$(A' \cup B') \cap (A' \cup B)$

$A' \cup (B' \cap B)$

ϕ

Date of Examination: 09/09/18

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department: Computer Science and Engineering

Program: Bachelor of Science in Computer Science and Engineering

Semester Final Examination: Spring 2018

Year: 1^{st} Semester: 2^{nd}

Course Number: CSE1203

Course Name: Discrete Mathematics

Time: 3 (Three) hours Full Marks: 70

[There are seven questions carrying a total of 14 marks each. **The question no.1 is compulsory** and answer any **four** from the rest.
Marks allotted are indicated in the right margin.]

$3 \times \frac{2}{2}$

1. a) What is a contradiction? Prove using laws of logical equivalences that 4

$$(\neg q \wedge (\neg p \wedge q)) \wedge (p \vee p) \equiv F.$$

b) There are m functions from a one-element set to the set {1, 2, ..., m}. How many functions are there from a two-element set to {1, 2, ..., m}? From a three-element set? Give a recurrence relation for the number T(n) of functions from an n-element set to {1, 2, ..., m}. Solve the recurrence relation. 3

c) Use mathematical induction to prove that if a > 1, then $a^n > 1$ for all n > 0. 4

d) Prove that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$. 3

$\frac{2 \times (2-1)}{2}$

K_n

2. a) Use rules of inference to determine the validity of the argument below: 4

The meeting can take place if all members are informed in advance and there is quorum (a minimum number of members are present). There is a quorum if at least 15 members are present. Members would have been informed in advance if there was no postal strike.

Therefore, if the meeting was canceled, then either there were fewer than 15 members present or there was a postal strike.

b) Prove using the laws of set operations that 3

$$((B' \cup A') \cap (A' \cup B)) \cup (A' \cap B) = A'.$$

c) Define the pigeonhole principle. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? 3

d) What is the chromatic number of a cycle C_n? Schedule the final exams for Math 1115, Math 1116, Math 1119, Math 1185, CSE 1101, CSE 1102, CSE 1203, and CSE 2105, using the fewest number of different time slots, if there are no students taking both Math 1115 and CSE 2105, both Math 1116 and CSE 2105, both Math 1119 and CSE 1101, both Math 1119 and CSE 1102, both Math 1115 and Math 1116, both Math 1115 and Math 1185, and both Math 1185 and Math 1119, but there are students in every other combinations of courses. 4

$p \rightarrow (s \vee t)$

C_8

$q \vee q = q$

$(\neg q \wedge p) \vee (\neg q \wedge p)$

3. a) Translate the following English sentences into logical expressions. 3

   i. No large birds live on honey.

   ii. Everyone in the class has taken exactly two courses from computer science courses.

b) Prove that if 3n+2 is odd, then n is odd using indirect proof and proof by contradiction. 5

c) Let m be a positive integer greater than 1. Show that the following relation is an equivalence relation on the set of integers. 4

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}.$$

d) Let K_n be the complete graph with n vertices. For what values of n does K_n admit an Eulerian circuit? 2

4. a) Write short notes on: 3

   i. Set difference

   ii. Cartesian product

   iii. Inverse functions

b) Show that a tree with n vertices has (n-1) edges. 2

c) The relation R on N is defined as (c, d) ∈ R if and only if (a + b) is even. 6

   i. Prove that R is an equivalence relation.

   ii. How many equivalence classes does R have?

d) Write a short note on planar graphs. Determine whether $K_{3,5}$ is a planar graph. 3

5. a) How many distinct ways are there to make a 5-letter word using the ENGLISH alphabet: 4

   i. with no restriction?

   ii. with only consonants?

   iii. with only vowels?

   iv. with a consonant as the first letter and a vowel as the second letter?

   v. if the vowels appear only at odd positions?

b) Show that inverse and converse of an implication are equivalent. 3

c) Describe the impact of the order of quantifiers using a suitable example. 4

d) Write down the Handshaking theorem and prove it. 3

6. a) What is a contingency? Show using a truth table whether the following expression is a tautology or not. 4

$$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$$

b) How many integers between 1 and 1000 are divisible by none of 2, 5 and 7? 3

c) Find out the total degree of a wheel W_n. 3

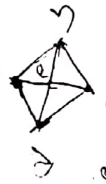
$p \to q$

$\neg q \to \neg p$

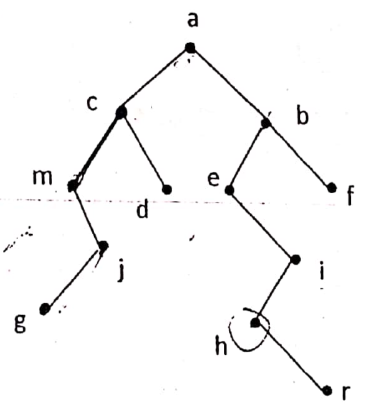
$n = 2k+1$

$3n+2 = 6k+3+2$

$= 6k+5$



d) Show the three types of traversal of the following tree. 4



PLR → ac m j g d b e i h r

LrR

m g j c d a e h r i b f

LRP

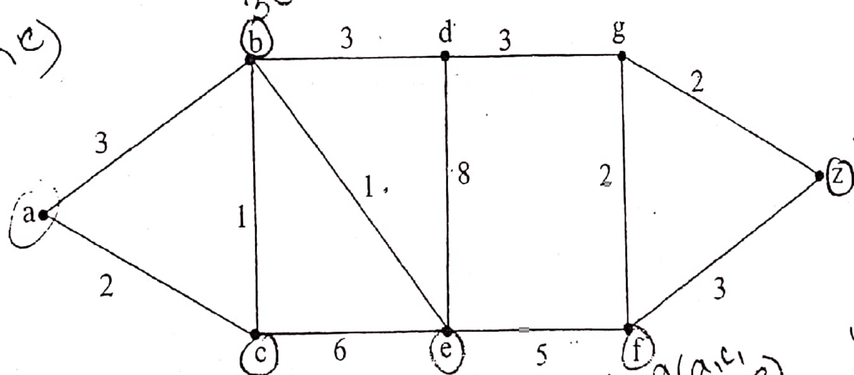
m g j d c e h r i f b

7\. a) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions and find out the non-recursive solution of this problem. 5

b) Is there a graph with 102 vertices, such that exactly 49 vertices have degree 5, and the remaining 53 vertices have degree 6? 2

c) What is wrong with the famous supposed "proof" that 1 = 2? 2

d) For the following weighted graph determine the shortest path between a and z; also calculate its length. Draw all the steps to calculate the shortest path. 5



$(a \cup b) \cap c$

$= (a \cap c) \cup (b \cap c)$

$2e = \deg$.

