9

DIFFRACTION

9.1 INTRODUCTION

It is a matter of common experience that the path of light entering a dark room through a hole in the window illuminated by sunlight is straight. Similarly, if an opaque obstacle is placed in the path of light, a sharp shadow is cast on the screen, indicating thereby that light travels in straight lines. Rectilinear propagation of light can be easily explained on the basis of Newton's corpuscular theory. But it has been observed that when a beam of light passes through a small opening (a small circular hole or a narrow slit) it spreads to some extant into the region of the geometrical shadow also. If light energy is propagated in the form of waves, then similar to sound waves, one would expect bending of a beam of light round the edges of an opaque obstacle or illumination of the geometrical shadow.

Each progressive wave, according to Huygens wave theory produces secondary waves, the envelope of which forms the secondary wavefront. In Fig. 9.1 (*a*), *S* is a source of monochromatic light and *MN* is a small aperture. *XY* is the screen placed in the path of light. *AB* is the illuminated portion of the screen and above *A* and below *B* is the region of the geometrical shadow. Considering *MN* as the primary wavefront, according to Huygens' construction, if secondary wavefronts are drawn, one would expect encroachment of light in the geometrical shadow. Thus, the shadows formed by small obstacles are not sharp. This bending of light round the edges of an obstacle or the encroachment of light within the geometrical shadow is called diffraction. Similarly, If an opaque obstacle *MN* is placed in the path of light [Fig. 9.1 (b), there should be illumination in the geometrical shadow region *AB* also. But the illumination in the geometrical shadow of an obstacle is not commonly observed because the light sources are not point sources and secondly the obstacles used are of very large size compared to the wavelength of light. If a shadow of an obstacle is cast by an extended source, say a frosted electric bulb, light from every point on the surface of the bulb forms its own diffraction pattern (bright and dark diffraction bands) and these overlap such that no single pattern can be identified. The term diffraction is referred to such problems in which one considers the resultant effect produced by a limited portion of a wavefront.

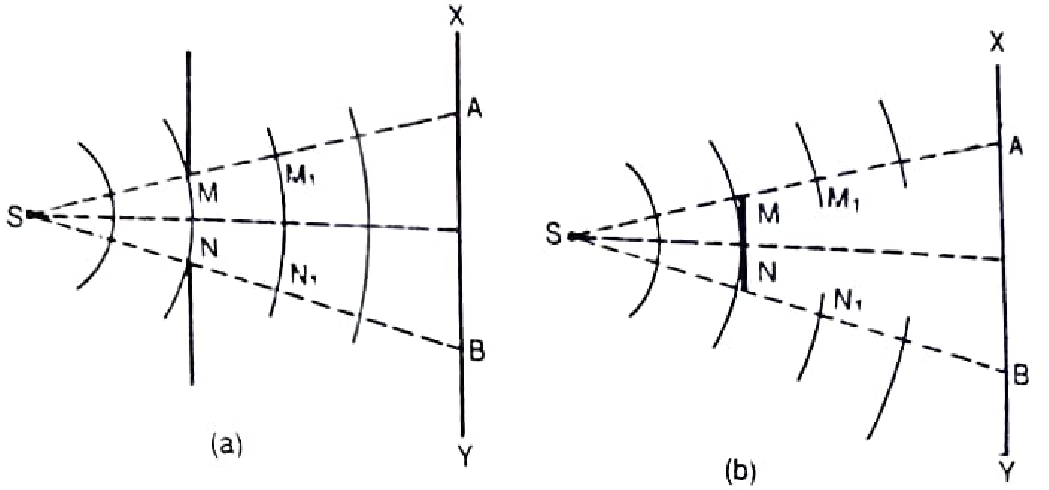


Fig. 9.1

Diffraction phenomena are part of our common experience. The luminous border that surrounds the profile of a mountain just before the sun rises behind it, the light streaks that one sees while looking at a strong source of light with half shut eyes and the coloured spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are all examples of diffraction effects.

Augustin Jean Fresnel in 1815, combined in a striking manner Huygens wavelets with the principle of interference and could satisfactorily explain the bending of light round obstacles and also the rectilinear propagation of light.

9.2 FRESNEL'S ASSUMPTIONS

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below :-

In Fig. 9.2, S is a point source of monochromatic light and MN is a small aperture. XY is the screen and SO is perpendicular to XY. MCN is the incident spherical wavefront due to the point source S. To obtain the resultant effect at a point P on the screen, Fresnel assumed that (1) a wavefront can be divided into a large number of strips or zones called Fresnal's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones ; (2) the effect at a point due to any particular zone will depend on the distance of the point from the zone ; (3) the effect at P will also depend on the obliquity of the point with reference to the zone under consideration, *e.g.* due to the part of the wavefront at C, the effect will be maximum at O and decreases with increasing obliquity., It is maximum in a direction radially outwards from C an it decreases in the opposite direction. The effect at a point due to the obliquity factor is proportional to $(1 + \cos\theta)$ where $\angle PCO = \theta$. Considering an elementary wavefront at C, the effect is maximum at O because $\theta = 0$ and $\cos\theta = 1$. Similarly, in a direction tangential to the primary wavefront at C (along CQ) the resultant effect is one half of that along

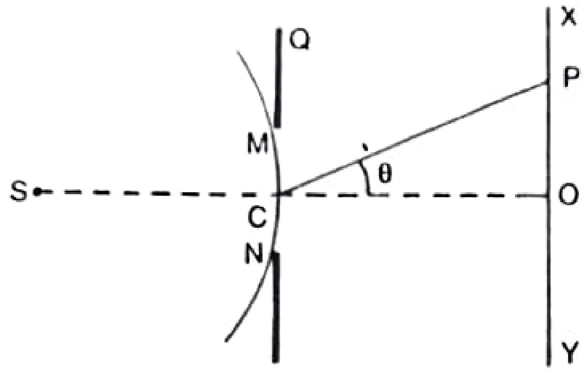


Fig. 9.2

CO because $\theta = 90^\circ$ and $\cos 90 = 0$. In this direction CS, the resultant effect is zero since $\theta = 180^\circ$ and $\cos 180 = -1$ and $1 + \cos 180 = 1 - 1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz., that if the secondary waves spread out in **all directions** from each point on the primary wavefront, they should give a wave travelling forward as well as backward. as the amplitude at the rear of the wave is zero there will evidently be no back wave.

