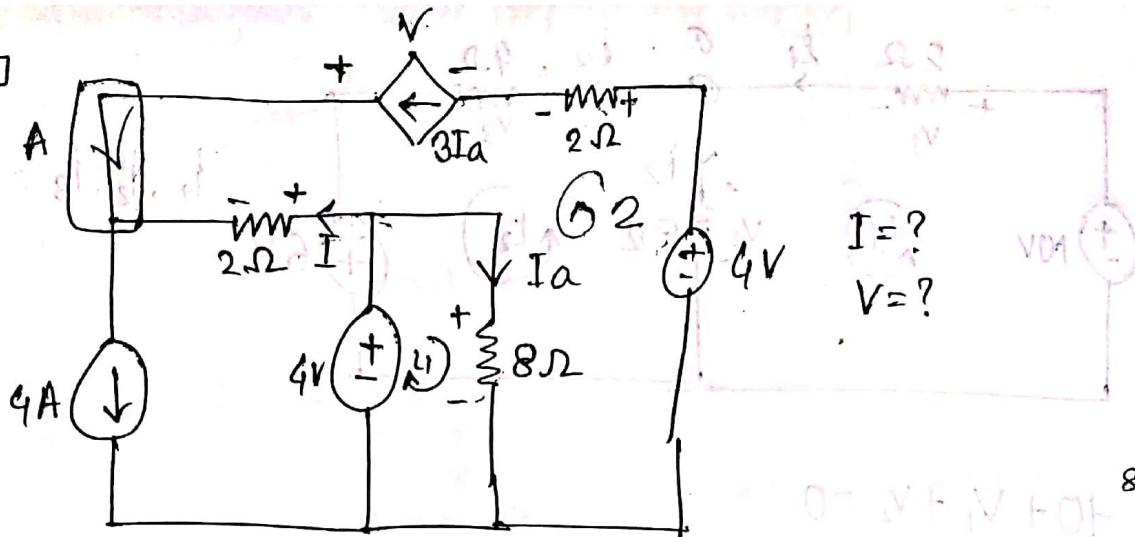
$$0 = 12V - 2V - 7V$$

$$V_{AB} = 2V$$

$$A \leftarrow \frac{2V}{8} = \frac{2}{8} = 0.25A$$

A short to ground

$$0 = 12V - 2V - 7V$$



In L1,

$$-4 + 8I_a = 0$$

$$\Rightarrow I_a = 0.5 \text{ A}$$

In node A, KCL

$$I + 3I_a - 4 = 0$$

$$\Rightarrow I = 4 - (3 \times 0.5)$$

$$= 2.5 \text{ A}$$

In loop 2 KVL,

$$-2I + V - (2 \times 3I_a) + 4 - (8 \times I_a) = 0$$

$$\Rightarrow V = -2V$$

$$8I_a - 4 + 3I_a = 0$$

$$11I_a = 4$$

$$I_a = \frac{4}{11}$$

$$0 = 2I - 8V + 4 - (-2V)$$

$$0 = 2I + 2V = 2(V + I)$$

$$0 = 2I - 2V = 2(I - V)$$

$$0 = \frac{V}{1} - \frac{2V}{3} - \frac{4V}{8}$$

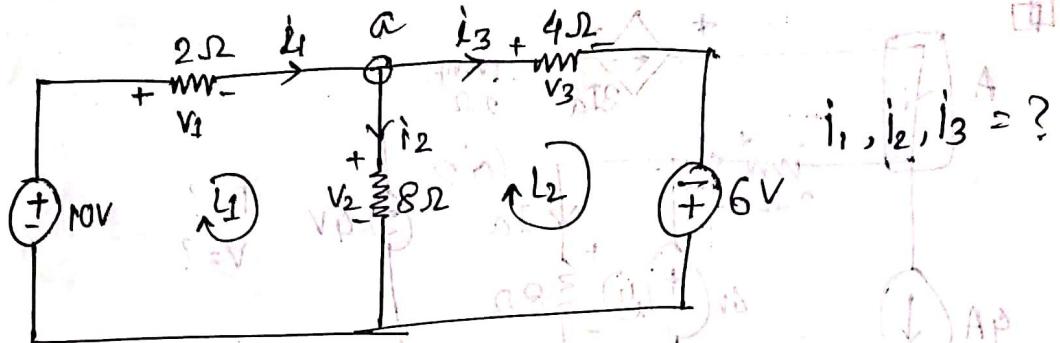
$$V = 3.2V$$

$$V_{01} = 3.2V$$

$$AB = \frac{V}{I} = 1.2$$

$$AB = V = 3.2$$

$$AB = 3 = V = 3.2$$



$$L_1, \quad -10 + V_1 + V_2 = 0$$

$$\Rightarrow V_1 = 10 - V_2 \quad \text{--- (I)}$$

$$L_2, \quad -V_2 + V_3 - 6 = 0$$

$$\Rightarrow V_3 = V_2 + 6 \quad \text{--- (II)}$$

KCL at 'a',

$$i_1 - i_2 - i_3 = 0$$

$$\Rightarrow \frac{V_1}{2} - \frac{V_2}{8} - \frac{V_3}{4} = 0$$

$$\Rightarrow \frac{10 - V_2}{2} - \frac{V_2}{8} - \left( -\frac{V_2 + 6}{4} \right) = 0$$

$$\Rightarrow V_2 = 4 \text{ V}$$

$V_2$  is value (I) & (II) & Now,

$$V_1 = 6 \text{ V}$$

$$V_3 = 10 \text{ V}$$

$$\therefore i_1 = \frac{V_1}{2} = 3 \text{ A}$$

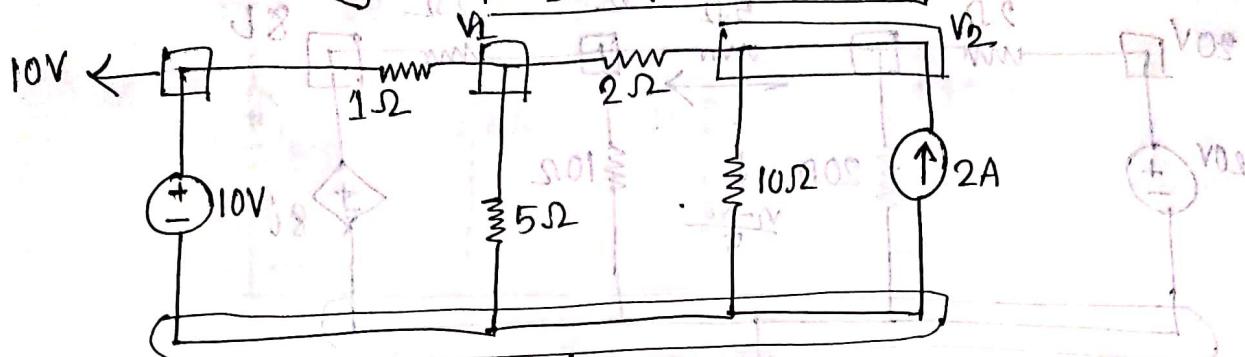
$$i_2 = \frac{V_2}{8} = 0.5 \text{ A}$$

$$i_3 = \frac{V_3}{4} = 2.5 \text{ A}$$

Quiz-01 [Equivalent Resistance KVL & KCL  
Next Monday (28.01.19)]

27.01.19

## Nodal Analysis: Very important topic



reference node (maximum num element connected)  
Voltage (0V)

For  $V_1$ ,

$$0 = \frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - 0}{5}$$

$$\Rightarrow V_1 \left(1 + \frac{1}{2} + \frac{1}{5}\right) + V_2 \left(-\frac{1}{2}\right) - \frac{10}{5} = 0$$

$$\Rightarrow 1.7 V_1 - 0.5 V_2 = 10 \quad (1) \quad (\text{eq 1})$$

For  $V_2$ ,  $V_2 = 10V$  (fixed)

$$0 = \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{10} - 2A \quad (2) \quad (\text{eq 2})$$

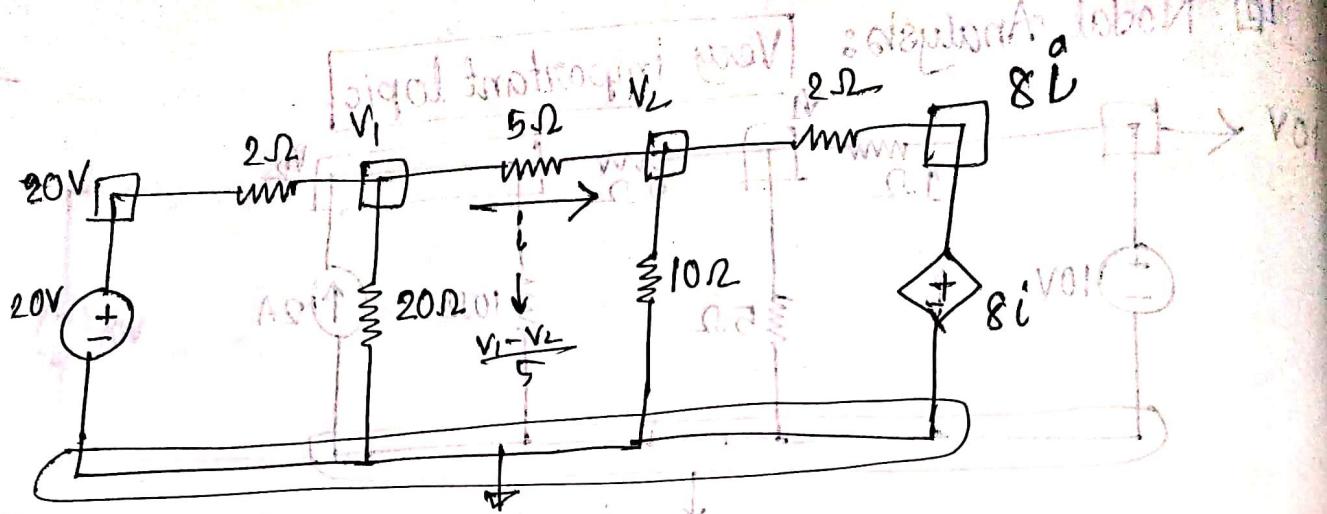
$$\Rightarrow V_2 \left(\frac{1}{2} + \frac{1}{10}\right) + V_1 (-\frac{1}{2}) = 2 \quad (1) \text{ sum } (2) \text{ gives}$$

$$\Rightarrow -0.5 V_1 + 0.6 V_2 = 2 \quad (11) \quad V_2 = 10V$$

Solving (1) and (2),

$$V_1 = 9.09V$$

$$V_2 = 10V$$



For  $V_1$ ,

$$\frac{V_1 - 20}{2} + \frac{V_1 - 20}{20} + \frac{V_1 - V_2}{5} = 0$$

$$\Rightarrow V_1 \left( \frac{1}{2} + \frac{1}{20} + \frac{1}{5} \right) - \frac{1}{5} V_2 - 10 = 0$$

$$\Rightarrow 0.75 V_1 - 0.2 V_2 = 10 \quad \text{(Equation 1)}$$

For  $V_2$ ,

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{10} + \frac{V_2 - 8i}{2} = 0$$

$$\Rightarrow -V_1 + 1.6 V_2 = 0 \quad \text{(Equation 2)}$$

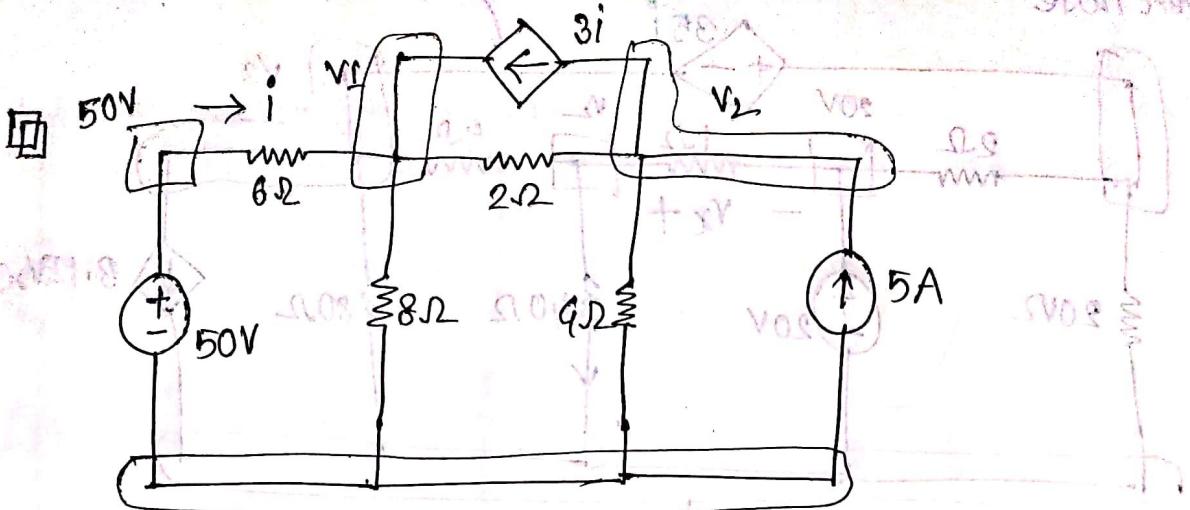
Solving (1) and (2),  $V_1 = 16V$ ,  $V_2 = 10V$

$$V_1 = 16V \quad \text{and} \quad V_2 = 10V$$

$$V_2 = 10V$$

$$V_{0.0} = 1V$$

$$V_{0.1} = 1V$$



For  $V_1$ ,  $\frac{V_1 - 50}{6} + \frac{V_1 - 0}{8} + \frac{V_1 - V_2}{2} - 3i = 0$

 $i = \frac{50 - V_1}{6}$

$\Rightarrow 1.29V_1 - 0.5V_2 = 33.33 \quad \textcircled{I}$

for  $V_2$ ,

$$3i + \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{8} - 5 = 0$$
 $\Rightarrow -V_1 + 0.75V_2 = -20 \quad \textcircled{II}$

By solving (I) and (II),

$V_1 = 32.08V$

$V_2 = 16.1V$

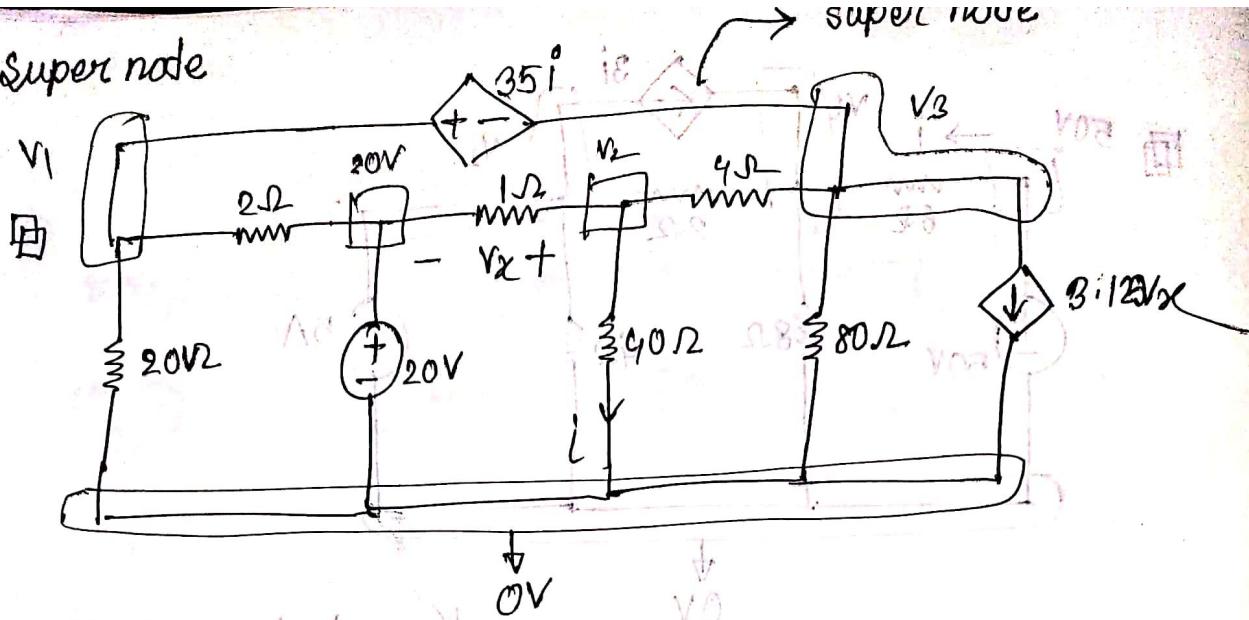
(III) previous eq.

$V_{10.88} = 1V$

$V_{08.81} = 2V$

$V_{0.81} = 3V$

~~#~~ Super node



For super node,

$$\frac{V_1 - 20}{2} + \frac{V_1 - 0}{20} + \frac{V_3 - V_2}{4} + \frac{V_3 - 0}{80} + 3.125/V_x = 0$$

$$\Rightarrow 0.55V_1 + 2.875V_2 + 0.25V_3 = 20 \quad \text{(I)}$$

For  $V_2$ ,

$$\frac{V_2 - 20}{1} + \frac{V_2 - 0}{40} + \frac{V_2 - V_3}{4} = 0$$

$$\Rightarrow 1.275V_2 - 0.25V_3 = 20 \quad \text{(II)}$$

$$V_1 - V_3 = 35^{\circ}$$

$$\Rightarrow V_1 - 0.875V_2 - V_3 = 0 \quad \text{(III)}$$

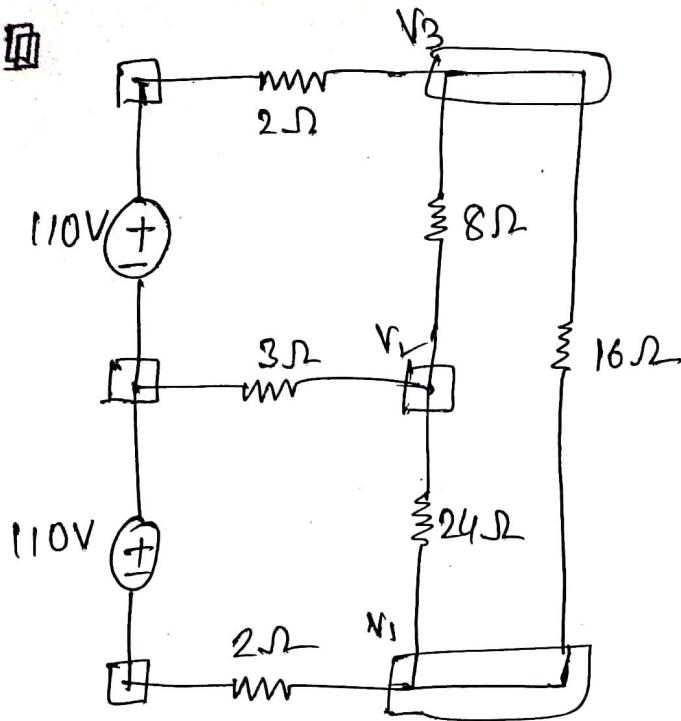
By solving (I), (II), (III)

$$V_1 = 29.54V$$

$$V_2 = 18.33V$$

$$V_3 = 13.5V$$

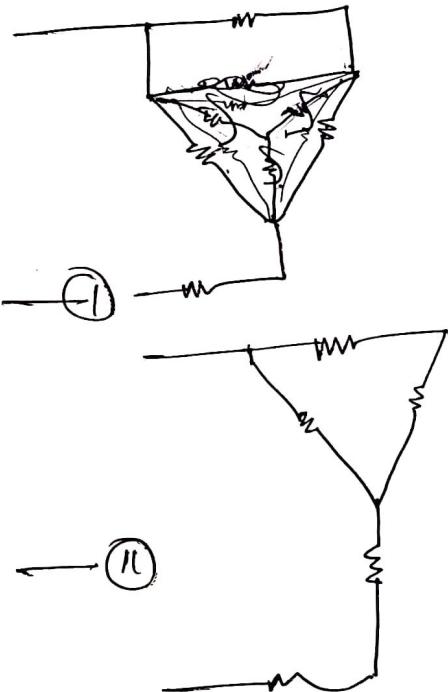
Super node: यही नोड एक वोल्टेज सर्स है जिसमें एक करेंट सर्स भी है और उसका ग्राफिकल रूप सुपर नोड होता है।



A set

$$I = 5.07$$

$$R_{eq} = 11.025$$



$$\text{For } V_1, \frac{V_1}{2} + \frac{V_1 - V_2}{24} + \frac{V_1 - V_3}{16} = 0$$

$$\Rightarrow 0.604V_1 - 0.042V_2 - 0.0625V_3 = 0 \quad \text{--- (I)}$$

$$\text{For } V_2, \frac{V_2 - V_1}{24} + \frac{V_2 - 110}{3} + \frac{V_2 - V_3}{8} = 0$$

$$\Rightarrow -0.042V_1 + 0.5V_2 - 0.125V_3 = 36.67 \quad \text{--- (II)}$$

$$\text{For } V_3, \frac{V_3 - 220}{2} + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{16} = 0$$

$$\Rightarrow -0.0625V_1 - 0.125V_2 + 0.6875V_3 = 110 \quad \text{--- (III)}$$

Solving (I), (II) and (III),

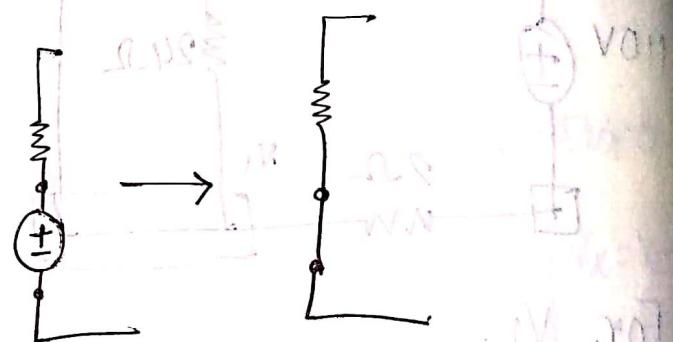
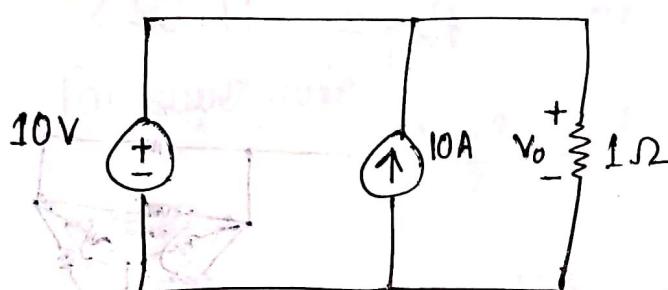
$$V_1 = 27.579V$$

$$V_2 = 121.82V$$

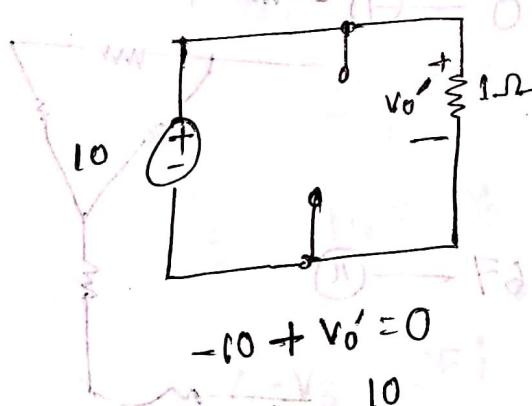
$$V_3 = 184.66V$$

## Superposition theorem:

The current through or voltage across any element at a network is equal to the algebraic sum of the currents and voltages produced independently by each source.



