~~$\tan^n x\, dx$~~

tan $\sec^{n-2} x$

~~$\sec^2 x\, dx$~~

Date of Examination : 04/09/2018

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department : Arts and Sciences

Program : Bachelor of Science in Computer Science and Engineering

Semester Final Examination: Spring 2018

Year: 1st **Semester: 2nd**

Course number: MATH 1219

Course Name: Mathematics II

Time: 3 (Three) hours **Full Marks: 70**

Instruction: There are 7 (**seven**) questions in group A and B. Answer 5 (**five**) questions, taking 3 (**three**) from **Group-A** and 2 (**two**) from **Group-B**. Marks allotted are indicated in the right margin.

2

Group-A

1. Evaluate the following indefinite integrals: [14]

(i) $\int \frac{x^2 - 6x + 9}{x + 3} dx$, (ii) $\int \tan^{-1} \frac{2x}{1 - x^2} dx$, (iii) $\int \frac{1}{x(x+1)^2} dx$, (iv) $\int \frac{1}{3\sin x - 4\cos x} dx$.

2. a. Evaluate the following definite integrals: [7]

(i) $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$, (ii) $\int_0^{\pi/2} \frac{1}{3 + 5\cos x} dx$.

b. Show that $\int_0^1 \frac{\log x}{\sqrt{1 - x^2}} dx = \frac{\pi}{2} \log \frac{1}{2}$ by using properties of definite integration. [7]

3. a. Obtain reduction formulae for (i) $I_n = \int \tan^n x\, dx$ and (ii) $J_n = \int_0^{\pi/4} \tan^n x\, dx$. [6]

b. Define Beta and Gamma function. Prove that $\Gamma(n+1) = \int_0^\infty e^{-y^{1/n}} dy$. [5]

c. Show that $\int_0^1 \frac{x dx}{\sqrt{(1 - x^5)}} = \frac{1}{5} \beta\left(\frac{2}{5}, \frac{1}{2}\right)$. [3]

4. a. Find the length of the whole cardioid $r = a(1 - \cos\theta)$. [4]

b. Find the area enclosed by the curve $a^4 y^2 = x^4 (a^2 - x^2)$. [5]

c. Find the volume generated by revolving the area cut off from the parabola $9y = 4(9 - x^2)$ by the line $4x + 3y = 12$ about the x axis. [5]

14

$3y = 3(12 - 4x)$

$\int_{\theta_1}^{\theta_2} \frac{2}{3}\pi r^3 \cos\theta\, d\theta$

$\tan x/2 = z$

$\frac{1}{2}\sec^2 x/2\, dx = dz$

$\sec^2 = 2dz$

$\tan x/2 = z$

$\frac{1}{2}\sec^2 x/2$

$\int \tan x =$

$y\frac{dy}{dx} = x+5$

Group-B

5. a. Define order and degree of a differential equation. Form the differential equation by eliminating A and B from the equation $y = e^{mx}(A\sin nx + B\cos nx)$. [5]

b. Solve the differential equation $x(y^2+1)dx + y(x^2+1)dy = 0$. [4]

c. Write the Lagrange's form of linear equation. Solve $\cos x\frac{dy}{dx} + y\sin x = 1$. [5]

6. a. Is $\frac{dy}{dx} = \frac{x-y-2}{x+y+6}$ homogeneous differential equation? Solve it. [5]

b. What is the necessary and sufficient condition that a first order and first degree differential equation to be exact? Solve $(2x-y+1)dx + (2y-x-1)dy = 0$. [4]

c. Solve the nonlinear differential equation $4xp^2 - 8yp - x = 0$, where $p = \frac{dy}{dx}$. [5]

7. a. What is Clairaut's equation? Solve the Clairaut's equation $y = 2px + p^2$, where $p = \frac{dy}{dx}$. [7]

b. Solve (i) $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y = 0$, (ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$. [7]

