# NATURE OF LIGHT

## 7.1 NEWTON'S CORPUSCULAR THEORY

The branch of optics that deals with the production, emission and propagation of light, its nature and the study of the phenomena of interference, diffraction and polarisation is called physical optics. The basic principles regarding the nature of light were formulated in the latter half of the seventeenth century. Until about this time, the general belief was that light consisted of a stream of particles called corpuscles. These corpuscles were given out by a light source (an electric lamp, a candle, sun etc.) and they travelled in straight lines with large velocities. The originator of the **emission or corpuscular theory** was Sir Isaac Newton. According to this theory, a luminous body continuously emits tiny, light and elastic particles called corpuscles in all directions. These particles or corpuscles are so small that they can readily travel through the interstices of the particles of matter with the velocity of light and they possess the property of reflection from a polished surface or transmission through a transparent medium. When these particles fall on the retina of the eye, they produce the sensation of vision. On the basis of this theory, phenomena like rectilinear propagation, reflection and refraction could be accounted for, satisfactorily. Since the particles are emitted with high speed from a luminous body, they, in the absence of other forces, travel in straight lines according to Newton's second law of motion. This explains rectilinear propagation of light.

## 7.2 REFLECTION OF LIGHT ON CORPUSCULAR THEORY

Let $SS'$ be a reflecting surface and $IM$ the path of a light corpuscle approaching the surface $SS'$. When the corpuscle comes within a very small distance from the surface (indicated by the dotted line $AB$) it, according to the theory, begins to experience a force of repulsion due to the surface (Fig. 7.1).

The velocity $v$ of the corpuscle at $M$ can be resolved into two components $x$ and $y$ parallel and perpendicular to the reflecting surface. The force of repulsion acts perpendicular to the surface $SS'$ and consequently the component $y$ decreases up to $O$ and becomes zero at $O$ the point of incidence on the surface $SS'$. Beyond $O$, the perpendicular component of the velocity increases up to $N$. its magnitude will be again $y$ at $N$ but in the opposite direction. The parallel component $x$ remains the same throughout. Thus at $N$, the corpuscle again possesses two components of velocity $x$ and $y$ and the resultant direction of the corpuscle is along $NR$. The velocity of the corpuscle will be $v$. Between the surfaces $AB$ and $SS'$, the path of the corpuscle is convex to the reflecting surface. Beyond the point $N$, the particle moves unaffected by the presence of the surface $SS'$.

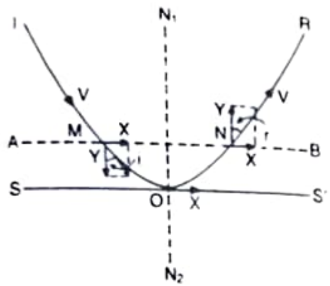



Fig. 7.1

$$x = v \sin i = v \sin r, \qquad \therefore\ i = r$$

Further, the angles between the incident and the reflected paths of the corpuscles with the normals at $M$ and $N$ are equal. Also, the incident and the reflected path of the corpuscle and the normal lie in the same plane viz. the plane of the paper.

## 7.3 REFRACTION OF LIGHT ON CORPUSCULAR THEORY

Newton assumed that when a light corpuscle comes within a very small limiting distance from the refracting surface, it begins to experience a force of attraction towards the surface. Consequently the component of the velocity perpendicular to the surface increases gradually from $AB$ to $A'B'$. $SS'$ is the surface separating the two media (Fig. 7.2). $IM$ is the incident path of the corpuscle travelling in the first medium with a velocity $v$ and incident at an angle $i$. $AB$ to $A'B'$ is a narrow region within which the corpuscle experiences a force of attraction. $NR$ is the refracted path of the corpuscle. Let $v \sin i$ and $v \cos i$ be the components of the velocity of the corpuscle at $M$ parallel and perpendicular to the surface. The velocity parallel to the surface increases by an amount which is independent of the angle of incidence, but which is different for different materials. Let $v$ and $v'$ be the velocity of the corpuscle in the two media and $r$ the angle of refraction in the second medium.



As the parallel component of the velocity remains the same,

$$v \sin i = v' \sin r$$

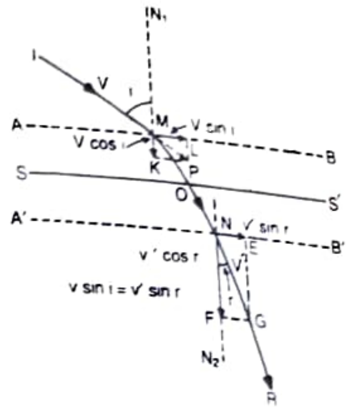



Fig. 7.2

or

$$\frac{\sin i}{\sin r} = \frac{v'}{v}$$

$$= \frac{\text{velocity of light in the second medium}}{\text{velocity of light in the first medium}}$$

$= {}_1\mu_2$ (refractive index of the second medium with reference to the first medium)

Thus, the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction. This is the well known Snell's law of refraction. If $i > r$, then $v' > v$. i.e., the velocity of lignt in a denser medium like water or glass is greater than that in a rarer medium such as air.

But the results of Foucault and Michelson on the velocity of light show that the velocity of light in a denser medium is less than that in a rarer medium. Newton's corpuscular theory is thus untenable. This is not the only ground on which Newton's theory is invalid. In the year 1800. Young discovered the phenomenon of interference of light. He experimentally demonstrated that under certain conditions, light when added to light produces darkness. The phenomena belonging to this class cannot be explained, if following Newton, it is supposed that light consists of material particles. Two corpuscles coming together cannot destroy each other

Another case considered by Newton was that of simultaneous reflection and refraction. To explain this he assumed that the particles had fits so that some were in a state favourable to reflection and others were in a condition suitable for transmission. No explanation of interference, diffraction and polarization was attempted because very little was known about these phenomena at the time of Newton. Further, the corpuscular theory has not given any plausible explanation about the origin of the force of repulsion or attraction in a direction normal to the surface.

## 7.4 ORIGIN OF WAVE THEORY

The test and completeness of any theory consists in its ability to explain the known experimental facts, with minimum number of hypotheses. From this point of view, the corpuscular theory is above all prejudices and with its halp rectilinear propagation, reflection and refraction could be explained.

By about the middle of the seventeenth century, while the corpuscular theory was accepted, the idea that light might be some sort of wave motion had begun to gain ground. In 1679, Christian Huygens proposed the wave theory of light. According to this, a luminous body is a source of disturbance in a hypothetical medium called ether. This medium pervades all is repea ... changes

## 7.12 HUYGENS PRINCIPLE

According to Huygens, a source of light sends out waves in all directions, through a hypothetical medium called ether. In Fig. 7.12 (*i*), *S* is a source of light sending light energy in the form of waves in all directions. After any given interval of time ($t$), all the particles of the medium on the surface $XY$ will be vibrating in phase. Thus, $XY$ is a portion of the sphere of radius $vt$ and centre $S$. $v$ *is the velocity of propagation of the wave.* $XY$ is called the primary wavefront. A wavefront can be defined as the locus of all the points of the medium which are vibrating in phase and are also displaced at the