9.3 RECTILINEAR PROPAGATION OF LIGHT

ABCD is a plane wavefront perpendicular to the plane of the paper

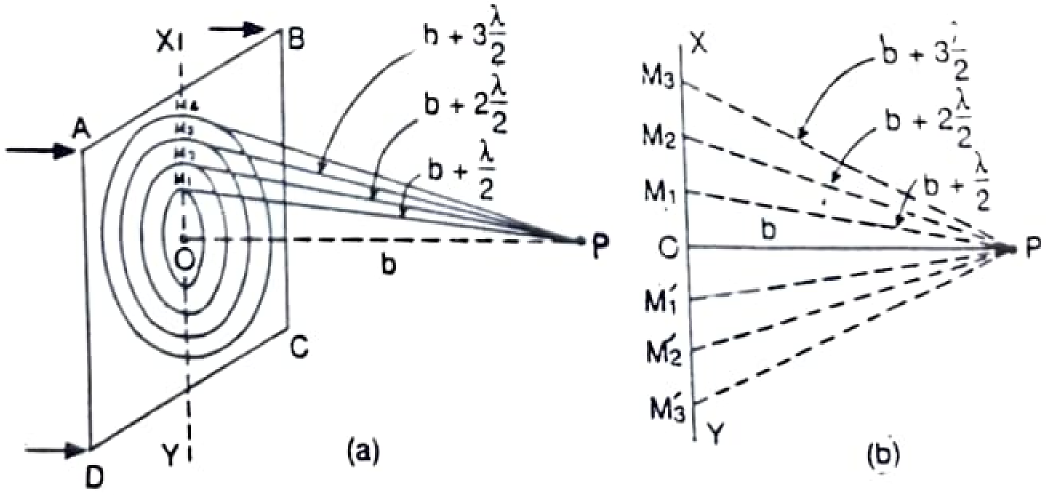


Fig. 9.3

For Newton' rings,

radius of the n th ring,

$$r_n = \sqrt{n \lambda R}$$

$$r_1 = \sqrt{\lambda R} \qquad ...(i)$$

For a zone plate, the principal focal length

$$f_1 = \frac{r_1^2}{\lambda} \qquad ...(ii)$$

From (i) and (ii)

$$f_1 = \frac{\lambda R}{\lambda} = R$$

But $\qquad R = 200 \text{ cm} = \mathbf{2\ m}$

$\therefore \qquad f_1 = \mathbf{2\ m}$

9.7 FRESNEL AND FRAUNHOFER DIFFRACTION

Diffraction phenomena can conveniently be divided into two groups viz, (i) Fresnel diffraction phenomena and (ii) Fraunhofer diffraction phenomena. In the Fresnel class of diffraction, the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction. In this case, the effect at a specific point on the screen due to the exposed incident wavefront is considered and no modification is made by lenses and mirrors. In such a case, the phenomenon observed on the screen is called Fresnel diffraction pattern. In the Fraunhofer class of diffraction phenomena, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction. Fraunhofer diffraction pattern can be easily observed in practice. The incoming light is rendered parallel with a lens and the diffracted beam is focussed on the screen with another lens. Observation of Fresnel diffraction phenomena do not require any lenses. Theoretical treatment of Fraunhofer diffraction phenomena is simpler. Fresnel class of diffraction phenomena are treated first in this chapter.

9.8 DIFFRACTION AT A CIRCULAR APERTURE

Let AB be a small aperture (say a pin hole) and S is a point source of monochromatic light. XY is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the screen. O is the centre of the aperture and r is the radius of the aperture. Let the distance of the source from the aperture be a ($SO = a$) and the distance of the screen from the aperture be b ($OP = b$). P_1OQ_1 is the incident spherical wavefront and with reference to the point P, O is the pole of the wavefront (Fig. 9.8). To consider the intensity at P, half period zones can be constructed with P as centre and radii $b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$ etc., on the exposed wavefront AOB. Depending on the distance of P from the aperture (*i.e.*, the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be constructed, then the intensity at P will be proportional to m_1^2 (where m_1 is the amplitude due to the first zone at P). On the other hand, if the whole of the wavefront is exposed to the point P, the resultant amplitude is $\frac{m_1}{2}$ or the intensity at P will be proportional to $\frac{m_1^2}{4}$. The position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only 2 zones are exposed, the resultant amplitude at $P = m_1 - m_2$ (minimum) and if 3 zones are exposed, the amplitude $= m_1 - m_2 + m_3$ (maximum) and so on. Thus, by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.

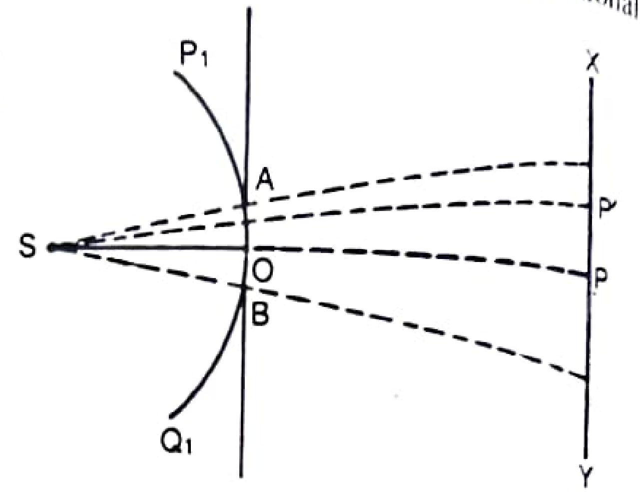


Fig 9.8

Now consider a point P' on the screen XY (Fig. 9.9) Join S to P'. The line SP' meets the wavefront at O'. O' is the pole of the wave-front

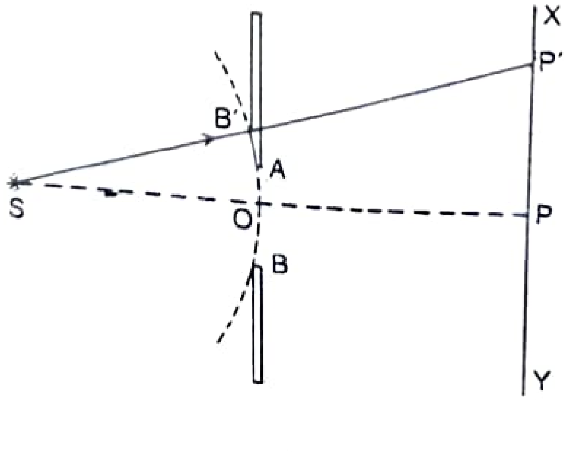


Fig. 9.9

with reference to the point P'. Construct half period zones with the point O' as the pole of the wavefront. The upper half of the wavefront is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O' and A and if only the 3 rd, 4 th and 5 th zones are exposed by the aperture AOB, then the intensity at P' will be maximum. Thus, if odd number of half period zones are exposed, point P' will be of maximum intensity and if even number of zones are exposed, the point P' will be of minimum intensity. As the distance of P' from P increases, the intensity of maxima and minima gradually decreases, because, with the point P' far removed from P, the most effective central half period zones are cut off by the obstacle between the points O' and A. With the outer zones, the obliquity increases with reference to the point P' and hence the intensity of maxima and minima also will be less. If the point P' happens to be of maximum intensity, then all the points lying on a circle of radius PP' on the screen will also be of maximum intensity. Thus, with a circular aperture, the diffraction pattern will be concentric bright and dark rings with the centre P bright or dark depending on the distance b. The width of the rings continuously decreases.