$y = px + f(p)$

$y = e^{mx}$

$= \frac{dy}{dx} = m e^{xm}$

$\int \frac{dx}{\sqrt{a^2 - x^2}}$

$\boxed{\int \sec^2 x = ?}$

$y^2+1 = q$

$2z = dq$

$\int \frac{dx}{a^2 - x^2} =$

$\int \frac{dx}{\sqrt{a^2-x^2}} =$

Date of Examination : 05.05.2019

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department : Arts and Sciences

Program : Bachelor of Science in Computer Science and Engineering

Semester Final Examination: Fall 2018

Year : 1st **Semester : 2nd**

Course Number : MATH 1219

Course Name : Mathematics II

Time : 3 (Three) hours **Full Marks : 70**

Instruction: There are **7 (Seven)** questions in this question paper. Answer **5 (Five)** questions taking **3 (Three)** from **Part-A** and **2 (Two)** from **Part-B**. Marks allotted are indicated in the right margin.

Use separate answer script for each Part

Part-A

1. a. Integrate the followings: (any three) [9]

(i) $\int x^4 e^{-3x}\,dx$, (ii) $\int \frac{x^3}{\sqrt{1-4x^2}}\,dx$, (iii) $\int \frac{3x^2-x+2}{x^2-x-6}\,dx$, (iv) $\int x^2 \tan^{-1} x\,dx$.

b. Derive a reduction formula for $\int \cot^n x\,dx$ and hence calculate $\int \cot^5 x\,dx$. [5]

2. a. State Walli's formula, hence compute $\int_0^{\pi/2} \sin^{10} x\,dx$. [3]

b. Show that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}}\,dy$, hence evaluate $\int_0^{\infty} \frac{x^{10}}{(1+x)^{15}}\,dx$. [5]

c. Evaluate (i) $\int_{-\pi}^{\pi} x^5 \cos 2x\,dx$, (ii) $\int_0^{\pi/2} \frac{1}{1+\cot^{3/2} x}\,dx$, (iii) $\int_0^{\pi/2} \sin^3 x \cos^5 x\,dx$. [6]

3. a. Find the volume of the solid generated by revolving the regions bounded by $y = 3\sin 2x,\ 0 \le x \le \pi/2$ about the x-axis. [4]

b. Calculate the area enclosed by the loop of the curve $a^2x^2 = y^3(2a-y)$. [5]

c. Test whether the following improper integrals converge or not: [5]

$(i)\ \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx,\quad (ii)\ \int_{-1}^{3} \frac{1}{x^2}\,dx.$

4. a. Classify and write the name of the following partial differential equations: [4]

(i) $\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial t} = 0$; (ii) $\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial^2 u(x,t)}{\partial t^2} = 0$. Hence describe the importance of the above partial differential equations in the field of computer science.

b. Derive a partial differential equation from the equation $z=\varphi(x+iy)+\psi(x-iy)$ where $i^2=-1$. Hence classify the obtained partial differential equation, and find one of a solution of it. [5]

c. Obtain the general integral of the equation $-2xyp+(x^2+z^2-y^2)q+2yz=0$, where $p=\partial z/\partial x$ and $q=\partial z/\partial y$. [5]

Part-B

5. a. Solve: $\left\{x+y\cos\left(\frac{y}{x}\right)\right\}dx-x\cos\left(\frac{y}{x}\right)dy=0$. [4]

b. Find the solution of $(2x+3y-5)dy+(3x+2y-5)dx=0$. [5]

c. When the differential equation $Mdx+Ndy=0$ is exact? [5]
Solve : $\left(x^2+y^2+x\right)dx+xy\,dy=0$.