$$V_o = ?$$



$$-10 + V_o' = 0$$

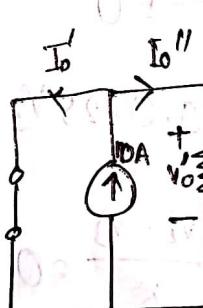
$$\Rightarrow I = \frac{10}{1}$$

$$= 10A$$

$$\therefore V_o' = I \times R$$

$$= 10 \times 1$$

$$= 10V$$



$$V_o'' = 10 \times 1$$

$$= 10V$$

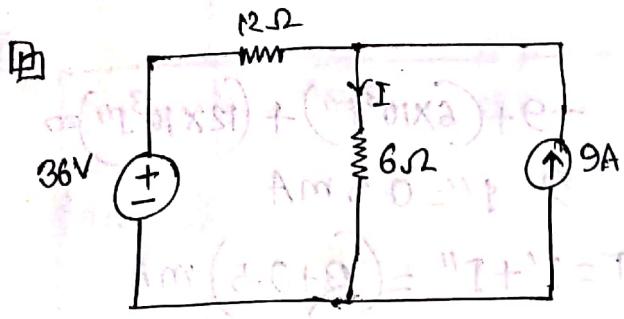
$$I_o'' = \frac{(1 \times 10A)}{6+1}$$

$$= 0A$$

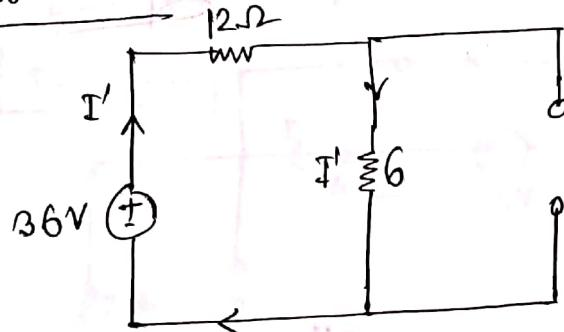
$$\therefore V_o'' = (0 \times 1) V$$

$$= 0V$$

(iii) from (ii), (ii) parallel



36V active  $I = ?$

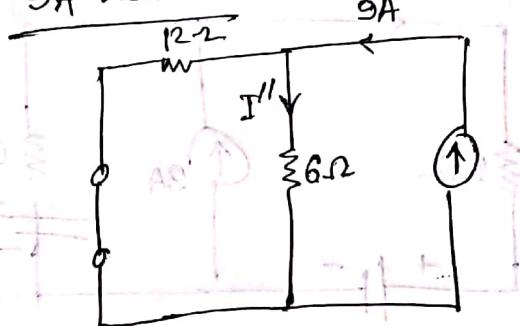


$$I' = \frac{36}{12+6}$$

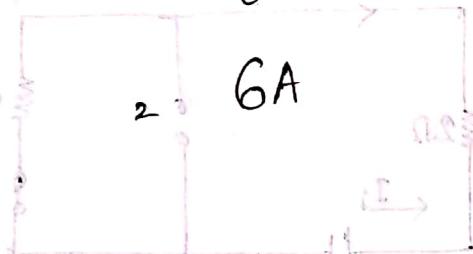
$$= 2A$$

$$\therefore I = I' + I'' = (2+6) = 8A.$$

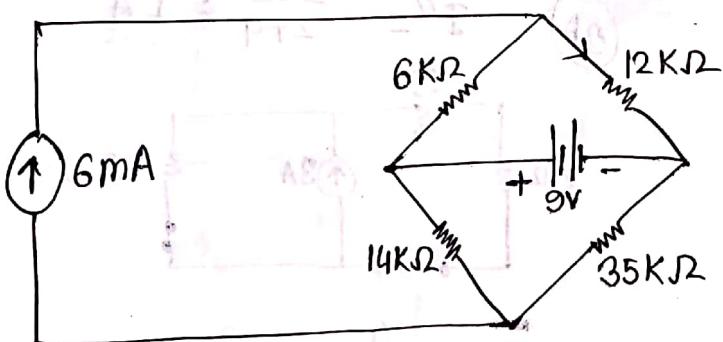
9A active  $I = ?$



$$I'' = \frac{12}{6+12} \times 9$$



Q2



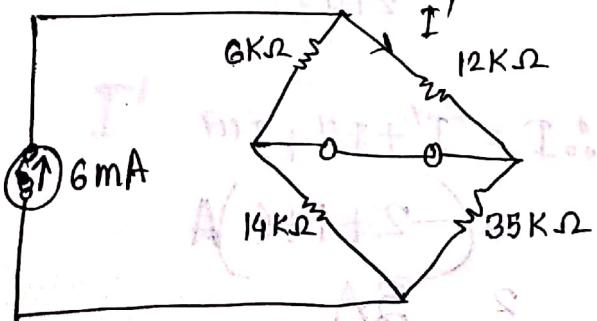
$$A = \frac{9}{12K\Omega} = 750$$

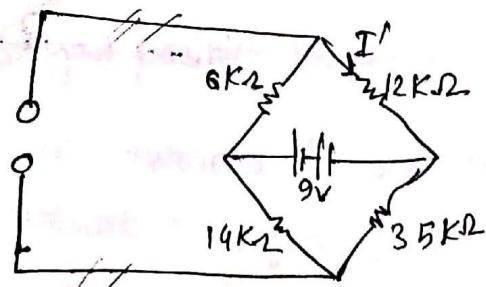
$A_{OL} =$

$A_{IL} = 2.75$

$$I' = \frac{6K\Omega}{(12+6)K\Omega} \times 6mA$$

$$= 2mA$$

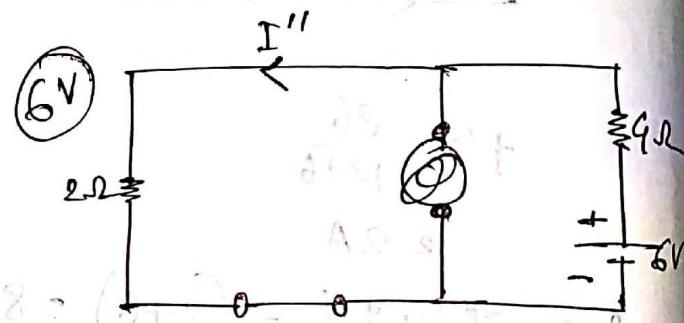
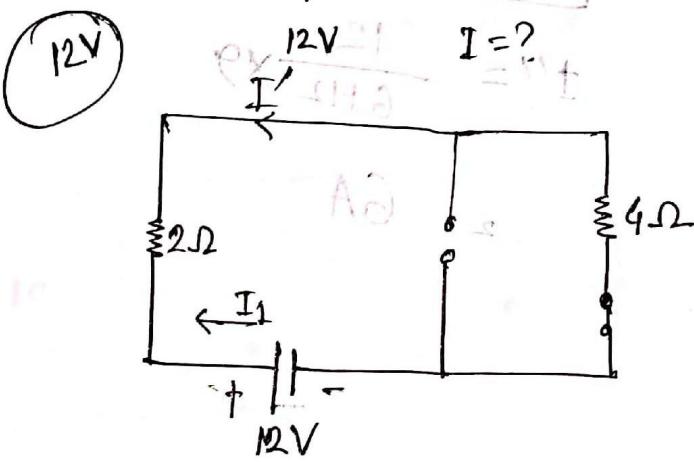
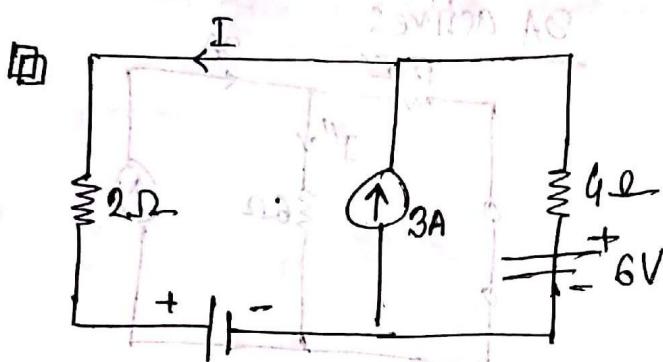




$$-9 + (6 \times 10^3 I'') + (12 \times 10^3 I'') = 0$$

$$\therefore I'' = 0.5 \text{ mA}$$

$$I = I' + I'' = (2 + 0.5) \text{ mA} = 2.5 \text{ mA}$$



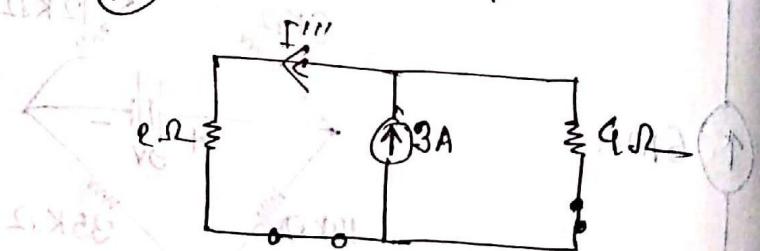
$$\therefore I_1 = \frac{12}{2+4} \text{ A} = 2 \text{ A}$$

$$\therefore I' = -2 \text{ A}$$

$$\text{Ansatz: } \frac{6}{2+4} = 1$$

$$\text{Ansatz: } =$$

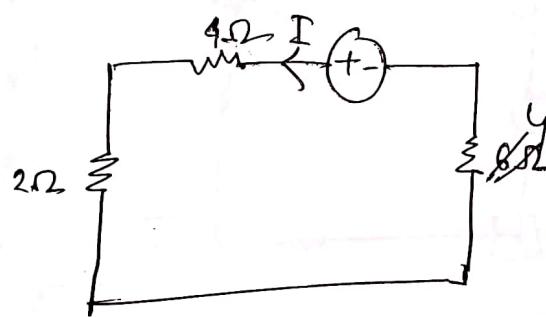
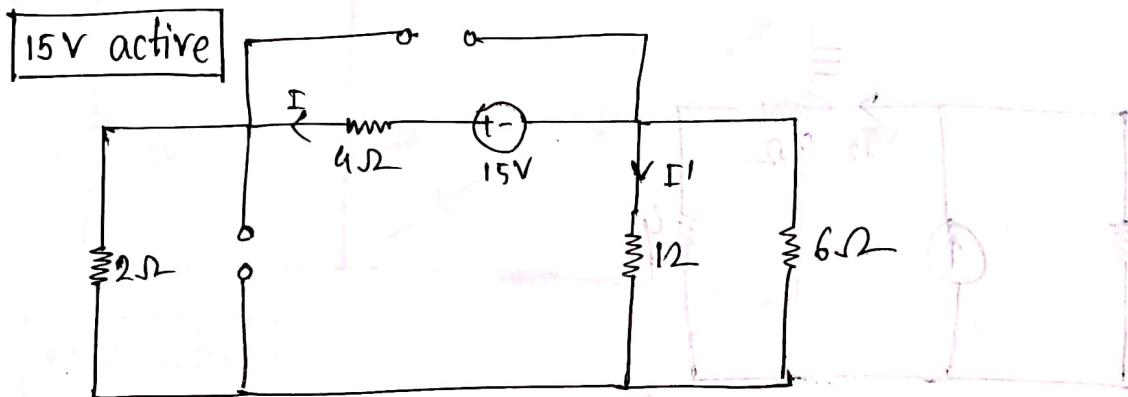
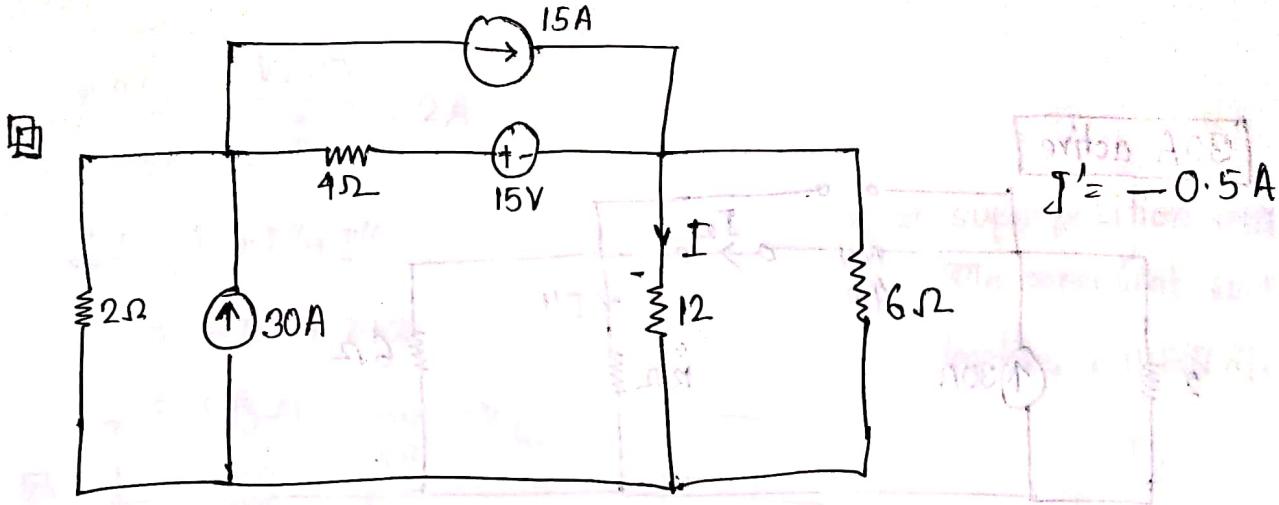
$$(3A) \quad I'' = \frac{6}{2+4} = 1 \text{ A}$$



$$I''' = \frac{4}{2+4} \times 3 = 1 \text{ A}$$

$$\therefore I = I' + I'' + I'''$$

$$2(-2 + 1 + 6) \text{ A} = 5 \text{ A}$$



$$I^0 = \frac{15V}{4+2+6}$$

$$= 1.5A$$

$$I' = -\frac{6}{12+6} \times 1.5$$

$$= -0.5A$$

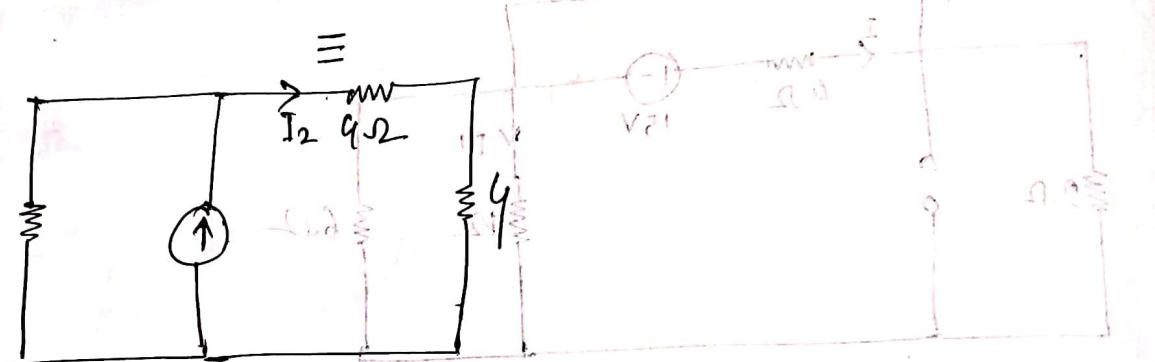
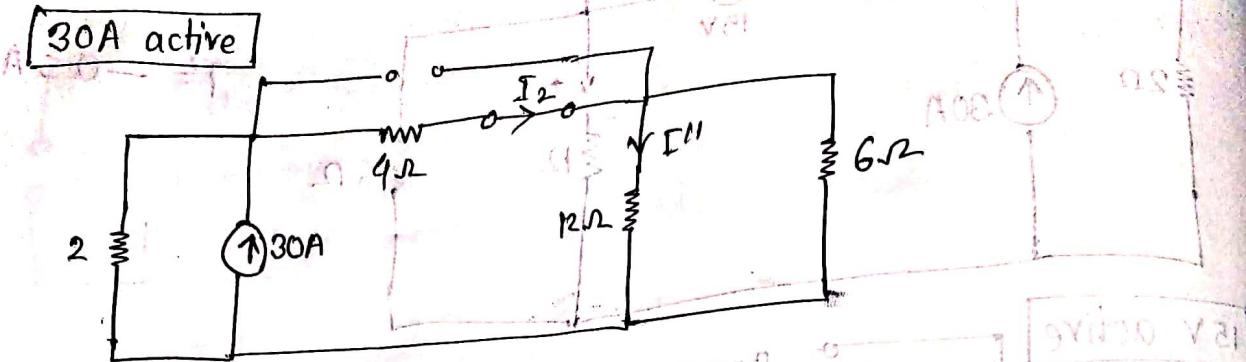
$$0 = 21 - \frac{15 - 6}{4 + 2 + 6} \times 2 + \frac{6 - 12}{12 + 6} \times 6 + \frac{12 - 6}{12 + 6} \times 6$$

$$0 = 21 - \frac{9}{12} \times 2 + \frac{-6}{18} \times 6 + \frac{6}{18} \times 6$$

$$0 = 21 - 1.5 + (-2) + 2$$

$$0 = 21 - 1.5 - 2$$

$$0 = 17.5$$

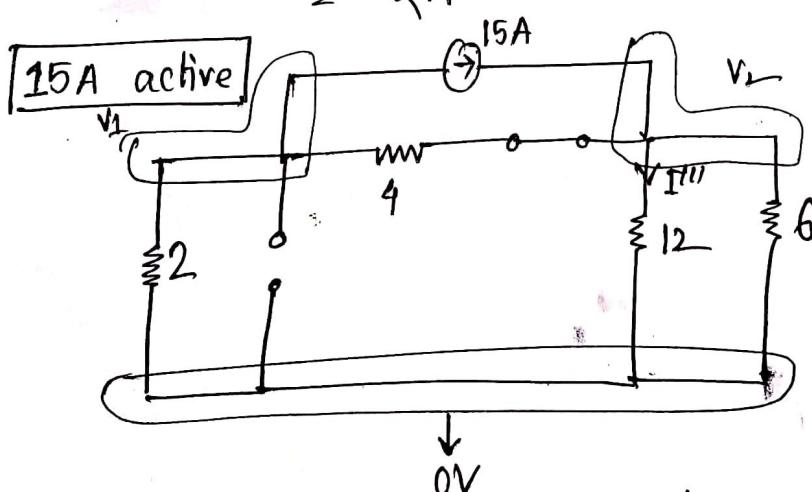


$$I_2 = \frac{2}{2+8} \times 30$$

$$= 6A$$

$$I''' = \frac{6}{12+6} \times 6$$

$$= 2A$$



$$\frac{V_1 - 0}{2} + \frac{V_1 - V_2}{4} + 15 = 0$$

$$V_1 = ()$$

$$V_2 = ()$$

$$\frac{V_2 - 0}{12} + \frac{V_2 - 0}{6} + \frac{V_2 - V_1}{4} - 15 = 0$$

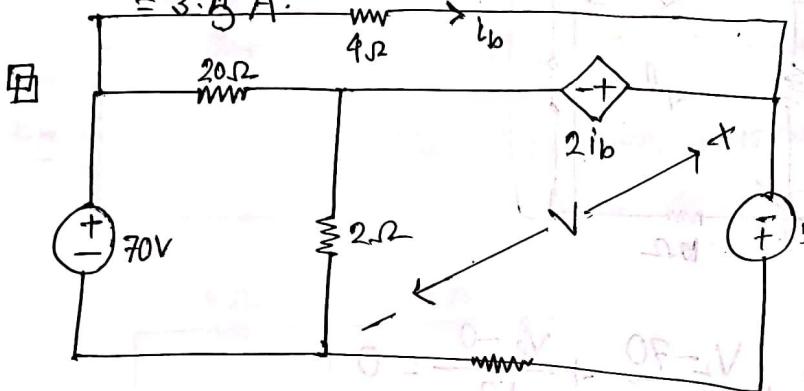
$$()$$

$$I''' = \frac{V_2 - 0}{1} = 2A$$

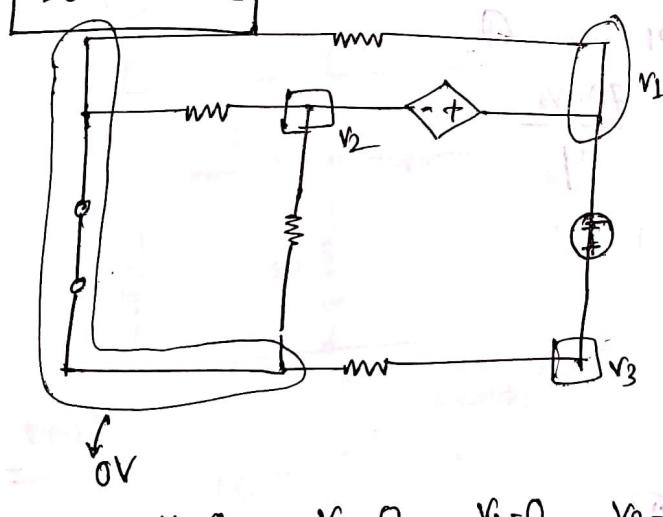
$$\therefore I = I' + I'' + I'''$$

$$= -0.5 + 2 + 2$$

$$= 3.5 A$$



50V active



$$\frac{V_1 - 0}{4} + \frac{V_2 - 0}{20} + \frac{V_2 - 0}{2} + \frac{V_3 - 0}{10} = 0$$

$$V_1 = -4.255V$$

$$V_2 = -6.38V$$

$$V_3 = 45.74V$$

$$V_1 = V_1 - 0 \\ = -4.255V$$

$$\therefore 0.25V_1 + 0.05V_2 + 0.1V_3 = 0 \quad \text{--- (1)}$$

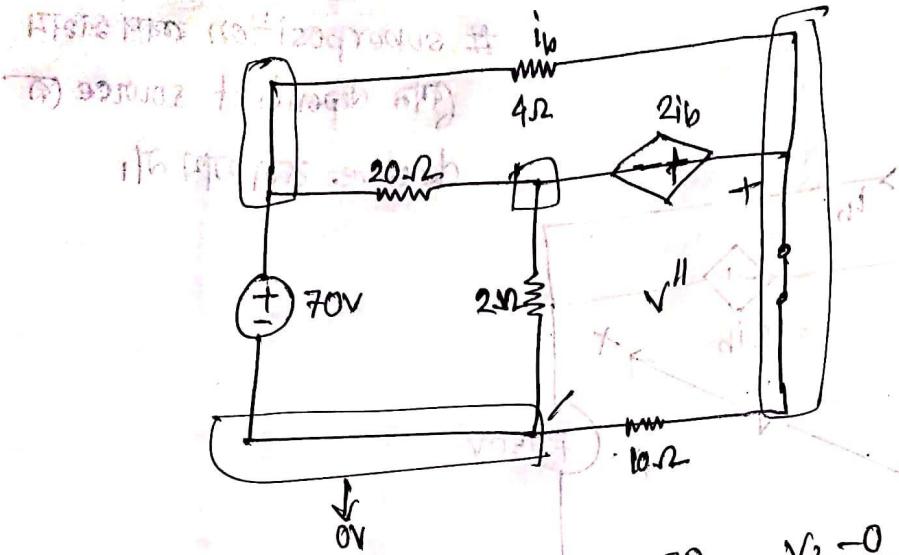
$$V_1 - V_2 = 2ib \quad \text{--- (II)}$$

$$\Rightarrow 1.5V_1 - V_2 = 0 \quad \text{--- (III)}$$

$$V_3 - V_1 = 50 \quad \text{--- (IV)}$$

~~$$V_1 = (?); V_2 = (?); V_3 = (?)$$~~

70V active



$$\frac{V_1 - 70}{20} + \frac{V_1 - 0}{2} + \frac{V_1 - 70}{4} + \frac{V_2 - 0}{10} = 0$$

$$\Rightarrow 0.55V_1 + 0.55V_2 = 21 \quad \text{(I)}$$

$$V_2 - V_1 = 2i_b \quad i_b = \frac{70 - V_2}{4}$$

$$\Rightarrow -V_1 + 1.5V_2 = 35 \quad \text{(II)}$$

Solving (I) and (II)  $\Rightarrow$

$$V_1 = 16.38V$$

$$V_2 = 34.25V$$

$$V' = V_2 - 0 \\ = 34.25V$$

$$V = V' + V'' \quad \cancel{V'' = V_1 - 0 + V_2 - 0 + V_3 - 0}$$

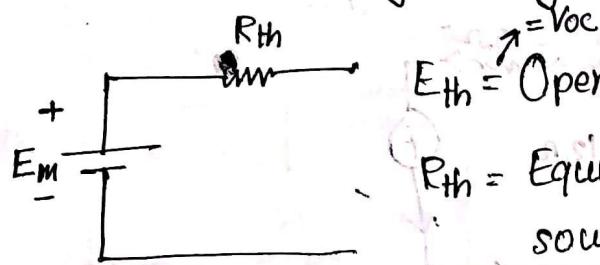
$$= (-4.255 + 34.255)V$$

$$= 30V$$

04.02.19

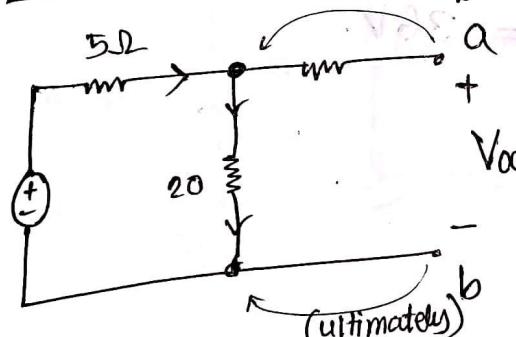
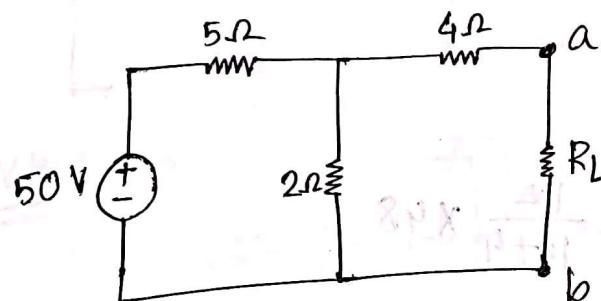
## Thermin's theorem

Any two terminal network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor.