9.9 MATHEMATICAL TREATMENT OF DIFFRACTION AT A CIRCULAR APERTURE

In Fig. 9.10, S is a point source of monochromatic light, AB is the circular aperture and P is a point on the screen. O is the centre of the circular aperture. The line SOP is perpendicular to the circular aperture AB and the screen at P. The screen is perpendicular to the plane of the paper.

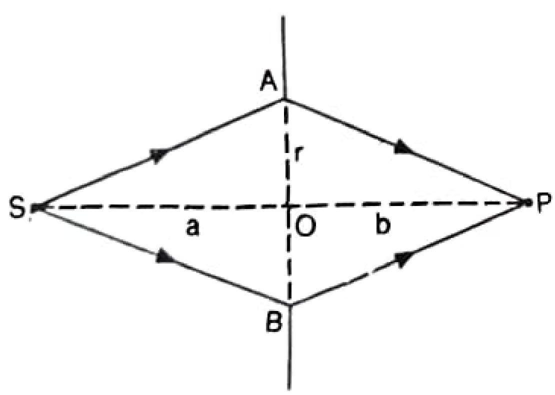


Fig. 9.10

Let δ be the path difference for the waves reaching P along the paths SAP and SOP.

$$SO = a; \quad OP = b; \quad OA = r$$

$$\delta = SA + AP - SOP$$

$$= (a^2 + r^2)^{1/2} + (b^2 + r^2)^{1/2} - (a+b)$$

$$= a\left(1 + \frac{r^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r^2}{b^2}\right)^{1/2} - (a+b)$$

$$= a\left(1+\frac{r^2}{2a^2}\right)+b\left(1+\frac{r^2}{2b^2}\right)-(a+b)$$

$$\delta = \frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$\therefore \qquad \frac{1}{a}+\frac{1}{b} = \frac{2\delta}{r^2} \qquad ...(i)$$

If the position of the screen is such that n full number of half period zones can be constructed on the aperture, then the path difference

$$\delta = \frac{n\lambda}{2} \quad \text{or} \quad 2\delta = n\lambda$$

Substituting this value of 2δ in (*i*)

$$\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2} \qquad ...(ii)$$

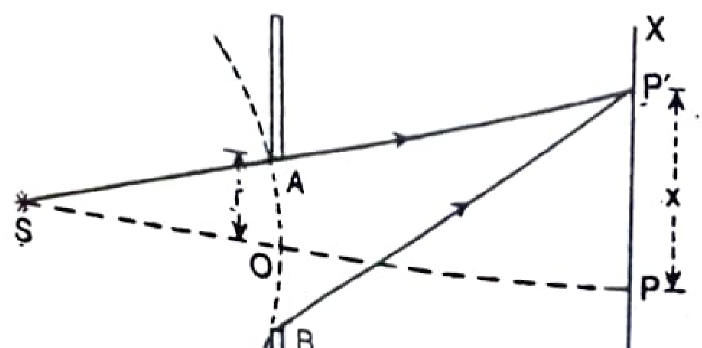
The point P will be of maximum or minimum intensity depending on whether n is odd or even. If the source is at infinite distance (for an incident plane wavefront), then $a = \infty$ and

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2} \qquad ...(iii)$$

If n is odd, P will be a bright point. The idea of focus at P does not mean that if is always a bright point.

9.10 INTENSITY AT A POINT AWAY FROM THE CENTRE

In Fig. 9.11, AB is a circular aperture and P and P' are two points on the screen. $PP' = x$ and $OP = b$. OP is perpendicular to the screen.



9.22 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

To obtain a Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

In Fig. 9.33, S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the

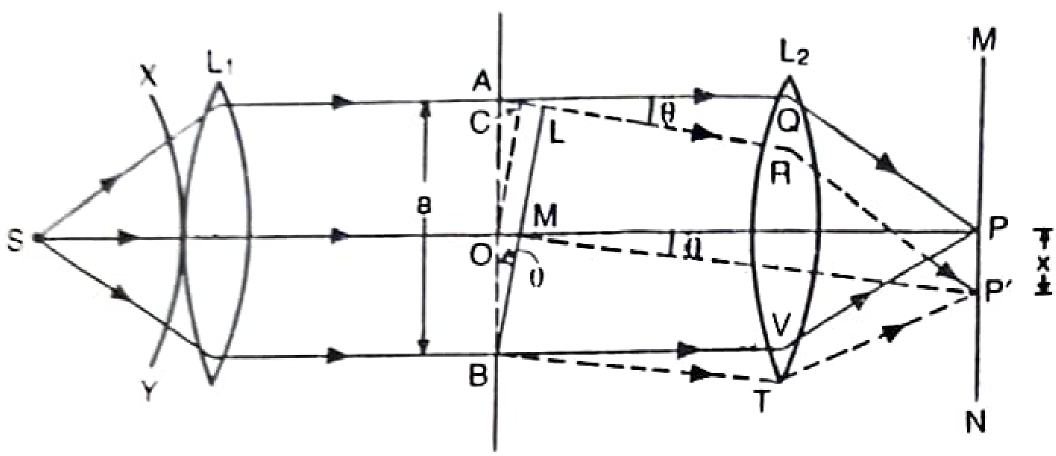


Fig. 9.33

plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wavefront is incident on the slit AB and each point on this wavefront is a source of secondary disturbance. The secondary waves travelling in the direction parallel to OP *viz.* AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the waterfront travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

Now, consider the secondary waves travelling in the direction AR, inclined at an angle θ to the direction OP. All the secondary wave travelling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw OC and BL perpendicular to AR.

Then, in the ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or
$$AL = a\sin\theta$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\frac{\lambda}{2}$. Thus, destructive interference takes place and the point P' will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so, because the secondary waves from the corresponding points of the lower half, differ in path by $\frac{\lambda}{2}$ and this again gives the position of minimum intensity. In general

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

where θ_n gives the direction of the n th minimum. Here n is an integer. If, however, the path difference is odd multiples of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In this case,

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or
$$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

where
$$n = 1, 2, 3 \text{ etc.}$$

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in Fig. 9.34.