$49 \times 5 = 2e$

$2e = 245$

$2e = 53 \times 6$

$\therefore 2e = 318$.

$p \to q$

$\neg q \to \neg p$

$\neg q \to p$

$\neg p \to q$

$n = 2k + 1$

$3n+2 = 3(2k+1) + 2$

$= 6k + 3 + 2 + 3$

$= 6k + 5$

$3n+2$ even

$3n+2 = 2k$

$n = (2k-2)/3$

$p \to q$

$\neg p \to$

$3n+2 = 2k$

$\cancel{3n}$

$n = 2k$

$6k+2$

$\cancel{2}$ $2(3k+1)$

$n = 2k+1$

$3(2k+1)+\cancel{4}$

$= 6k+\cancel{3}+2$

$= 6k+5$

$= 2(\cancel{6}\,3k+2)+1$

$= 2m+1$

$6k+4$

$2m.$

$=$

$2k+$

$n = 2k+1$

$3n+2 = 6k+3+2$

$3n+2 = 6k+5$

Date of Examination: 21/05/2019

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department: Computer Science and Engineering

Program: Bachelor of Science in Computer Science and Engineering

Semester Final Examination: Fall 2018

Year: 1st Semester: 2nd

Course Number: **CSE1203**

Course Name: **Discrete Mathematics**

Time: 3 (Three) hours Full Marks: 70

[*There are seven questions carrying a total of 14 marks each. Answer **any five** questions. Marks allotted are indicated in the right margin.*]

1. a) What is a proposition? Illustrate the concept of conjunction of propositions. [1+3]

   b) Show that the converse of a conditional statement is not equivalent to it, while the contrapositive is. [3]

   c) Explain the concept of existentially quantified predicates. [3]

   d) Prove using propositional equivalences that $((\neg q \vee p) \wedge \neg(p \wedge q)) \vee (q \wedge (q \vee r))$ is a tautology. [4]

2. a) How do you explain the concept of inference? Derive $p \vee q$ from the propositions $r \rightarrow p$, $\neg s \vee r$ and $s \wedge t$ using propositional inference rules. [1+3]

   b) Narrate the major steps of proof by mathematical induction. [2]

   c) Cite an illustrative example of universal modus ponens. [2]

   d) Propose and correspond bit strings to the subsets of {b, c, g} for indexing them. [2]

   e) Verify one of the associative laws of set operations. [4]

3. a) Consider the two relations given below and answer the questions that follow.
   - $\leq$ on Z
   - R on English words such that $(x, y) \in R$ if and only if both x and y have 'at' at their ends.

   i) Demonstrate that the 1st relation given above is not symmetric and is antisymmatric. [2]

   ii) When a relation is said to be an equivalence relation? Establish that the 2nd relation given above is an equivalence relation. [1+3]

   iii) List 3 representatives of each of the equivalence classes with respect to the 2nd relation. [2]

   b) Draw a graph of the function, $f(x) = |x| + 1$ defined from Z to N, and determine its properties. [3]

   c) What important facts about bijective functions do you know? [3]

4. a) Find out the number of alphanumeric strings of length 3 to 5 that have uppercase vowels at the central one or two places, and digits at other places. Also, describe one of the counting principles that you involve. [3+2]

b) Assume that the maximum size of the largest group of students in a class who have birthday on the same day of a week is 25. Determine the class size with sufficient explanation. [4]

c) Elaborate the fact that there is always a scope of discovering a new prime. [3]

d) Justify with an example the naming of the principle of inclusion and exclusion. [2]

5. Given below are adjacency lists of 3 graphs. Study the representations and answer the questions that follow.

G_1:

Vertex	Adjacent Vertices
1	2, 5, 6
2	1, 3, 6
3	2, 4, 6
4	3, 5, 6
5	1, 4, 6
6	1, 2, 3, 4, 5

G_2:

Vertex	Adjacent Vertices
x	y, z
y	y, z
z	w, w
w	x, y

G_3:

Vertex	Adjacent Vertices
a	b, d, f
b	a, c, e
c	b, d, f
d	a, c, e
e	b, d, f
f	a, c, e

a) Discuss about the types of each of the graphs. [5]

b) Determine the number of edges of each graph involving the information about its type. [5]

c) Find Eulerian circuits and / or Hamiltonian circuits, as applicable, in the undirected ones of those given graphs. [4]

6. a) What is a Tree? Write down the properties of Trees. [4]

b) Two algebraic systems are shown below. Study the systems and answer the questions that follow.

- $\langle \mathbf{P(A)}, \cup, \cap, ', \varnothing, \mathbf{A} \rangle$
- $\langle \{\mathbf{F}, \mathbf{T}\}, \vee, \wedge, \neg, \mathbf{F}, \mathbf{T} \rangle$

i) Describe the components of the systems mentioning their role in the respective system. [5]

ii) Carryout a component-by-component comparison of the systems. [3]

iii) Draw conclusions based on the analysis above. [2]

7. a) Write short notes on any three of the following: [2×3]

i) Universal instantiation;

ii) Set difference operation;

iii) Gödel number function;

iv) Recurrence relations.

b) Explain with examples the counter example and one example methods of proof. [3]

c) Suppose, f: X → Y, for finite X and Y. You may also suppose that f may be an injection, while it may or may not be a surjection. Find the expressions for calculating the number of such functions. [5]

Date: 19/03/2017

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester **Final Examination, Fall 2016**

Course No: **CSE 1203** Course Title: **Discrete Mathematics**

Time: **3 Hours** Full Marks: **70**

[There are 7(Seven) questions. The question no.1 is compulsory and answer any four from the rest.]

