# ODE Summary

~~$y'' - 4y' + 4y = (x+1)e^{2x}$~~

(*) **Derive differential equation** — differentiate (x) times the no. of arbitary variable.

(*) **Solve ODE** → ① **Seperation variables.**

$$x\,dx = y\,dy$$

② **Homogenous**

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

(*) **Reducible to homogenous.**

$$\frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1}$$

let, $x = X+h$

$y = Y+K$

then, $$\frac{dY}{dX} = \frac{X+Y+(h+K+c)}{X_1+Y_1+(h_1+K_1+c_1)}$$

$h, k$-এর মান বের করব ~~they~~ then solve.

$$Y = VX \;;\; \frac{dY}{dX} = V + X\frac{dV}{dX}$$

③ **First Order Linear Equation**

$$\frac{dy}{dx} + p(x)y = q(x)$$

or, $$\frac{dx}{dy} + p(y)x = q(y)$$

IF বের করব → $\boxed{e^{\int p\,dx} \,/\, e^{\int p\,dy}}$

G.S বানাব → $ye^{\int p\,dx} = \int q e^{\int p\,dx}\,dx$

or, $xe^{\int p\,dy} = \int q e^{\int p\,dy}\,dy$

⇒ RL circuit solve করতে পারব $L\frac{dI}{dt} + RI = E$

$$\Rightarrow I\,(IF) = \int E(IF)\,dt$$

(*) **Bernolli** → if $\frac{dy}{dx} + p(x)y = q(x)y^n$

let, $z = \frac{1}{y}$ ; $\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx}$

then equation-এ বসাব, $-\frac{1}{z^2}\frac{dz}{dx} - \frac{P(x)}{z} = -\left(\frac{1}{z}\right)^n q(x)$.

(solve).

④ **Exact DE**: $Mdx + Ndy = 0$. | bool yes = true;

check $\rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

if (yes)
{ G.S $\Rightarrow \int M\cdot dx + \int (x \text{ free terms from } N)\, dy$ }.

else
{ it is not exact, $\left[\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}\right]$

exact জোর করে বানাব। IF বের ~~হবে~~ করে multiply করব।

4 ways:

non-homogenous:

① $g(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$. | যদি N → ছোট হয় (Common sense).

$IF = e^{\int g(x)dx}$.

$\boxed{IF \times Mdx + IF \times Ndy = 0}$

② $f(y) = \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ | যদি M ছোট হয়

$IF = e^{-\int f(y)dy}$

$\boxed{IF \times Mdx + IF \times Ndy = 0}$

③ যদি M আর N-এ xy mixed থাকে

$IF = \frac{1}{xM - yN}$

$\boxed{IF \times Mdx + IF \times Ndy = 0}$

④ If homogenous,

$IF = \frac{1}{xM + yN}$

$\boxed{IF \times Mdx + IF \times Ndy = 0}$

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GS $\rightarrow \int IF \times Mdx + \int (N - x \text{ free terms})\, dy = 0$ ✓

| | |
|---|---|
| $r_1, r_2$ | $y = C_1 x^{r_1} + C_2 x^{r_2}$ |
| $r_1 = r_2$ | $y = x^{r}(C_1 + C_2 \ln x)$ |
| $\alpha \pm \beta i$ | $y = x^{\alpha}(C_1 \cos(\beta \ln x) + C_2 \sin \beta \ln x)$ |

← For Euler

HODE → GS table

| | |
|---|---|
| $r_1, r_2$ | $y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$ |
| $r_1 = r_2$ | $y = e^{rx}(C_1 + C_2 x)$ |
| $r = \alpha \pm i\beta$ | $y = e^{\alpha x}(C_1 \sin \beta x + C_2 \cos \beta x)$ |

## ✱ HODE

→ Check —
- homogenous কি না.
- Higher order কি না
- Const. coefficient আছে কি না (0 হবে)

If(yes).

{ suppose $y'' - 5y' + 6y = 0$.

$(D^2 - 5D + 6)y = 0$

trial solution নিব $y = e^{rx}$.

degree wise diff করব $y' = re^{rx}$

$y'' = r^2 e^{rx}$

equation -এ বসাব,

$\boxed{r^2 - 5r + 6 = 0}$

auxillary / characteristic equation.

$r$ – বের করব.

then G.S বানাব box wise }

(Complimentary equation == GS).

else

{ Non-homogenous থেকে homogenous বের করব।

suppose $y'' + 2y' + y = 2x + x^2$.

Homogenous → $y'' + 2y' + y$.

Characteristic equation → $r^2 + 2r + 1 = 0$.

Complimentary equation → $y_c = e^{-x}(C_1 + C_2 x)$

Particular integral বের করব,

$y_p = p + qx + rx^2$

(degree wise diff) $y_p' = q + 2rx$

$y_p'' = 2r$

sin-হলে
box-হলে
trial solution
wise $y_p$ ধরব

equation-এ বসাব → $\boxed{y'' + 2y' + y = 2x + x^2}$

$$2r + 2(q + 2rx) + (p + qx + rx^2) = 2x + x^2$$

$$\Rightarrow 2r + 2q + 4rx + p + qx + rx^2 = 2x + x^2$$

$$\Rightarrow (2r + 2q + p) + x(4r + q) + rx^2 = 2x + x^2$$

Co-efficient wise মান বের করব,

For $x^2$;

$r = 1$.