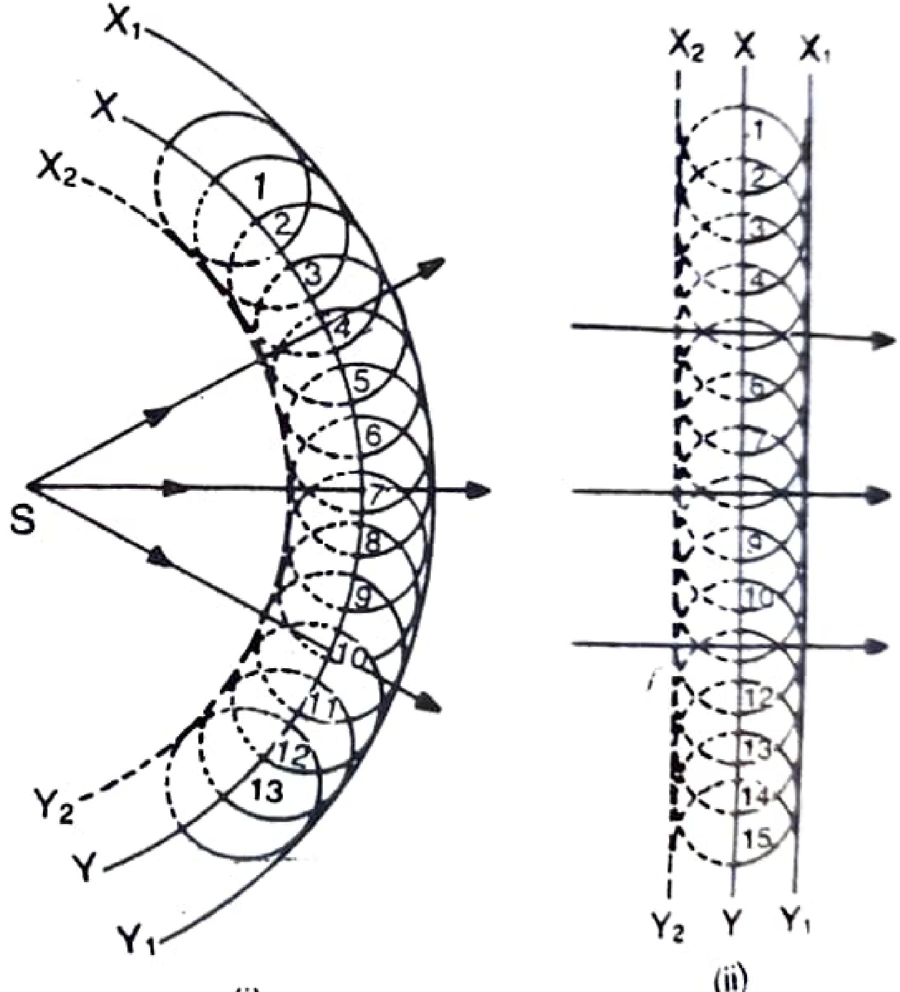



Fig. 7.12

same time. If the distance of the source is small [Fig. 7.12 (*i*)] the wavefront is spherical. When the source is at a large distance, then any small portion of the wavefront can be considered plane [Fig. 7.12 (*ii*)]. Thus rays of light diverging from or converging to a point give rise to a spherical wavefront and a parallel beam of light gives rise to a plane wave front.

According to Huygens principle, all points on the primary wavefront (1, 2, 3 etc., Fig. 7.12) are sources of secondary disturbance. These secondary waves travel through space with the same velocity as the original wave and the envelope of all the secondary wavelets after any given interval of time gives rise to the secondary wavefront. In Fig. 7.12 (*i*), $XY$ is the primary spherical wavefront and in Fig. 7.12 (*ii*) $XY$ is the primary plane wavefront. After an interval of time $t'$, the secondary waves travel a distance $vt'$. With the points 1, 2, 3 etc. as centres, draw spheres of radii $vt'$. The surfaces $X_1Y_1$ and $X_2Y_2$ refer to the secondary wavefront. $X_1Y_1$ is the forward wavefront and $X_2Y_2$ is the backward wavefront. But according to Huygens principle, the secondary wavefront is confined only to the forward wavefront $X_1Y_1$ and not the backward wavefront $X_2Y_2$. However, no explanation to the absence of backward wavefront was given by Huygens.

## 7.13 REFLECTION OF A PLANE WAVE FRONT AT A PLANE SURFACE

Let $XY$ be a plane reflecting surface and $AMB$ the incident plane wavefront. All the particles on $AB$ will be vibrating in phase. Let $i$ be the angle of incidence (Fig. 7.13).

In the time the disturbance at $A$ reaches $C$, the secondary waves from the point $B$ must have travelled a distance $BD$ equal to $AC$. With the point $B$ as centre and radius equal to $AC$ construct a sphere. From the point $C$, draw tangents $CD$ and $CD'$. Then $BD = BD'$.

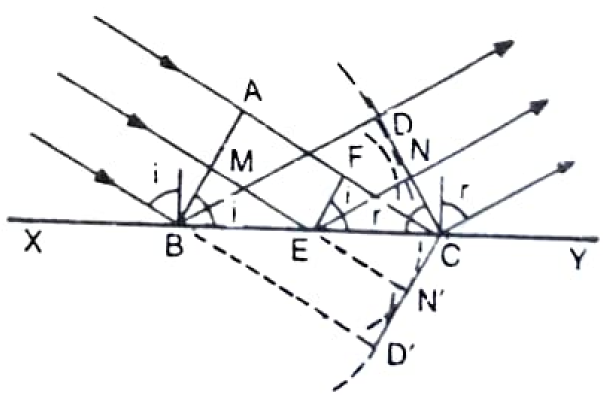


Fig. 7.13

In the $\Delta$ s $BAC$ and $BDC$

$$BC \text{ is common}$$

$$BD = AC$$

and
$$\angle BAC = \angle BDC = 90^\circ$$

$\therefore$ The two triangles are congruent,

$$\therefore \quad \angle ABC = i = \angle BCD = r.$$

$$\therefore \quad i = r$$

Thus, the angle of incidence is equal to the angle of reflection. Hence, $CD$ forms the reflected plane wavefront. It can be shown that all the points on $CD$ form the reflected plane wavefront. In the time the disturbance from $F$ reaches the point $C$, the secondary waves from $E$ must have travelled a distance $EN = FC$. With $E$ as centre and radius $FC$ draw a sphere and draw tangents $CN$ and $CN'$ to the sphere. It can be shown that the triangles $EFC$ and $ENC$ are congruent.

$$AC = AF + FC$$

But
$$AF = ME$$

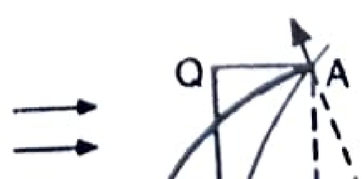
and
$$FC = EN$$

$$\therefore \quad AC = ME + EN$$

Thus, all the secondary waves from different points on $AB$ reach the corresponding points on $CD$ at the same time. Therefore, $CD$ forms the reflected plane wavefront and also the angle of incidence is equal to the angle of reflection.

## 7.14 REFLECTION OF A PLANE WAVEFRONT AT A SPHERICAL SURFACE

Let $APB$ be a convex reflecting surface and $QPR$ the incident plane wavefront (Fig. 7.14). By the time the disturbance at $Q$ and $R$ reaches the points $A$ and $B$ on the reflecting surface, the secondary waves from $P$ must have travelled a distance $PK$ back into



# 8

# *INTERFERENCE*

## 8.1 INTRODUCTION

The phenomenon of interference of light has proved the validity of the wave theory of light. Thomas Young successfully demonstrated his experiment on interference of light in 1802. When two or more wave trains act simultaneously on any particle in a medium, the displacement

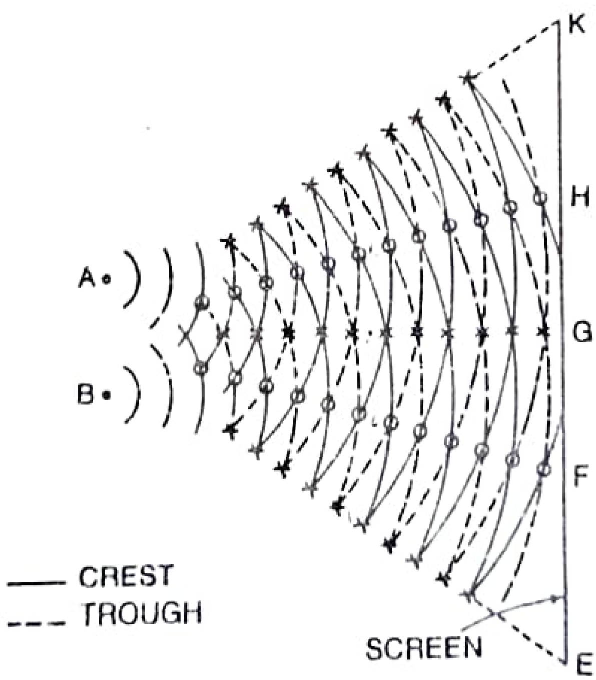



Fig. 8.1

of the particle at any instant is due to the superposition of all the wave trains. Also, after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent. This principle was explained by Huygens in 1678.

The phenomenon of interference of light is due to the superposition of two trains within the region of cross over. Let us consider the waves produced on the surface of water. In Fig. 8.1 points $A$ and $B$ are the two sources which produce waves of equal amplitude and constant phase difference. Waves spread out on the surface of water which are circular in shape. At any instant, the particle will be under the action of the displacement due to both the waves. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The points shown by crosses in the diagram will have maximum displacement because, either the crest of one will combine with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of either of the waves. Therefore, at these points the waves **reinforce** with each other. As the intensity (energy) is directly proportional to the square of the amplitude ($I \propto A^2$) the intensity at these points is four times the intensity due to one wave. It should be remembered that **there is no loss of energy due to interference.** The energy is only transferred from the points of minimum displacement to the points of maximum displacement.

## 8.2 YOUNG'S EXPERIMENT

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole $S$ and then at some distance away on two pinholes $A$ and $B$ (Fig. 8.2).