[*Marks allotted are indicated in the right marginwithin '[ ]'.*]

1. a) Argue that $\exists x \forall y P(x, y) \leftrightarrow \forall x \exists y P(x, y)$ is (or is not) a tautology. [2]

b) Show that an implication and its contrapositive are equivalent. [2]

c) Use mathematical induction to prove that one can solve the Tower of Hanoi problem with n disks in $2^n - 1$ moves. [3]

d) Prove that for any integer x, the integer $x(x + 1)$ is even. [2]

e) Let K_n be the complete graph with n vertices. For what values of n, K_n admits an Eulerian circuit? [3]

2. a) State and prove De Morgan's laws for sets. [4]

b) Write what you know about planar graphs. Show that $K_{2,4}$ is a planar graph, where K_5 is not. [3]

c) Define the generalized pigeonhole principle. [2]

d) Prove that, "if n is an integer and $n^3 + 7$ is odd then n is even", using – [5]
   i. Indirect Proof
   ii. Proof by Contradiction

3. a) Given the following statements as premises, all referring to an arbitrary meal: [4]

If he takes coffee, he does not drink milk.
He eats crackers only if he drinks milk.
He does not take soup if he eats crackers.
At noon today, he had coffee.

Draw the conclusion whether he took soup at noon today or not, using rules of inference.

b) Write short notes on: [4]
   i. Spanning tree
   ii. Binary search tree

c) Suppose that the number of bacteria in a colony triples every hour. Set up a recurrence relation for the number of bacteria after n hours have elapsed. [2]

d) Prove using mathematical induction that $1 + 5 + 9 + \ldots + (4n+1) = (n+1)(2n+1)$. [4]

4. a) How many eight digit numbers are there that contain a 5 and a 6? Explain. [4]

b) What is the cardinality of each of the following sets? [3]

i. **Z**
ii. **N-Z$^+$**
iii. **A-B**

c) Determine whether $(p \wedge (p \vee q)) \leftrightarrow (p \vee (p \wedge q))$ is a tautology using laws of logic. [3]

d) What is the value of each of these expressions: [4]

i. **- * + - 5 3 2 1 4**
ii. **5 2 1 - - 3 1 4 + + ***

5. a) Let A = {1, 2, 3, 4, 5}, determine the truth value of the following: [2]

i. $(\forall x \in A)(x + 3 = 7)$
ii. $(\exists x \in A)(x + 3 < 5)$

b) What are the equivalence classes of 1 and 2 for congruence modulo 4? [3]

c) Write a short note on bipartite graphs. Distinguish between Hamiltonian and Eulerian graphs. [4]

d) Show that a full **m**-ary tree with **i** internal vertices has **m*i** edges. [3]

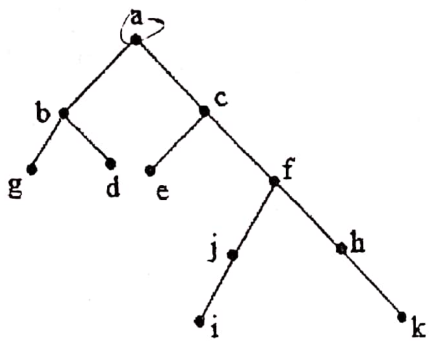
e) Let A = {1, 2, 3}, B = {p, q} and C = {a, b}. Let f: A → B is f = {(1, p), (2, p), (3, a)} and g: B → C is given by {(p, b), (q, b)}. What is the composition of f and g? [2]

6. a) Let F(x, y) be the statement "x can fool y", where the universe of discourse consists of all people in the world. Use quantifiers to express each of these statements. [4]

i. Everybody can fool somebody.
ii. There is exactly one person whom everybody can fool.
iii. No one can fool himself or herself.

b) Determine whether $f(x)=3x+2$ is a bijection from **R** to **R**. [2]

c) Show the three types of traversal for the following tree. [4]



d) Write short notes on: [4]

i. Equivalence relation
ii. Equivalence class

7. a) Consider the following conditional statement: [3]

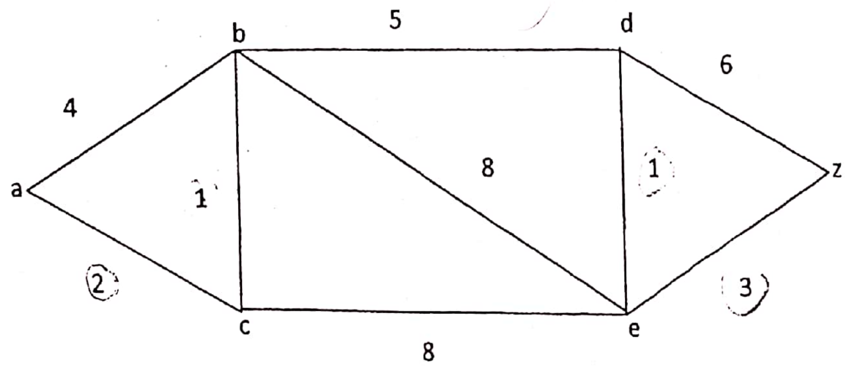
"If the flood destroy my house or the fires destroy my house, then my insurance company will pay me."