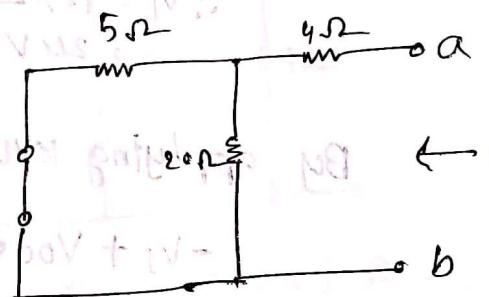


$E_{th} = \text{Open Circuit voltage at the two terminal ab.}$

$R_{th} = \text{Equivalent resistance with all the independent sources deactivated.}$



$$V_{oc} = E_{th}$$



$$R_{th} = \frac{5 \times 20}{5 + 20} + 4$$

$V_{th}$

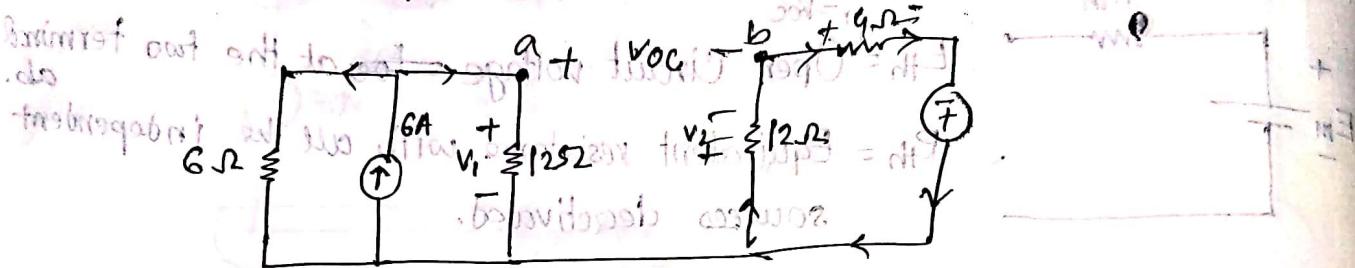
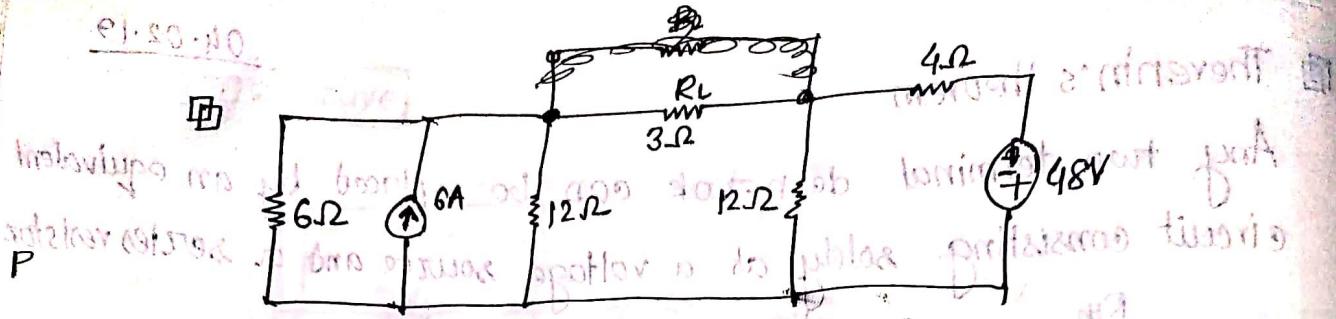
$$V_{oc} = \frac{20}{20+5} \times 50$$

$$= 40V$$

$$R_{th} =$$



$$\frac{40^2}{5+2} + \frac{40^2}{2+4} = 112W$$



$V_{TH}:$

$$I_s = \frac{6}{6+12} \times 6$$

$$I_s = 2A$$

$$\therefore V_1 = 12 \times 2 \\ = 24V$$

$$V_2 = \frac{12}{12+4} \times 48 \\ = 36V$$

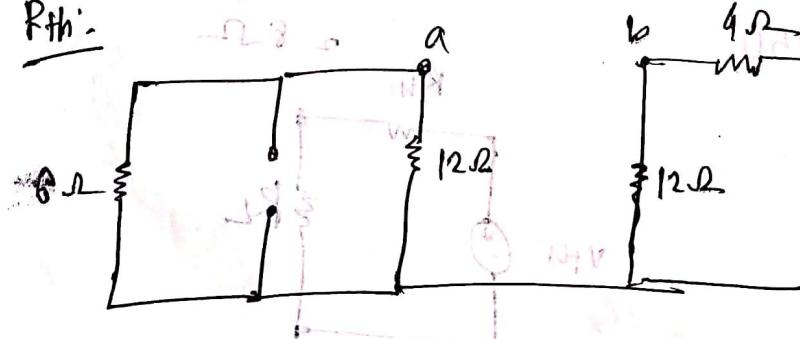
Now → By applying KVL,

$$-V_1 + V_{OC} - V_2 = 0$$

$$\therefore V_{OC} = 24 + 36$$

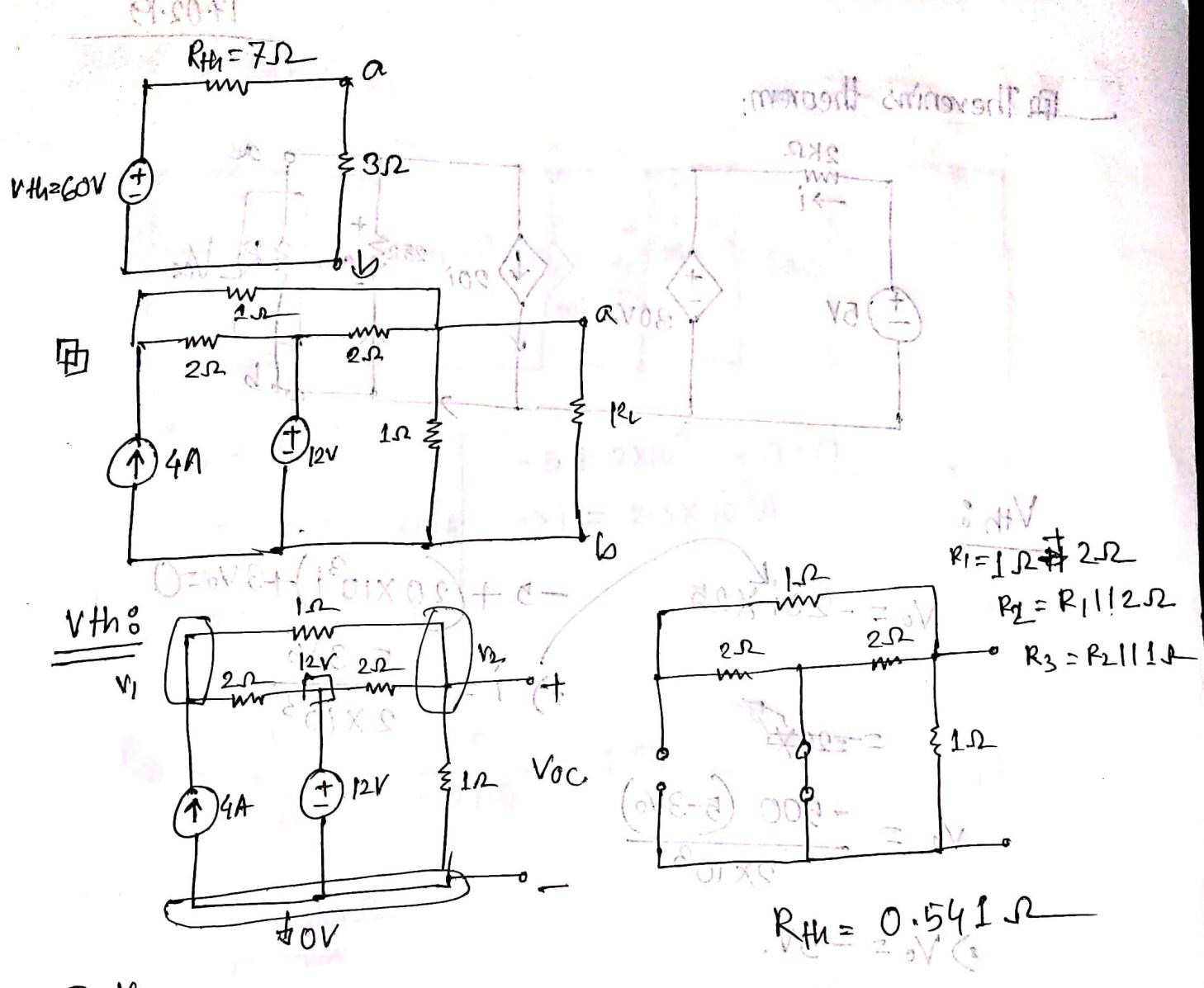
$$\therefore V_{TH} = 60V$$

$R_{TH}:$



$$R_{TH} = \frac{6 \times 12}{6+12} + \frac{4 \times 12}{4+12}$$

$$= 9 + 3 \\ = 12\Omega$$



For  $V_1$

$$\frac{V_1 - V_2}{1} + \frac{V_1 - 12}{2} - 4 = 0$$

$$2) 1.5V_1 - V_2 = 10 \quad \textcircled{1}$$

For  $V_2$ ,

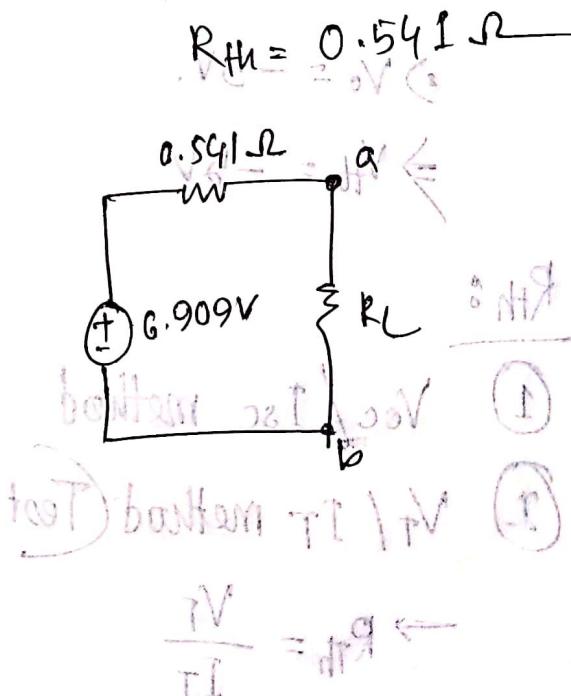
$$\frac{V_2 - 12}{2} + \frac{V_2 - V_1}{1} + \frac{V_2 - 0}{1} = 0$$

$$2) -V_1 + 2.5V_2 = 6 \quad \textcircled{II}$$

$V_1 = 11.27V$

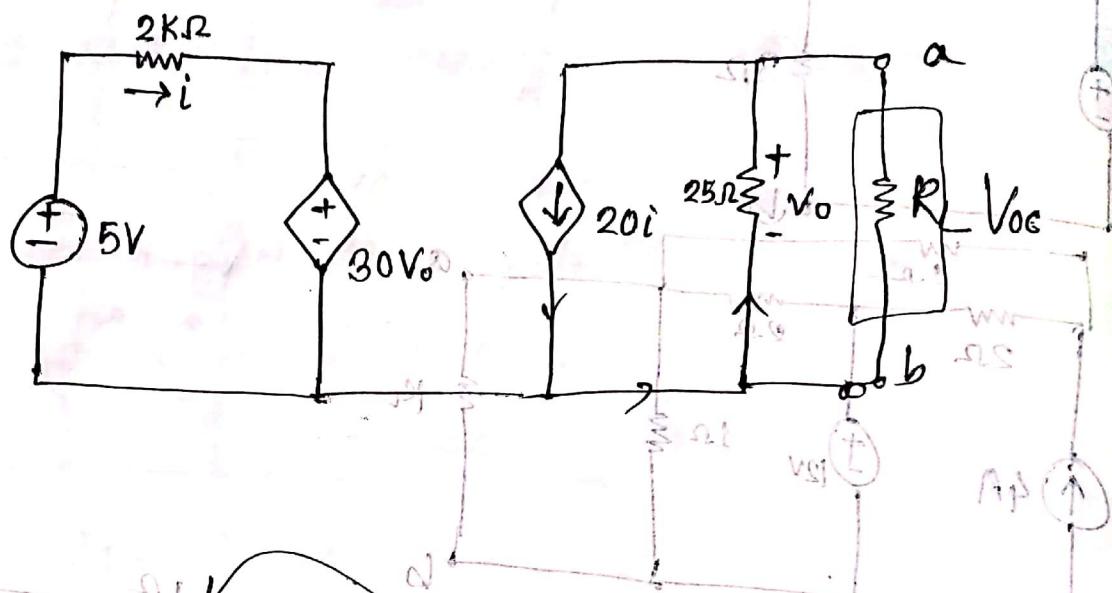
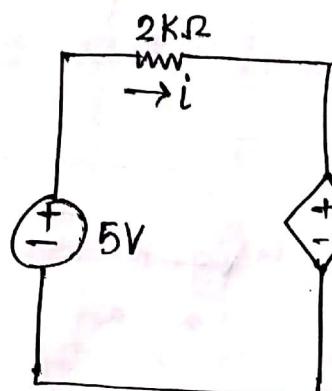
$V_2 = 6.909V$

$\therefore V_{oc} = V_2 - 0 = 6.909V$



17.02.19

## Thevenin's theorem:



$$V_{th} :$$

$$V_o = -20i \times 25$$

$$= -20 \times 25$$

$$V_o = \frac{-500(5-3V_o)}{2 \times 10^3}$$

$$-5 + (20 \times 10^3 i) + 3V_o = 0$$

$$i = \frac{5-3V_o}{2 \times 10^3}$$

$$\Rightarrow V_o = -5V$$

$$\Rightarrow V_{th} = -5V$$

$$R_{th} :$$

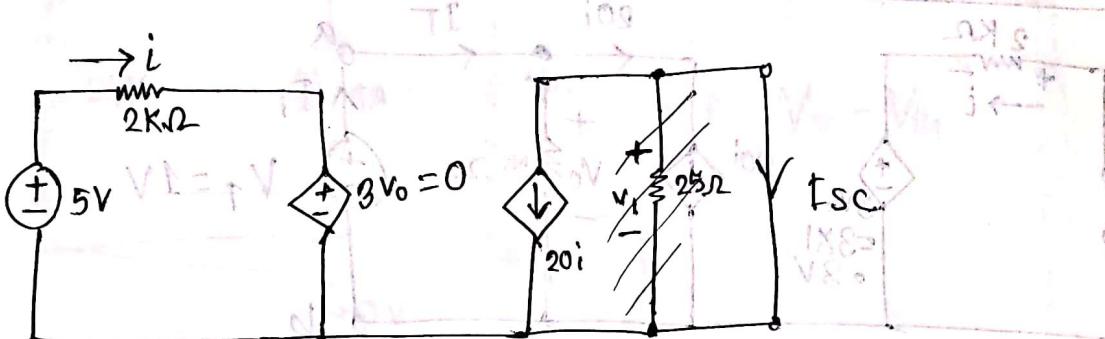
①  $V_{oc}$  /  $I_{sc}$  method

②  $V_T$  /  $I_T$  method (Test Voltage Method)

$$\Rightarrow R_{Th} = \frac{V_T}{I_T}$$

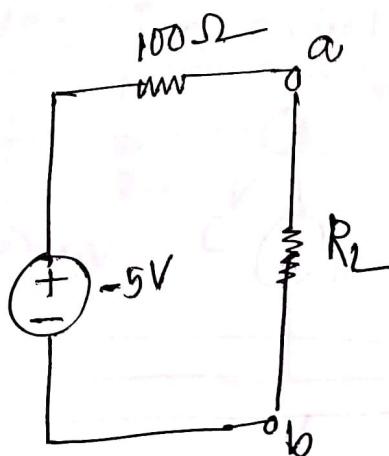
$$V_{FB, II} = 1V$$

Method: 1<sup>o</sup>  $V_{OC}$  /  $I_{SC}$

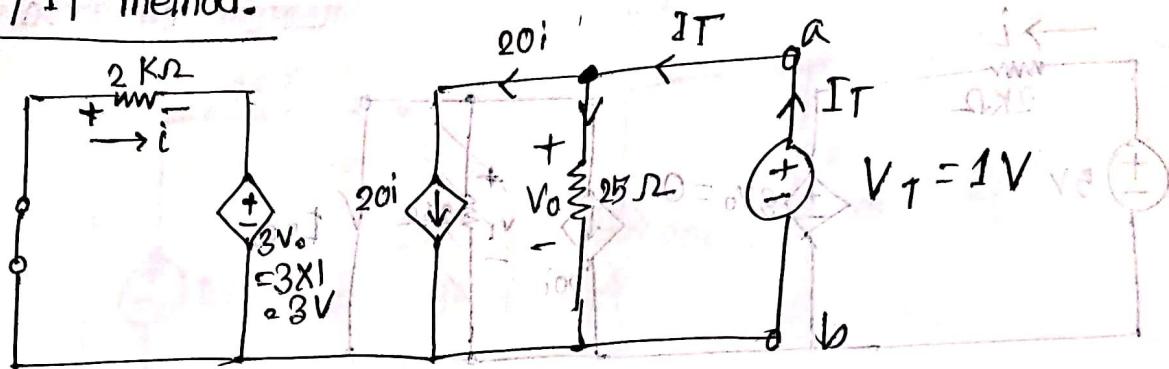


$$I_{SC} = -20i \quad | \quad -5 + 2 \times 10^3 i + 0 = 0 \\ -20 \times 2.5 \times 10^{-3} A \quad | \quad \Rightarrow i = 2.5 \times 10^{-3} A \\ = -50 \times 10^{-3} A \quad | \quad A_{01} \times 2.5 \times 10^{-3} A \\ = -0.05 A \quad | \quad A_{10,0} = T_1 \quad |$$

$$R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{-5}{-0.05} = 100\Omega \quad \frac{V}{T_1} = A_{10,0}$$



② VT/IT method:

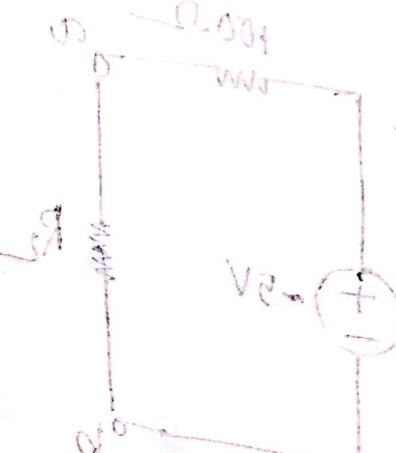


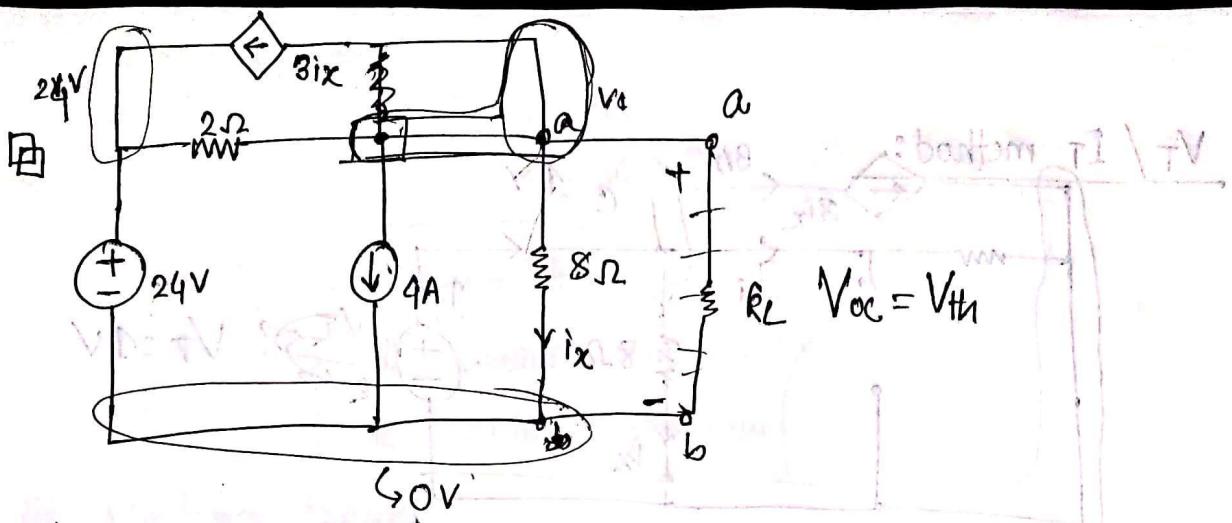
$$I_T = -\frac{1}{25} = -20i \quad \Rightarrow \quad 2 \times 10^3 i + 3 = 0$$

$$\Rightarrow i = -\frac{3}{2 \times 10^3} = -1.5 \times 10^{-3} \text{ A}$$

$$\therefore I_T = 0.01 \text{ A}$$

$$R_{th} = \frac{V_T}{I_T} = \frac{1}{0.01} = \frac{100}{0.01} = 100 \Omega$$





V<sub>th</sub>:

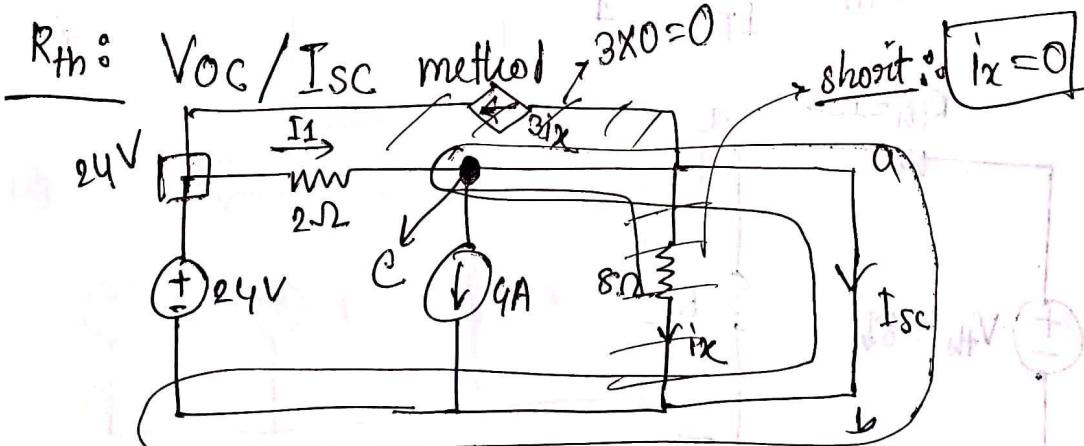
$$\frac{V_1 - 24}{2} + 4 + \frac{V_1}{8} + 3ix = 0 \quad | \quad ix = \frac{V_1 - 0}{8}$$