6. a. Write down the Bernouli's equation of first order and first degree differential equation. Obtain the solution of $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$. [5]

b. Find the general solution of $p^2+2p\cot x-y^2=0$, where $p=\frac{dy}{dx}$. [4]

c. Compute the complete and singular solution of $y=-px+x^4p^2$, where $p=\frac{dy}{dx}$. [5]

7. a. Solve the differential equation $(D^2-2D+5)y=e^{2x}\sin x$, where $D=\frac{d}{dx}$. [7]

b. Write down Cauchy-Euler equation. Find the solution of $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\ln x$. [7]

Date: 13/03/2017

Ahsanullah University of Science and Technology

Final Examination of Fall Semester 2016

Department of Arts and Sciences

Program: B. Sc. in Computer Science and Engineering

2nd Semester of 1st Year

Course No: MATH -1219 **Course Title:** Mathematics-II

Time: 03 (three) hours **Full Marks:** 70

There are 7 (seven) questions. Answer 5 (five) questions, taking any 3(three) from **PART- A** and **2 (two)** from **PART- B.** Marks allotted are indicated in the right margin.

[Use separate Answer Script for each part]

PART- A

1. (a) Integrate the followings: (any three) [9]

(i) $\int x^4 \cos 2x\, dx$, (ii) $\int \frac{dx}{x\sqrt{4-9(\ln x)^2}}$, (iii) $\int \frac{x^2-2x+1}{x^3-3x^2-x+3}dx$,

(iv) $\int x^3\sqrt{9+4x^2}\, dx$.

(b) Find the reduction formula for $\int \cos^m x\, dx$ and hence calculate $\int \cos^6 x\, dx$. [5]

2. (a) Evaluate (i) $\int_{-\pi}^{\pi} t^5 \sin^2 3t\, dt$, (ii) $\int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}dx$. [4]

(b) Define Beta and Gamma function. Hence prove that [5]

$$\int_0^{\pi/2} \sin^p x \cos^q x\, dx = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}.$$

(c) Calculate (i) $\int_0^{\infty} \frac{x^9}{(1+x)^{16}}dx$, (ii) $\int_0^{\infty} \frac{x^4}{(1+x^2)^4}\, dx$. [5]

3. (a) A loop of the curve $(x-4a)\, y^2 = ax\,(x-3a)$ is revolved about x axis, find its volume. [4]

(b) Compute the area of the region bounded by the curves $y = \sin x$ and $y = \cos x$, from $x = 0$, to $x = \pi/2$. [5]

(c) Determine the arc length of the curve $y = 2\sqrt{a}\sqrt{x}$ from $x = 0$ to $x = 1$. [5]

4. (a) Classify and name of the following partial differential equations: [4]

(i) $\alpha \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial t} = 0$; (ii) $c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}$. Hence describe the importance of the above partial differential equations in the field of Computer Science and Engineering.

(b) Derive a partial differential equation from the equation $z = e^{(ax+by)} f(ax-by)$. [5]

(c) Obtain a general integral of the equation $(y^2+z^2-x^2)p-2xyq+2zx=0$, where $p=\partial z/\partial x,\ q=\partial z/\partial y$. [5]

PART-B

5. (a) Define order and degree of a differential equation. Find order, degree and linearity of the differential equation $\sqrt[4]{\frac{d^6y}{dx^6}+\frac{d^3y}{dx^3}} = \sqrt[3]{\frac{d^6y}{dx^6}+2}$. [3]

(b) Find the differential equations of all circles in the XY-plane passing through the origin and having their centres on Y axis. [3]

(c) Find the general solution of the following differential equations. [8]

(i) $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$; (ii) $\frac{dy}{dx} = \frac{2x-6y+7}{x-3y+4}$.

6. (a) Reduce the equation $\frac{dy}{dx} = \frac{-x+y+1}{x+y-5}$ to homogeneous form and hence find the general solution. [5]

(b) Write the general form of Bernoulli's differential equation. Solve the initial value problem $x\frac{dy}{dx}+2y = x\ln x,\ y(1)=0$. [5]

(c) State and prove the necessary and sufficient condition for $M\,dx+N\,dy=0$ to be exact. [4]

7. (a) Determine whether $(y^4+2y)\,dx+(xy^3+2y^4-4x)\,dy=0$ an exact differential equation or not. If not, calculate the integrating factor of the differential equation to make it an exact and hence solve the equation. [6]

(b) Find the general solution of the following differential equations: [8]

(i) $(D^3-4D)y = \sinh 2x + x^3 - 3x - 1$;

(ii) $(D^3+3D^2+3D+1)y = e^{-x}+\cos 2x$.