Write the converse, inverse and contrapositive of the statement.

b) For what numbers m do we have that m^2-1 is a prime number? [2]

c) How many bit strings of length six are possible if they start with 1 or end with 0 or have 1s in the two central places? [4]

d) For the following weighted graph determine the shortest path from z to a and also calculate its length. Draw all the steps to calculate the shortest path. [5]



Date: 26/09/16

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester **Final Examination, Spring 2016**

Course No: **CSE1203** Course Title: **Discrete Mathematics**

Time: **3 Hours** Full Marks: **70**

[There are <u>7(Seven)</u> questions. <u>The question no.1 is compulsory</u> and answer any four from the rest.]

[Marks allotted are indicated in the right margin within '[ ]'.]

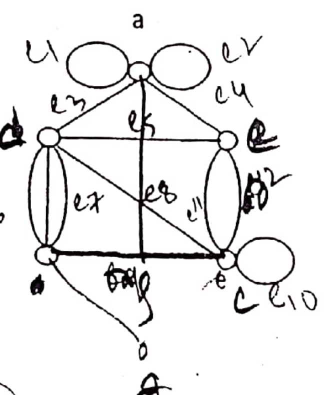
1. a) There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying same thing? Explain your answer with propositional logic. [3]

b) Prove that if n and m are positive even integers, then n*m is divisible by 4. [2]

c) There are m functions from a one-element set to the set {1, 2, ..., m}. How many functions are there from a two-element set to {1, 2, ..., m}? Give a recurrence relation for the number T(n) of functions from an n-element set to {1, 2, ..., m}. Solve the recurrence relation. [3]

d) Prove that the maximum number of edges in a simple graph with n vertices is n(n-1)/2. [4]

e) Represent using adjacency matrix the graph G_1 given below, and construct graph G_2 represented below using adjacency lists. [2]



G_1

G_2

a: d, e, f

b: d, e, f, g

c: e, f, g

d: a, b

e: a, b, c

~~f: a, b, e~~

~~g: b, e~~

2. a) What is a contingency? Show using a truth table whether the following expression is a tautology or not. [4]

$$(p \rightarrow q) \land (q \rightarrow r) \rightarrow (p \rightarrow r)$$

$\Rightarrow ml = mn - n + 1$

$\Rightarrow$

b) Show that a full **m-ary** tree with n vertices has **i = (n-1)/m** internal vertices and **l = [(m-1)n + 1]/m** leaves. [3]

c) Write short notes on: [3]

i. Power set,
ii. Set difference,
iii. Cartesian product.

d) Use mathematical induction to show that (n^3-n) is divisible by 3 for all positive integer n. [4]

3. a) Derive **r** from the given set of facts { **t ∨ r, s ∨ ¬ p, t→q, ¬ s ∨ r, ¬ q ∧ u** } using propositional rules of inference. [4]

b) Draw a binary tree that represents $(((z + 5) / 3) \wedge x - 5) + (x * 5)$. Now find the prefix and postfix notation of this expression. [4]

c) Consider the function f: X→Y on the sets X={a, b, c, d} and Y={1, 2, 3, 4}, where f(a) = 2, f(b) = 1, f(c) = 2, f(d) = 3. Determine whether the function is Injective, Surjective and Bijective. If so then why? [3]

d) Write down the Handshaking theorem and prove it. [3]

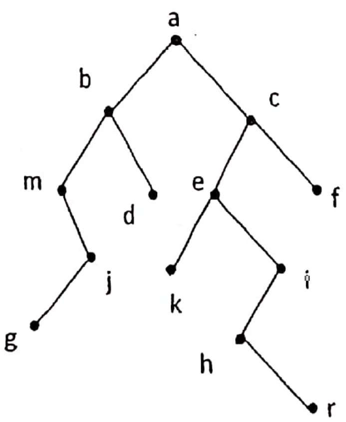
$t \vee r$
$s \vee \neg p$
$t \rightarrow q$
$\neg s \vee r$
$\neg q \wedge u$

4. a) Write short notes on: [3]

i. Quasi order,
ii. Partitions,
iii. Inverse functions.

b) Show the three types of traversal of the following tree. [4]

$t \vee r$
$s \vee \neg p$



$n = mi + 1$

$= ml = mn + n - 1$

c) Define the pigeonhole principle. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? [3]

d) Let **m** be a positive integer greater than 1. Show that the following relation is an equivalence relation on the set of integers. [4]

$$R = \{(a, b) \mid a \equiv b \pmod m\}$$

$l = n(m+i)$

$l = n + i$
$= n + \frac{n-1}{m}$
$= \frac{mn + n - 1}{m}$

$l = mi - i + 1$
$i(m-1) + 1$
$= \left(\frac{n-1}{m}\right)(m-1) + 1$
$\frac{(n-1)(m-1) \cdot 1}{m}$

d) Consider the following relations on {0, 1, 2, 3}: [4]

i. $R_1 = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$
ii. $R_2 = \{(0, 0), (1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$
iii. $R_3 = \{(0, 0), (1, 1), (1, 3), (3, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$
iv. $R_4 = \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Determine whether the relations are Reflexive, Symmetric, Anti-symmetric or Transitive. Which of these relations are equivalence relations?

7. a) Draw the following graphs: [3]

i. C_5
ii. W_6
iii. $K_{5,2}$

b) Prove that if 5n+4 is odd, then n is odd by giving a proof by contradiction. [3]

c) Prove using the laws of set operations that [4]

$$((B' \cup A') \cap (A' \cup B)) \cup (A' \cap B) = A'.$$

d) Describe Euler circuit and Hamiltonian circuit with examples. [4]

$\Rightarrow A'B' \cup A \cup (A' \cap B)$

$A'B' \cup A' \cap B$

$\Rightarrow A'B'$

$\Rightarrow A'$

$(a+b) \times (a-b)$

$p' \vee q'$

5. a) Translate the following English sentences into logical expressions. [4]

i. All your friends are perfect.

ii. Someone in your class has exactly one best friend.

b) Write short note on planar graph. [2]

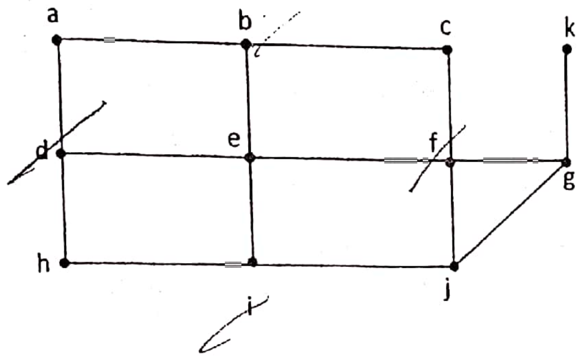
c) How many identifiers of length at most 4 are possible under the following conditions? [4]

i. An identifier starts with an upper case or lower case letter.

ii. There are 30 reserved words of length 4, each containing only lowercase letters which cannot be used as identifiers.

iii. Digits can also be used.