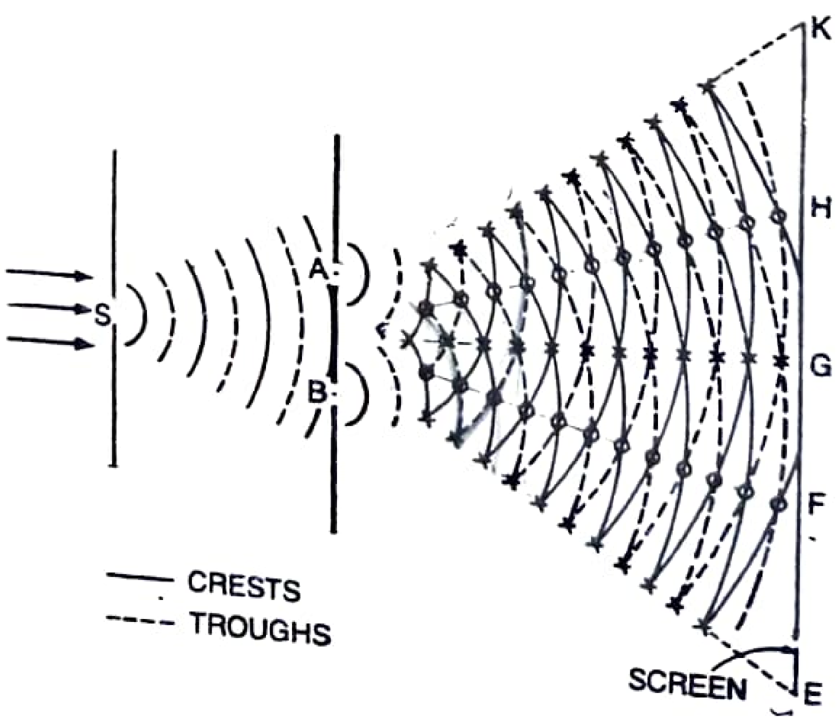



Fig. 8.2

*A* and *B* are equidistant from *S* and are close to each other. Spherical waves spread out from *S*. Spherical waves also spread out from *A* and *B*. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as *E* are bright because the crest due to one wave coincides with the crest due to the other and therefore they reinforce with each other. The points such as *F* are dark because the crest of one falls on the trough of the other and they neutralize the effect of each other. Points, similar to *E*, where the trough of one falls on the trough of the other, are also bright because the two waves reinforce.

It is not possible to show interference due to two independent sources of light, because a large number of difficulties are involved. The two sources may emit light waves of largely different amplitude and wavelength and the phase difference between the two may change with time.

## 8.3 COHERENT SOURCES

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. Methods have been devised where (*i*) interference of light takes place between the waves from the real source and a virtual source (*ii*) interference of light takes place between waves from two sources formed due to a single source. In all such cases, the two sources will act, as if they are perfectly similar in all respects.

Since the wavelength of light waves is extremely small (of the order of $10^{-5}$ cm), the two sources must be narrow and must also be close to each other. Maximum intensity is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of $2\pi$ or the path difference between the two waves is a whole number multiple of wavelength. For minimum intensity at a point, the phase difference between the two waves reaching the point should be an odd number multiple of $\pi$ or the path difference between the two waves should be an odd number multiple of half wavelength.

## 8.4 PHASE DIFFERENCE AND PATH DIFFERENCE

If the path difference between the two waves is $\lambda$, the phase difference $= 2\pi$.

Suppose for a path difference $x$, the phase difference is $\delta$

For a path difference $\lambda$, the phase difference $= 2\pi$

$\therefore$ For a path difference $x$, the phase difference $= \frac{2\pi x}{\lambda}$

Phase difference $\quad \delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times (\text{pathdifference})$

## 8.5 ANALYTICAL TREATMENT OF INTERFERENCE

Consider a monochromatic source of light $S$ emitting waves of wavelength $\lambda$ and two narrow pinholes $A$ and $B$ (Fig. 8.3). $A$ and $B$ are equidistant from $S$ and act as two virtual coherent sources. Let $a$ be the amplitude of the waves. The phase difference between the two waves reaching the point $P$, at any instant, is $\delta$.

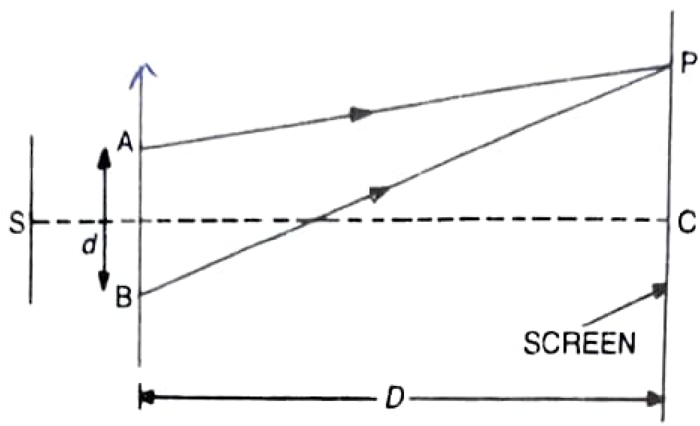



Fig. 8.3

If $y_1$ and $y_2$ are the displacements

$$y_1 = a \sin \omega t$$

$$y_2 = a \sin (\omega t + \delta)$$

$$\therefore \qquad y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$$

$$y = a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$

$$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta.$$

Taking $\quad a(1 + \cos \delta) = R \cos \theta \qquad \text{...}(i)$

and $\quad a \sin \delta = R \sin \theta \qquad \text{...}(ii)$

$$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

$$y = R \sin (\omega t + \theta) \qquad \text{...}(iii)$$

which represents the equation of simple harmonic vibration of amplitude $R$.

Squaring $(i)$ and $(ii)$ and adding,

$$R^2 \sin^2 \theta + R^2 \cos^2 \theta = a^2 \sin^2 \delta + a^2 (1 + \cos \delta)^2$$

or
$$R^2 = a^2 \sin^2 \delta + a^2 (1 + \cos^2 \delta + 2 \cos \delta)$$
$$R^2 = a^2 \sin^2 \delta + a^2 + a^2 \cos^2 \delta + 2 a^2 \cos \delta$$
$$= 2a^2 + 2a^2 \cos \delta = 2a^2 (1 + \cos \delta)$$
$$R^2 = 2a^2 . 2 \cos^2 \frac{\delta}{2} = 4a^2 \cos^2 \frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$$\therefore \qquad I = R^2$$

or
$$I = 4a^2 \cos^2 \frac{\delta}{2} \qquad ...(iv)$$

**Special cases :** (*i*) When the phase difference $\delta = 0, 2\pi, 2(2\pi), \ldots n(2\pi)$, or the path difference $x = 0, \lambda, 2\lambda, \ldots n\lambda$.

$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of $2\pi$ or the path difference is a whole number multiple of wave-length.

(*ii*) When the phase difference, $\delta = \pi, 3\pi, \ldots (2n + 1)\pi$, or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (2n + 1)\frac{\lambda}{2}$,

$$I = 0$$

Intensity is minimum when the path difference is an odd number multiple of half wavelength.

(*iii*) **Energy distribution**. From equation (*iv*), it is fount that the intensity at bright points is $4a^2$ and at dark points it is zero. According to

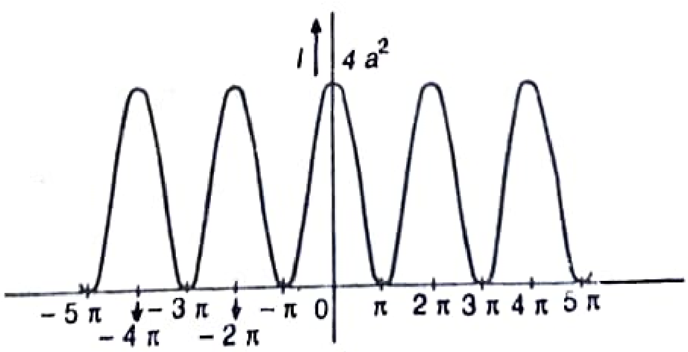



Fig. 8.4

the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. For, at bright points, the intensity due to the two waves should be $2a^2$ but actually it is $4a^2$. As shown in Fig. 8.4 the intensity varies from 0 to $4a^2$, and the average is still $2a^2$. It is equal to the uniform intensity $2a^2$ which will be present in the absence of the interference phenomenon due to the two waves. Therefore, the formation of interference fringes is in accordance with the law of conservation of energy.

## 8.6 THEORY OF INTERFERENCE FRINGES

Consider a narrow monochromatic source $S$ and two pinholes $A$ and $B$, equidistant from $S$. $A$ and $B$ act as two coherent sources separated by a distance $d$. Let a screen be placed at a distance $D$ from the coherent

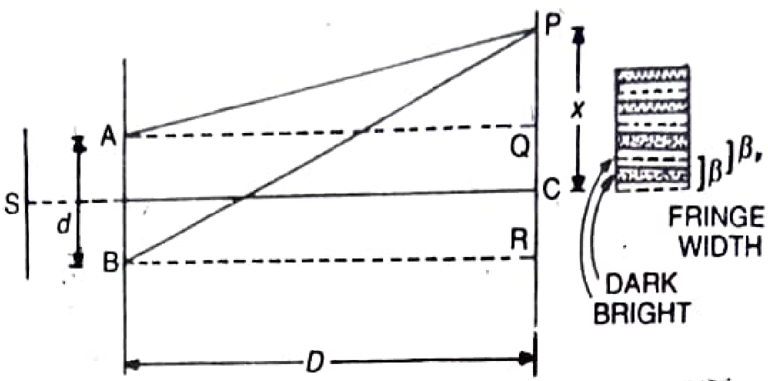



Fig. 8.5

sources. The point $C$ on the screen is equidistant from $A$ and $B$. Therefore, the path difference between the two waves is zero. Thus, the point $C$ has maximum intensity.

