AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
Department: Computer Science and Engineering
Program: Bachelor of Science in Computer Science and Engineering
Semester Final Examination: Spring 2019
Year: 1ˢᵗ   Semester: 2ⁿᵈ
Course Number: **CSE1203**
Course Name: **Discrete Mathematics**

Time: 3 (Three) hours   Full Marks: 70

[*There are seven (07) questions carrying a total of 14 marks each. Answer **any five (05)** questions. Marks allotted are indicated in the right margin.*]

1. a) Determine whether $(Q \rightarrow \neg P) \leftrightarrow (Q \wedge P)$ is a contradiction. [5]

    b) Prove using laws of set operations that - [5]

    $$A \cup (A \cap B) = A$$

    c) Illustrate the Pigeonhole Principle including its generalized form. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? [4]

2. a) Translate the following expressions into English, where **bought (x, y)** is "**x bought item y**". [5]

    i) $\exists_x$ bought (Rahim, x)
    ii) $\forall_x$ (bought (Rahim, x) → bought (Karim, x))
    iii) $\forall_x$ (bought (Rahim, x) → $\forall_x$ bought(Karim, x))
    iv) $\forall_x \exists_y$ bought (x, y)
    v) $\exists_x \forall_y$ bought(x, y)

    b) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." [5]

    c) When we call a graph **Eulerian**? Is it possible to solve **Königsberg Bridge Problem** by Euler graph. [4]

3. a) Use mathematical induction to show that for all positive integers n- [5]

    $$n < 2^n$$

Scanned by CamScanner

b) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f? [5]

c) Draw a binary tree that represents the following expression. Also find the prefix and postfix notation of this expression. [4]

$$(((x-7)/3)^{\wedge}y-5)+(x*3)$$

4. a) For the following weighted graph identify the shortest path between **A** and **H**, and also calculate its length by using Dijkstra's Algorithm. [5]
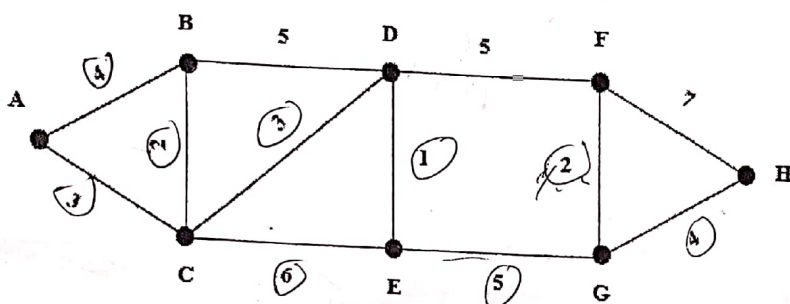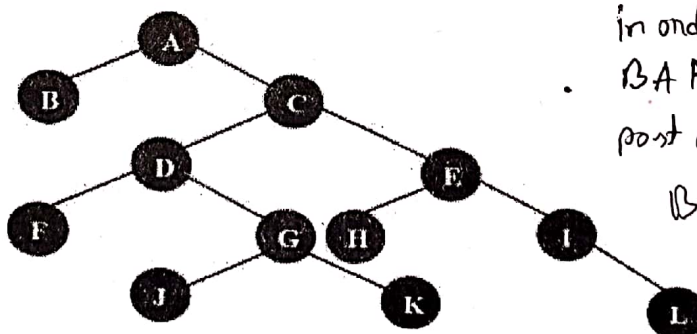

b) What is the value of each of these expressions? [5]
   i) $- * + - 5 \, 3 \, 2 \, 1 \, 4$
   ii) $5 \, 2 \, 1 - - 3 \, 1 \, 4 + + *$

c) Define **Axiom** and **Corollary**. State the **Converse**, **Contrapositive** and **Inverse** of the implication, [4]

   "When I go to the beach, it is necessary that I am free"

5. a) Show the in-order and post-order traversals of the following tree. [5]

In order:
BAFDJGKCHEIL

post order:
BFJKGOHLIE
CA

b) Prove that $\sqrt{2}$ is irrational by using the contradiction method. [5]

c) Show that a full m-ary tree with [4]

    i) N vertices has (N–1) / M internal vertices and [(M - 1) N + 1] / M leaves.

    ii) L leaves has (ML - 1)/(M -1) vertices and (L - 1)/(M-1) internal vertices.

6. a) Find the number of strings of length 8 of letters of the alphabet, with repeated letters [5] allowed,

    i)   that begin with A and end with Z.

    ii)   that begin with A or end with Z.

    iii)   that does not begin with A and does not end with Z.

    iv)   that begin with A but does not end with Z.

b) Assume that the number of bacteria in a colony triples every hour. Set up a recurrence [5] relation for the number of bacteria after **n** hours have elapsed.

c) Explain the Handshaking theorem with an example. [4]

7. a) Design a combinational logic circuit that will detect the prime numbers from the [5] range 0-15.

b) Consider the following relations on a set, {1, 2, 3, 4}. [5]

    $R_1 = \{(a, b) \mid a \leq b\}$

    $R_2 = \{(a, b) \mid a = b+1\}$

    $R_3 = \{(a, b) \mid a > b\}$

    $R_4 = \{(a, b) \mid a + b \leq 3\}$

Determine whether the relations are Reflexive, Symmetric, Anti-symmetric and/or Transitive.

c) Write what you know about Planar Graphs? Show that, $K_{2,5}$ is a Planar Graph, where [4] K_5 is not.