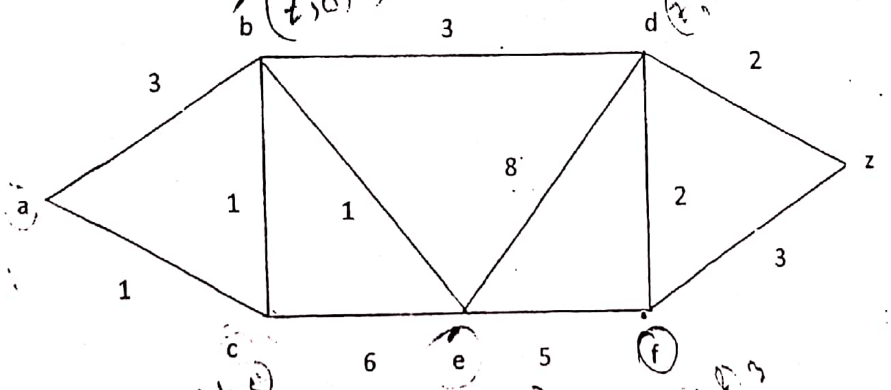
d) What is a spanning tree? Use breadth-first search to find a spanning tree for the graph shown below. (Choose the vertex e for the root.) [4]



6. a) What is wrong with the famous supposed "proof" that 1 = 2? [2]

b) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. [4]

c) For the following weighted graph determine the shortest path from z to a and also calculate its length using Dijkstra's algorithm. Draw all the steps to calculate the shortest path. [4]



Date: 15/03/16

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester — Final Examination, Fall 2015

Course No: CSE 1203 — Course Title: Discrete Mathematics

Time: 3 Hours — Full Marks: 70

[There are 7(Seven) questions. The question no.1 is compulsory. Answer any four from the rest.]

[Marks allotted are indicated in the right margin within '[ ]'.]

1. a) A set S is defined recursively by [2]

Basis: $0 \in S$

Recursive step: if $a \in S$ then $a + 3 \in S$ and $a + 5 \in S$.

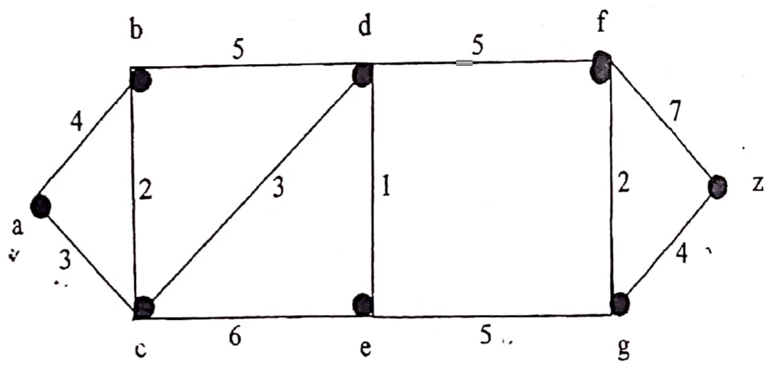
Determine the set $S \cap \{a \in Z \mid 0 < a < 12\}$.

b) Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent using laws of logical equivalences. [2]

c) Use mathematical induction to show that $2^n > n^2$ where $n>4$. [4]

d) A perfect number is a positive integer n such that the sum of the factors of n is equal to 2*n (1 and n are considered factors of n). So 6 is a perfect number as the sum of the factors of 6 is 1+2+3+6 =12=2*6. Now prove that no prime number is a perfect number. [2]

e) For the following weighted graph identify the shortest path between a and z, and also calculate its length. [4]



acd

acdfgz (17)

acdegz (16)

acdegfz

2. a) Suppose that on an island there are three types of people, knights, knaves and normals. Knights always speak the truth, knaves always lie and normals sometimes lie and sometimes tell the truth. Detectives questioned three inhabitants of the island- X, Y, and Z as part of the investigation of a crime. The detectives knew that one of the three committed the crime, but not which one. They also knew that the criminal was a knight, but the other two were not knights. Additionally the detectives recorded these statements: [4]

X: "I am innocent."
Y: "What X says is true."
Z: "Y is not a normal."

After analyzing their information, the detectives identified the guilty party. Who was it? Justify your answer.

b) Translate the following English sentences into logical expressions using predicates and quantifiers. [3]

i. Every guest gets exactly one gift.

ii. ~~There is a person in your school who is not happy.~~

c) Consider the following relations on a set of integers {1, 2, 3,4}. [4]

R1= $\{(a, b) \mid a \geq b\}$
R2= $\{(a, b) \mid a = b - 1\}$
R3= $\{(a, b) \mid a < b\}$
R4= $\{(a, b) \mid a + b \leq 4\}$.