$\frac{4a^2}{8} - \frac{1}{4a} \cdot \frac{8a^3}{2 \cdot 12} = \frac{16a^2}{8} - \frac{8a^2}{6}$

Date: 24/09/2016

Ahsanullah University of Science and Technology

Final Examination of Spring Semester 2016

Department of Arts and Sciences

Program: B. Sc. in Computer Science and Engineering

2nd Semester of 1st Year

Course No: Math-1219, Course Title: Mathematics-II

Time: 03 (three) hours | **Full Marks: 70**

There are 7(Seven) questions. Answer 5(Five) questions, taking 2(Two) from group-A and 3(Three) from group-B.

(Marks allotted are indicated in the right margin.)

$x = a\sin^2\theta$

$dx = 2a\sin\theta\cos\theta\, d\theta$

Group-A

1. (a) Evaluate the following integrals (any two): (3.5 x 2 = 7)

(i) $\int \frac{4x+3}{3x^2+3x+1}dx$, (ii) $\int_0^{\pi/2} \frac{dx}{a+b\cos x}$, (iii) $\int_1^{\infty} \frac{dx}{x(x+1)}$.

(b) Obtain a reduction formula for $\int \cos^n x\,dx$. Hence find $\int \cos^7 x\,dx$. (7)

2. (a) Determine the length of an arc of the following curve (5)

$x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ measured from $\theta = 0$ to $\theta = \pi$.

(b) Obtain the intrinsic equation of the curve $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ (5) taking the vertex as the fixed point and the tangent at that point as the fixed line.

(c) Prove that $\int_0^{\pi/2} \sin^p\theta\cos^q\theta\,d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$. (4)

$4x^2 = 4y$

3. (a) Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$. (4)

(b) Find the area between the curve $y^2(2a-x) = x^3$ and its asymptote. (5)

$x = a\sin$

(c) Find the volume of the solid generated when the region between the curve $y^2 = 4x$ and the line $y = 2x$, is revolved about x-axis. (5)

$\tan\frac{x}{2} = z$

$\frac{1}{a-b}\left[\frac{1}{A} \cdot \tan^{-1}\right.$

$= \frac{1}{a-b}\left[\frac{1}{A}\right.$

$\frac{\pi}{2} \cdot 2$

Group-B

4. Solve the following differential equations:

(a) $x^2 y\, dx - (x^3 + y^3) dy = 0$. (5)

(b) $x(x^2 + y^2 - a^2)\, dx + y(x^2 - y^2 - b^2)\, dy = 0$. (4)

(c) $x(x-1)\dfrac{dy}{dx} - y = x^2 (x-1)^2$. (5)

5. (a) Reduce the equation $\dfrac{dy}{dx} + \dfrac{2}{x} y = \dfrac{y^3}{x^3}$ to linear form and then solve it. (5)

(b) Solve the following linear differential equations: (4+5)

(i) $\dfrac{d^3 y}{dx^3} + 3\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + y = e^{-x}$

(ii) $\dfrac{d^4 y}{dx^4} + 2\dfrac{d^2 y}{dx^2} + y = x^2 \cos^2 x$.

6. (a) Define homogeneous linear differential equation. (2)

(b) Solve the homogeneous linear equation $x^3 \dfrac{d^3 y}{dx^3} - x^2 \dfrac{d^2 y}{dx^2} + 2x\dfrac{dy}{dx} - 2y = x^3 + 3x$. (6)

(c) Reduce the equation $(px - y)(x - yp) = 2p$ to Clairaut's form, where $p = \dfrac{dy}{dx}$ (6)

and hence find its solution.