Consider a point $P$ at a distance $x$ from $C$. The waves reach at the point $P$ from $A$ and $B$.

Here, $$PQ = x - \frac{d}{2}, \qquad PR = x + \frac{d}{2}$$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(BP)^2 - (AP)^2 = 2xd$$

$$BP - AP = \frac{2xd}{BP + AP}$$

But $$BP = AP = D \quad \text{(approximately)}$$

$$\therefore \text{ Path difference } = BP - AP = \frac{2xd}{2D} = \frac{xd}{D} \qquad ...(i)$$

$$\text{Phase difference } = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) \qquad ...(ii)$$

(*i*) **Bright fringes**. If the path difference is a whole number multiple of wavelength $\lambda$, the point $P$ is bright.

$$\therefore \qquad \frac{xd}{D} = n\lambda$$

where $\qquad n = 0, 1, 2, 3 \ldots$

or $$x = \frac{n\lambda D}{d} \qquad \ldots(iii)$$

This equation gives the distances of the bright fringes from the point $C$. At $C$, the path difference is zero and a bright fringe is formed.

When $\qquad n = 1, \quad x_1 = \frac{\lambda D}{d}$

$$n = 2, \quad x_2 = \frac{2\lambda D}{d}$$

$$n = 3, \quad x_3 = \frac{3\lambda D}{d}$$

$$x_n = \frac{n\lambda D}{d}$$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d} \qquad \ldots(iv)$$

(*ii*) **Dark fringes**. If the path difference is an odd number multiple of half wavelength, the point $P$ is dark.

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2} \quad \text{where } n = 0, 1, 2, 3 \ldots$$

or $$x = \frac{(2n+1)\lambda D}{2d} \qquad \ldots(v)$$

This equation gives the distances of the dark fringes from the point $C$.

When, $\qquad n = 0, \quad x_0 = \frac{\lambda D}{2d}$

$$n = 1, \quad x_1 = \frac{3\lambda D}{2d}$$

$$n = 2, \quad x_2 = \frac{5\lambda D}{2d}$$

and $$x_n = \frac{(2n+1)\lambda D}{2d}$$

The distance between any two consecutive dark fringes,

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \qquad ...(vi)$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of $C$. Moreover, from equations ($v$) and ($vi$), it is clear that the width of the bright fringe is equal to the width of the dark fringe. All the fringes are equal in width and are independent of the order of the fringe. The breadth of a bright or a dark fringe is, however, equal to half the fringe width and is equal to $\frac{\lambda D}{2d}$. The fringe width $\boxed{\beta = \frac{\lambda D}{d}}$

Therefore, ($i$) the width of the fringe is directly proportional to the wavelength of light, $\beta \propto \lambda$. ($ii$) The width of the fringe is directly proportional to the distance of the screen from the two sources, $\beta \propto D$. ($iii$) the width of the fringe is inversely proportional to the distance between the two sources, $\beta \propto \frac{1}{d}$. Thus, the width of the fringe increases ($a$) with increase in wavelength ($b$) with increase in the distance $D$ and ($c$) by bringing the two sources $A$ and $B$ close to each other.

**Example 8.1.** *Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.*

[Delhi B.Sc. (Hons.)]

$x = \frac{n\lambda D}{d}$

$$\beta = \frac{\lambda D}{d}$$

Here

$$\lambda = 5100 \times 10^{-8} \text{ cm}, \qquad d = ?$$

$$D = 200 \text{ cm} \qquad x = 2 \text{ cm.}$$

$$10\,\beta = 2 \text{ cm}$$

or

$$\beta = 0.2 \text{ cm}$$

$$d = \frac{\lambda D}{\beta}$$

$$d = \frac{5100 \times 10^{-8} \times 200}{0.2}$$

$$d = \mathbf{0.051\ cm}$$

$x = \frac{n\lambda D}{d}$, $d = \frac{n\lambda D}{x}$

**Example 8.2.** *Two coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.*

Here, $D = 80$ cm, $d = 0.18$ mm $= 0.018$ cm

$n = 4$, $x = 10.8$ mm $= 1.08$ cm, $\lambda = ?$

$n\beta = x$

$n \times \frac{\lambda D}{d} = x$

$$x = \frac{n\lambda D}{d}$$

or

$$\lambda = \frac{xd}{nD} = \frac{1.08 \times 0.018}{4 \times 80} = 6075 \times 10^{-8} \text{ cm}$$

$$= \mathbf{6075\ \text{Å}}$$

**Example 8.3.** *In Young's double slit experiment the separation of the slits is 1.9 mm and the fringe spacing is 0.31 mm at a distance of 1 metre from the slits. Calculate the wavelength of light.*

Here

$$\beta = 0.31 \text{ mm} = 0.031 \text{ cm}$$

$$d = 1.9 \text{ mm} = 0.19 \text{ cm}$$

$$D = 1 \text{ m} = 100 \text{ cm}$$

$$\beta = \frac{\lambda D}{d}$$

or

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.031 \times 0.19}{100}$$

$$\lambda = 5890 \times 10^{-8} \text{ cm} = \mathbf{5890\ \text{Å}}$$

$\beta = 0.31$ mm

$d = 1.9$ mm $= 0.19$ cm

$D = 1$ m $= 100$ cm

$\beta = \frac{\lambda D}{d}$

$\lambda = 1.9 \times .31$

**Example 8.4.** *Two straight and narrow parallel slits 1 mm apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of 100 cm from the slits are 0.50 mm apart. What is the wavelength of light ?* [Delhi 1977]

Here

$$\beta = 0.50 \text{ mm} = 0.05 \text{ cm}$$

$$d = 1 \text{ mm} = 0.1 \text{ cm}$$

$$D = 100 \text{ cm}$$

$$\beta = \frac{\lambda D}{d}$$

or

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.05 \times 0.1}{100}$$

$$\lambda = 5 \times 10^{-5} \text{ cm}$$

$$\lambda = \mathbf{5000\ \text{Å}}$$

$d = 1$ mm $= .1$ cm

$D = 100$ cm

$\beta = 0.5$

**Example 8.5**. *A Young's double slit experiment is arranged such that the distance between the centers of the two slits is d and the source slit, emitting light of wavelength λ, is placed at a distance x from the double slit. If now the source slit is gradually opened up, for what width will the fringes first disappear ?* [Delhi (Hons) 1992]

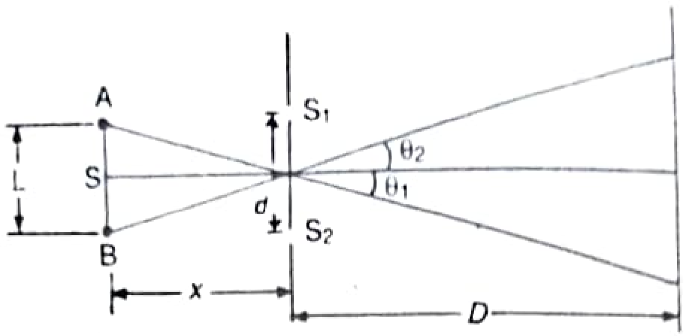



Fig. 8.6

$A$ and $B$ are two extreme points of the source $S$ separated by distance $L$.