2)  $V_1 = 8V$

$$\Rightarrow V_{OC} = V_1 - 0 - \frac{0+0}{2} = \frac{1}{2} \cdot 8 = 4V$$

$$= 8V - 0$$

$$= 8V$$



$$\therefore I_1 = \frac{24-0}{2}$$

$$= 12A$$

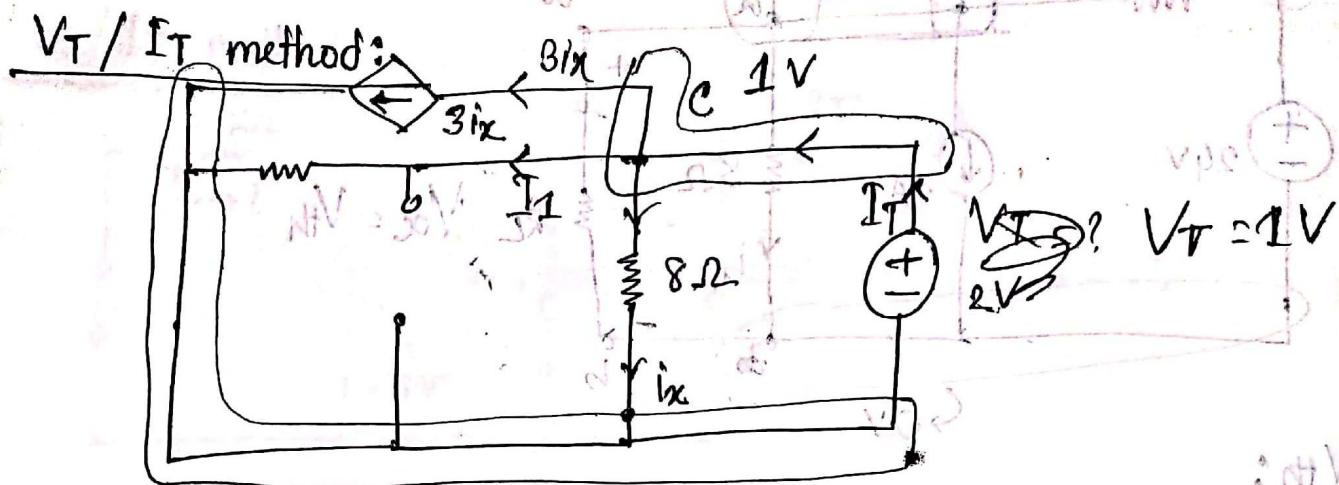
KCL at node C,

$$I_1 - 4 - I_{SC} = 0$$

$$\Rightarrow 12 - 4 - I_{SC} = 0$$

$$\Rightarrow I_{SC} = 8A$$

$$\therefore R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8}{8} = 1\Omega$$



KCL at node C,

$$I_T - i_x - I_1 - 3i_x = 0$$

$$1A - i_x - \frac{30}{8} - 3i_x = 0$$

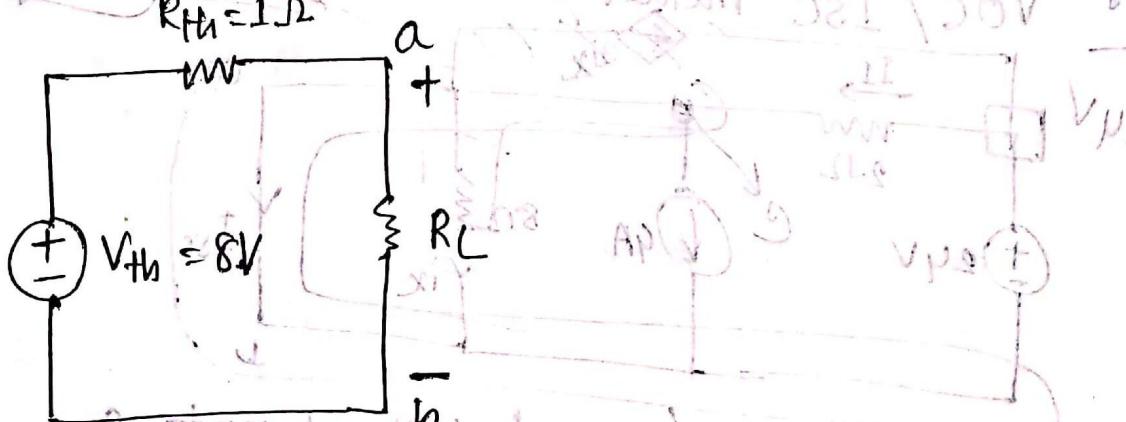
$$i_x = \frac{1}{8} A$$

$$\therefore I_T = \frac{1}{8} + \frac{1-0}{2} - 3 \cdot \frac{1}{8} = 0.125A = 12.5mA$$

$$\therefore I_T = 1A$$

$$R_{Th} = \frac{V_T}{I_T} = \frac{1}{1} = 1\Omega$$

$R_{Th} = 1\Omega$



$$0 = 32L - p - 11$$

$$0 = 32L - p - 11$$

$$A8 = 32L$$

$$8 = 20V \Rightarrow 4A$$

$$\frac{0-18}{8} = 1.5V$$

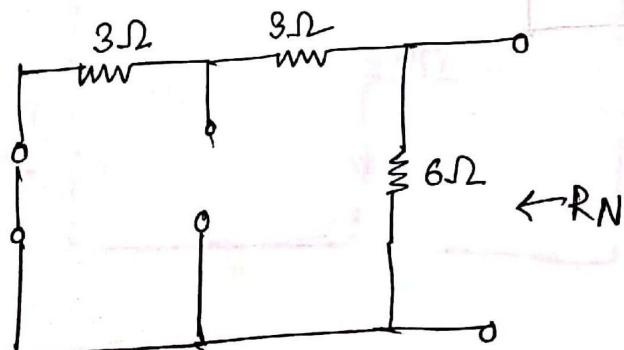
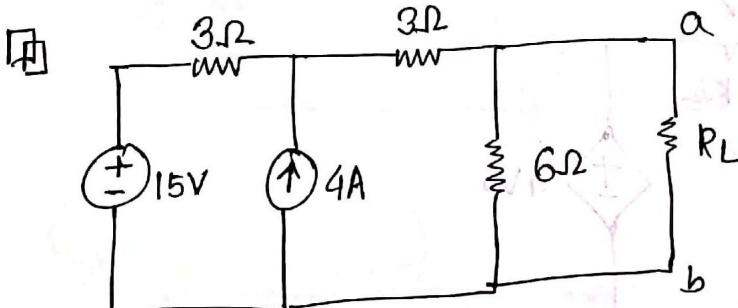
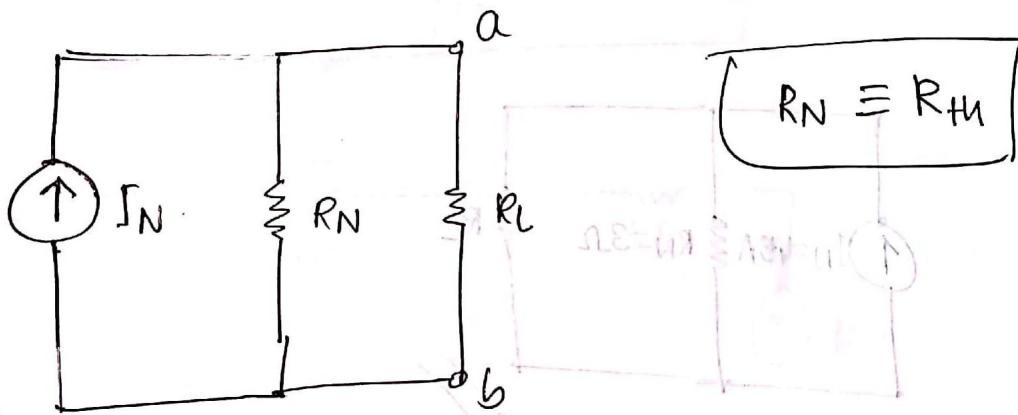
## Quiz - 02

4/03/2019

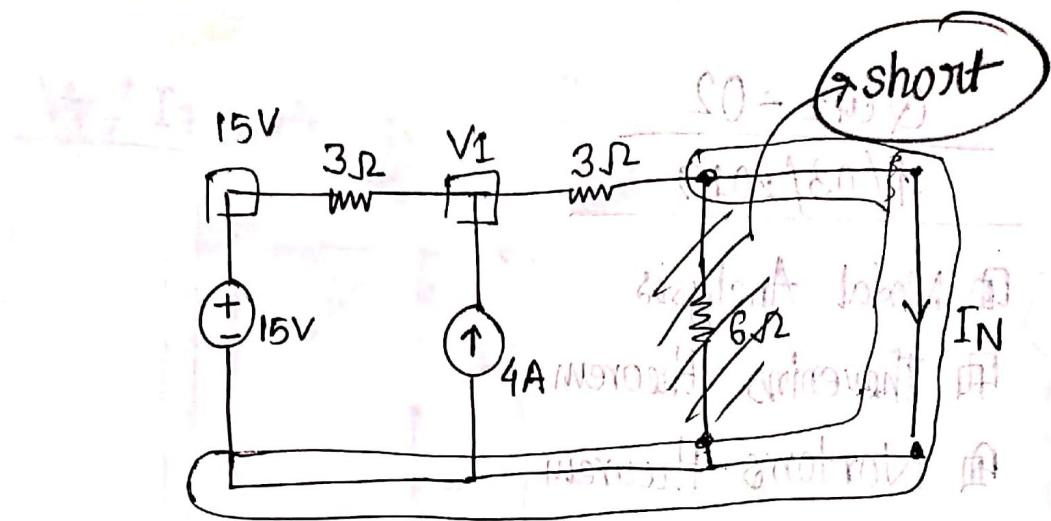
- Nodal Analysis
- Thevenin's theorem
- Norton's theorem

### ■ Norton's theorem:

Any two terminal de-network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.



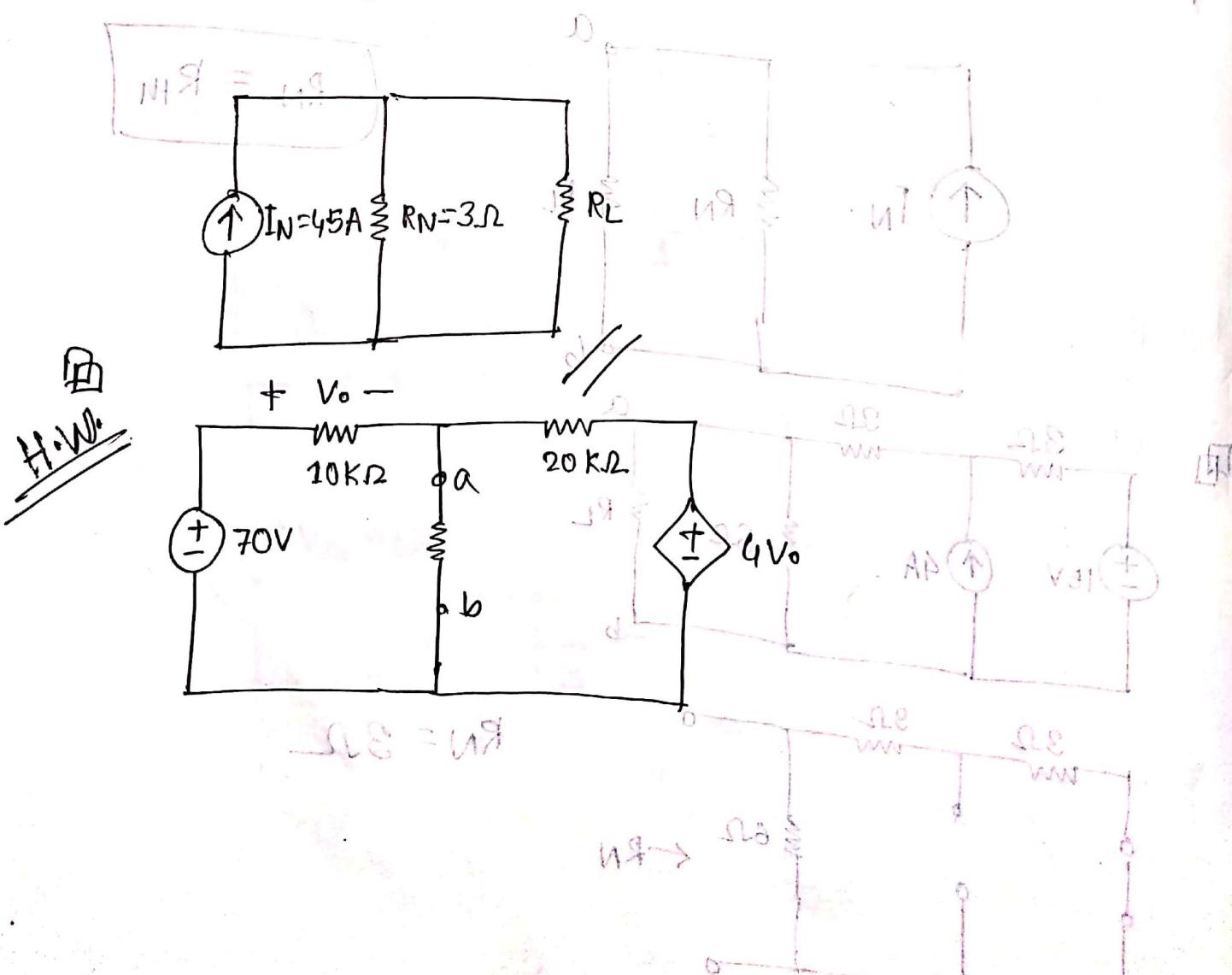
$$R_N = 3\Omega$$



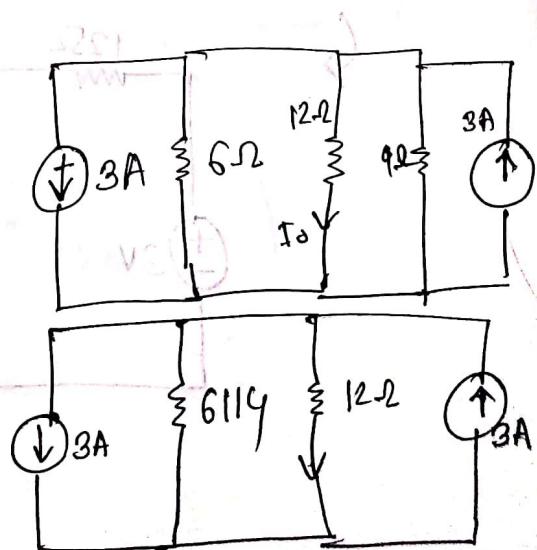
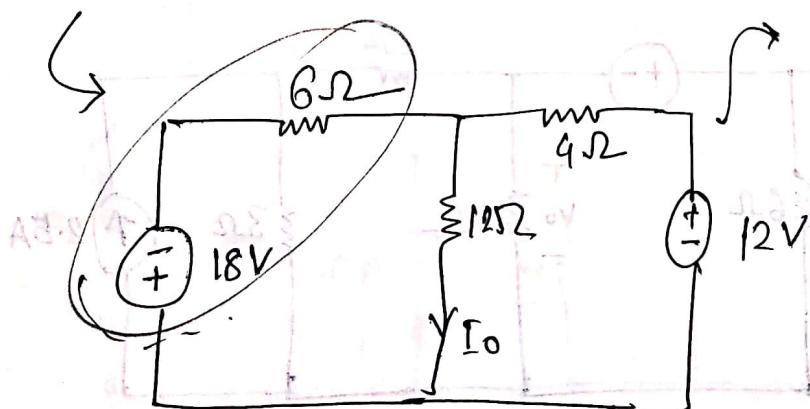
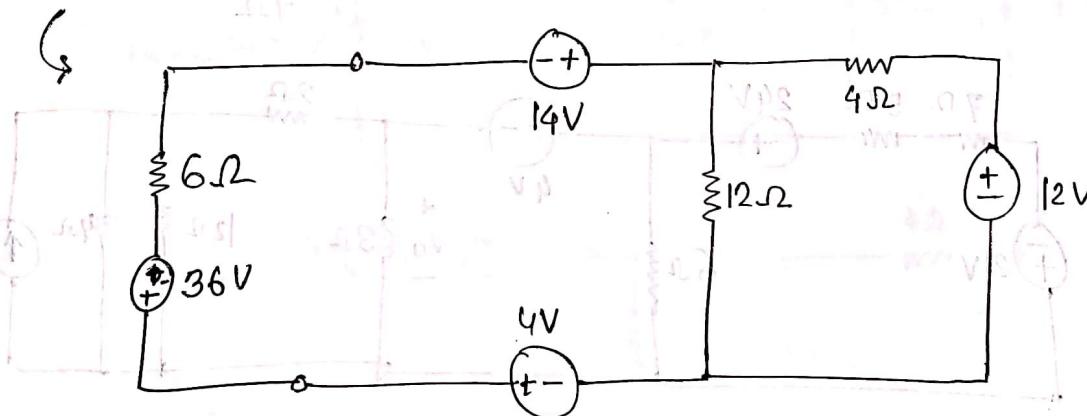
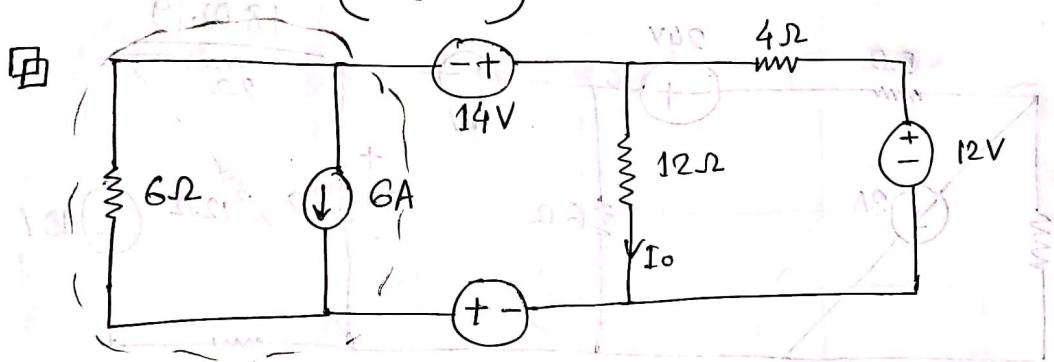
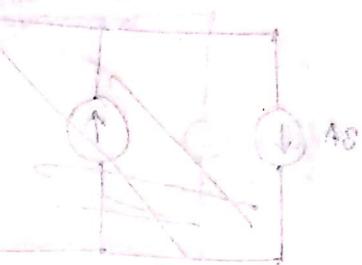
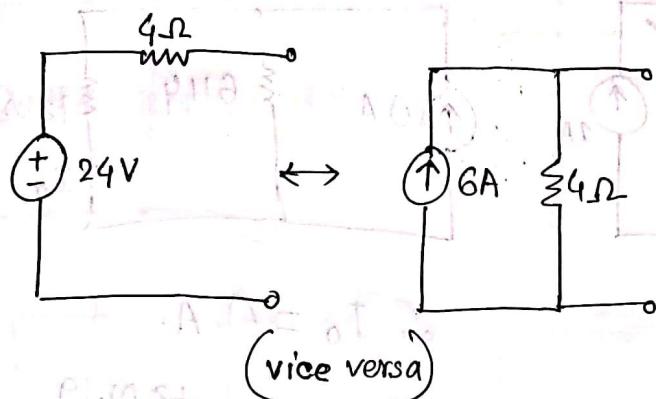
$$\frac{V_1 - 15}{3} + \frac{V_1}{3} - 4 = 0$$

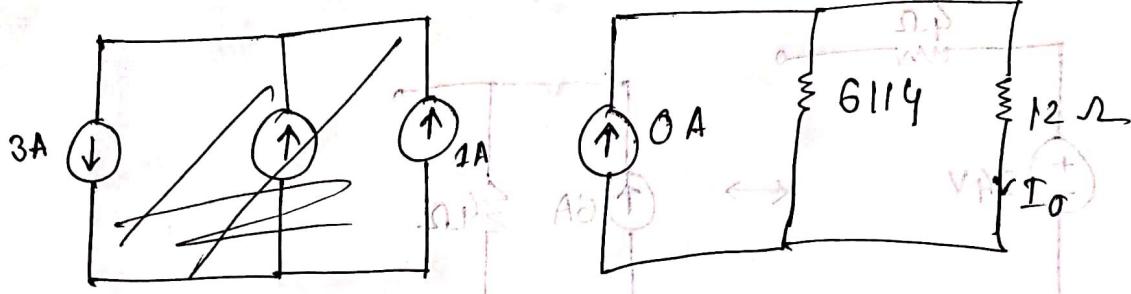
$\Rightarrow V_1 = 13.63$  V to calculate finding following solution following

$$I_N = \frac{V_1 - 0}{3} = 4.5 \text{ A}$$

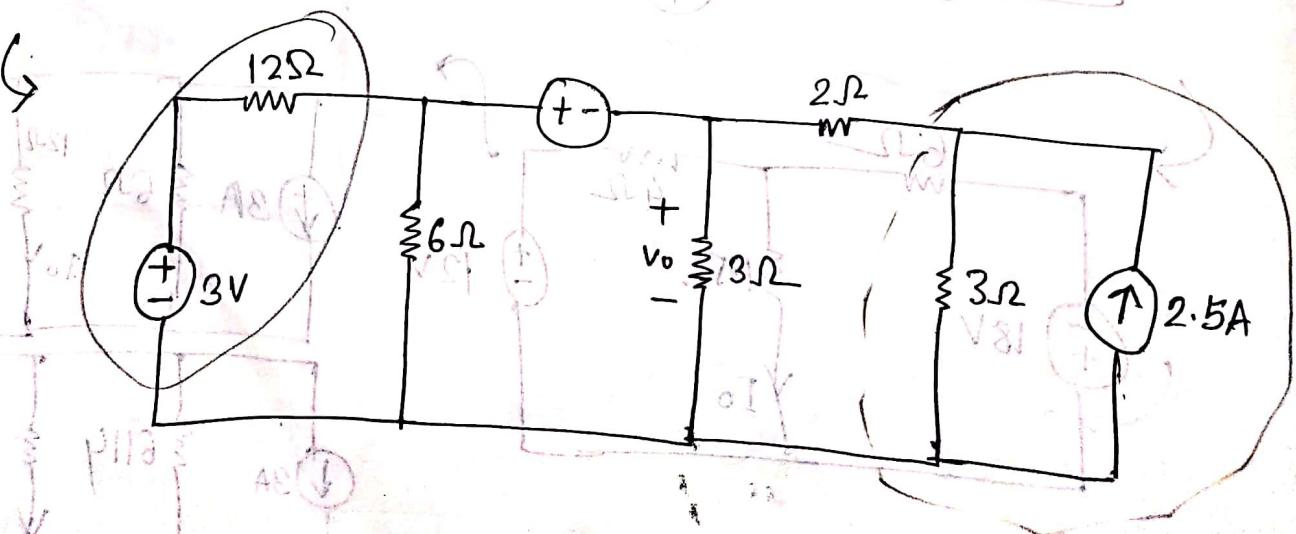
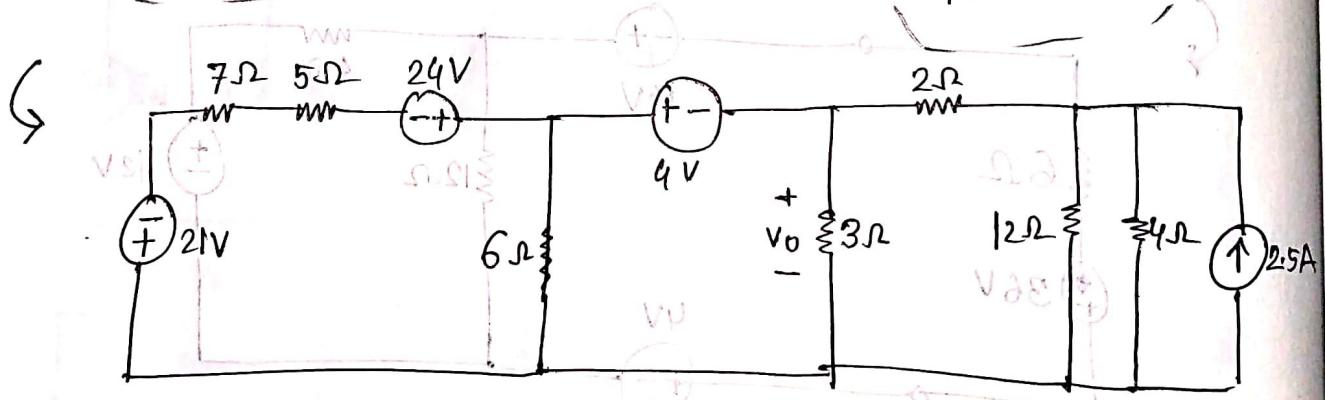
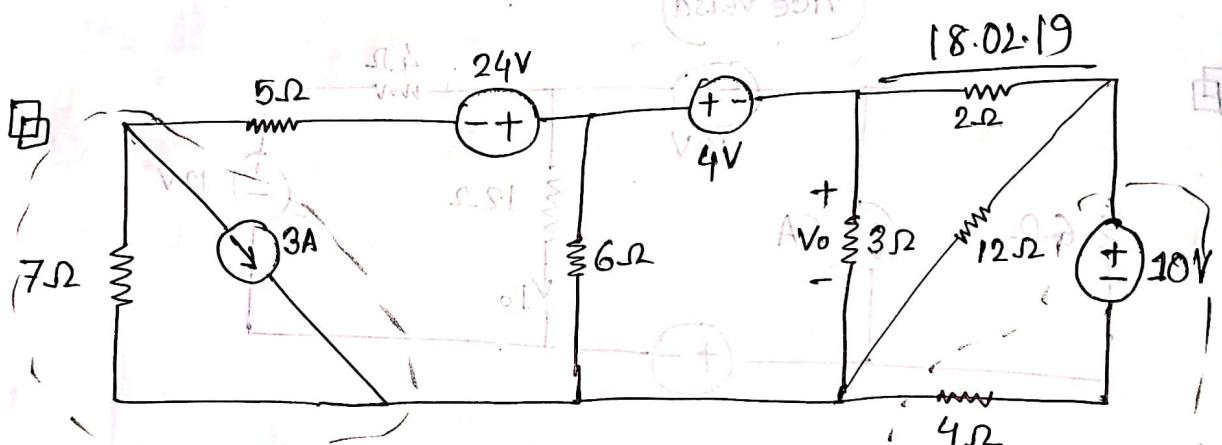