7. (a) Solve the linear partial differential equation $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ (7)

using Lagrange's method, where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

(b) Find the complete integral of the equation $q = px + p^2$ using Charpit's (7)

method, where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

$I_n - (n-1) I_{n-2} \cos x$

$I_n + I_{n-2} \cos x$

Date: 12/03/16

Ahsanullah University of Science and Technology

Final Examination of Fall Semester 2015

Department of Arts and Sciences

Program: B. Sc. in Computer Science and Engineering

2nd Semester of 1st Year

Course No: Math-1219 **Course Title: Mathematics-II**

Time: 03 (three) hours **Full Marks: 70**

There are 7(seven) questions. Answer 5(five) questions, taking 2(two) from Group-A and 3(three) from Group-B.

(Marks allotted are indicated in the margin.)

Group-A

(Marks)

1. (a) Evaluate the following integrals (any two): (3.5 x 2 = 7)

(i) $\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$, (ii) $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x}\,dx$, (iii) $\int_0^{\pi/2} \frac{dx}{1+\sin x}$.

(b) Obtain a reduction formula for $\int \cos^n x\,dx$. Hence find $\int \cos^7 x\,dx$. (7)

2. (a) Determine the length of an arc of the curve $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ measured from $\theta = 0$ to $\theta = \pi$. (5)

(b) Obtain the intrinsic equation of the Catenary $y = c\cosh\frac{x}{c}$ in the form $s = c\tan\psi$. (5)

(c) Prove that $\int_0^{\pi/2} \sqrt{\cot\theta}\,d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$. (4)

3. (a) Find the area of the curve $a^2y^2 = x^3(2a - x)$. (7)

(b) A loop of a curve $(x-4a)y^2 = ax(x-3a)$ is revolved about the x- axis, find its volume. (7)

Group-B

4. Solve the following differential equations:

(a) $x^2y\,dx - (x^3 + y^3)dy = 0$. (5)

(b) $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$. (4)

(c) $dx+x\,dy=e^{-y}\ln y\,dy$. (5)

5. (a) Reduce the equation $\frac{dy}{dx}\left(x^2y^3+xy\right)=1$ to linear form and then solve it. (7)

(b) Solve the differential equation $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3x^2e^{2x}\sin 2x$. (7)

---

6. (a) Solve the equation $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$. (7)

(b) Solve the homogeneous linear equation $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. (7)

7. (a) Reduce the equation $(px-y)(x-yp)=2p$ to Clairaut's form, where $p=\frac{dy}{dx}$ (7)

and hence solve the equation.

(b) Solve the linear partial differential equation $z-xp-yq=a\sqrt{(x^2+y^2+z^2)}$ (7)

by using Lagrange's method, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

Ahsanullah University of Science and Technology

Final Examination of Spring Semester 2015
Department of Arts and Sciences
Program: B. Sc. in Computer Science and Engineering
2^{nd} Semester of 1^{st} year

Course No: MATH-1219
Course Title: Mathematics-II
Time: 03 (three) hours
Full Marks: 70

There are 7 **(Seven)** questions in group A and B. Answer 5 **(Five)** questions, taking 3 **(Three)** from **Group-A** and 2 **(Two)** from **Group-B**.

Marks allotted are indicated in the right margin

Group-A

1. Evaluate the following indefinite integrals: [14]

(i) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$, (ii) $\int \tan^{-1} \frac{2x}{1-x^2} dx$, (iii) $\int \frac{x^2}{(x+1)(x+2)^2} dx$.

2. a) Evaluate the following definite integrals: [8]

(i) $\int_0^1 \frac{x^3 \sin^{-1} x}{\sqrt{1-x^2}} dx$, (ii) $\int_0^{\pi/2} \frac{1}{3+5\cos x} dx$

15.73

b) Show that $\int_0^{\pi} x \log(\sin x)\, dx = \frac{\pi^2}{2} \log \frac{1}{2}$. [6]

3. a) If $U_n = \int_0^{\pi/2} x^n \sin x \, dx$ and $n>0$, prove that $U_n + n(n-1)U_{n-2} = n\left(\frac{1}{2}\pi\right)^{n-1}$. [6]

b) Define Beta and Gamma function. Prove that (i) $\Gamma(1)=1$ and (ii) $\Gamma(n+1)=n!$. [4]

c) Evaluate $\int_0^1 x^6 \sqrt{1-x^2}\, dx$ using Beta and Gamma function. [4]