P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B

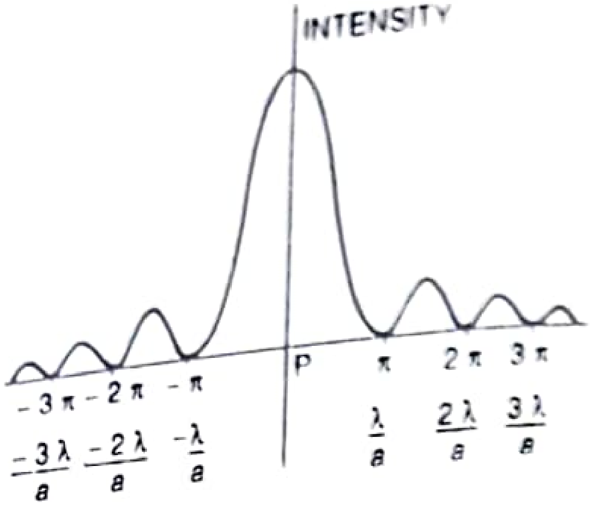


Fig. 9.34

is λ, 2λ etc., correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then

$$\sin\theta = \frac{x}{f}$$

where f is the focal length of the lens L_2

But, $$\sin\theta = \frac{\lambda}{a} \qquad \text{...(ii)}$$

$\therefore$ $$\frac{x}{f} = \frac{\lambda}{a}$$

or $$x = \frac{f\lambda}{a}$$

where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum $= 2x$.

or $$2x = \frac{2f\lambda}{a} \qquad \text{...(iii)}$$

The width of the central maximum is proportional to λ, the wavelength of light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of the central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction bands are coloured. From equation (*ii*), if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results a distinct diffraction maxima and minima on both the sides of P.

Example 9.9. *Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width* 12×10^{-5} cm *when the slit is illuminated by monochromatic light of wavelength* 6000 Å.

Here $$\sin\theta = \frac{\lambda}{a}$$

where θ is half angular width of the central maximum.

$$a = 12 \times 10^{-5} \text{ cm}, \lambda = 6000 \text{ Å} = 6 \times 10^{-5} \text{ cm}.$$

$$\therefore \quad \sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50$$

or $$\theta = \mathbf{30^\circ}$$

Example 9.10. *In Fraunhofer diffraction due to a narrow slit a screen is placed* 2 m *away from the lens to obtain the pattern. If the slit width is 0.2* mm *and the first minima lie* 5 mm *on either side of the central maximum, find the wavelength of light.* [*Delhi (Sub) 1977*]

In the case of Fraunhofer diffraction at a narrow rectangular aperture,

$$a\sin\theta = n\lambda$$

$$n = 1$$

$$\therefore \quad a\sin\theta = \lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \quad \frac{ax}{D} = \lambda$$

$$\lambda = \frac{ax}{D}$$

Here

$$a = 0.2 \text{ mm} = 0.02 \text{ cm}$$

$$x = 5 \text{ mm} = 0.5 \text{ cm}$$

$$D = 2 \text{ m} = 200 \text{ cm}$$

$$\therefore \quad \lambda = \frac{0.02 \times 0.5}{200}$$

$$\lambda = 5 \times 10^{-5} \text{ cm}$$

$$\lambda = \mathbf{5000 \text{ Å}}$$

Example 9.11. *Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.*

The first dark fringe is on either side of the central bright fringe.

Here
$$n = \pm 1,\ D = 2\text{ m}$$
$$\lambda = 6000\text{ Å} = 6 \times 10^{-7}\text{ m}$$
$$\sin\theta = \frac{x}{D}$$
$$a = 0.30\text{ mm} = 3 \times 10^{-4}\text{ m}$$
$$a\sin\theta = n\lambda$$
$$\frac{ax}{D} = n\lambda$$

(*a*)
$$x = \frac{n\lambda D}{a}$$
$$x = \pm\left[\frac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right]$$
$$x = \pm \mathbf{4 \times 10^{-3}\ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

(*b*) The width of the central bright fringe,
$$y = 2x$$
$$= 2 \times 4 \times 10^{-3}$$
$$= \mathbf{8 \times 10^{-3}\ m}$$
$$= \mathbf{8\ mm}$$

Example 9.12. *A single slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2 m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light used.*

(*IAS*, 1990)

In the case of Fraunhofer diffraction at a narrow rectangular slit,
$$a\sin\theta = n\lambda$$

Here θ gives the directions of the minimum
$$n = 2$$
$$\lambda = ?$$

$$a = 0.14 \text{ mm} = 0.14 \times 10^{-3} \text{ m}$$

$$D = 2 \text{ m}$$

$$x = 1.6 \text{ cm} = 1.6 \times 10^{-2} \text{ m}$$

$$\sin\theta = \frac{x}{D} = \frac{n\lambda}{a}$$

$$\therefore \quad \lambda = \frac{xa}{nD}$$

$$= \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2 \times 2}$$

$$= \mathbf{5.6 \times 10^{-7}\ m}$$

$$= \mathbf{5600\ Å}$$

Example 9.13. *A screen is placed 2 m away from a narrow slit which is illuminated with light of wavelength 6000 Å. If the first minimum lies 5 mm on either side of the central maximum, calculate the slit width.*

(Delhi, 1990)

In the case of Fraunhofer diffraction at a narrow slit,

$$a\sin\theta = n\lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \quad \frac{ax}{D} = n\lambda$$

Here, width of the slit $= a = ?$

$$x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$$D = 2 \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

$$n = 1$$

$$a = \left(\frac{n\lambda D}{x}\right)$$

$$a = \left(\frac{1 \times 6 \times 10^{-7} \times 2}{5 \times 10^{-3}}\right)$$

$$a = \mathbf{2.4 \times 10^{-4}\ m}$$

$$a = \mathbf{0.24\ mm}$$

Example 9.14. *Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

(Kanpur, 1990)

Here

$$\sin\theta = \frac{\lambda}{a}$$

where θ is the half angular width of the central maximum

$$a = 12 \times 10^{-5} \text{ cm} = 12 \times 10^{-7} \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

$$\sin\theta = \frac{6 \times 10^{-7}}{12 \times 10^{-7}} = 0.5$$

$$\theta = \mathbf{30^\circ}$$

Angular width of the central maximum,

$$2\theta = \mathbf{60^\circ}$$

Example 9.15. *Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength = 4890 Å.*

[Kanpur, 1991]

