To find ϕ

Ex: 76

$$\bar{F} = (y^2 + 2xz^2)\hat{i} + (2xy - z)\hat{j} + (2x^2z - y + 2z)\hat{k}$$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz = \nabla\phi \cdot d\bar{r} = \bar{F}\cdot d\bar{r}$$

$$= (y^2 + 2xz^2)\,dx + (2xy - z)\,dy + (2x^2z - y + 2z)\,dz$$

$$\Rightarrow \frac{\partial \phi}{\partial x} = y^2 + 2xz^2$$

Also

$$\frac{\partial \phi}{\partial z} = 2x^2z - y + 2z \quad \text{--- (A)}$$

Int.

$$\boxed{\phi = xy^2 + x^2z^2 + f(y,z)} \quad \text{--- (1)}$$

$$\frac{\partial \phi}{\partial y} = 2xy - z \quad \text{--- (2)}$$

Now diff. (1) w.r.to y so as to compare with (2)

$$\frac{\partial \phi}{\partial y} = 2xy + 0 + f_y(y,z) \quad \text{--- (3)}$$

Compare (2) and (3)

$$f_y(y,z) = -z \quad \text{--- (4)}$$

Int w.r.to y

(4) $\Rightarrow$ $\cancel{f(y,z) = -yz + \text{--- 5}}$

$$f(y,z) = -yz + g(z) \quad \text{--- (5)}$$

Substitute (5) in (1)

$$\boxed{\phi = xy^2 + x^2z^2 + g(z) - yz} \quad \text{--- (6)}$$

Now to compare with (A) and to find $g(z)$

diff. (6) w.r.to z

$$\frac{\partial \phi}{\partial z} = 0 + 2x^2z + g'(z) - y \quad \text{--- (7)}$$

Compare 7 with (A)

$\cancel{2x^2z} + g'(z) \cancel{-y} = \cancel{2x^2z} \cancel{-y} + 2z.$

$g'(z) = 2z \quad \text{--- (8)}$

Int (8) w.r.to z $\quad g(z) = 2\frac{z^2}{2} + c$

$\boxed{g(z) = z^2 + c}$

Substitute $g(z)$ in 6

$\boxed{\phi = xy^2 + x^2z^2 - yz + z^2 + c}$

Ans.