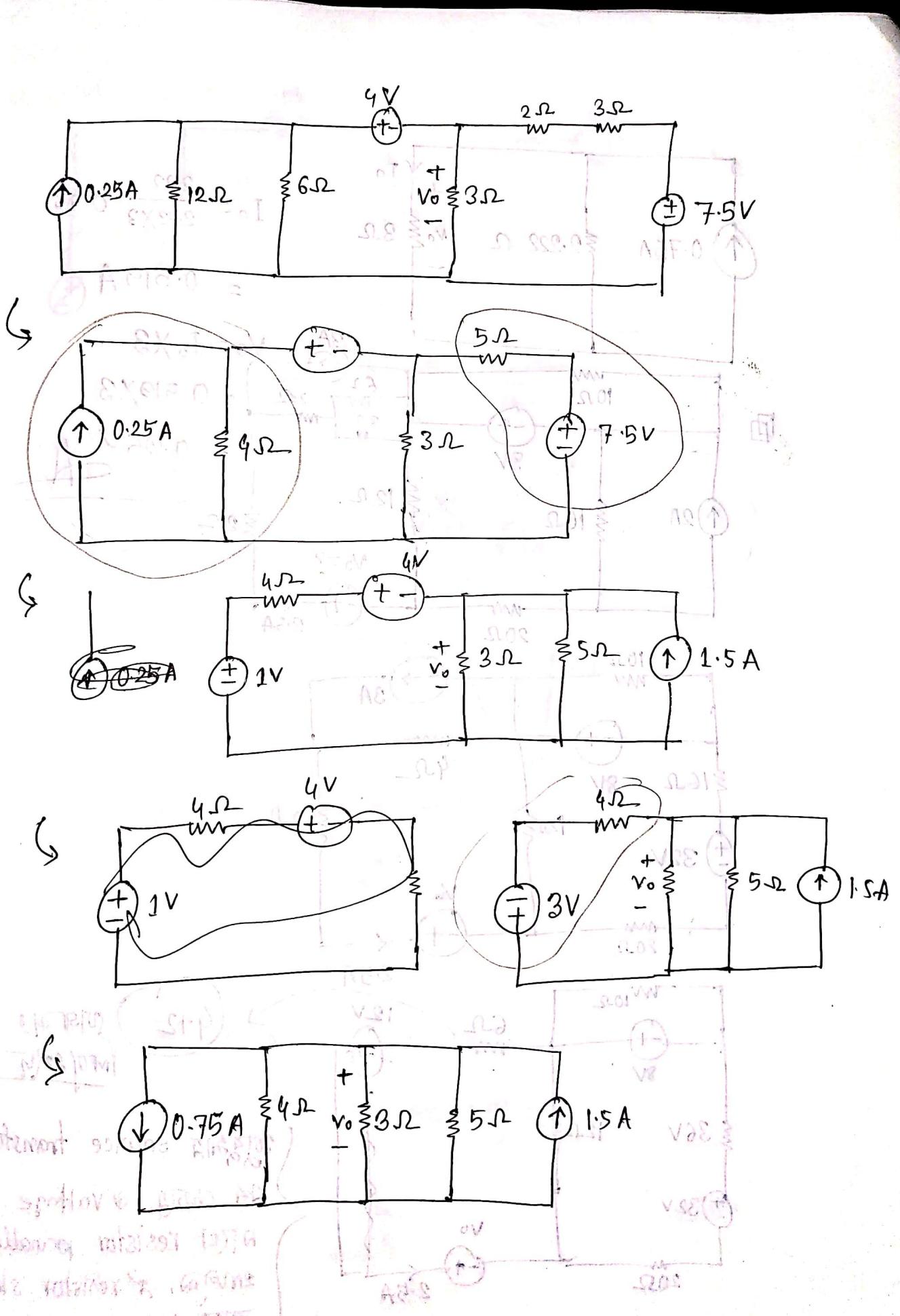
## Source Transformation:

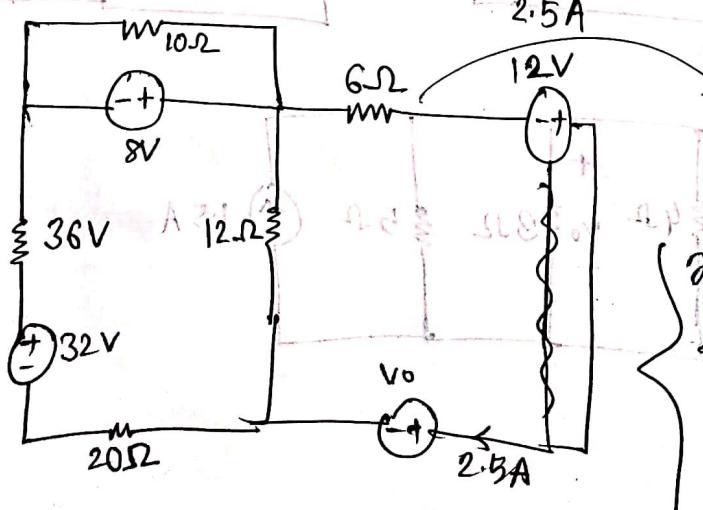
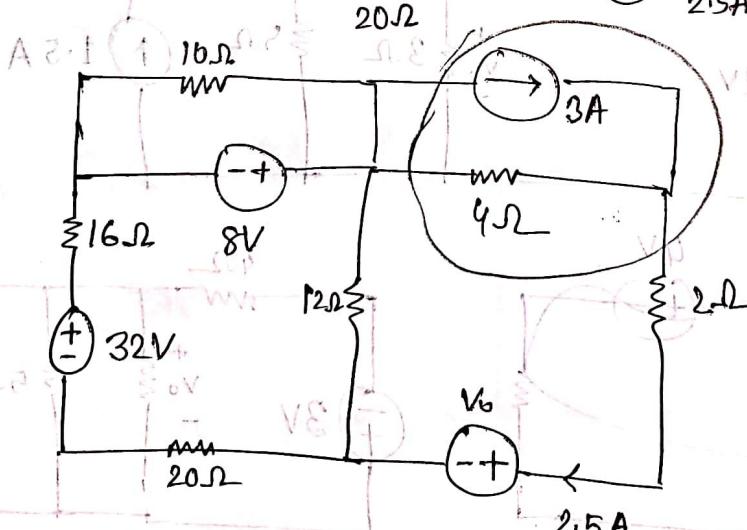
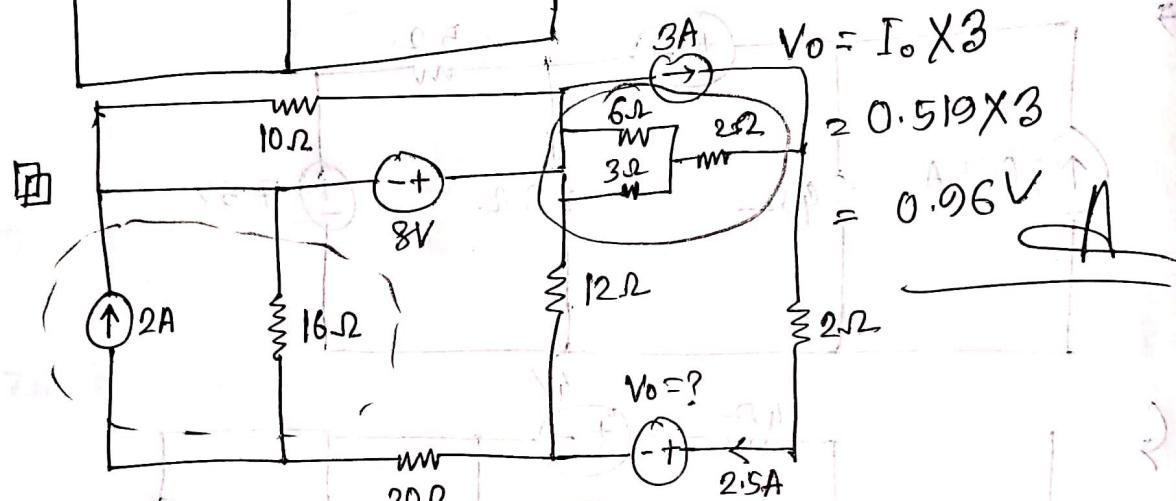
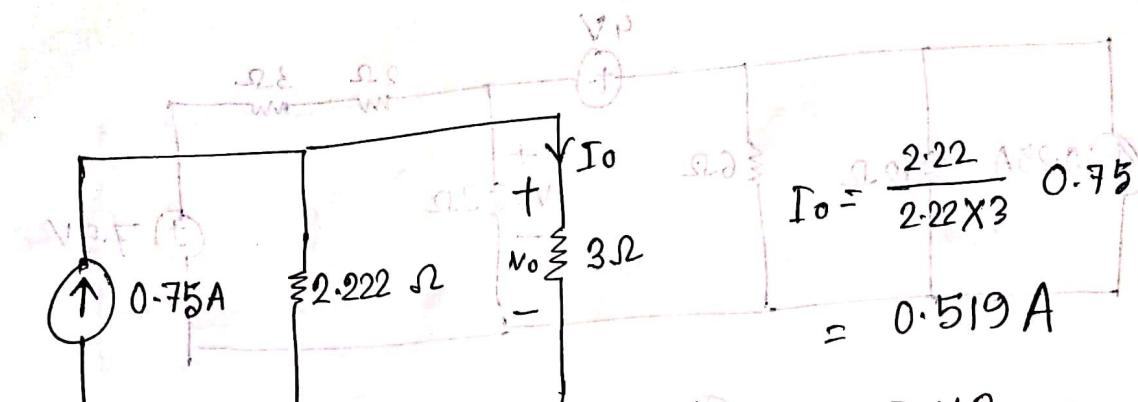




$$\therefore I_0 = 0 \text{ A.}$$

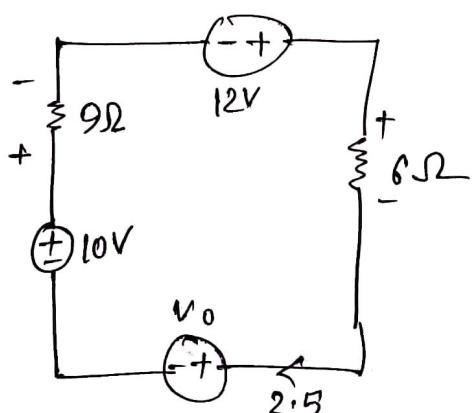
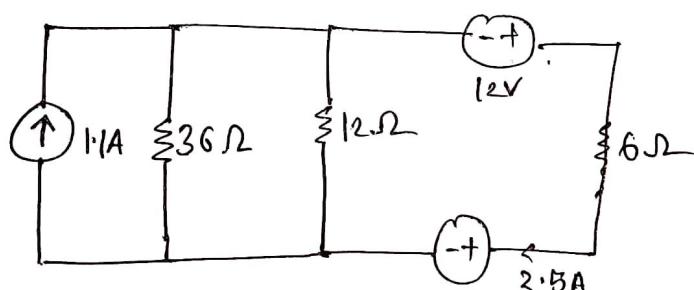
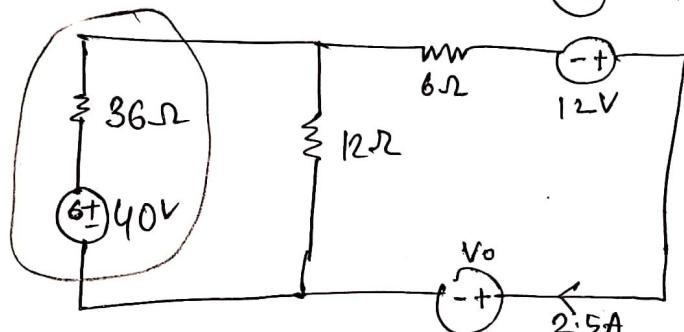
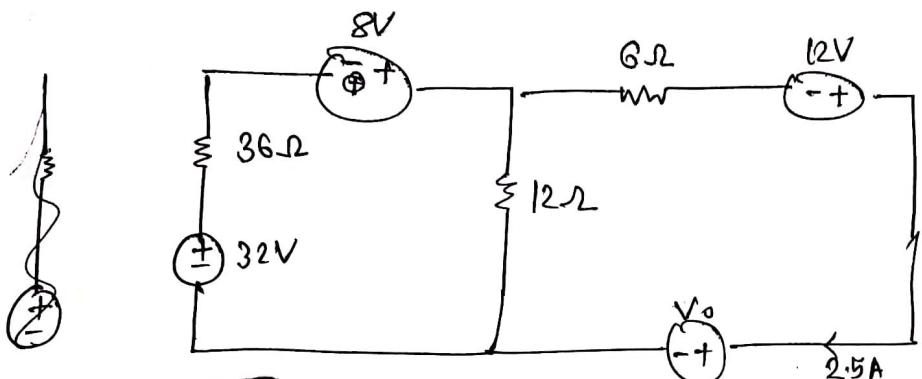






प्रृष्ठापात्र source transformation

प्रृष्ठा कम है, अतः voltage नहीं  
NTC resistor parallel  
मात्रा कम है, अतः resistor skip  
करका 2MΩ.



$$V_o - 10 + (9 \times 2.5) - 12 + (6 \times 2.5) = 0$$

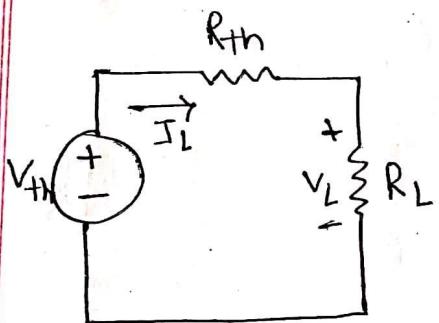
2)  $V_o = -15.5V$

A

Date: 03/03/2019

## Maximum Power Transfer Theorem

A load will receive maximum power from a network when its' resistance is exactly equal to the Thévenin's resistance of the network applied to the load.



$$I_L = \frac{V_{th}}{R_{th} + R_L} \quad \text{--- (i)}$$

$$\text{Now, } V_L = I_L R_L = \frac{V_{th}}{R_{th} + R_L} \cdot R_L \quad \text{--- (ii)}$$

$$P_L = V_L I_L$$

$$= \frac{V_{th}}{(R_{th} + R_L)} R_L \times \frac{V_{th}}{R_{th} + R_L}$$

$$= \frac{V_{th}^2}{(R_{th} + R_L)^2} \times R_L$$

$$(R_{th} + R_L)^2 - 2R_L(R_L + R_{th})$$

$$\text{Now, } \frac{dP_L}{dR_L} = V_{th}^2 \frac{(R_{th} + R_L)^2 - 2R_L(R_L + R_{th})}{(R_L + R_{th})^3}$$

For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$\Rightarrow V_{th}^2 \cdot \frac{(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)}{(R_L + R_{th})^4} = 0$$

$$\Rightarrow (R_{th} + R_L)^2 - 2R_L(R_L + R_{th}) = 0$$

$$\Rightarrow (R_{th} + R_L)(R_{th} + R_L - 2R_L) = 0$$

$$\Rightarrow (R_{th} + R_L)(R_{th} - R_L) = 0$$

Hence,  $R_{th} + R_L \neq 0$

$$\therefore R_{th} - R_L = 0$$

$$\Rightarrow R_L = R_{th}$$

When,  $R_L = R_{th}$

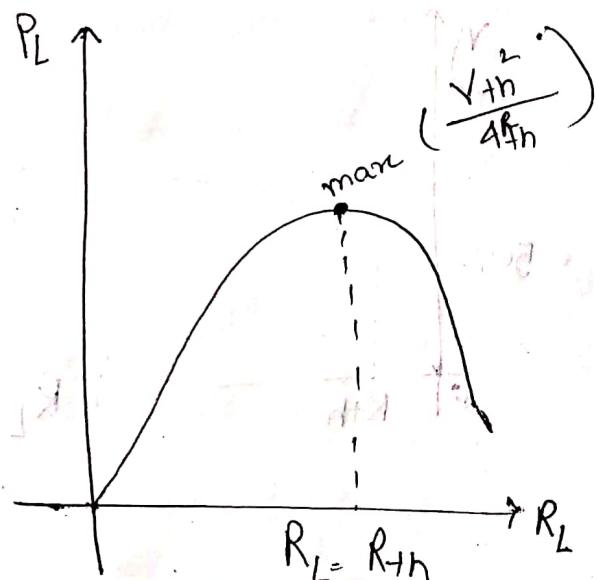
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$= \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{2R_{th}}$$

$$P_{L\max} = I_L^2 R_L$$

$$= \frac{V_{th}^2}{4R_{th}^2} \times$$

$$R_{th} = \frac{V_{th}^2}{4R_{th}}$$



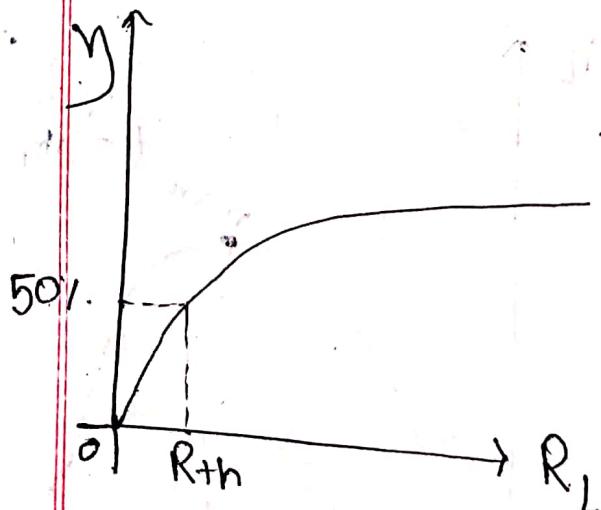
## Efficiency

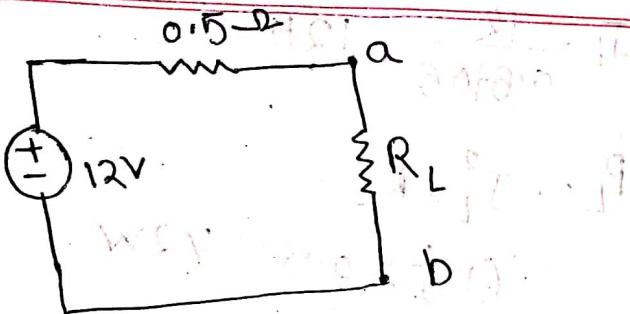
$$\eta = \frac{P_L}{RP_S} \times 100\%$$

$$\eta = \frac{I_L^2 R_L}{I_L^2 (R_L + R_{th})} \times 100$$

When  $R_L = R_{th}$

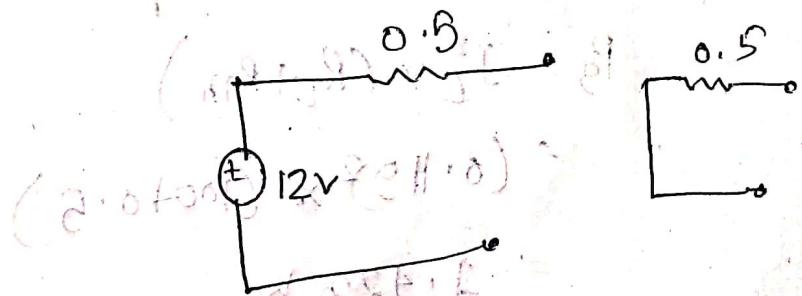
$$\eta = \frac{I_L^2 R_{th}}{I_L^2 (R_{th} + R_{th})} = \frac{1}{2} \times 100 = 50\%$$



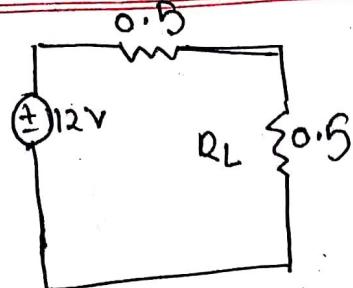


- (a) Determine  $R_L$  for maximum power transfer.
- (b) Under maximum power conditions, what are the current level and the power to the load?
- (c) What is the efficiency of the system?
- (d) If a load of  $100\Omega$  is connected to the terminals, what will be the efficiency and transferred power to the load?
- (e) Determine  $R_L$  for obtaining efficiency of 75%.

$$(a) R_L = 0.5 \Omega = R_{th}$$



(b)



$$I_L = \frac{12}{0.5 + 0.5} = 12A$$

$$P_L = I_L^2 \times R_L$$

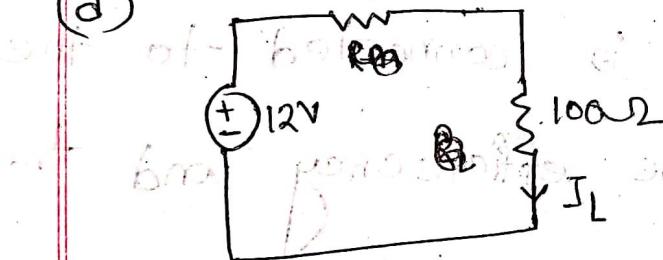
$$= (12)^2 \times 0.5 = 72W$$

$$P_{L\max} = \frac{V_{th}^2}{4R_{th}}$$

$$= \frac{(12)^2}{4 \times 0.5} = 72W$$

$$\eta = 50\%$$

$$(d) \quad 0.5 \Omega$$



$$P_L = I_L^2 \times R_L$$

$$= (0.119)^2 \times 100$$

$$= 1.4161W$$

$$I_2 = \frac{12}{0.5 + 100}$$

$$= 0.119 A$$

$$P_S = I_L^2 \times (R_L + R_{th})$$

$$= (0.119)^2 \times (100 + 0.5)$$

$$= 1.423 W$$

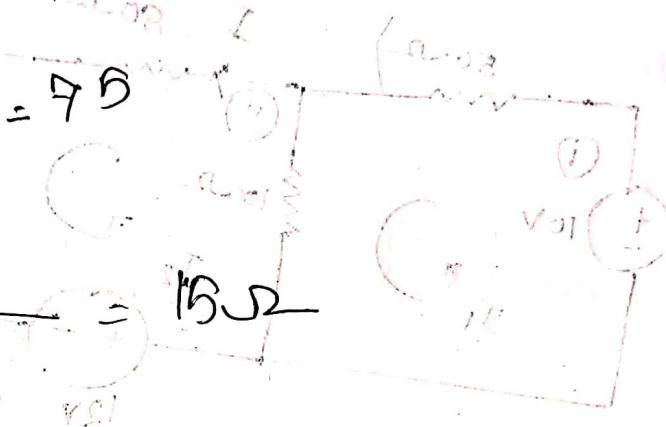
$$\eta = \frac{P_L}{P_S} \times 100 = \frac{1.4161}{1.423} \times 100 = 99.31\% \text{ dear 1A}$$