Determine whether the relations are reflexive, irreflexive, Symmetric, Anti-symmetric and/or transitive.

d) Write down the Handshaking Theorem and prove it. [3]

3. a) A group of ten people begin a chain letter, with each person sending the letter to five other people. Each of these people sends the letter to five additional people. [4]

i. Find a recurrence relation for the number of letters sent at the nth stage of this chain letter, if no person ever receives more than one letter.

ii. What are the initial conditions for the recurrence relation?

iii. How many letters are sent at the nth stage of the chain letter?

b) Let f and g be the functions from the set of integers to the set of integers defined by **f(y) = 5y+9** and **g(y) = 2y+5**. What is the composition of f and g? What is the composition of g and f? [3]

c) Write what you know about Planar Graphs? Show that, $\mathbf{k_{2,5}}$ is a Planar Graph, where $\mathbf{k_5}$ is not. [3]

d) Prove that, "if n is an integer and 3n + 2 is even then n is even", using – [4]

a) Indirect Proof
b) Proof by Contradiction

4. a) What is Implication of propositions? State the Converse, Contra-positive and Inverse of the following implication: [3]

"If it snows tonight, then I will stay at home".

b) Verify the distributive properties of set theory using a membership table. [4]

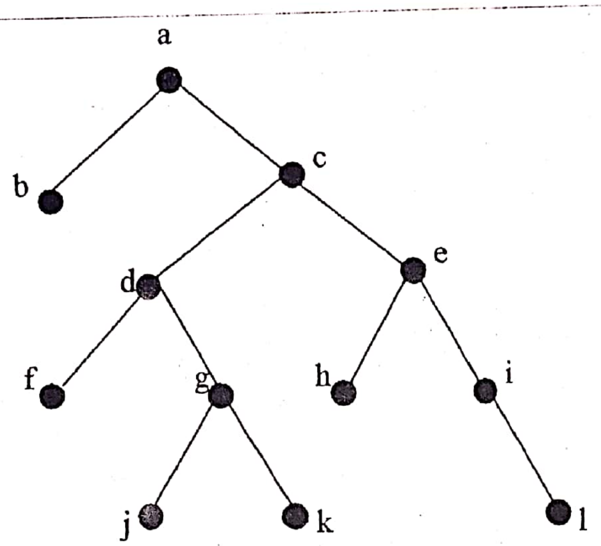
c) Write short notes on: [3]

i. Congruence Modulo,

ii. Partial order,

iii. Equivalence classes.

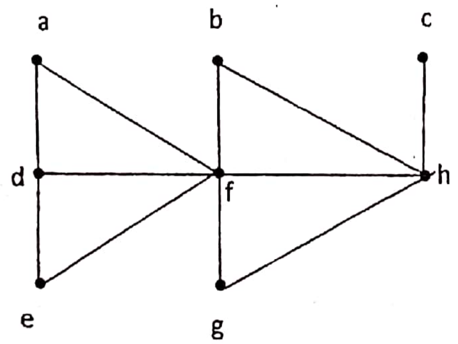
d) Demonstrate the three types of traversals on the following tree. [4]



5. a) Define the "Tower of Hanoi" problem using a recurrence relation with initial conditions. [4]

b) Illustrate the Pigeonhole Principle including its generalized form. [3]

c) What is a spanning tree? Construct two spanning trees using DFS and BFS on the following graph. Consider vertext 'f' as the root of thr tree. [6]



d) If $A \subset B$, then what is $|A \cap B|$? [1]

6. a) Prove using laws of set operations that $(A \cap B) \cup (A \cap B') = A$. [4]

b) Draw a binary tree that represents ((x – 3) +((x/4) + (x – y). ^ 5)). Now find the prefix and postfix forms of the expression. [5]

c) There are 21 different letters in the Italian alphabet, of which 5 are vowels. How many words of 10 letters you can form using the Italian alphabet? How many words of length 8 you can form which contain exactly one vowel? [4]

d) What is a binary search tree? [1]

~~7. a) Show that $\neg p \rightarrow \neg q$ and its contrapositive are equivalent.~~ [2]

b) Is it possible to solve the Konigsberg-bridge problem? Justify your answer. [4]

c) How many edges do the following graphs have: C_n, W_n, K_n, $K_{n,m}$? Explain your answer. [4]

d) Show that a full m-ary tree with [4]

i. n vertices has (n–1)/m internal vertices and [(m–1)n + 1]/m leaves .
ii. k leaves has (mk-1) / (m-1) vertices and (k-1) / (m-1) internal vertices.

Date: 14/10/15

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester — Final Examination, Spring 2015

Course No: CSE 1203 — Course Title: Discrete Mathematics

Time: 3 Hours — Full Marks: 70

[There are 7(Seven) sets. The set no.1 is compulsory and answer any four sets from the rest six sets]

[*Marks allotted are indicated in the right margin within '[ ]'.*]

1\. a) What is a contradiction? Show by using a truth table whether the expression $(\neg p \lor q) \leftrightarrow (p \land \neg q)$ is a contradiction or not. [2]

b) Express the following statement using predicates and quantifiers- [1]

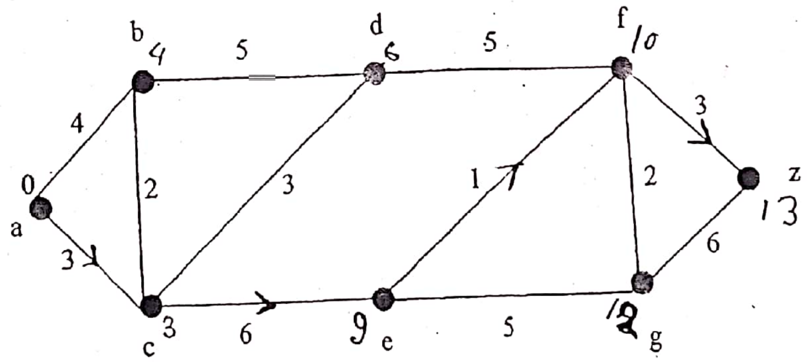
"Everyone has exactly one best friend"

c) Consider the function f: A→ B on the sets A= {a, b, c} and B= {1, 2, 3, 4}, where f (a) = 3, f (b) = 4, f(c) =1. Determine whether the function is Injective, Surjective and Bijective. If so then why? [3]

d) Use mathematical induction to show that- [4]

$$1^2+2^2+3^2+\ldots\ldots+n^2 = n^2(n+1)^2/4$$

e) For the following weighted graph identify the shortest path between a and z. Also calculate the length of the path. [4]