Here $$\theta_1 = -\left(\frac{L}{2x}\right) \quad \text{when } x >> L$$

$$\theta_2 = \left(\frac{L}{2x}\right)$$

The fringe pattern first disappears when the central maximum of one pattern overlaps on the first minimum of the second pattern. The first minimum occurs at a distance given by

$$y = \pm\frac{\lambda D}{2d}$$

Also $$\frac{y}{D} = \theta = \pm\frac{\lambda}{2d}$$

For source $A$, these minima occur at an angle

$$\theta_1 \pm \frac{\lambda}{2d}$$

The fringe width is very large when $d$ is very small. As $d$ increases, the first minimum of $S_1$, moves towards the zeroeth maximum of $S_2$. These two meet when $d = d_0$

Here $$\theta_2 = \theta_1 + \frac{\lambda}{2d_0}$$

or $$\frac{L}{2x} = -\frac{L}{2x} + \frac{\lambda}{2d_0}$$

$$d_0 = \left(\frac{\lambda x}{2L}\right)$$

$$L = \left[\frac{\lambda x}{2\, d_0}\right]$$

**Example 8.6.** *A light source emits light of two wavelengths* $\lambda_1 = 4300$ Å and $\lambda_2 = 5100$ Å. *The source is used in a double slit interference experiment. The distance between the sources and the screen is 1.5* m *and the distance between the slits is 0.025* mm. *Calculate the separation between the third order bright fringes due to these two wavelengths.*

Here

$nB = x$

$\frac{nD}{d} = x$

$$D = 1.5\,\text{m}$$

$$d = 0.025 \text{ mm} = 25 \times 10^{-6} \text{ m}$$

$$\lambda_1 = 4300 \text{ Å} = 4.3 \times 10^{-7} \text{ m}$$

$$\lambda_2 = 5100 \text{ Å} = 5.1 \times 10^{-7} \text{ m}$$

$$n = 3$$

$$x_1 = \frac{n \lambda_1 D}{d}$$

$$x_2 = \frac{n \lambda_2 D}{d}$$

$$x_2 - x_1 = \left(\frac{n \lambda_2 D}{d}\right) - \left(\frac{n \lambda_1 D}{d}\right)$$

$$= \frac{nD}{d}\left[\lambda_2 - \lambda_1\right]$$

$$= \left(\frac{3 \times 1.5}{25 \times 10^{-6}}\right)\left[5.1 \times 10^{-7} - 4.3 \times 10^{-7}\right]$$

$$= \mathbf{0.0144 \text{ m}}$$

$$= \mathbf{1.44 \text{ cm}}$$

Hence, the separation between the two fringes is 1.44 cm.

**Example 8.7.** *Two coherent sources of monochromatic light of wavelength 6000* Å *produce an interference pattern on a screen kept at a distance of 1* m *from them. The distance between two consecutive bright fringes on the screen is 0.5* mm. *Find the distance between the two coherent sources.* [IAS]

Here

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

$$D = 1 \text{ m}$$

$$\beta = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$$

$$d = ?$$

$$\beta = \frac{\lambda D}{d}$$

$$d = \frac{\lambda D}{\beta}$$

$$d = \frac{6 \times 10^{-7} \times 1}{5 \times 10^{-4}}$$

$$d = \mathbf{1.2 \times 10^{-3}\ m}$$

$$d = \mathbf{1.2\ mm}$$

**Example 8.8.** *Light of wavelength 5500 Å from a narrow slit is incident on a double slit. The overall separation of 5 fringes on a screen 200 cm away is 1 cm, calculate (a) the slit separation and (b) the fringe width.*

D    2 β

d

Here $$x = \frac{n \lambda D}{d}$$

$$n = 5$$

$$D = 200 \text{ cm} = 2 \text{ m}$$

$$\lambda = 5500 \text{ Å} = 5.5 \times 10^{-7} \text{ m}$$

$$x = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$d = ?$$

(*a*) $$d = \frac{n \lambda D}{x}$$

$$d = \frac{5 \times 5.5 \times 10^{-7} \times 2}{10^{-2}}$$

$$d = \mathbf{5.5 \times 10^{-4}\ m}$$

$$d = \mathbf{0.055\ cm}$$

(*b*) $$\beta = \frac{x}{n}$$

$$\beta = \frac{1}{5} \text{ cm}$$

$$\beta = \mathbf{0.2\ cm}$$

## 8.7 FRESNEL'S MIRRORS

Fresnel produced the interference fringes by using two plane mirrors $M_1$ and $M_2$ arranged at an angle of nearly 180° so that their surfaces are nearly (not exactly) coplanar (Fig. 8.7).

A monochromatic source of light $S$ is used. The pencil of light from $S$ incident on the two mirrors, after reflection, appears to come from two virtual sources $A$ and $B$ at some distance $d$ apart. Therefore, $A$ and $B$ act

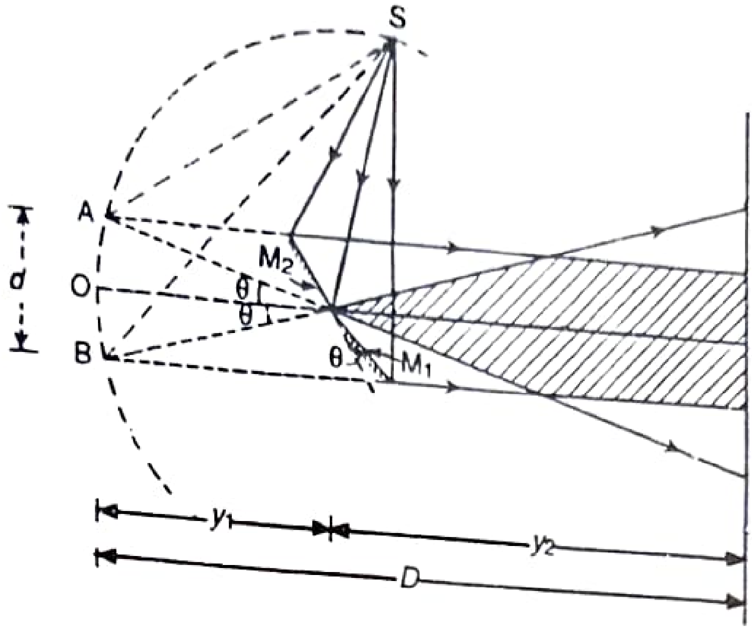



Fig. 8.7

as two virtual coherent sources and interference fringes are obtained on the screen. These fringes are of equal width and are alternately dark and bright.

**Theory**. $A$ and $B$ are two coherent sources at a distance $d$ apart. The screen is at a distance $D$ from the virtual sources. The two reflected beams from the mirrors $M_1$ and $M_2$ overlap between $E$ and $F$ (shown as shaded in the diagram) and interference fringes are formed.

(For complete theory read Article 8.6)

Here, $D = Y_1 + Y_2$

Fringe width $\beta = \dfrac{\lambda D}{d}$

A point on the screen will be at the centre of a bright fringe, if its distance from $C$ is $\dfrac{n\lambda D}{d}$ where $n = 0, 1, 2, 3$ *etc*, and it will be at the centre of a dark fringe, if its distance from $C$ is

$$\frac{(2n+1)\lambda D}{2d}$$

where $n = 0, 1, 2, 3, \ldots$ *etc.*

For the fringes to be formed, the following conditions must be satisfied. The two mirrors $M_1$ and $M_2$ should be made from optically flat glass and silvered on the front surfaces. No reflection should take place from the back of the mirrors. The polishing should extend up to the line of intersection of the two mirrors and the line of intersection must be parallel to the line source (slit).

The distance between the two virtual sources $A$ and $B$ can be calculated as follows. Suppose the distance between the points of intersection of the mirrors and the source $S$ is $Y_1$.

$\theta$ is known. The angle of separation between $A$ and $B$ is $2\theta$.

$$\therefore \qquad d = 2\theta\, y_1$$

When white light is used the central fringe $C$ is white whereas the other fringes on both sides of $C$ are coloured because the fringe width ($\beta$) depends upon the wavelength. Only the first few coloured fringes are observed and the other fringes overlap. Therefore, the number of fringes seen in the field of view with a monochromatic source of light are more, than with white light.

## 8.8 FRESNEL'S BIPRISM

Fresnel used a biprism to show interference phenomenon. The biprism *abc* consists of two acute angled prisms placed base to base. Actually, it is constructed as a single prism of obtuse angle of about 179° (Fig. 8.7A). The acute angle $\alpha$ on both sides is about 30′. The prism is placed with its refracting edge parallel to the line source $S$ (slit) such that $Sa$ is normal to the face $bc$ of the prism. When light falls from $S$ on the lower portion of the biprism it is bent upwards and appears to come from

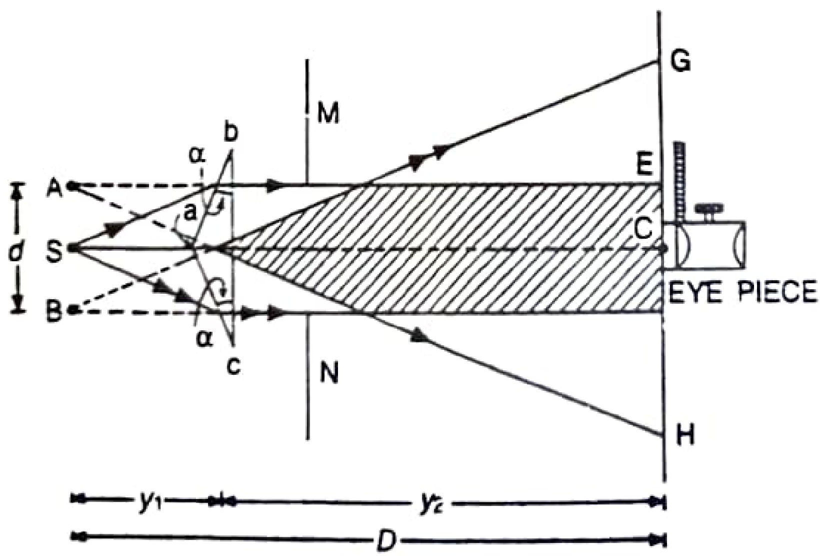



Fig. 8.7A

thevirtual source $B$. Similarly light falling from $S$ on the upper portion of the prism is bent downwards and appears to come from the virtual source $A$. Therefore $A$ and $B$ act as two coherent sources. Suppose the distance between $A$ and $B$ = $d$. If a screen is placed at $C$, interference fringes of equal width are produced between $E$ and $F$ but beyond $E$ and $F$ fringes of large width are produced which are due to diffraction. $MN$ is a stop to limit the rays. To observe the fringes, the screen can be replaced by an eye-piece or a low power microscope and fringes are seen in the field of view. If the point $C$ is at the principal focus of the eyepiece, the fringes are observed in the field of view.