For minimum intensity

$$a \sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{x_1}{f}, \quad n = 1$$

$$\frac{x_1}{f} = \frac{\lambda}{a}$$

Here

$$\lambda = 4890 \text{ Å} = 4890 \times 10^{-10} \text{ m}$$

$$a = 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$$

$$f = 40 \text{ cm} = 0.4 \text{ m}$$

$$x_1 = \frac{f\lambda}{a}$$

$$x_1 = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}$$

$$x_1 = \mathbf{3.912 \times 10^{-5} \text{ m}}$$

For secondary maximum

$$a \sin\theta_n = \frac{(2n+1)\lambda}{2}$$

For the first secondary maximum

$$n = 1$$

$$\sin \theta_n = \frac{x_2}{f}$$

$$\frac{x_2}{f} = \frac{3\lambda}{2a}$$

$$x_2 = \frac{3\lambda f}{2a}$$

$$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-3}}$$

$$x_2 = \mathbf{5.868 \times 10^{-5}}\ \text{m}$$

Difference,

$$x_2 - x_1 = 5.868 \times 10^{-5} - 3.912 \times 10^{-5}$$

$$= \mathbf{1.956 \times 10^{-5}\ m}$$

$$= \mathbf{1.596 \times 10^{-2}\ mm}$$

9.23 INTENSITY DISTRIBUTION IN THE DIFFRACTION PATTERN DUE TO A SINGLE SLIT

The intensity variation in the diffraction pattern due to a single slit can be investigated as follows. The incident plane wavefront on the slit *AB* (Fig. 9.33) can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points *A* and *B* is $a \sin \theta$ where a is the width of the slit and $\angle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can be taken to be the same. As one considers the secondary waves in a direction inclined

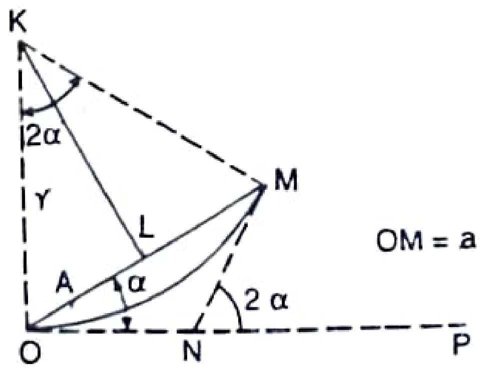


Fig. 9.35

where x is the radius of the Airy's disc. But actually, ... first dark ring is slightly more than that given by equation (v). According to Airy, it is given by

$$x = \frac{1.22 f\lambda}{d} \qquad ...(vi)$$

The discussion of the intensity distribution of the bright and dark rings is similar to the one given for a rectangular slit. With increase in the diameter of the aperture, the radius of the central bright ring decreases.

Example 9.16. *In Fraunhofer diffraction pattern due to a single slit, the screen is at a distance of 100 cm from the slit and the slit is illuminated by monochromatic light of wavelength 5893 Å. The width of the slit is 0.1 mm. Calculate the separation between the central maximum and the first secondary minimum.* *(Mysore)*

For a rectangular slit,

$$x = \frac{f\lambda}{d}$$

Here $f = 100 \text{ cm}, \ \lambda = 5893 \text{ Å}$

$$= 5893 \times 10^{-8} \text{ cm},$$

$$d = 0.1 \text{ mm} = 0.01 \text{ cm}, \ x = ?$$

$$\therefore \quad x = \frac{100 \times 5893 \times 10^{-8}}{0.01} = \mathbf{0.5893 \text{ cm}}$$

9.26 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

In Fig. 9.40, *AB* and *CD* are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and *MN* is a screen perpendicular to the plane of the paper. P is a point on the screen such that *OP* is perpendicular to the screen. Let a plane wavefront be incident on the surface of *XY*. All the secondary waves travelling in a direction parallel to *OP* come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

In this case, the diffraction pattern has to be considered in two parts : (*i*) the interference phenomenon due to the secondary waves emanating from th corresponding points of the two slits and (*ii*) the diffraction pattern due to the secondary waves from the two slits individually. For calculating

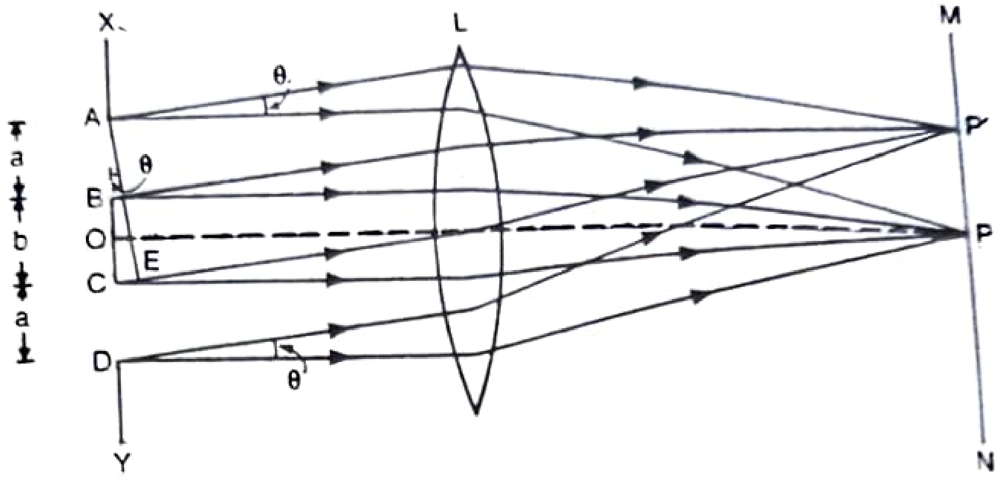


Fig. 9.40

the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

(*i*) **Interference maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle θ with the initial direction.

In the $\Delta\, ACN$ (Fig. 9.41)

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or

$$CN = (a+b)\sin\theta$$

If this path difference is equal to odd multiples of $\frac{\lambda}{2}$, θ gives the direction of minima due to interference of the secondary waves from the two slits.

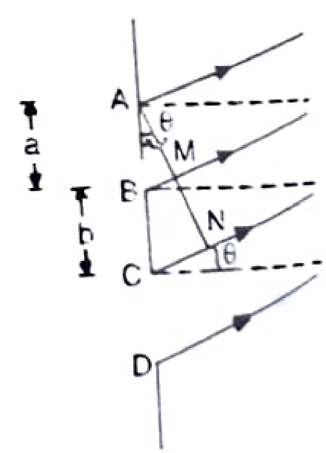


Fig. 9.41

$$\therefore \qquad CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad \text{...}(i)$$

Putting $n = 1, 2, 3$ etc., the values of $\theta_1, \theta_2, \theta_3$ etc., corresponding to the directions of minima can be obtained.