2\. a) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \lor r)$ are logically equivalent using laws of logical equivalence. [3]

b) Find the number of strings of length 8 of letters of the English alphabet, with repeated letters allowed and without repeated letters, [4]

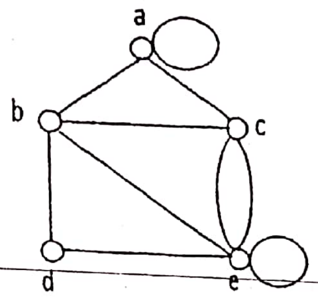
i. that begin with A and end with Z.

ii. that begin with Z or end with Z.

c) Prove using the laws of set operations that [4]

$$(A \cap A) \cup (B \cap (B' \cup A))' = U$$

d) Represent using adjacency matrix the graph G_1 given below, and construct graph G_2 represented below using adjacency lists. [3]



G_1

G_2
a: d, e, f
b: d, e, f, g
c: e, f, g
d: a, b
e: a, b, c
f: a, b, c
g: b, c

3\. a) Let F(x, y) be the statement "x can fool y", where the universe of discourse consists of all people in the world. Use quantifiers to express each of these statements. [3]

i. Everybody can fool somebody.
ii. There is exactly one person whom everybody can fool.
iii. No one can fool himself or herself.

b) Draw the graph of the function $f(x) = \lceil 2x \rceil$. [3]

c) Represent the expression $((y - 7) \uparrow 5) * (x - (5+y)) + 9$ using a binary tree. Write this Expression in prefix and postfix notation. [4]

d) Let m be a positive integer greater than 1. Show that the following relation is an equivalence relation on the set of integers. [4]

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}$$

4\. a) Draw the following graphs: [3]
i. C_5 ii. W_6 iii. K_5

b) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. Show an iterative approach to solve this recurrence relation. [6]

c) Determine whether the relations on {0, 1, 2, 3} are equivalence relations? [2]
i. {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}
ii. {(0,0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

d) Derive **p** applying rules of inference over the set of propositions given below. [3]
$\{ t \rightarrow q,\ r \rightarrow t,\ \neg q \wedge s, \text{ and } r \vee p \}$

5. a) Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes 6, 7, 8 and the man's shoe comes in sizes 8, 9, 10, 11, and 12. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black. Determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women using counting rules. [3]

b) Show that a full m-ary tree with [4]

   i. n vertices has $(n-1)/m$ internal vertices and $[(m-1)n + 1]/m$ leaves.
   ii. k leaves has $(mk-1) / (m-1)$ vertices and $(k-1) / (m-1)$ internal vertices.

c) ~~Let f and g be the functions from the set of integers to the set of integers defined by $f(x)$~~ $= 2x+3$ and $g(x) = 3x+2$. What is the composition of f and g? What is the composition of g and f? [3]

d) Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n >= 4$. [4]

6. a) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if [4]

   i. a is taller than b.
   ii. a and b were born on the same day.

b) Verify the associative property of set theory using a membership table. [4]

c) Write a short note on bipartite graphs. Distinguish between Hamiltonian and Eulerian graphs. [2]

d) Show that $\neg p \rightarrow q$ and its contrapositive are equivalent. [2]

e) Write short notes on: [2]

   i. Power Set
   ii. Cartesian product

7. a) Prove that, "if **n** is an integer and $n^3 + 5$ is odd then **n** is even", using – [5]

   i. Indirect Proof
   ii. Proof by Contradiction

b) Write short notes on: [2]

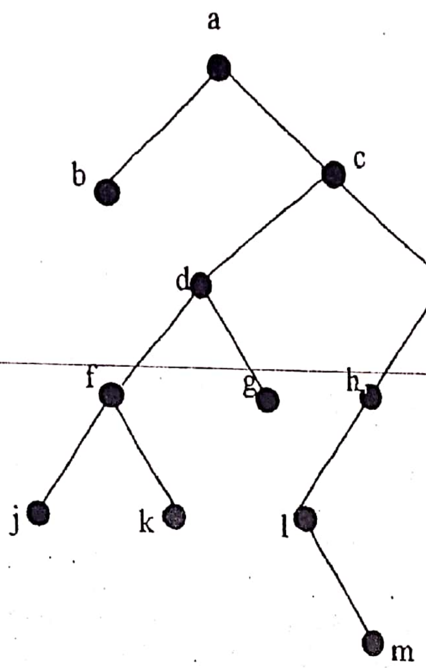
   i. Partitions
   ii. Partial Order

c) A bowl contains ten red balls and ten blue balls. A man selects balls out at random without looking at them. [2]

   i. How many balls must be selected to be sure that he has at least three balls of the same color?
   ii. How many balls must be selected to be sure that he has at least three red balls?

6)

d) Show the three types of traversals of the following tree. [3]



e) Write down the Handshaking Theorem and prove it. [2]

$\neg p \to (q \to r)$

$p \vee q$ $\quad$ $q \vee p$

$\neg p \to (\neg q \vee r)$

$\neg(\neg p) \vee (\neg q \vee r)$

$p \vee (\neg q \vee r)$

$\neg q \vee (p \vee r)$

$\cancel{\neg q \to}$ $q \to p$.

Date: 21-4-15

Ahsanullah University of Science & Technology
Department of Computer Science and Engineering
Year: 1st, Semester: 2nd, Final Examination (Fall 2014)

Course No: CSE 1203
Full Marks: 70

Course Title: Discrete Mathematics
Time: 3 Hours

[There are Seven (7) Questions. Answer any Five (5).]
[Marks allotted are indicated in the right margin.]