**Theory**. For complete theory refer to Article 8.6. The point $C$ is equidistant from $A$ and $B$. Therefore, it has maximum intensity. On both sides of $C$, alternately bright and dark fringes are produced. The width of the bright fringe or dark fringe, $\beta = \frac{\lambda D}{d}$. Moreover, any point on the screen will be at the centre of a bright fringe if its distance from $C$ is $\beta = \frac{n\lambda D}{d}$, where $n = 0, 1, 2, 3$ *etc.* The point will be at the centre of a dark fringe if its distance from $C$ is

$$\frac{(2n+1)\lambda D}{2d},$$

where $n = 0, 1, 2, 3$ *etc.*

**Determination of wavelength of light**. Fresnel's biprism can be used to determine the wavelength of a given source of monochromatic light.

A fine vertical slit $S$ is adjusted just close to a source of light and the refracting edge is also set parallel to the slit $S$ such that $bc$ is horizontal

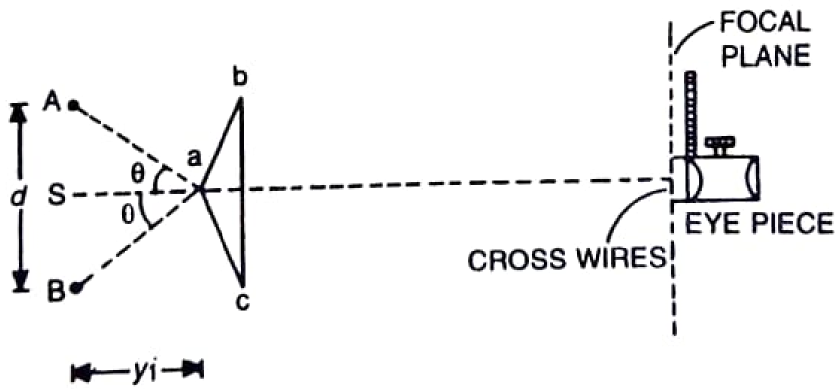



Fig. 8.8

(Fig. 8.8). They are adjusted on an optical bench. A micrometer eyepiece is placed on the optical bench at some distance from the prism to view the fringes in its focal plane (at its cross wires).

Suppose the distance between the source and the eyepiece = $D$ and the distance between the two virtual sources $A$ and $B$ = $d$. The eyepiece is moved horizontally (perpendicular to the length of the bench) to determine the fringe width. Suppose, for crossing 20 bright fringes from the field of view, the eyepiece has moved through a distance $l$.

Then the fringe width, $\beta = \frac{l}{20}$

But the fringe width $\beta = \frac{\lambda D}{d}$

$$\therefore \qquad \lambda = \frac{\beta d}{D} \qquad ...(i)$$

In equation (*i*) $\beta$ and $D$ are known. If $d$ is also known, $\lambda$ can be calculated.

**Determination of the distance between the two virtual sources (*d*).** For this purpose, we make use of the displacement method. A convex lens is placed between the biprism and the eyepiece in such a position, that the images of the virtual sources $A$ and $B$ are seen in the field of view of the eyepiece. Suppose the lens is in the position $L_1$ (Fig. 8.9). Measure the distance between the images of $A$ and $B$ as seen in the eyepiece. Let it be $d_1$

In this case,

$$\frac{d_1}{d} = \frac{v}{u} = \frac{n}{m} \qquad ...(ii)$$

Now move the lens towards the eyepiece and bring it to some other position $L_2$, so that again the images of $A$ and $B$ are seen clearly in the

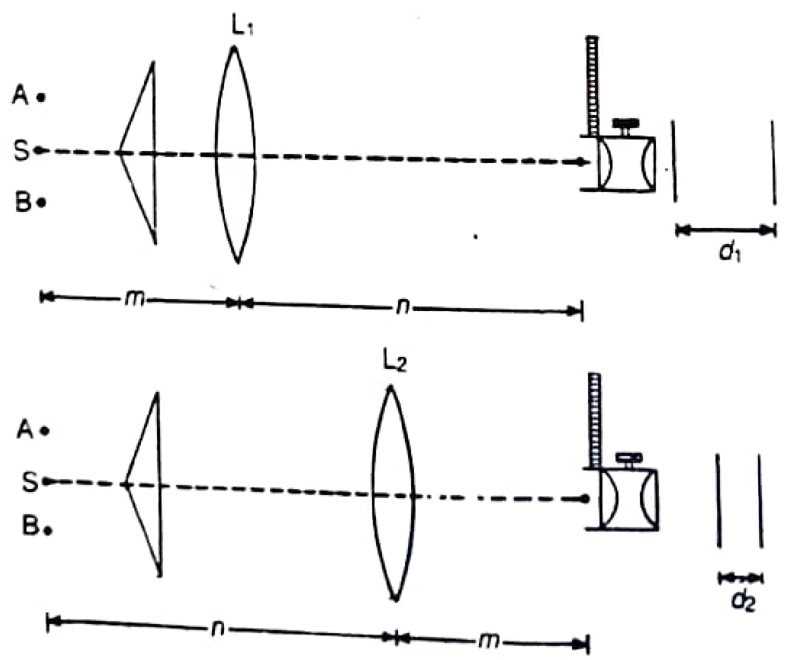



Fig. 8.9

field of view of the eyepiece. Measure the distance between the two images in this case also. Let it be equal to $d_2$.

Here, $$v = m \text{ and } u = n,$$

$$\therefore \qquad \frac{d_2}{d} = \frac{v}{u} = \frac{m}{n} \qquad ...(iii)$$

From equations (*ii*) and (*iii*),

$$\frac{d_1 d_2}{d^2} = 1$$

or $$d = \sqrt{d_1 d_2}$$

Here $d_1$ will be greater than $d_2$ and $d$ is the geometrical mean of $d_1$ and $d_2$. Therefore $d$ can be calculated. Substituting the value of $d$, $\beta$ and $D$ in equation (*i*), the wavelength of the given monochromatic light can be determined.

The second method to find $d$ is to measure accurately the refracting angle $\alpha$. As the angle is small, the deviation produced $\theta = (\mu - 1)\,\alpha$. Therefore the total angle between $Aa$ and $Ba$ is $2\,\theta = 2\,(\mu - 1)\,\alpha$. If the distance between the prism and the slit $S$ is $y_1$ then $d = 2\,(\mu - 1)\,\alpha\, y_1$. Therefore $d$ can be calculated.

## 8.9 FRINGES WITH WHITE LIGHT USING A BIPRISM

When white light is used, the centre of the fringe at $C$ is white while the fringes on both sides of $C$ are coloured because the fringe width ($\beta$) depends upon wavelength. Moreover, the fringes obtained in the case of a biprism using white light are different from the fringes obtained with Fresnel's mirrors. In a biprism, the two coherent virtual sources are produced by refraction and the distance between the two sources depends upon the refractive index, which in turn depends upon the wavelength of light. Therefore, for blue light the distance between the two apparent sources is different to that with red light. The distance of the $n$ th fringe from the centre (with monochromatic light)

$$x = \frac{n\,\lambda D}{d}, \qquad \text{where } d = (2\mu - 1)\,\alpha y_1$$

$$\therefore \qquad x = \frac{n\,\lambda D}{2\,(\mu - 1)\,\alpha y_1}$$

Therefore for blue and red rays, the $n$ th fringe will be,

$$x_b = \frac{n\lambda_b\, D}{2\,(\mu_b - 1)\,\alpha y_1} \qquad ...(i)$$

$$x_r = \frac{n\lambda_r\, D}{2\,(\mu_r - 1)\,\alpha y_1} \qquad ...(ii)$$

**Example 8.9**. *A biprism is placed 5* cm *from a slit illuminated by sodium light* ($\lambda = 5890$ Å). *The width of the fringes obtained on a screen 75* cm *from the biprism is* $9.424 \times 10^{-2}$ cm. *What is the distance between the two coherent sources ?* (*Nagpur 1984*)