(6)

$$(e) \eta = \frac{P_L}{P_S} \times 100$$

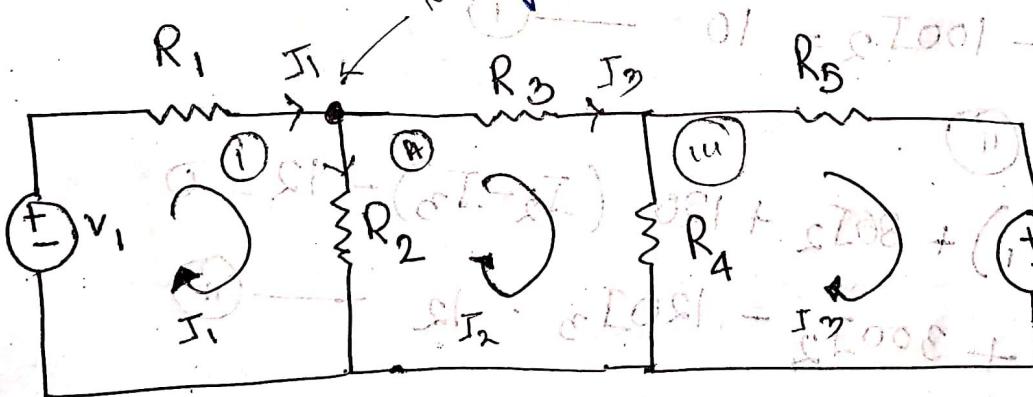
$$\Rightarrow \frac{I_L^2 R_L \times 100}{I_L^2 (R_L + R_{L2})}$$

$$\Rightarrow R_L = \frac{0.75 \times 0.5}{1 - 0.75}$$



(D) dear 1A

Mesh current analysis



At mesh ①,

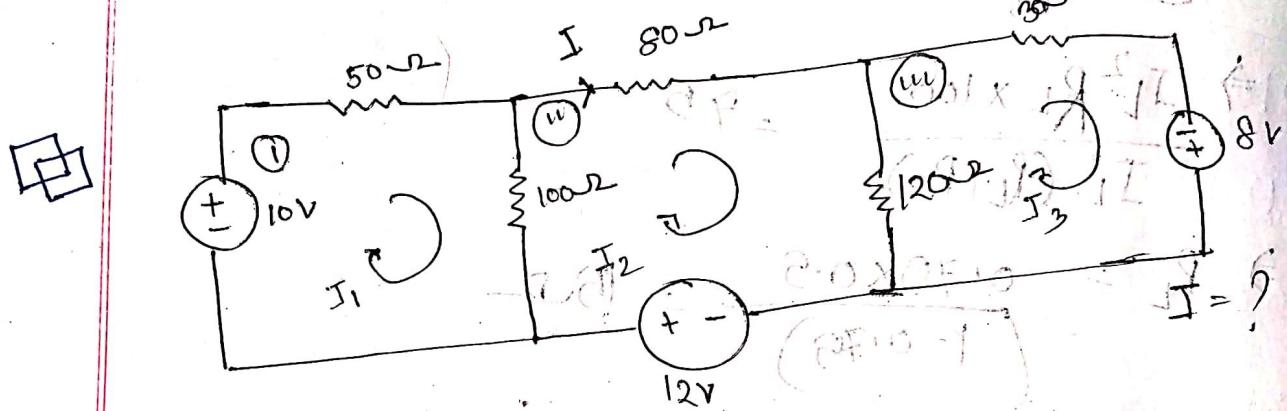
$$-v_1 + I_1 R_1 + (I_1 - I_2) R_2 = 0 \quad \text{(1)}$$

$$\text{At mesh } ③, \quad (I_2 - I_1) R_2 + R_3 I_2 + R_4 (I_2 - I_3) = 0 \quad \text{(3)}$$

(61)

At mesh (ii)

$$R_4(J_3 - J_2) + R_5 J_3 + V_2 = 0$$



At mesh (i)

$$-10 + 50J_1 + 100(J_1 - J_2) = 0 \quad \text{at mesh (i)}$$

$$\Rightarrow 150J_1 - 100J_2 = 10 \quad \text{--- (i)}$$

At mesh (ii)

$$100(J_2 - J_1) + 80J_2 + 120(J_2 - J_3) - 12 = 0$$

$$\Rightarrow -100J_1 + 300J_2 - 120J_3 = 12 \quad \text{--- (ii)}$$

At mesh (iii)

$$20J_3 + 120(J_3 - J_2) - 8 = 0 \quad \text{at mesh (iii)}$$

$$\Rightarrow -120J_2 + 150J_3 = 8 \quad \text{--- (iii)}$$

Solving ①, ② & ③

$$I_1 = 0.188 \text{ A} \quad (1) \quad L + (R - \epsilon L) S + (R + \epsilon L) I_1 + I_2 = 0$$

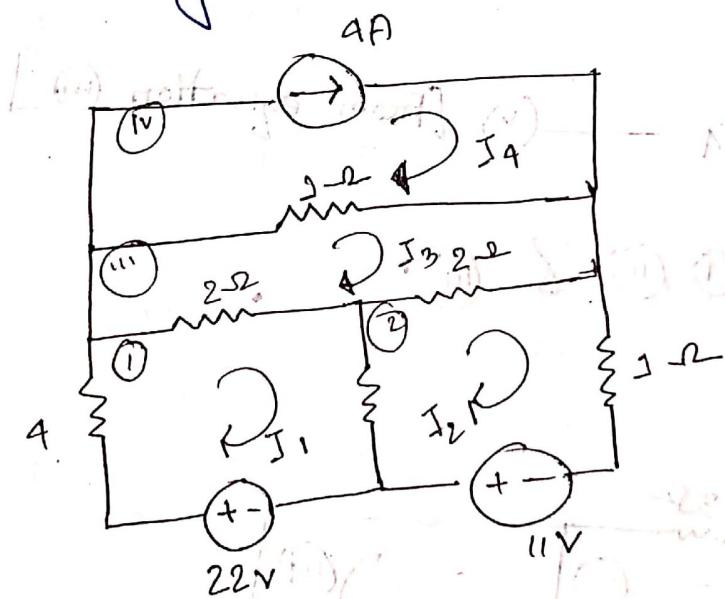
$$I_2 = 0.1825 \text{ A} \quad (2) \quad -L + (R - \epsilon L) S + I_1 + I_2 = 0$$

$$I_3 = 0.1993 \text{ A} \quad (3) \quad \text{desire IA}$$

$$I = I_2 = 0.1825 \text{ A} \quad (4) \quad S + (R - \epsilon L) C + (R + \epsilon L) S$$

Date: 10/03/2019

### Mesh Analysis



At mesh ①

$$-22 + 4I_1 + 2(I_1 - I_3) + 2(I_1 - I_2) = 0 \quad A$$

$$\Rightarrow 8I_1 - 2I_2 - 2I_3 = 22 \quad \rightarrow ①$$

At mesh (II)

$$-1I + 2(I_2 - I_1) + 2(I_2 - I_3) + 1(I_2) = 0 \quad (II)$$

$$\Rightarrow -2I_1 + 5I_2 - 3I_3 = 11 \quad (II)$$

At mesh (III)

$$2(I_3 - I_1) + 1(I_3 - I_4) + 2(I_3 - I_2) = 0 \quad (III)$$

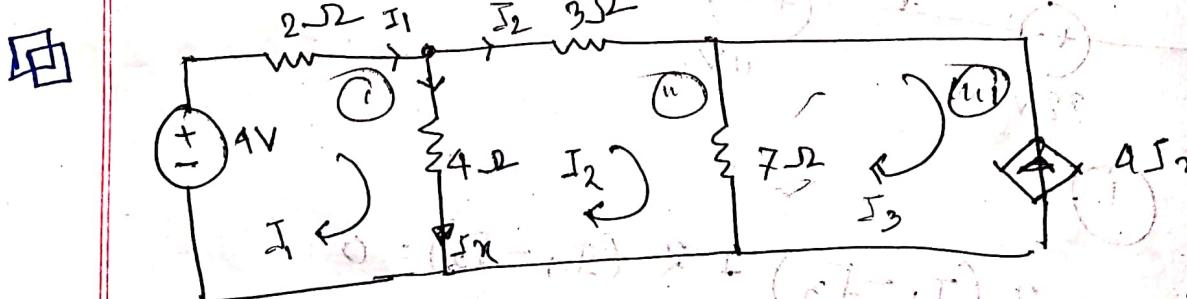
At mesh (IV)

$$I_4 = 4 \quad (IV)$$

$$I_4 = 4 \quad (III) \Rightarrow$$

$$-2I_1 - 2I_2 + 5I_3 = 4$$

$$\left. \begin{array}{l} I_1 = 6A \\ I_2 = 7A \\ I_3 = 6A \end{array} \right\} \text{Solving } (I), (II) \text{ & } (III)$$



At mesh (I)

$$-4 + 2I_1 + 4(I_1 - I_2) = 0$$

$$\Rightarrow 6I_1 - 4I_2 = 4 \quad (I)$$

64

At mesh (A)

$$4(J_2 - J_1) + 3J_2 + 7(J_2 - J_3) = 0 \quad (I) \text{ about A}$$

$$\Rightarrow -4J_1 + 14J_2 - 7J_3 = 0 \quad (II) \text{ about A}$$

At mesh (III)

(III) about III

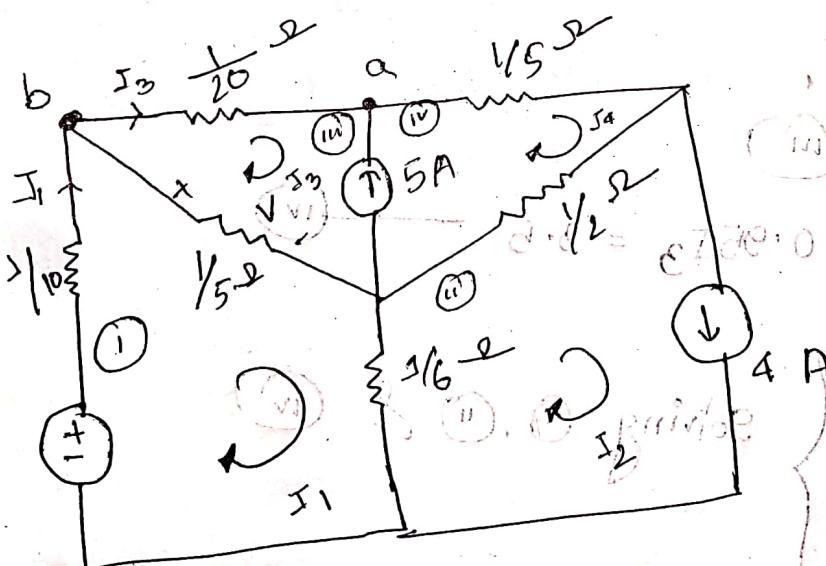
$$J_3 = -4J_x \quad \rightarrow J_1 - J_2 - J_x = 0 \quad (III) \text{ about III}$$

$$\Rightarrow 4J_1 - 4J_2 + J_3 = 0 \quad (III) \text{ about III}$$

$$J_1 = -4.67 \text{ A}$$

$$J_2 = 8 \text{ A}$$

$$J_3 = -13.33 \text{ A}$$



(65)

At mesh (i)

$$-6 + \frac{1}{10} I_1 + \frac{1}{5} (I_1 - I_3) + \frac{1}{6} (I_1 - I_2) = 0 \quad (i)$$

$$\Rightarrow 0.47 I_1 - 0.167 I_2 - 0.2 I_3 = 6 \quad (ii)$$

At mesh (iv)

$$I_2 = 40$$

$$\text{At superc. mesh, } \frac{1}{5} (I_3 - I_1) + \frac{1}{20} I_3 + \frac{1}{5} I_4 + \frac{1}{2} (I_4 - I_2) = 0 \quad (iii)$$

k<sub>EL</sub> at node a,

$$I_3 + 5 - I_4 = 0$$

$$\Rightarrow I_4 = I_3 + 5$$

From equation (iii)

$$-0.2 I_1 - 0.5 I_2 + 0.95 I_3 = 3.5$$

$$I_1 = 37.375 \text{ A}$$

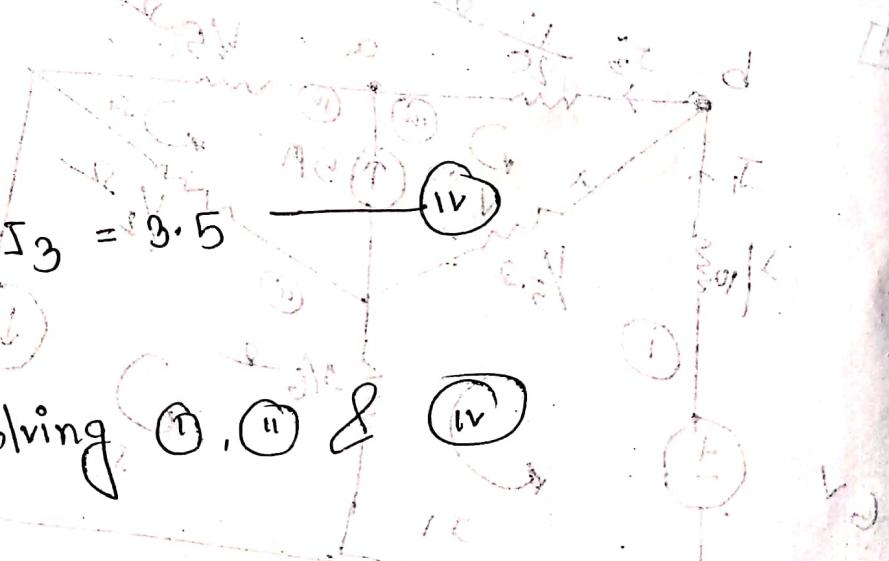
$$I_2 = 40 \text{ A}$$

$$I_3 = 25.316 \text{ A}$$

$$I_4 = 30.316 \text{ A}$$

Solving (i), (ii) &amp; (iv)

}



KCL at node (b)

$$\sum I = I_1 - I_3 = 0$$

$$\Rightarrow I = I_1 - I_3$$

$$V = \frac{1}{5} \times I$$

$$= \frac{1}{5} (I_1 - I_3) = \frac{1}{5} (37.75 - 25.316) = 2.4868 V$$

$$\frac{I}{I_1} = 11$$

## Magnetic Circuits

$$\text{Effect} = \frac{\text{Cause}}{\text{Opposition}}$$

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}}$$

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}}$$

$$\Phi = \frac{F}{R}$$

Unit:

$$\frac{\text{At}}{\text{At/Hb}} = \text{Hb (Weber)}$$

$$\text{mmf} = N \times I$$

Magnetizing force / Magnetic field strength / Magnetic field intensity

The mmf per unit length is called magnetizing force

$$H = \frac{F}{l}$$

Unit : At/m

$$H = \frac{F}{l} = \frac{NI}{l}$$

Flux density

$$B = \frac{\Phi}{A} \quad [\text{Flux per unit area}]$$

We know,

$$H = \frac{NI}{l}; \quad R = \frac{l}{\mu A}$$

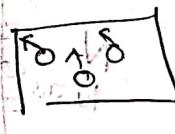
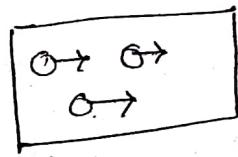
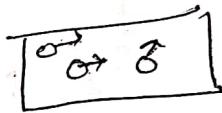
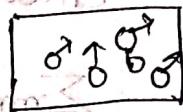
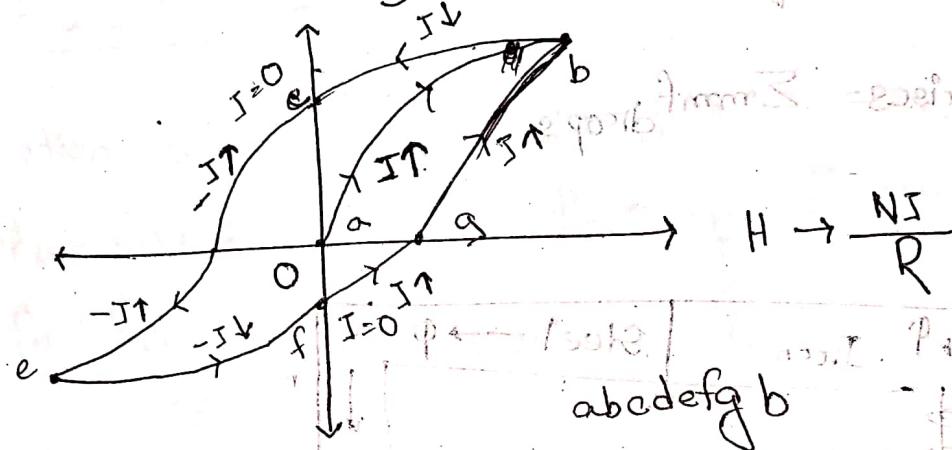
$$\begin{aligned} \text{Now, } B &= \frac{F/R}{A} = \frac{F/l}{\mu A} \\ &= \frac{F}{A \cdot l/\mu A} = \frac{Fu}{l} = \frac{NIu}{l} = uH \end{aligned}$$

Unit : Wb/m<sup>2</sup> (Tesla) (T)

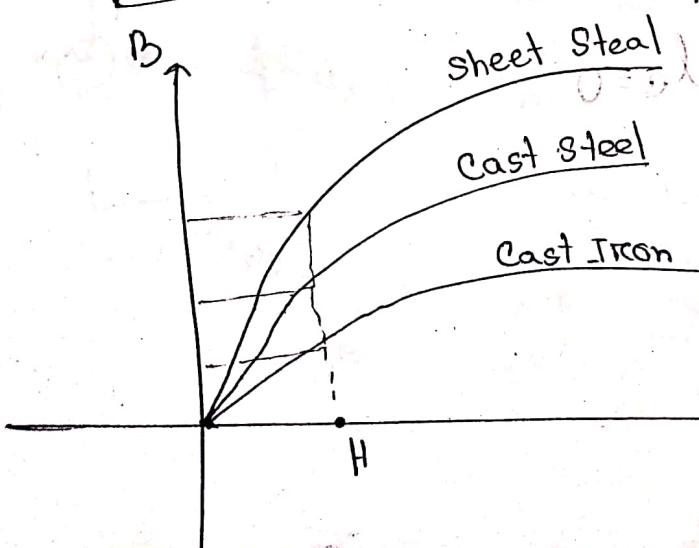
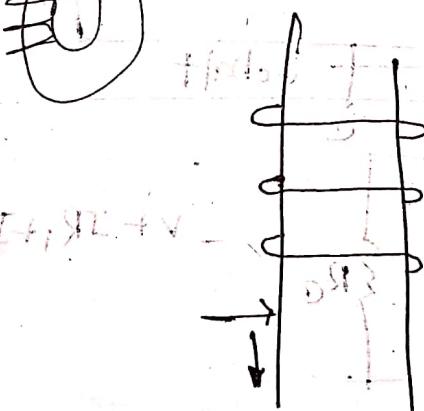
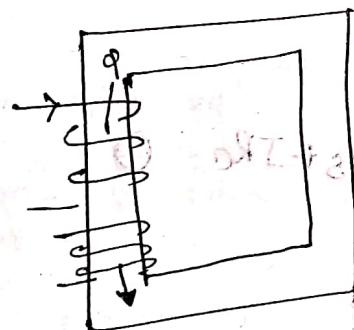
## Magnetic Circuit & flux produced

to support the coils with no iron saturation  
Hysteresis → to avoid heating due to hysteresis loss

→ to avoid loss of loops in flux path



$J = 0$

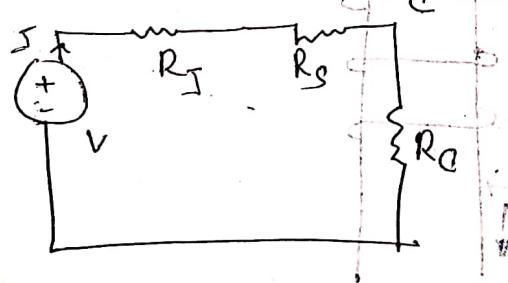
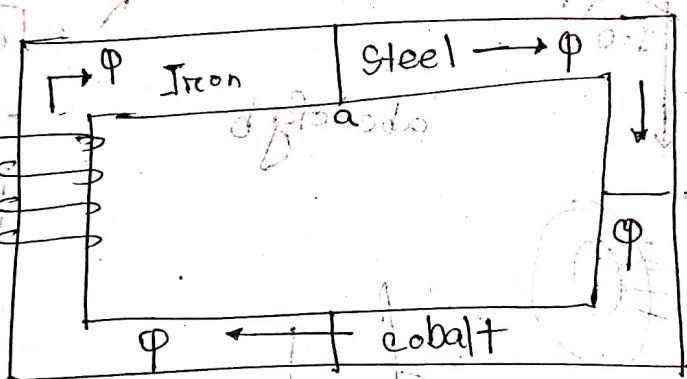




### Ampere's Circuital Law

The algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero.