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1. a) Write a brief essay on quantified predicates and their negations. [4]

   b) Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology using truth table and laws of logical equivalences. [3+3]

   c) Write short notes on 'Counter example method' and 'Modus ponens'. [2+2]

2. a) Translate the following English sentences into logical expressions. [2+2]
      i. Every guest gets at least one gift.
      ii. There is a person in your school who is not happy.

   b) Define any two rules of interference for quantified predicates. Show the following argument is valid using the rules of interference. [2+5]
   There is a student such that if he knows programming, then he knows Java. All students know programming. Therefore, there is a student that knows either Java or C++.

   c) Prove that if $3n+2$ is odd, then n is odd by giving a proof by contradiction. [3]

3. a) Use set builder notation and logical equivalences to show that [3]

   $$A \cap (A \cup B) = A.$$

   A∩(A∪B)

   b) Prove using the laws of set operations that [4]

   $$(A \cap A) \cup (B \cap (B' \cup A))' = U.$$

   c) What is wrong with the famous supposed "proof" that 1 = 2? [3]

   d) Prove using mathematical induction that $1+5+9+\ldots\ldots+(4n+1) = (n+1)(2n+1)$. [4]

4. a) Let R and S be relations on a set A represented by the matrices [6]

   $$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

   Find the matrix that represent $M_R \odot M_S$. Determine whether the relations represented by the matrices are reflexive, symmetric and/or transitive.

   b) What are the equivalence classes of 1 and 2 for congruence modulo 4? [2]

c) Consider the function f: X→Y on the sets X={a, b, c, d} and Y={1, 2, 3, 4}, where f(a) = 2, f(b) = 1, f(c) = 2, f(d) = 3. Determine whether the function is Injective, Surjective and Bijective. If so then why? [4]

d) Draw the graph of the function $f(x) = \lceil x \rceil$. [2]

5\. a) How many positive integers between 100 and 999 are divisible by 3 or 5 or 7? [4]

b) Define the inclusion-exclusion principle with example. [3]

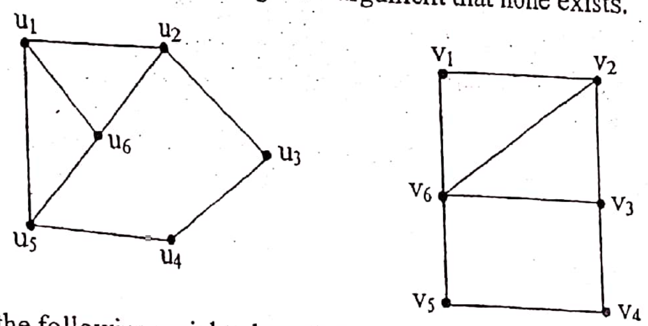
c) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. [[illegible]]

d) Define the generalized pigeonhole principle. What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$? [[illegible]]

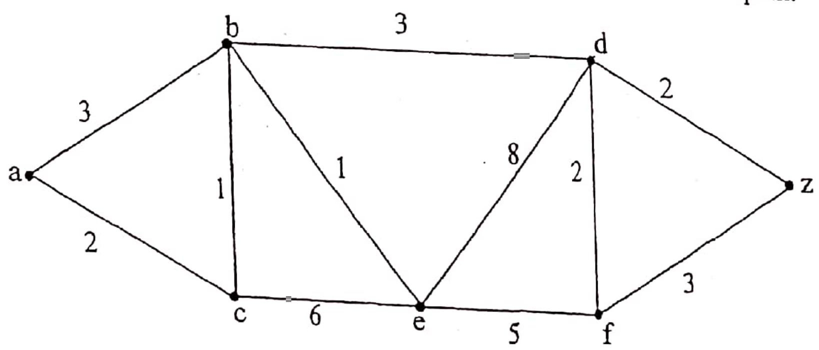
6\. a) Draw a graph which consists of 7 vertices and which chromatic number is 2. [2]

b) Write a short note on planar graphs. Distinguish between a strongly connected and a weakly connected graphs. [1+1]

c) What is graph isomorphism? What is Adjacency matrix and Incidence matrix? Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. [1+2+2]



d) For the following weighted graph determine the shortest path between a and z; also calculate its length. Draw all the steps to calculate the shortest path. [5]



$H_n = 2 \cdot H_{n-1} + 1$

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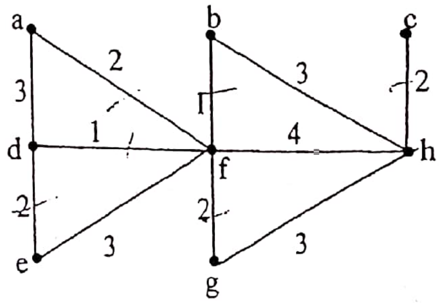
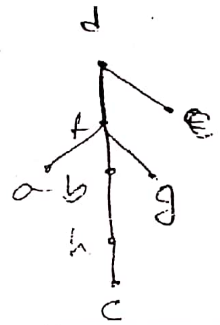
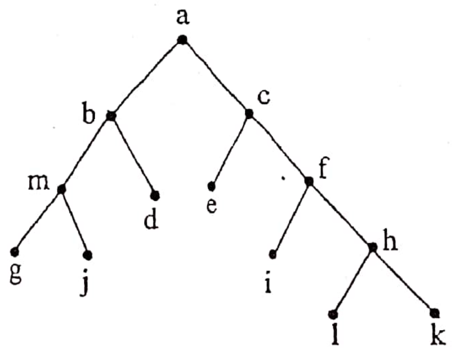
7. a) Use Prim's algorithm to find a minimum spanning tree in the weighted graph shown below. [4]





b) Show the three types of traversal of the following tree. [6]



d, f - 1

f, b - 1

d, e - 2

f, a - 2

f, g - 2

b, h - 3

h, c - 2

c) Show that a full *m-ary* tree with *n* vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n + 1]/m$ leaves. [4]

a——bmd c

a b mgjd