Here
$$\lambda = 5890 \times 10^{-8} \text{ cm}$$
$$d = ?, \quad \beta = 9.424 \times 10^{-2} \text{ cm}$$
$$D = 5 + 75 = 80 \text{ cm}$$
$$\beta = \frac{\lambda D}{d}$$

or
$$d = \frac{5890 \times 10^{-8} \times 80}{9.424 \times 10^{-2}}$$

or
$$d = \mathbf{0.05 \text{ cm}}$$

**Example 8.10.** *The inclined faces of a glass prism* ($\mu = 1.5$) *make an angle of* $1^\circ$ *with the base of the prism. The slit is 10* cm *from the biprism and is illuminated by light of* $\lambda = 5900$ Å. *Find the fringe width observed at a distance of 1* m *from the biprism.*

[*Delhi B.Sc.(Hons.) 1991*]

$$\beta = \frac{\lambda D}{d}$$
$$d = 2(\mu - 1)\alpha y_1$$

Here
$$\mu = 1.5$$
$$\alpha = 1^\circ = \frac{\pi}{180} \text{ radian}$$
$$y_1 = 10 \text{ cm}; \quad y_2 = 100 \text{ cm}$$
$$D = y_1 + y_2, = 10 + 100 = 110 \text{ cm}$$
$$\lambda = 5900 \times 10^{-8} \text{ cm.}$$
$$\beta = ?$$

$\therefore$
$$\beta = \frac{5900 \times 10^{-8} \times 110 \times 180 \times 7}{2(1.5 - 1) \times 22 \times 10}$$
$$= \mathbf{0.037 \text{ cm}}$$

**Example 8.11.** *In a biprism experiment with sodium light, bands of width 0.0195* cm *are observed at 100* cm *from the slit. On introducing a convex lens 30* cm *away from the slit, two images of the slit are seen*

0.7 cm apart, at 100 cm distance from the slit. Calculate the wave length of sodium light. [Rajasthan, 1985]

$$\beta = \frac{\lambda D}{d}$$

or
$$\lambda = \frac{\beta d}{D}$$

Here
$$\beta = 0.0195 \text{ cm}$$
$$D = 100 \text{ cm.}$$

For a convex lens

$$\frac{I}{O} = \frac{v}{u}, \quad u + v = 100 \text{ cm}$$

$$u = 30 \text{ cm}$$

or
$$\frac{0.7}{O} = \frac{70}{30} \text{ cm}$$

or
$$O = 0.30 \text{ cm}$$

*i.e.* Distance between the two coherent sources,

$$d = O = 0.30 \text{ cm}$$

$\therefore$
$$\lambda = \frac{0.0195 \times 0.30}{100} = 5850 \times 10^{-8} \text{ cm}$$

or
$$\lambda = \mathbf{5850 \text{ Å}}$$

**Example 8.12**. *Interference fringes are observed with a biprism of refracting angle 1° and refractive index 1.5 on a screen 80 cm away from it. If the distance between the source and the biprism is 20 cm, calculate the fringe width when the wavelength of light used is (i) 6900 Å and (ii) 5890 Å* (Kanpur, 1986)

$$\beta = \frac{\lambda D}{d}$$

$$d = 2(\mu - 1)\alpha\, y_1$$

Here
$$\mu = 1.5$$

$$\alpha = 1^\circ = \frac{\pi}{180} \text{ radian}$$

$$y_1 = 20 \text{ cm}; \quad y_2 = 80 \text{ cm}$$

$$D = y_1 + y_2 = 20 + 80 = 100 \text{ cm}$$

(i) $\lambda = 6900\ \text{Å}$ or $6900 \times 10^{-8}$ cm

$$\therefore \qquad \beta = \frac{\lambda D}{2(\mu - 1)\alpha y_1}$$

$$\beta = \frac{6900 \times 10^{-8} \times 100 \times 180 \times 7}{2(1.5 - 1) \times 22 \times 20}$$

$$\beta = \mathbf{0.01976\ cm}$$

(ii) $\lambda = 5890\ \text{Å}$

or $x = 5890 \times 10^{-8}$ cm

$$\beta = \frac{\lambda D}{2(\mu - 1)\alpha y_1}$$

or
$$\beta = \frac{5890 \times 10^{-8} \times 100 \times 180 \times 7}{2(1.5 - 1) \times 22 \times 20}$$

or
$$\beta = \mathbf{0.01687\ cm}$$

**Example 8.13**. *A biprism is placed at a distance of 5 cm in front of a narrow slit, illuminated by sodium light ($\lambda = 5890 \times 10^{-8}$ cm) and the distance between the virtual sources is found to be 0.05 cm. Find the width of the fringes observed in an eyepiece placed at a distance of 75 cm from the biprism.*

*(Mysore 1981)*

Here $\lambda = 5890 \times 10^{-8}$ cm, $d = 0.05$ cm

$$D = 5 + 75 = 80 \text{ cm}$$

Width of the fringe

$$\beta = \frac{\lambda D}{d} = \frac{5890 \times 10^{-8} \times 80}{0.05}$$

$$\beta = \mathbf{9.424 \times 10^{-8}\ cm}$$

**Example 8.14**. *In a biprism experiment the eyepiece was placed at a distance of 120 cm from the source. The distance between the two virtual sources was found to be 0.075 cm. Find the wavelength of light of the source if the eyepiece has to be moved through a distance 1.888 cm for 20 fringes to cross the field of view.*

Here, $n = 20$

$$l = 1.888 \text{ cm}$$

$\therefore$ Fringe width $\beta = \dfrac{l}{n} = \dfrac{1.888}{20}$ cm

$$d = 0.075 \text{ cm}, \qquad D = 120 \text{ cm}$$

$$\lambda = \frac{\beta d}{D} = \frac{1.888}{20} \times \frac{0.075}{120} = 5900 \times 10^{-8} \text{ cm}$$

$$= \mathbf{5900 \text{ Å}}$$

**Example 8. 15.** *In an experiment with Fresnel's biprism, fringes for light of wavelength* $5 \times 10^{-5}$ cm *are observed* 0.2 mm *apart at a distance of 175* cm *from the prism. The prism is made of glass of refractive index 1.50 and it is at a distance of* 25 cm *from th illuminated slit. Calculate the angle at the vertex of the biprism.*

Here $\quad y_1 = 25 \text{ cm}, \quad y_2 = 175 \text{ cm}$

$$\beta = 0.2 \text{ mm} = 0.02 \text{ cm}$$

$$\lambda = 5 \times 10^{-5} \text{ cm}$$

$$\mu = 1.50$$

$$\alpha = ?$$

$$d = 2(\mu - 1)\,\alpha \cdot y_1 \qquad ...(i)$$

But $\quad \beta = \dfrac{\lambda D}{d}$

or $\quad d = \dfrac{\lambda D}{\beta} \qquad ...(ii)$

From equations (*i*) and (*ii*)

$$\frac{\lambda D}{\beta} = 2(\mu - 1)\,\alpha \,.\, y_1$$

Also $\quad D = y_1 + y_2$

$\therefore \quad \dfrac{\lambda (y_1 + y_2)}{\beta} = 2(\mu - 1)\,\alpha \,.\, y_1$

or $\quad \alpha = \dfrac{\lambda (y_1 + y_2)}{2\beta (\mu - 1)\, y_1} = \dfrac{5 \times 10^{-5} (25 + 175)}{2 \times 0.02 (1.5 - 1)\, 25}$

$$= 0.02 \text{ radian}$$

The vertex angle $\theta = (\pi - 2\alpha)$ radian $= (\pi - 0.04)$ radian

$$\theta = \mathbf{177^{\circ}\ 42'}$$

**Example 8.16.** *Calculate the separation between the coherent sources formed by a biprism whose inclined faces make angles of 1 degree with its base. The slit source is 20* cm *away from the biprism and* $\mu$ *of the biprism material = 1.5.*

$$d = 2(\mu - 1)\,\alpha y_1$$

Here $\mu = 1.5, \quad \alpha = 1^\circ = \dfrac{\pi}{180}$ radian

$$y_1 = 20 \text{ cm}$$

$$d = \frac{2(1.5-1)\,\pi \times 20}{180} = \frac{2 \times 0.5 \times 22 \times 20}{7 \times 180}$$

$$= \mathbf{0.35 \ cm}$$

**Example 8.17.** *Calculate the separation between the coherent sources formed by a biprism whose inclined faces make angles of* $2^\circ$ *with its base, the slit source being 10* cm *away from the biprism* ($\mu$ = 1.50.) *(Delhi 1974, 1977)*

$$d = 2(\mu - 1)\,\alpha\, y_1$$

Here $\mu = 1.50$

$$\alpha = 2^\circ = \frac{2 \times \pi}{180} = \frac{\pi}{90} \text{ radian}$$

$$y_1 = 10 \text{ cm}$$

$$d = \frac{2(1.5-1) \times 10}{90} = \frac{2 \times 0.5 \times \pi \times 10}{90}$$

$$= \mathbf{0.35 \ cm}$$

**Example 8.18**. *In a biprism experiment, the eye-piece is placed at a distance of 1.2* m *form the source. The distance between the virtual sources was found to be 7.5* × $10^{-4}$ m. *Find the wavelength of light, if the eye-piece is to be moved transversely through a distance of 1.888* cm *for 20 fringes.* (Delhi 1985)

$$\beta = \frac{\lambda D}{d}\ ; \quad \beta = \frac{l}{n}$$

$$\frac{l}{n} = \frac{\lambda D}{d}$$

$$\lambda = \frac{l\,d}{nD}$$

$$l = 1.888 \text{ cm} = 0.01888 \text{ m}$$

$$d = 7.5 \times 10^{-4} \text{ m}$$

$n = 2.0$

$D = 1.2$ m

$\lambda = \dfrac{0.01888 \times 7.5 \times 10^{-4}}{20 \times 1.2}$

$= 5900 \times 10^{-10}$ m

$= 5900$ Å.