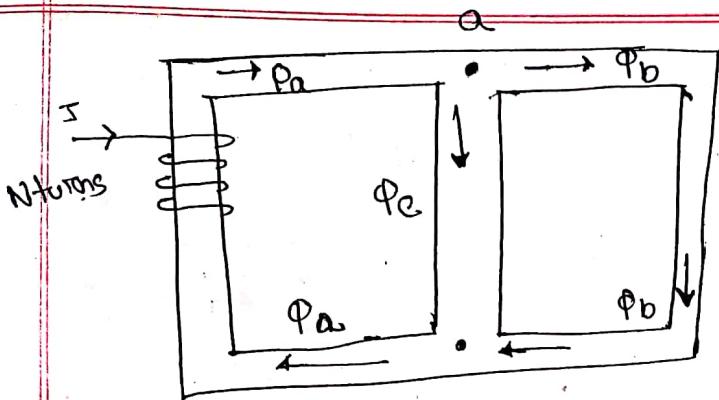
$$\sum \text{mmf rises} = \sum \text{mmf drops}$$



$$-NI + H_I l_I + H_S l_S + H_C l_C = 0$$

Left & long

more long



At junction a,

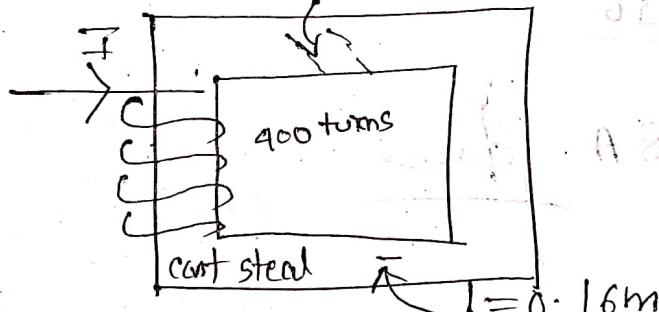
$$\Phi_a - \Phi_b - \Phi_c = 0$$

$$\Rightarrow \Phi_a = \Phi_b + \Phi_c$$

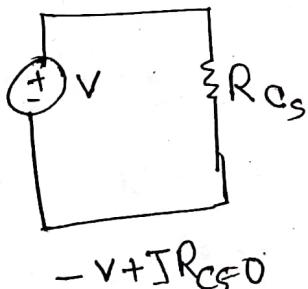
At junction b,

$$\Phi_a + \Phi_b - \Phi_b = 0$$

$$\Rightarrow \Phi_a = \Phi_b + \Phi_c$$



Find the value of I required to develop a magnetic flux of  $\Phi = 4 \times 10^{-4}$  Wb



By applying Amperes circuital law,

$$-N\mathbf{I} + \mathbf{H}_{cs} l_{cs} = 0$$

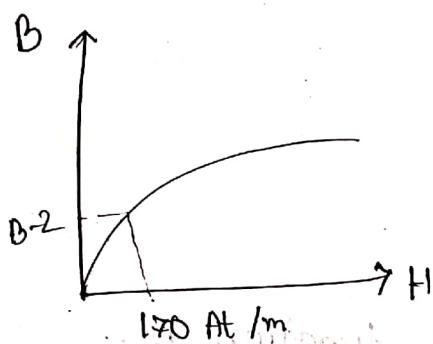
$$\Rightarrow I = \frac{H_{cs} l_{cs}}{N}$$

$$B = \frac{\Phi}{A}$$

$$= \frac{4 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 0.2 T$$

From B-H curve



From B-H curve

$$H_{es} \approx 170 \text{ At/m}$$

$$I = \frac{170 \times 0.16}{400}$$

$$= 0.068 \text{ A}$$



question 5. Calculate the current in each branch

$\Delta H = \mu_0 I + \mu_0 P - \mu_0 P$  To self magnetism

Induced emf in each branch

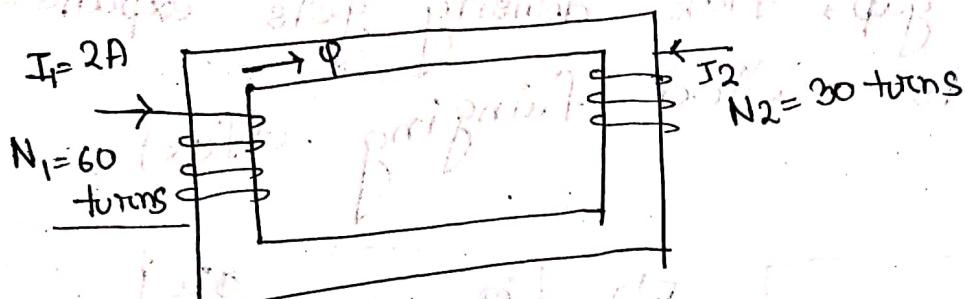
$$\frac{P}{A}$$

$$A_{tot} \times \frac{P}{A}$$

Since

$$T_B = 0.5 \quad \text{and} \quad I = \frac{\Delta H}{\mu_0} = \frac{0.5 \times 10^6}{4 \pi \times 10^{-7}}$$

Date: 11/03/2019

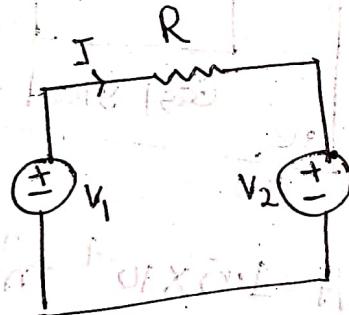


$$\Phi = 1.5 \times 10^{-5} \text{ WB}$$

$$A = 0.15 \times 10^{-3} \text{ m}^2$$

$$l = 0.16 \text{ m} \quad I_2 = ?$$

$$-V_1 + IR + V_2 = 0$$

 $\Rightarrow$ 

By applying Ampere's Circuital law

$$-N_1 I_1 + Hl + N_2 I_2 = 0$$

$$\Rightarrow N_2 I_2 = N_1 I_1 - Hl$$

$$\Rightarrow I_2 = \frac{N_1 I_1 - Hl}{N_2}$$

$$= 3.89 \text{ A}$$

$$B = \frac{\Phi}{A} = \frac{1.5 \times 10^{-5}}{0.15 \times 10^{-3}}$$

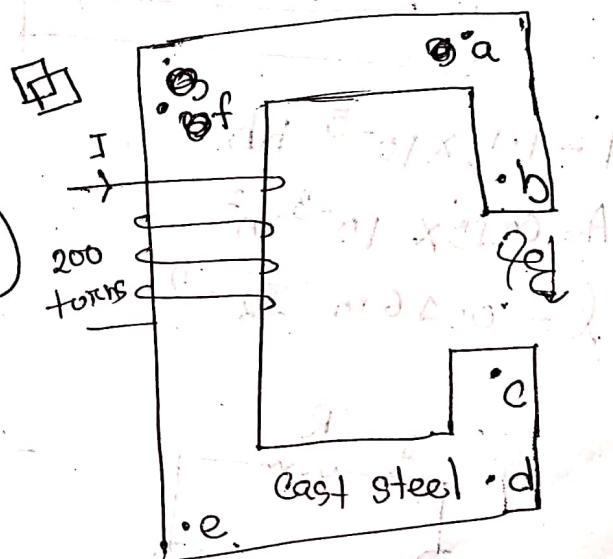
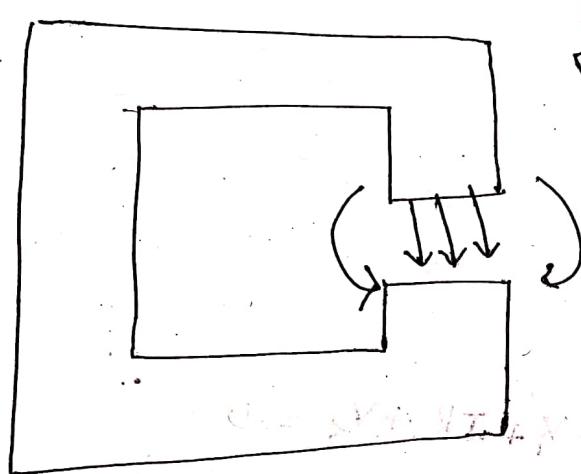
$$= 0.1 \text{ T}$$

From B-H curve,

$$H \approx 20 \text{ At/m}$$

### Fringing Effect:

In air gap, flux density gets expanded.  
This effect is called fringing effect.

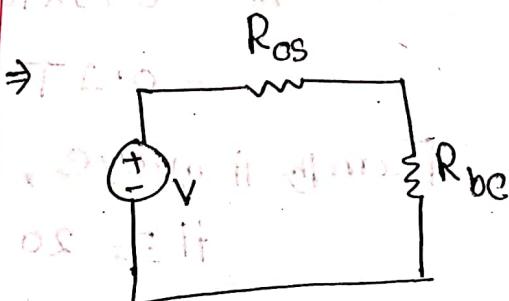


$$A = 1.5 \times 10^{-4} \text{ m}^2$$

$$\Phi = 0.75 \times 10^{-4} \text{ Wb}$$

$$l_{defab} = 100 \times 10^{-3} \text{ m}$$

$$l_{be} = 2 \times 10^{-3} \text{ m}$$



$$-V + IR_{os} + IR_{bc} = 0$$

$$\Rightarrow -NI + H_{cs} l_{cs} + H_{bc} l_{bc} = 0$$

74

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-9}}{1.5 \times 10^{-9}} = 0.5 T$$

From B-H curve,  $H_{es} \approx 6280 \text{ At/m}$

For air,  $B = \mu H$

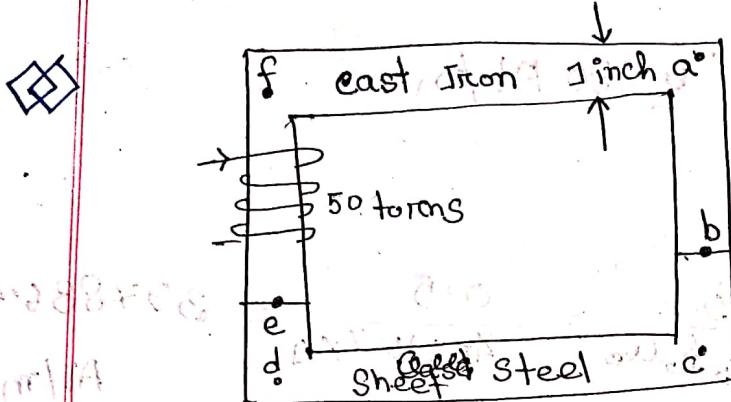
$$\mu_{bc} = \frac{B}{H} = \frac{0.5}{4\pi \times 10^{-7} \times 6280} = 397886.43 \text{ At/m}$$

$$\therefore NJ = H_{es} l_{es} + H_{bc} l_{bc}$$

$$\Rightarrow J = \frac{H_{es} l_{es} + H_{bc} l_{bc}}{N}$$

$$= 4.119 A$$

Date: 18/03/2019



$$I = ?$$

$$A = 1 \text{ inch}^2$$

$$l_{ab} = l_{cd} = l_{fe} = l_{fc} = 4 \text{ inch}$$

$$l_{be} = l_{de} = 0.5 \text{ inch}$$

$$\Phi = 3.5 \times 10^{-4} \text{ wb}$$

$$A = 1 \text{ inch}^2$$

$$= (2.54 \times 10^{-2})^2$$

$$= 6.4516 \times 10^{-4} \text{ m}^2$$

$$\text{Now } B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4}}{6.4516 \times 10^{-4}} = 0.53 \text{ T}$$

From B-H curve

$$H_{st} \equiv 70 \text{ At/m}; H_c = 1550 \text{ At/m}$$

Now,

$$l_{st} = l_{bc} + l_{cd} + l_{de} = 0.5 + 4 + 0.5$$

$$= 5 \text{ inch}$$

$$= 5 \times 2.54 \times 10^{-2} = 0.127 \text{ m}$$

$$l_{ci} = l_{ef} + l_{fu} + l_{ab}$$

$$= 4 + 4 + 4$$

$$= 12 \text{ inch}$$

$$= 0.3048 \text{ m}$$

By applying Ampere's Circuital law, we get

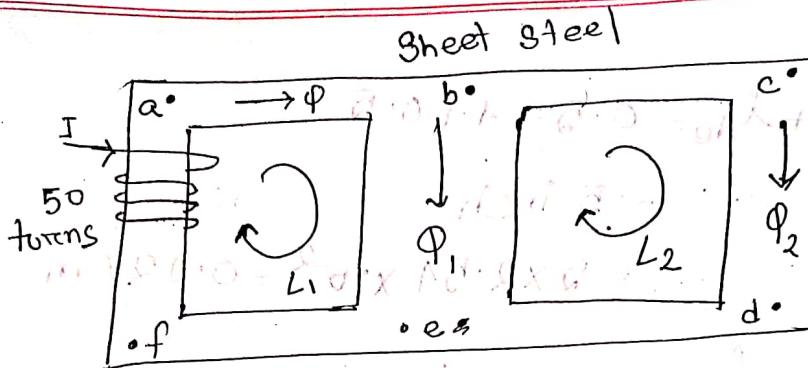
$$NI - H_{st} l_{st} - H_{ci} l_{ci} = 0$$

$$\Rightarrow I = \frac{H_{st} l_{st} + H_{ci} l_{ci}}{N}$$

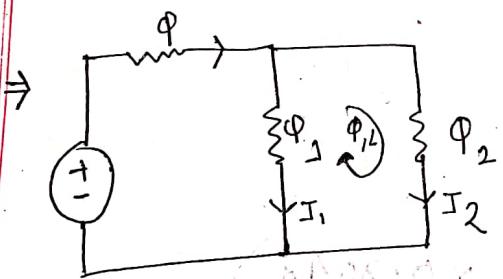
$$I = \frac{(70 \times 0.127) + (1550 \times 0.3048)}{50}$$

$$= 9.627 \text{ A}$$

Q



Find  $I$ ,  $H$ ,  $B$  in each section.



$$B_2 = \frac{\phi_2}{A}$$

$$= \frac{1.5 \times 10^{-4}}{(6 \times 10^{-4})}$$

$$= 0.25 \text{ T}$$

From B-H curve

$$H_{bede} \approx 40 \text{ At/m}$$

By applying Amperes circuital law around  $L_2$

$$-H_{bc}l_{bc} + H_{bede}l_{bede} = 0$$

$$\Rightarrow H_{bc} = \frac{H_{bede}l_{bede}}{l_{bc}}$$

$$= \frac{40 \times 0.2}{0.05}$$

$$= 160 \text{ At/m}$$

From B-H curve

$$B_1 \approx 0.97 \text{ T}$$

$$B_1 = \frac{\Phi_1}{A}$$

$$\Rightarrow \Phi_1 = B_1 \times A = \frac{0.97 \times 6 \times 10^{-4}}{0.576 \text{ A}} = 5.82 \times 10^{-4} \text{ Wb}$$

$$\Phi = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4} + 1.5 \times 10^{-4} = 7.32 \times 10^{-4} \text{ Wb}$$

$$B = \frac{\Phi}{A} = \frac{7.32 \times 10^{-4}}{6 \times 10^{-4}} = 1.22 \text{ T}$$

From B-H curve,

$$H_{\text{efab}} \approx 400 \text{ At/m}$$

Ampere's circuital law around  $L_1$ ,

$$-NI + H_{\text{efab}} l_{\text{efab}} + H_{\text{be}} l_{\text{be}} = 0$$

$$\Rightarrow I = \frac{400 \times 0.2 + 160 \times 0.05}{L \times 0.50}$$

$$= 1.76 \text{ A}$$

From section bede

$$B = \mu H$$

$$\Rightarrow \mu = \frac{B}{H} = \frac{0.25}{40} = 6.25 \times 10^{-3} \text{ wb/A.m}$$

$$\mu = \mu_0 \mu_r$$

$$\Rightarrow \mu_r = \frac{\mu}{\mu_0} = \frac{6.25 \times 10^{-3}}{4\pi \times 10^{-7}} = 4972.2$$

For section be,

$$\mu = \frac{B}{H}$$

$$= \frac{0.97}{360}$$

$$= 6.06 \times 10^{-3} \text{ wb/A.m}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.06 \times 10^{-3}}{4\pi \times 10^{-7}} = 4821$$

For section efab

$$\mu = \frac{B}{H} = \frac{1.22}{400} = 3.05 \times 10^{-3} \text{ wb/A.m}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{3.05 \times 10^{-3}}{4\pi \times 10^{-2}} = 2426.41$$

## ■ Alternating Current

An alternating current is such type of signal that goes through a series of different values both positive and negative in a period of time, after which it repeats the same series of values in cyclic manner.

application repetition

## ■ Instantaneous value:

The magnitude of a wave form at any instant of time.

## ■ Amplitude / Peak value:

The maximum value of a wave form.

$$V_{p-p} = 2 V_p$$

## ■ Peak to peak value:

Full voltage between positive and negative peaks of the waveform.

## ■ Period (T):

The time needed for one complete cycle of vibration to pass in a given point.

## ■ Cycle:

The portion of a waveform continued in one period of time.

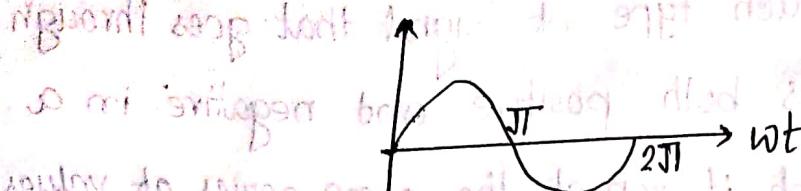
## ■ Frequency:

The number of cycles that occurs in 1 sec.

$$f = \frac{1}{T}$$

Unit: Cycle per second or ~~Hz~~ Hertz (Hz)

## ■ Angular velocity

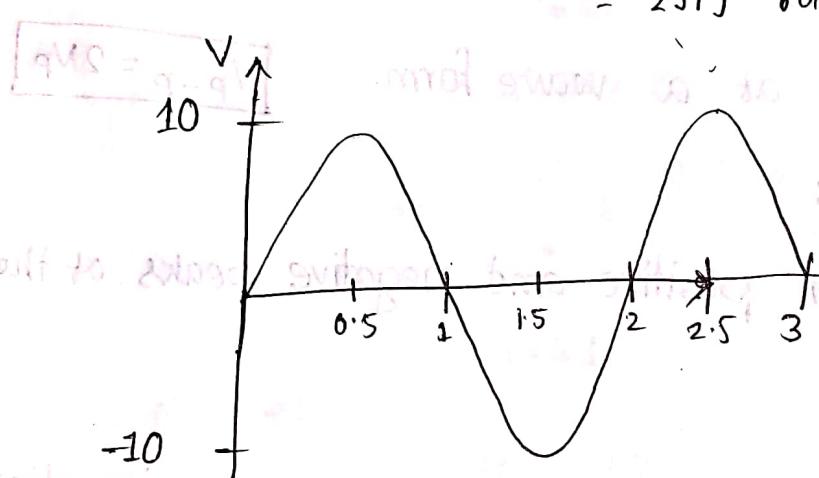


$$\text{complete cycle} = 2\pi$$

Time for a complete cycle =  $T$

$$\text{Angular velocity} = \frac{2\pi}{T} \text{ rad/s}$$

■



$$\begin{aligned} v &= V_m \sin \theta \\ &= V_m \sin wt \end{aligned}$$

$$\begin{aligned} f &= \frac{1}{T} = \frac{1}{2 \times 10^3} \text{ Hz} \\ &= 500 \text{ Hz} \end{aligned}$$

$$= V_m \sin 2\pi ft$$

$$= 10 \sin (2\pi \times 500t)$$

$$= 10 \sin (18000\pi t)$$

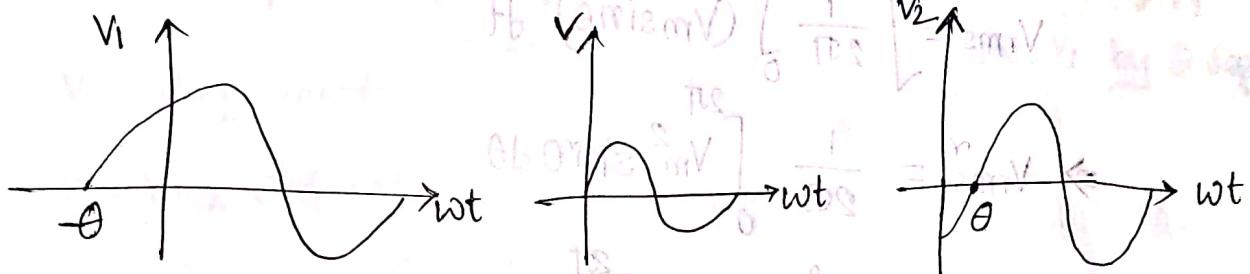
=

$$-\frac{1}{T} = 1$$

(st)

## Phase:

Phase is the fractional part of a period through which time or associated time angle  $\omega t$  has advanced from an arbitrary reference.



$$V_1 = V_m \{ \sin \omega t - (-\theta) \} \\ = V_m \sin (\omega t + \theta)$$

Phase angle =  $-\theta$

phase angle =  $\theta$

Average value  $\theta = \frac{\text{Area under the curve}}{\text{Period}}$

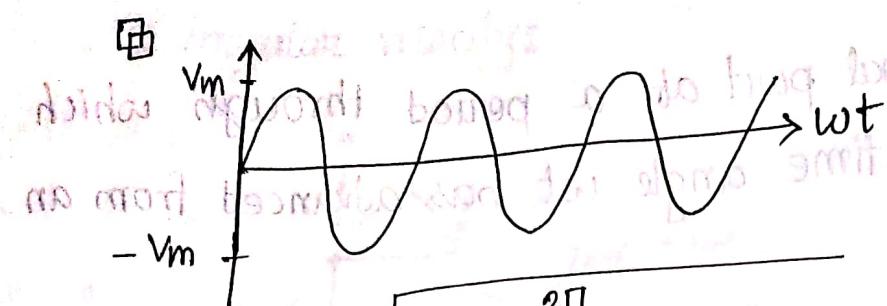
$$V_{avg} = \frac{1}{T} \int_0^T v dt$$

Effective value / Root mean square (rms) value

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}}$$

$$\text{peak factor} = \frac{V_{avg}}{V_{rms}}$$



$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \theta)^2 dt}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \theta d\theta}$$

$$(0 = \omega t) \text{ min. } V = \sqrt{\frac{V_m^2}{2\pi}} \cdot \frac{1}{2} \int_0^{2\pi} 2 \sin^2 \theta d\theta$$

$$\theta = \sin^{-1} \theta$$

$$\frac{V_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta$$

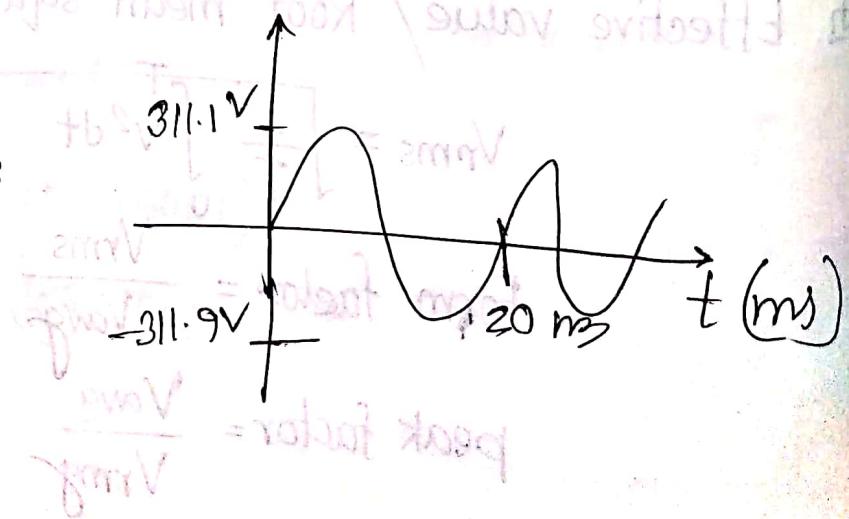
$$\Rightarrow V_{rms} = \frac{V_m^2}{4\pi} \left[ \theta - \frac{1}{2} \sin \theta \right]_0^{2\pi}$$

$$\Rightarrow V_{rms} = \frac{V_m^2}{4\pi} [2\pi - 0 + 0 + 0]$$

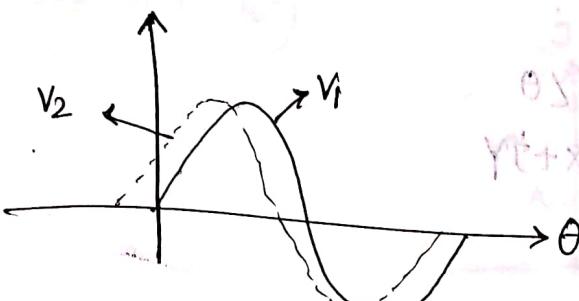
$$\Rightarrow V_{rms} = \frac{V_m^2}{2}$$

$$\Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\Rightarrow V_m = \sqrt{2} V_{rms}$$

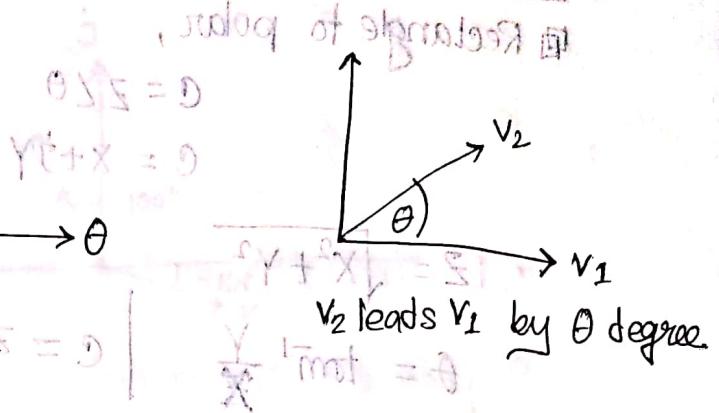


## Phase relations:



$$V_1 = V_{m1} \sin(\omega t)$$

$$\begin{aligned} V_2 &= V_{m2} \sin(\omega t - (-\theta)) \\ &= V_{m2} \sin(\omega t + \theta) \end{aligned}$$



$$\cos \alpha = \sin(\alpha + 90^\circ)$$

$$\sin \alpha = \cos(\alpha - 90^\circ)$$

$$-\sin \alpha = \sin(\alpha \pm 180^\circ)$$

$$-\cos \alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$

$$V = V_m \sin(\omega t + \theta)$$

$$V = V_{rms} \angle \theta$$

Polar form

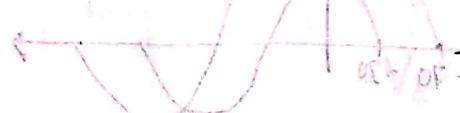
Example

$$V = 10 (\sin \omega t + 10^\circ)$$

$$V = \frac{10}{\sqrt{2}} \angle 10^\circ$$

$$V = A + jB$$

Rectangular form



Conversion:

④ Rectangle to polar,

$$C = Z \angle \theta$$

$$C = X + jY$$

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1} \frac{Y}{X}$$

$$C = Z \angle \theta$$

$$\rightarrow \text{tanda } \mu mV = N$$

④ Polar to rectangle:

$$X = Z \cos \theta$$

$$Y = Z \sin \theta$$

$$C = X + jY$$

$$((\theta) - j\omega) \text{ reac. } \sin V = \phi$$

$$(\theta + j\omega) \text{ reac. } \sin V =$$

$$(\theta + j\omega) \text{ reac. } = 10200$$

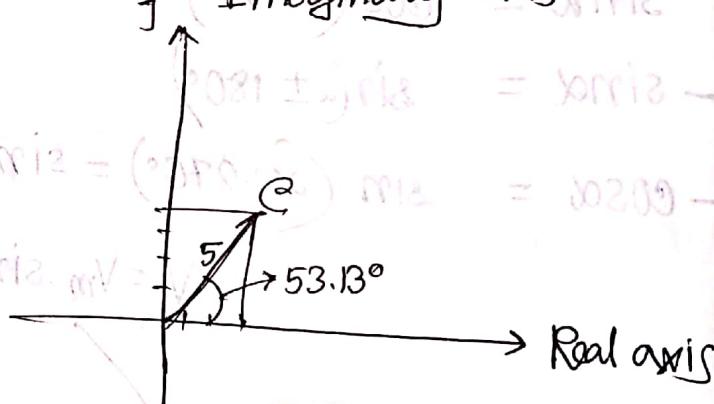
$$C = 3 + j4$$

$$Z = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \tan^{-1} \frac{4}{3} = 53.13^\circ$$

$$C = 5 \angle 53.13^\circ$$

Imaginary axis



④

$$V = 10 \sin(\omega t + 30^\circ) = 10 \sin \omega t$$

$$i = 5 \sin(\omega t + 70^\circ)$$

Find out the relationship between  $V$  &  $i$ .