From equation (*i*)

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \qquad ...(ii)$$

On the other hand, if the secondary waves travel in a direction θ' such that the path difference is even multiples of $\frac{\lambda}{2}$, then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore \qquad CN = (a+b)\sin\theta'_n = 2n.\frac{\lambda}{2}$$

or

$$\sin\theta'_n = \frac{n\lambda}{(a+b)} \qquad ...(iii)$$

Putting $n = 1, 2, 3$ etc., the values $\theta'_1, \theta'_2, \theta'_3$ etc., corresponding to the directions of the maxima can be obtained.

From equation (*ii*)

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and

$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \qquad \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a+b} \qquad ...(iv)$$

Thus, the angular separation between any two consecutive minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular separation is inversely proportional to $(a + b)$, the distance between the two slits.

(*ii*) **Diffraction maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle ϕ with the initial direction of the incident light.

If the path difference BM is equal to λ the wavelength of light used, then ϕ will give the direction of diffraction minimum (Fig. 9.41). That is, the path difference between the secondary waves emanating from the extremities of a slit (*i.e.*, points A and B) is equal to λ. Considering the wavefront on AB to be made up of two halves, the path difference between the corresponding points of the upper and the lower halves is equal to $\frac{\lambda}{2}$. The effect at P' due to the wavefront incident on AB is zero. Similarly for the same direction of the secondary waves, the effect at P' due to the wavefront incident on th slit CD is also zero. In general,

$$a \sin \phi_n = n\lambda \qquad ...(v)$$

Putting $n = 1, 2$ etc., the values of ϕ_1, ϕ_2 etc., corresponding to the directions of diffraction minima can be obtained.

9.27 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT CALCULUS METHOD)

The intensity distribution due to Fraunhofer diffraction at double slit (two parallel slits) can be obtained by integrating the expression for dy (vide single slit) for both the slits.

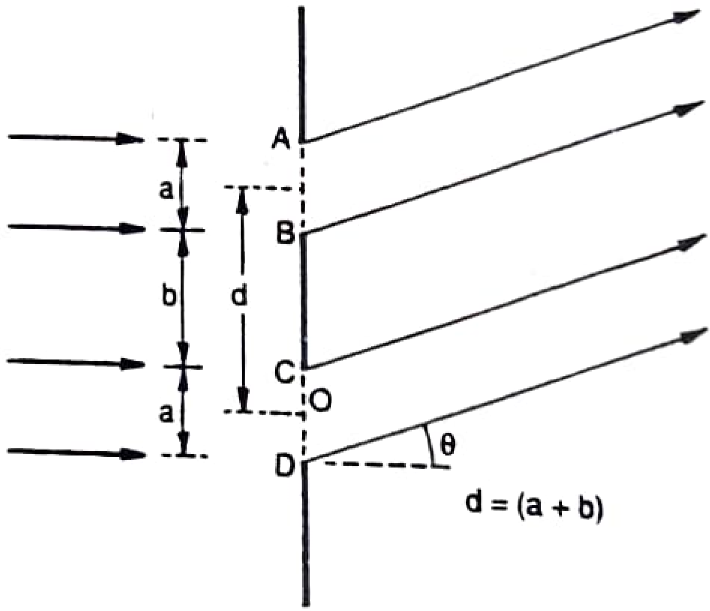


Fig. 9.42

Here

$$y = K\left[\int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right)\right] dz + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right)\right] dz\right]$$

$$\therefore \qquad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right]$$

$$-\frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]_{d-\frac{a}{2}}^{d+\frac{a}{2}}$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$-\frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}+\frac{a\sin\theta}{2\lambda}\right)\right.$$

$$\left.-\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}-\frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$+\frac{K\lambda}{\pi\sin\theta}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)\right]$$

But $\qquad \alpha = \dfrac{\pi a\sin\theta}{\lambda}$

$$\therefore \qquad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right.$$

$$\left.+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\right]$$

$$y = 2\,Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right)\cos\frac{\pi d\sin\theta}{\lambda}$$

Let $\qquad \dfrac{\pi d\sin\theta}{\lambda} = \beta$

$$\therefore \qquad y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta\sin 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right]$$

The intensity at a point P' is given by

$$I = 4K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

$$\therefore \quad I_0 = K^2a^2$$

$$\therefore \quad I = 4I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

The intensity of the central maximum $= 4I_0$ when $\alpha = 0$ and $\beta = 0$

In Fig. 9.43, the dotted curve represents the intensity distribution due to diffraction pattern due to double slit and the thick line curve represents the intensity distribution due to interference between the light from both the slits. The pattern consists of diffraction maxima within each diffraction maximum.

The intensity distribution due to Fraunhofer diffraction at two parallel slits is shown in Fig. 9.43. The full line represents equally spaced interference maxima and minima and the dotted curve represents the diffraction maxima and minima. In the region originally occupied by the

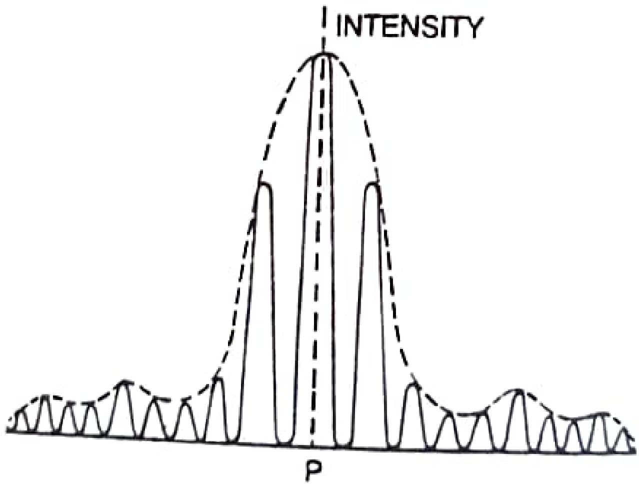


Fig. 9.43

central maximum of the single slit diffraction pattern, equally spaced interference maxima and minima are observed . The intensity of the central interference maximum is four times the intensity of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum gradually decreases. In the region of the secondary maxima due to diffraction at a single slit, equally spaced interference maxima of low intensity are observed. The intensity distribution shown in Fig. 9.43 corresponds to $2a = b$ where a is the width of each slit and b is the opaque spacing between the two slits. Thus, the pattern due to diffraction at a double slit consists of a diffraction pattern due to the individual slits of width a each and the interference maxima and minima of equal spacing. The spacing of the interference maxima and minima is dependent on the values of a and b.