For $x$,

$4r + q = 2$.

Constant,

$2r + 2q + p = 0$.

as, $r = 1$,

So, $4(1) + q = 2$.

$\Rightarrow q = -2$.

again, $2(1) + 2(-2) + p = 0$.

$\Rightarrow 2 - 4 + p = 0$.

$\Rightarrow p = 2$.

এবার $p, r, q$ -এর মান বসায় দিব $y_p$-তে,

$\cancel{y_p = 2(1) + 2(-2 + 2.1.x) + (2 - 2x + 1x^2)}$

$\cancel{\Rightarrow y_p = 2 - 4 + 4x + 2 - 2x + x^2}$

$\cancel{= 2 - 2x + x^2}$

$$y_p = 2 - 2x + x^2.$$

$$G.S = y_p + y_c$$

$$\Rightarrow y = 2 - 2x + x^2 + e^{-x}(c_1 + c_2 x).$$

Non-homogeneous — trial particular solutions;

| $g(x)$ | |
|---|---|
| ① $1$ (const) | $\longrightarrow A$. |
| ② $(5x+7)$ | $\longrightarrow Ax+B$. |
| ③ $(3x^2-2)$ | $\longrightarrow Ax^2+Bx+C$. |
| ④ $(x^3-x+1)$ | $\longrightarrow Ax^3+Bx^2+Cx+D$. |
| ⑤ $\sin ax$ | $\longrightarrow A\cos ax + B\sin ax$. |
| ⑥ $\cos ax$ | $\longrightarrow A\cos ax + B\sin ax$ |
| ⑦ $e^{mx}$ | $\longrightarrow Ae^{mx}$ |
| ⑧ $(9x+2)e^{5x}$ | $\longrightarrow (Ax+B)e^{5x}$ |
| ⑨ $x^2e^{5x}$ | $\longrightarrow (Ax^2+Bx+C)e^{5x}$ |
| ⑩ $e^{mx}\sin ax$ | $\longrightarrow Ae^{mx}\cos ax + Be^{mx}\sin ax$. |
| ⑪ $5x^2\sin ax$ | $\longrightarrow (Ax^2+Bx+C)\cos ax + (Ex^2+Fx+G)\sin ax$ |
| ⑫ $xe^{mx}\cos ax$ | $\longrightarrow (Ax+B)e^{mx}\cos ax + (Ex+F)e^{mx}\sin ax$ |

$\boxed{\text{Variation of Parameters;}}$

① Complementary function বের করব $(y_c)$

② Wronskian method-এ particular equation ~~বের করব~~ $(y_p)$

$$\cancel{y_p = -y_1\int \frac{y_2\,g(x)}{W(y_1,y_2)}}$$

~~Let, $y_1$ ; $y_2$~~

Let, $y_c = e^x c_1 + xe^x c_2$

So, $y_1 = e^x$ $\quad y_2 = xe^x$ $\quad$ [$c_1$, $c_2$-হল coefficient]

Wronskian method,

$$W(x,y) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix} = \text{solve করব}$$

এরপর formula, $y_p = -y_1 \int \frac{y_2}{W(y_1, y_2)} \times g(x)\,dx + y_2 \int \frac{y_1 \; g(x)}{W(y_1, y_2)}\,dx$

$$\boxed{GS = y_c + y_p.}$$

**PDE**

**Laplace** $\rightarrow$ $\dfrac{\partial^2 u(x,y,z)}{\partial x^2} + \dfrac{\partial^2 u(x,y,z)}{\partial y^2} + \dfrac{\partial^2 u(x,y,z)}{\partial z^2} = 0.$

Application: ⊛ Describes steady state distribution of heat ~~in a body~~ and electrical charge in a body.

**Heat Equation** $\rightarrow$ $\dfrac{\partial u(x,y,z,t)}{\partial t} = \alpha \left( \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} \right).$

Appli: The function $u(x,y,z,t)$ represents the temp. at time $t$ in a body at a point with coordinates $(x,y,z)$.

$\alpha$ is thermal diffusivity. Normally $\alpha = 1$.

Simpler $\rightarrow$ $\dfrac{\partial T(x,t)}{\partial t} = \dfrac{\partial^2 T(x,t)}{\partial t^2}.$

temp at time $t$ at point $x$ of a thin rod.

**Wave Equation** $\rightarrow$ $\dfrac{\partial^2 u(x,y,z,t)}{\partial t^2} = c^2 \left( \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} \right).$

Appli: $u(x,y,z,t)$ represents the displacement at time $t$ of a particle whose position at rest is $(x,y,z)$.

$c$ is the propagation speed of the wave.

$$Au_{xx} + Bu_{xy} + Cu_{yy} + D = 0 .$$

shortcuts

$B^2 - 4AC < 0$ Elliptical

$B^2 - 4AC = 0$ parabola - (parallel).

$B^2 - 4AC > 0$ Hyperbolic - (High)

~~Euler~~

- DUNA -