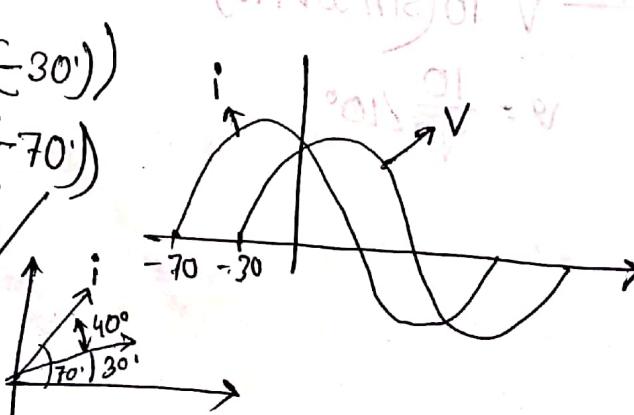
Soh:

$$V = 10 \sin(\omega t - (-30^\circ))$$

$$i = 5 \sin(\omega t - (-70^\circ))$$

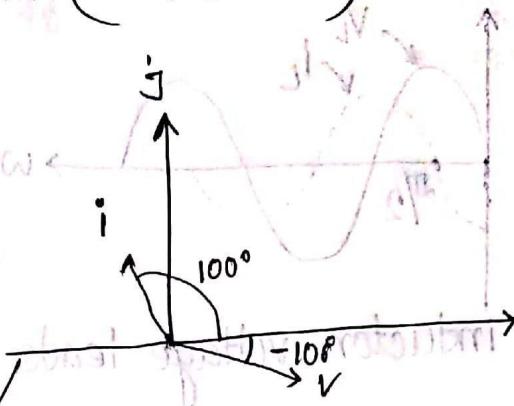
$i$  leads  $V$  by  $40^\circ$

$V$  lags  $i$  by  $40^\circ$



$$\boxed{\text{Q}} \quad i = 2 \cos(\omega t + 10^\circ) = 2 \sin(\omega t + 10^\circ + 90^\circ) = 2 \sin(\omega t + 100^\circ)$$

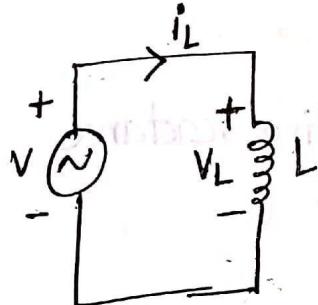
$$v = 3 \sin(\omega t - 10^\circ)$$



i leads v by  $110^\circ$   
v lags i by  $110^\circ$

31/03/2019

### Inductor



The term  $L$  represents inductance. It opposes the rate of change of current and for this reason it is called electric inertia.

$$V_L = L \frac{di_L}{dt}$$

$$i_L = \frac{1}{L} \int V_L dt$$

$$\Rightarrow V_m \sin \omega t = L \frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_m}{L} \sin \omega t$$

$$\Rightarrow \int di_L = \int \frac{V_m}{L} \sin \omega t dt$$

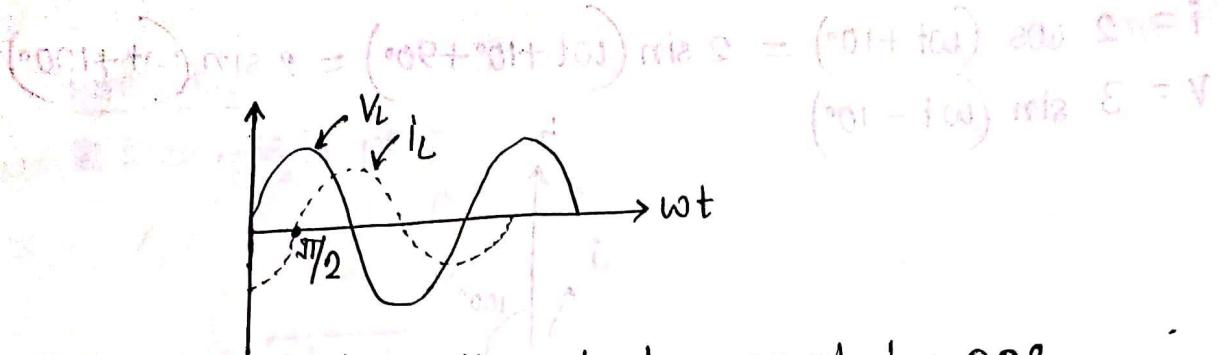
$$\Rightarrow i_L = -\frac{V_m}{WL} \cos \omega t$$

$$\Rightarrow i_L = \frac{V_m}{WL} (\sin \omega t - 90^\circ)$$

$$\Rightarrow i_L = I_m \sin(\omega t - 90^\circ) \quad \text{where, } I_m = \frac{V_m}{WL}$$

$$V_L = V_{rms} \angle 90^\circ$$

$$i_L = I_{rms} \angle -90^\circ$$



$\therefore$  For inductor, voltage leads current by  $90^\circ$ .

Ampedance: The impedance at the L branch is

$$Z_L = \frac{V_L}{I_L} = \frac{V_{rms} \angle 0^\circ}{I_{rms} \angle -90^\circ} = \frac{\sqrt{2} V_m \angle 0^\circ}{\sqrt{2} I_m \angle -90^\circ} = \frac{V_m}{I_m} \angle 90^\circ = \omega L \angle 90^\circ$$

$$= j\omega L = j2\pi f L$$

$$\therefore Z_L = \omega L \angle 90^\circ \quad | \quad X_L = \omega L = \text{Inductive reactance}$$

$$= jX_L \angle 90^\circ \quad | \quad = 2\pi f L$$

Unit of Inductance: Henry (H)

Unit of reactance & impedance: Ohm ( $\Omega$ ).

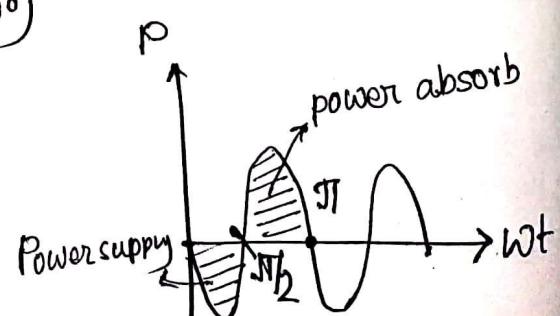
Power:

$$P = V_L I_L$$

$$= V_m \sin \omega t \times I_m \sin (\omega t - 90^\circ)$$

$$= -\frac{V_m I_m}{2} \times 2 \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$



Energy delivered to the inductor during a quarter of a cycle:

P.T.O

$$W_L = \int_{\pi/4}^{\pi/2} -\frac{V_m I_m}{2} \sin 2\omega t dt$$

$$= \frac{V_m I_m}{2(2\omega)} [\cos 2\omega t]_{\pi/4}^{\pi/2}$$

$$= \frac{V_m I_m}{4 \frac{2\pi}{T}} [\cos \frac{4\pi t}{T}]_{\pi/4}^{\pi/2}$$

$$= \frac{V_m I_m}{8\pi/T} [\cos 2\pi - \cos \pi]_{\pi/4}^{\pi/2}$$

$$= \frac{V_m I_m}{8\pi/T} [1 - (-1)]$$

$$= \frac{V_m I_m}{8\pi/T} 2$$

$$= \frac{V_m I_m}{2\omega}$$

$$= \frac{(WL) I_m}{2\omega}$$

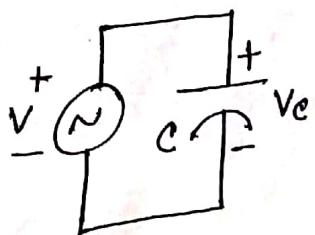
$$\therefore W_L = \frac{1}{2} L I_m^2$$

Capacitors

current through  $C = i_x$



## ■ Capacitor:



$C$  = capacitor

$$i_C = C \frac{dV_C}{dt}$$

$$V_C = \frac{1}{C} \int i_C dt$$

$$\text{Let, } V = V_m \sin \omega t$$

$$\text{Here, } V = V_C$$

$$\text{Now, } i_C = C \frac{dV_C}{dt}$$

$$= C \frac{dV}{dt}$$

$$= C \frac{d}{dt} (V_m \sin \omega t)$$

$$= C V_m \omega \cos \omega t$$

$$= \frac{V_m}{1/\omega C} \sin(\omega t + 90^\circ)$$

$$= I_m \sin(\omega t + 90^\circ) \quad \text{Here, } \frac{V_m}{1/\omega C} = I_m$$

A graph illustrating the phase relationship between current  $i_C$  and voltage  $V_C$ . The vertical axis represents  $V_C$  and the horizontal axis represents time. The voltage  $V_C$  is shown as a sine wave starting at zero. The current  $i_C$  is shown as a sine wave starting at its maximum value at  $t = -\pi/2$ , leading the voltage by  $90^\circ$ .

$\therefore$  current leads voltage by  $90^\circ$ .

Impedance:

$$Z_C = \frac{V}{i_C} = \frac{V_{rms} \angle 0^\circ}{I_{rms} \angle 90^\circ}$$

$$= \frac{\sqrt{2} V_m \angle 0^\circ}{\sqrt{2} I_m \angle 90^\circ}$$

$$= \frac{V_m}{I_m} \angle -90^\circ$$

$$= \frac{1}{\omega C} \angle -90^\circ$$

$$\therefore Z_C = \frac{1}{\omega C} \angle -90^\circ$$

$$= -j X_C \quad | \quad X_C = \text{Capacitive reactance.}$$

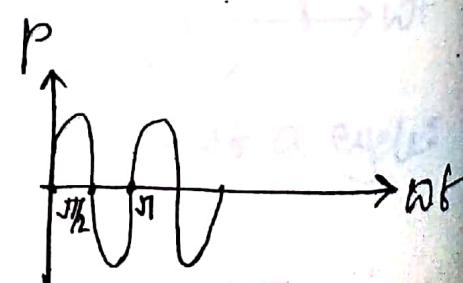
Instantaneous power:

$$P = V_C i_C$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t + 90^\circ)$$

$$= \frac{V_m I_m}{2} 2 \sin \omega t \cdot \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$



Energy received by a capacitor during a quarter cycle:

$$W_C = \int_0^{\pi/4} \frac{V_m I_m}{2} \sin \omega t \sin \omega t dt$$

$$= \frac{V_m I_m}{2} \left[ -\frac{\cos 2\omega t}{2\omega} \right]_0^{\pi/4}$$

$$= \frac{V_m I_m}{2 \times 2\omega} \left[ -\cos \frac{4\pi t}{T} \right]_0^{\pi/4}$$

$$= \frac{V_m I_m}{4\omega} \left[ -\cos \pi + \cos 0 \right]$$

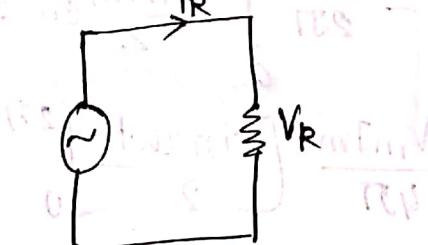
$$= \frac{V_m I_m}{2\omega}$$

$$= \frac{V_m (\omega C V_m)}{2\omega}$$

$$= \frac{1}{2} V_m^2 C$$

$$\therefore W_C = \frac{1}{2} V_m^2 C$$

The R branch



$$V = V_m \sin \omega t$$

$$i_R = \frac{V_R}{R} = \frac{V}{R}$$

$$= \frac{V_m \sin \omega t}{R}$$

$$= I_m \sin \omega t \quad ; \quad I_m = \frac{V_m}{R}$$

Impedance: ~~group~~ ~~parallel combination of load and source impedances~~

$$Z_R = \frac{\frac{V_m}{\sqrt{2}} \angle 0^\circ}{\frac{I_m}{\sqrt{2}} \angle 0^\circ}$$

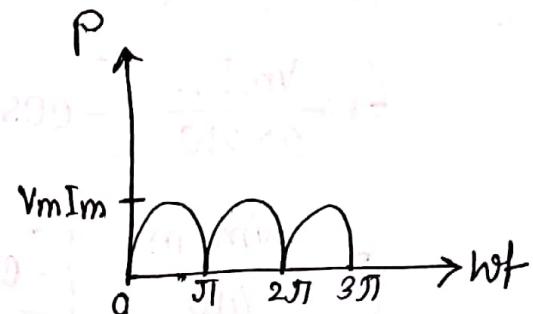
$$= R \angle 0^\circ$$

$$\therefore Z_R = R$$

Power:  $P = V_R I_R$

$$= V_m \sin \omega t \cdot I_m \sin \omega t$$

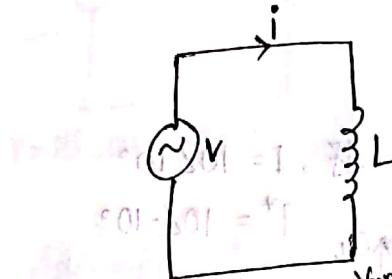
$$= V_m I_m \sin^2 \omega t$$



Average power:

$$\begin{aligned} P_{avg} &= \frac{1}{2\pi} \int_0^{2\pi} P d\omega t \\ &= \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \sin^2 \omega t d\omega t \\ &= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} (1 - \cos 2\omega t) d\omega t \\ &= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} d\omega t - \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \cos 2\omega t d\omega t \\ &= \frac{V_m I_m}{4\pi} \left[ \omega t \right]_0^{2\pi} - \frac{V_m I_m}{4\pi} \left[ \frac{\sin 2\omega t}{2} \right]_0^{2\pi} \\ &= \frac{V_m I_m}{4\pi} \times 2\pi \\ &= \frac{V_m I_m}{2} \end{aligned}$$
$$\therefore P_{avg} = \frac{V_m^2}{2} = \frac{V_m I_m}{\sqrt{2} \cdot \sqrt{2}} = V_{rms} I_{rms}$$

If  $L = 10 \text{ mH}$ ,  $f = 60 \text{ Hz}$  and applied voltage is  $V = 100 \sin \omega t$ ,  
Find  $X_L$  &  $i$  for the following circuit.



$$X_L = \omega L$$

$$= 2\pi f L$$

$$= 2\pi \times 60 \times 10 \times 10^{-3}$$

$$= 3.77 \Omega$$

$$V = 10 \frac{100}{\sqrt{2}} \angle 0^\circ$$

$$Z_L = j X_L$$

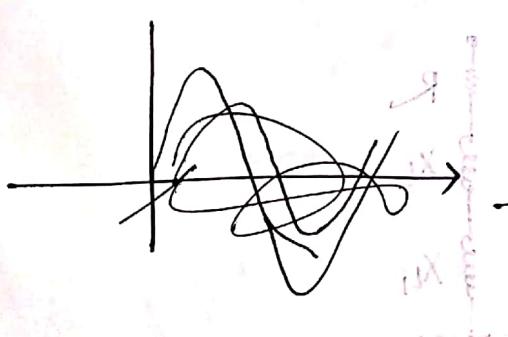
$$= 3.77 j = 3.77 L 90^\circ$$

$$i = \frac{V}{Z_L} = \frac{100 \angle 0^\circ}{3.77 L 90^\circ}$$

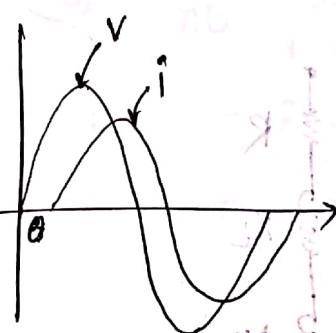
$$= -18.75 j / -18.75 \angle -90^\circ = 12 \angle 0^\circ$$

Answer

Power factor:



Cosine of  $\theta$



$$PF = \cos \theta$$



Phase difference between voltage and current is power factor.

$$P_{avg} = V_{rms} I_{rms} \cos \theta$$

## Power triangle

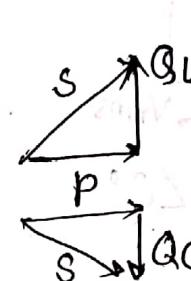
Real/Average power  $\rightarrow P = V_{rms} I_{rms} \cos\theta$

Reactive power  $\rightarrow Q = V_{rms} I_{rms} \sin\theta$

Apparent power  $\rightarrow S = V I^*$

$S = P + jQ_L$  [for inductive load]

$S = P - jQ_C$  [for capacitive load]

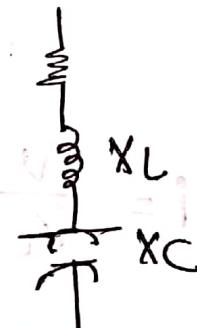


$$I = 10 \angle 10^\circ$$

$$I^* = 10 \angle -10^\circ$$

$x_L > x_C$  the circuit is inductive

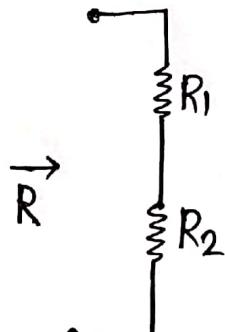
$x_C > x_L$  the circuit is capacitive



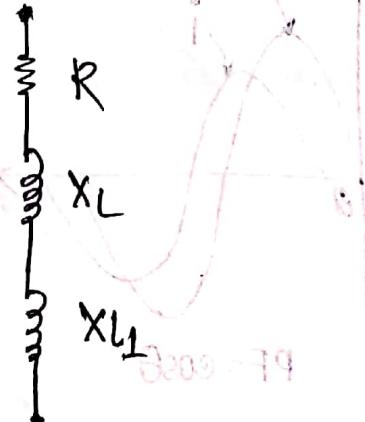
$$\therefore |S| = \sqrt{Q^2 + P^2}$$

Inductor consumes  $Q$

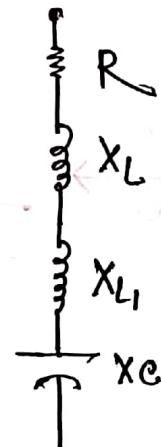
Capacitor supplies  $Q$



$$R = R_1 + R_2$$

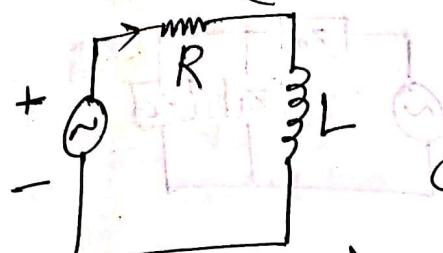


$$Z = R + jX_L$$



$$Z = R + jX_L + jX_C + (-jX_C)$$

$$i = 5 \sin(314t + 120^\circ)$$



$$V = 15 \sin(314t + 150^\circ)$$

- a) What is the value of the impedance at the circuit?
- b) What is the value of resistance?
- c) What is the value of the inductance?
- d) Find the power factor.

Answers

a)  $Z = \frac{V}{I}$

$$= \frac{\frac{15}{\sqrt{2}} \angle 150^\circ}{\frac{5}{\sqrt{2}} \angle 120^\circ}$$

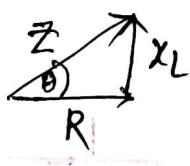
$$= 3 \angle 30^\circ \text{ impe}$$

b)  $\text{PF} = \cos(150^\circ - 120^\circ)$

$$= 0.86 \text{ (Lagging)} \rightarrow \text{If the circuit is inductive}$$

\* capacitive ~~atm~~ (Leading)

c)



$$\cos \theta = \frac{R}{|Z|}$$

$$R = |Z| \cos \theta$$

$$= 3 \times 0.86$$

$$= 2.59 \Omega$$

d)  $\sin \theta = \frac{X_L}{|Z|}$

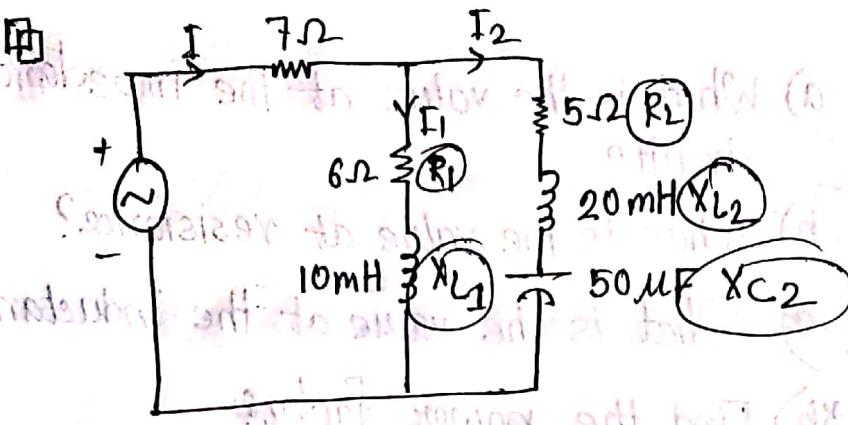
$$= (3 \times 2 \times X_L) / (3 \sqrt{3})$$

$$= 3 \times \sin 30^\circ$$

$$= 1.5 \Omega$$

$$X_L = \omega L$$

$$\Rightarrow L = \frac{X_L}{\omega} = \frac{1.5}{314} = 4.7 \times 10^{-3} \text{ H}$$



$$V = 141.4 \sin 100\pi t$$

Find the current through each branch and the equivalent impedance connected to the source. Also find the power factor.

$$Z_0 = 7\Omega$$

$$Z_1 = R_1 + jX_{L1}$$

$$= 6 + j\omega L_1$$

$$= 6 + j(100\pi \times 10 \times 10^{-3})$$

$$= 6 + 3.14j$$

$$= 6.77 \angle 27.62^\circ \Omega$$

$$Z_2 = R_2 + jX_{L2} - jX_{C2}$$

$$= 5 + j\omega L_2 - j\frac{1}{\omega C_2}$$

$$= 5 + j(100\pi \times 20 \times 10^{-3}) - j\left(\frac{1}{100\pi \times 50 \times 10^{-6}}\right)$$

$$= 57.59 \angle -85.02^\circ \Omega$$

$$Z = Z_0 + Z_1 Z_2$$

$$= Z_0 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= 27 + \frac{(6.77 \angle 27.62^\circ) \times (57.59 \angle -85.02^\circ)}{(6.77 \angle 27.62^\circ) + (57.59 \angle -85.02^\circ)}$$

$$= 13.81 \angle 10.6^\circ \Omega$$

Equivalent impedance

$$I = \frac{V}{Z}$$

$$= \frac{141.4}{\sqrt{2}} \angle 0^\circ$$

$$= \frac{141.4}{13.81} \angle 10.6^\circ$$

$$= 7.24 \angle -10.6^\circ A$$

$$I_1 = I \times \frac{Z_2}{Z_1 + Z_2}$$

$$= 7.24 \angle -17.1^\circ A$$

$$I_2 = I \times \frac{Z_1}{Z_1 + Z_2}$$

$$= 0.886 \angle 95.6^\circ A$$

$$\therefore PF = \cos \left\{ 0 - (-10.6^\circ) \right\}$$

$$= 0.98$$

*lagging*