4. a) Find the area of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ between the major and minor axes. [5]

b) Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$. [5]

c) Find the volume of the solid generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about the initial line. [4]

$(x+2)^{-2}$ k,

-2k,

Group-B

5. a) Define order and degree of a differential equation. Form the differential equation of Simple Harmonic Motion given by $y = A\cos(nt + \alpha)$. [5]

b) Solve: $x(y^2+1)dx + y(x^2+1)dy = 0$. [4]

c) When a differential equation is said to be homogeneous? Is $(x^2+y^2)dy = xydx$ homogeneous? Solve it. [5]

---

6. a) What is Bernoulli's equation? Solve: $\frac{dy}{dx} + 2y\tan x = \sin x$. [5]

b) Is the differential equation $(2x - y + 1)dx + (2y - x - 1)dy = 0$ exact? Solve it. [4]

c) What is Clairaut's equation? Solve: $y = 2px + p^2$, where $p = \frac{dy}{dx}$. [5]

7. a) Solve: $(D^3 + 3D^2 + 3D + 1)y = 0$, where $D \equiv \frac{d}{dx}$. [3]

b) Solve: $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2\sin 3x$. [5]

c) Solve: $x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$. [6]

29/04/15

Ahsanullah University of Science and Technology

Final Examination of Fall Semester 2014

Department of Arts and Sciences

Program: B. Sc. in Computer Science and Engineering

1^{st} Year 2^{nd} Semester

Course No: Math-1219, Course Title: Mathematics-II

Time: 03 (three) hours Full Marks: 70

There are 7(Seven) questions. Answer 5(Five) questions, taking 2(Two) from group-A and 3(Three) from group-B.

(Marks allotted are indicated in the right margin.)

Group-A

(Marks)

1. (a) Evaluate the following integrals (any two): (3.5 x 2 = 7)

(i) $\int \frac{4x+3}{3x^2+3x+1}dx$ (ii) $\int \frac{11\cos x - 16\sin x}{2\cos x + 5\sin x}dx$ (iii) $\int_0^5 \frac{dx}{\sqrt{25-x^2}}$

(b) Obtain a reduction formula for $\int \tan^n x\,dx$. Hence find $\int \tan^5 x\,dx$. (7)

2. (a) Determine the length of an arc of the following curve (7)

$x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ measured from $\theta = 0$ to $\theta = \pi$.

(b) Obtain the intrinsic equation of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ taking the vertex as the fixed point and the tangent at that point as the fixed line. (7)

3. (a) Find the area between the curve $y^2(2a-x) = x^3$ and its asymptote. (7)

(b) A loop of a curve $(x-4a)y^2 = ax(x-3a)$ is revolved about the x- axis, find its volume. (7)

Group-B

4. Solve the following differential equations:

(a) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$ (5)

(b) $(1+e^{x/y})\,dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0.$ (4)

(c) $x\dfrac{dy}{dx}+2y=x^2\ln x$ (5)

5. (a) Reduce the equation $\dfrac{dy}{dx}=x^3y^3-xy$ to linear form and then solve it. (7)

(b) Solve the differential equation $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+4y=3x^2e^{2x}\cos 2x$. (7)

6. (a) Solve the equation $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$. (7)

(b) Reduce the equation $axyp^2+\left(x^2-ay^2-b\right)p-xy=0$ to Clairaut's form, (7)

where $p=\dfrac{dy}{dx}$ and hence solve the equation.

7. (a) Solve the linear partial differential equation $\left(y^2+z^2-x^2\right)p-2xyq+2xz=0$ (7)

by using Lagrange's method, where $p=\dfrac{\partial z}{\partial x}$ and $q=\dfrac{\partial z}{\partial y}$.

(b) Find the complete integral of the equation $pxy+pq+qy-yz=0$ by using (7)

Charpit's method, where $p=\dfrac{\partial z}{\partial x}$ and $q=\dfrac{\partial z}{\partial y}$.