9.28 DISTINCTION BETWEEN SINGLE SLIT AND DOUBLE SLIT DIFFRACTION PATTERNS

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima within the central maximum. The intensity of the central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with an opaque screen, the pattern observed is similar to the one observed with a single slit. The spacing of the diffraction maxima and minima depends on a, the width of the slit and the spacing of the interference maxima and minima depends on the value of a and b where b is the opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

9.29 MISSING ORDERS IN A DOUBLE SLIT DIFFRACTION PATTERN

In the diffraction pattern due to a double slit discussed earlier, the slit width is taken as a and the separation between the slits as b. If the slit width a is kept constant, the diffraction pattern remains the same. Keeping a constant, if the spacing b is altered the spacing between the interference maxima changes. Depending on the relative values of a and b certain orders of interference maxima will be missing in the resultant pattern.

The directions of interference maxima are given by the equation

$$(a + b) \sin \theta = n\lambda \qquad ...(i)$$

The direction of diffraction minima are given by the equation,

$$a \sin \theta = p\lambda \qquad ...(ii)$$

In equations (i) and (ii) n and p are integers. If the values of a and b are such that both the equations are satisfied simultaneously for the same values of θ, then the positions of certain interference maxima correspond to the diffraction minima at the same position on the screen.

Thus, ... tensity is proportional to N^2.

$$I = N^2 I_0 \left(\frac{\sin^2 \alpha}{\alpha^2} \right)$$

These maxima are intense and are called principal maxima.

9.33 PLANE DIFFRACTION GRATING

A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits side by side. The slits are separated by opaque spaces. When a wavefront is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a transmission grating. The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhofer used the first gratings which consisted of a large number of parallel fine wires stretched on a frame. Now, gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point. The space in between any two lines is transparent to light and the lined portion is opaque to light. Such surfaces act as transmission gratings. If, on the other hand, the lines are drawn on a silvered surface (plane or concave) then light is reflected from the positions of the mirror in between any two lines and such surfaces act as reflection gratings.

If the spacing between the lines is of the order of the wave length of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain 10,000 lines per cm. Gratings, with originally ruled surfaces are only few. For practical purposes, replicas of the original grating are prepared. On the original grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then, the film of collodion is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single original ruled surface.

9.34 THEORY OF THE PLANE TRANSMISSION GRATING

In Fig. 9.44, XY is the grating surface and MN is the screen, both perpendicular to the plane of the paper. The slits are all parallel to one another and perpendicular to the plane of the paper. Here AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b. Let a plane wavefront be incident on the grating surface. Then all the secondary waves travelling in the same direction as that of the incident light will come to focus at the point P on the screen. The screen is placed at the focal plane of the collecting lens. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

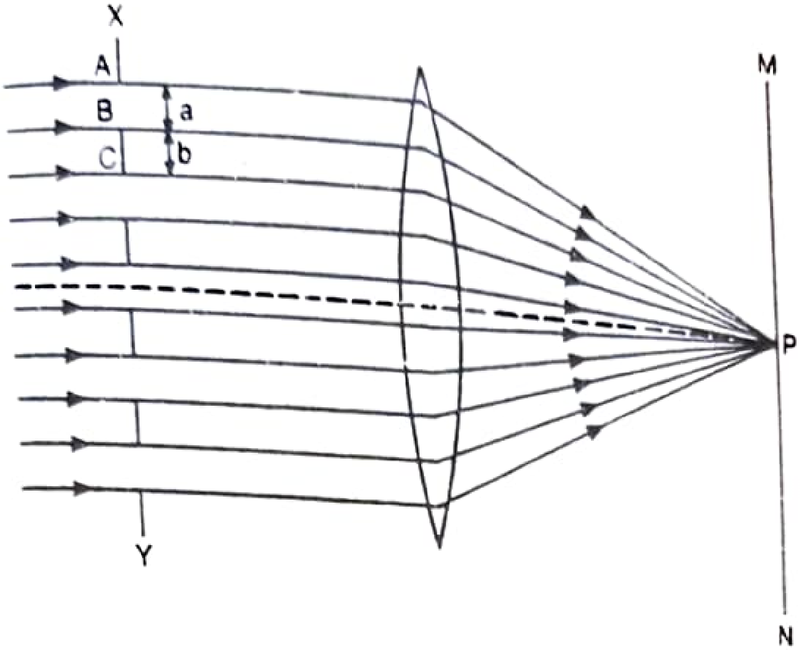


Fig. 9.44

Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light (Fig. 9.45). The collecting lens also is suitably rotated such that the axis of the lens is

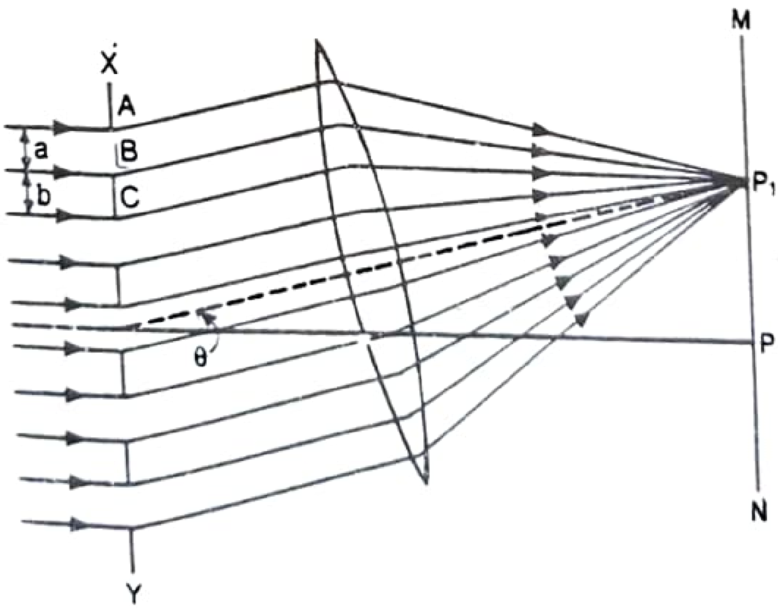


Fig. 9.45

parallel to the direction of the secondary waves. These secondary waves come to focus at the point P_1 on the screen. The intensity at P_1 will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In Fig. 9.45, $AB = a$ and $BC = b$. The path difference between the secondary waves starting from A and C is equal to $AC \sin \theta$. (This will be clear from Fig. 9.41)

But $$AC = AB + BC = a + b$$

$\therefore$ Path difference $= AC \sin \theta$

$$= (a + b) \sin \theta$$

The point P_1 will be of maximum intensity if this path difference is equal to integral multiples of λ where λ is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighouring slits reinforce one another and the angle θ gives the direction of maximum intensity. In general

$$(a + b) \sin \theta_n = n\lambda \quad \text{...(i)}$$

where θ_n is the direction of the n th principal maximum. Putting $n = 1$, 2, 3 etc., the angles $\theta_1, \theta_2, \theta_3$ etc. corresponding to the directions of the principal maxima can be obtained.

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelengths will be different. Let λ and $\lambda + d\lambda$ be two nearby wavelengths present in the incident light and θ and $(\theta + d\theta)$ be the angles of diffraction corresponding to these two wavelengths. Then, for the first order principal maxima

$$(a + b) \sin \theta = \lambda$$

and $$(a + b) \sin (\theta + d\theta) = \lambda + d\lambda$$

Thus, is any order, the number of principal maxima corresponds to the number of wavelengths present. A number of parallel slit images corresponding to the different wavelengths will be observed on the screen. In equation (i), $n = 1$ gives the direction of the first order image, $n = 2$ gives the direction of the second order image and so on. When white light is used , the diffraction pattern on the screen consists of a white central bright maximum and on both the sides of this maximum a spectrum corresponding to the different wavelengths of light present in the incident beam will be observed in each order.

Secondary maxima and minima. The angle of diffracting θ_n corresponding to the direction of the n th principal maximum is given by the equation

$$(a + b) \sin \theta_n = n\lambda$$

In this equation, $(a + b)$ is called the **grating element**. Here a is the width of the slit and b is the width of the opaque portion. For a grating with 15,000 lines per inch the value of

$$(a+b) = \frac{2.54}{15000} \text{ cm}$$

Now, let the angle of diffraction be increased by a small amount $d\theta$ such that the path difference between the secondary waves from the points A and C (Fig. 9.45) increases by $\frac{\lambda}{N}$. Here N is the total number of lines on the grating surface. Then, the path difference between the secondary waves from the extreme points of the grating surface will be $\frac{\lambda}{N} \cdot N = \lambda$. Assuming the whole wavefront to be divided into two halves, the path difference between the corresponding points of the two halves will be $\frac{\lambda}{2}$ and all the secondary waves cancel one another's effect. Thus, $(\theta_n + d\theta)$ will give the direction of the first secondary minimum after the

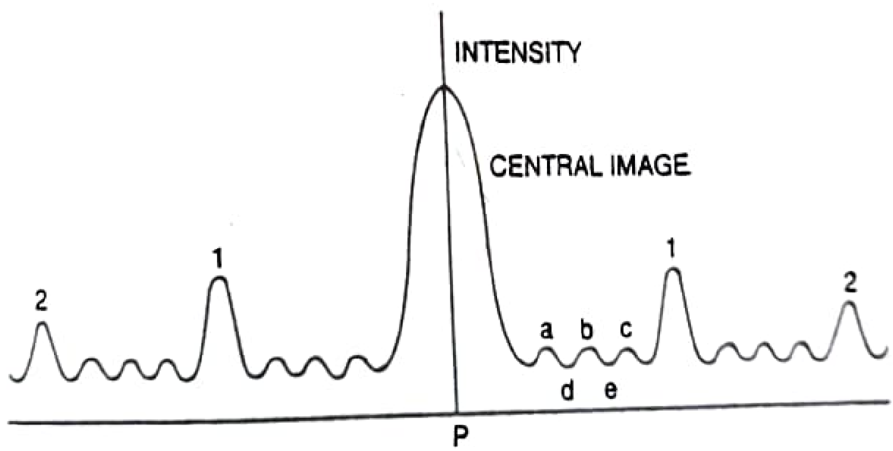


Fig. 9.46

n th primary maximum. Similarly, if the path difference between the secondary waves from the points A and C is $\frac{2\lambda}{N}, \frac{3\lambda}{N}$ etc. for gradually increasing values of $d\theta$, these angles correspond to the directions of 2 nd, 3 rd etc. secondary minima after the n th primary maximum. If the value is $\frac{2\lambda}{N}$, then the path difference between the secondary waves from the extreme points of the grating surface is $\frac{2\lambda}{N} \times N = 2\lambda$ and considering the wavefront to be divided into 4 portions, the concept of the 2 nd secondary minimum can be understood. The number of secondary minima in between any two primary maxima is $N - 1$ and the number of secondary maxima is $N - 2$.

The intensity distribution of the screen is shown in Fig. 9.46. P corresponds to the position of the central maxima and 1, 2 etc. on the two sides of P represent the 1st, 2nd etc. principal maxima. a, b, c etc. are secondary maxima and d, e etc. are the secondary minima. The intensity as well as the angular spacing of the secondary maxima and minima are so small in comparison to the principal maxima that they cannot be observed. It results in uniform darkness between any two principal maxima.

9.35 WIDTH OF PRINCIPAL MAXIMA

The direction of the n th principal maximum is given by

$$(a+b)\sin\theta_n = n\lambda \qquad ...(i)$$

Let $\theta_n + d\theta$ and $\theta_n - d\theta$ give the directions of the first secondary minima on the two sides of the n th primary maxima (Fig. 9.47).

Then.

$$(a+b)\sin[\theta_n \pm d\theta] = n\lambda \pm \frac{\lambda}{N} \qquad ...(ii)$$

where N is the total number of lines on the grating surface.

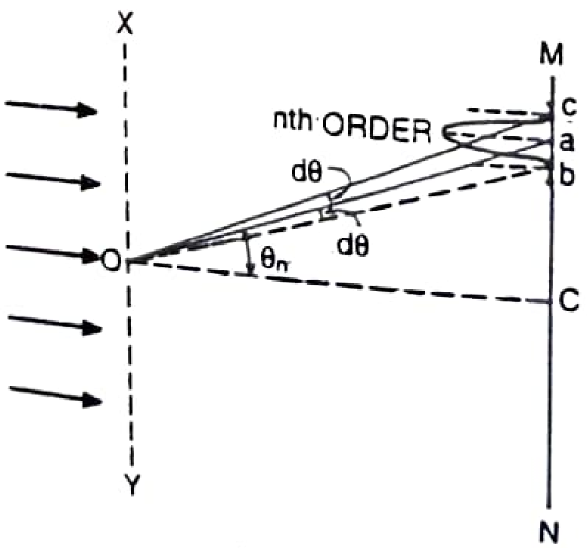


Fig. 9.47

Dividing (ii) by (i)

$$(a+b)\sin(\theta \ldots \lambda$$