$$n = 2.0$$

$$D = 1.2 \text{ m}$$

$$\lambda = \frac{0.01888 \times 7.5 \times 10^{-4}}{20 \times 1.2}$$

$$= 5900 \times 10^{-10} \text{ m}$$

$$= \mathbf{5900 \text{ Å}}$$

**Example 8.19**. *The inclined faces of a biprism of refractive index 1.50 make angle of 2° with the base. A slit illuminated by a monochromatic light is placed at a distance of 10* cm *from the biprism. If the distance between two dark fringes observed at a distance of 1* cm *from the prism is 0.18* mm, *find the wavelength of light used.*

[*Delhi* (*Hons*) *1991*]

Here, $\beta = \frac{\lambda D}{d} \quad \therefore \quad \lambda = \frac{\beta d}{D}$

$$\beta = 0.18 \text{ mm} = 0.18 \times 10^{-3} \text{ m}$$

$$d = 2(\mu - 1)\alpha y_1$$

$$m = 1.5$$

$$\alpha = 2^\circ = \frac{2 \times \pi}{180} = \frac{\pi}{90} \text{ radian}$$

$$y_1 = 10 \text{ cm} = 0.1 \text{ m} ; \quad y_2 = 1 \text{ m}$$

$$D = y_1 + y_2 = 0.1 + 1 = 1.1 \text{ m}$$

$$\lambda = ?$$

$$d = \frac{2(1.5 - 1)\ \pi \times 0.1}{90} = 3.49 \times 10^{-3}$$

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.18 \times 10^{-3} \times 3.49 \times 10^{-3}}{1.1} = 5.711 \times 10^{-7} \text{ m}$$

$$\lambda = \mathbf{5711 \text{ Å}}$$

## 8.10 DETERMINATION OF THE THICKNESS OF A THIN SHEET OF TRANSPARENT MATERIAL

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material *e.g.*, glass or mica.

Suppose $A$ and $B$ are two virtual coherent sources. The point $C$ is equidistant from $A$ and $B$. When a transparent plate $G$ of thickness $t$ and refractive index $\mu$ is introduced in the path of one of the beams (Fig. 8.10),

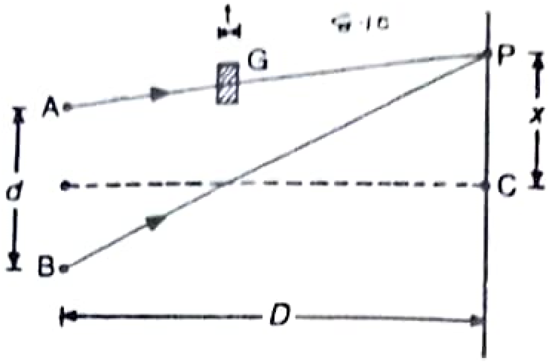



Fig. 8.10.

the fringe which was originally at $C$ shifts to $P$. The time taken by the wave from $B$ to $P$ in air is the same as the time taken by the wave from $A$ to $P$ partly through air and partly through the plate. Suppose $c_0$ is the velocity of light in air and $c$ its velocity in the medium

$$\therefore \qquad \frac{BP}{c_0} = \frac{AP-t}{c_0} + \frac{t}{c}$$

or

$$BP = AP - t + \frac{c_0}{c}\, t. \qquad \text{But } \frac{c_0}{c} = \mu$$

$$\therefore \qquad BP - AP = \mu t - t = (\mu - 1)t$$

If $P$ is the point originally occupied by the $n$ th fringe, then the path difference

$$BP - AP = n\lambda$$

$$\therefore \qquad (\mu - 1)\, t = n\lambda \qquad \ldots(i)$$

Also the distance $x$ through which the fringe is shifted

$$= \frac{n\lambda D}{d}$$

where

$$\frac{\lambda D}{d} = \beta, \text{ the fringe width.}$$

$$\therefore \qquad x = \frac{n\lambda D}{d}$$

Also,
$$n\lambda = \frac{xd}{D}$$

or
$$(\mu - 1)t = \frac{xd}{D} \qquad ...(ii)$$

Therefore, knowing $x$, the distance through which the central fringe is shifted, $D$, $d$ and $\mu$, the thickness of the transparent plate can be calculated. If a monochromatic source of light is used, the fringes will be similar and it is difficult to locate the position where the central fringe shifts after the introduction of the transparent plate. Therefore, white light is used. The fringes will be coloured but the central fringe will be white. When the cross wire is at the central white fringe without the transparent plate in the path, the reading is noted. When the plate is introduced, the position to which the central white fringe shifts is observed. The difference between the two positions on the micrometer scale of the eyepiece gives the value of the shift which is equal to $x$. Now, with the monochromatic source of light, the micrometer eyepiece is moved through the same distance x and the number of fringes that cross the field of view is observed. Suppose $n$ fringes cross the field of view. Then from the relation
$$(\mu - 1)t = n\lambda$$

the value of $t$ can be calculated. The value of $t$ can also be calculated from equation (*ii*). However, if $t$ is known, $\mu$ can be calculated.

This experiment also shows that light travels more slowly in a medium of refractive index $\mu > 1$, than in air because the central fringe shifts towards the side where the transparent plate is introduced. Had it been opposite, the shift should have been to the other side. The optical path in air $= \mu \times t$, for a medium of thickness $t$ and refractive index $\mu$.

**Example 8.20.** *When a thin piece of glass $3.4 \times 10^{-4}$ cm thick is placed in the path of one of the interfering beams in a biprism arrangement, it is found that the central bright fringe shifts through a distance equal to the width of four fringes. Find the refractive index of the piece of glass. Wavelength of light used is $5.46 \times 10^{-5}$ cm.*

[*Delhi (Hons.)*]

Here
$$t = 3.4 \times 10^{-4} \text{ cm}$$
$$n = 4$$
$$\lambda = 5.46 \times 10^{-5} \text{ cm}$$
$$\mu = ?$$
$$(\mu - 1)\, t = n\lambda$$

$$\mu = \frac{n\lambda}{t} + 1 = \frac{4 \times 5.46 \times 10^{-5}}{3.4 \times 10^{-4}} + 1$$

$$\mu = 0.6424 + 1 = \mathbf{1.6424}$$

**Example 8.21**. *Fringes are produced with monochromatic light of $\lambda = 5450$ Å. A thin plate of glass of $\mu = 1.5$ is then placed normally in the path of one of the interfering beams and the central band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. calculate the thickness of the glass plate.*

*(Delhi B.Sc. Hons.)*

Here $\quad (\mu - 1)\, t = n\lambda$

$$\mu = 1.5, \quad n = 3,$$

$$\lambda = 5450 \times 10^{-8} \text{ cm}$$

$$t = \frac{n\lambda}{(\mu - 1)} = \frac{3 \times 5450 \times 10^{-8}}{(1.5 - 1)}$$

$$= \mathbf{0.000237\ cm}$$

**Example 8.22**. *A transparent plate of thickness $10^{-3}$ cm is placed in the path of one of the interfering beams of a biprism experiment using light of wavelength 5000 Å. If the central fringe shifts by a distance equal to the width of ten fringes, calculate refractive index of the material of the plate.*

*(Mysore)*

$$(\mu - 1) = n\lambda$$

Here, $\quad t = 10^{-3} \text{ cm}, \quad n = 10$

$$\lambda = 5000 \times 10^{-8} \text{ cm}, \quad \mu = ?$$

$$(\mu - 1)\, 10^{-3} = 10 \times 5000 \times 10^{-8}$$

$$\mu = \mathbf{1.5}$$

**Example 8.23**. *A thin sheet of a transparent material ($\mu_D = 1.60$) is placed in the path of one of the interfering beams in a biprism experiment using sodium light, $\lambda = 5890 \times 10^{-8}$ cm. The central fringe shifts to a position originally occupied by the 12 th bright fringe. Calculate the thickness of the sheet.*

Here, $\quad \mu = 1.60, \ \lambda = 5890 \times 10^{-8} \text{ cm}, \quad n = 12$

$$(\mu - 1)\, t = n\,\lambda$$

$$t = \frac{n\,\lambda}{(\mu - 1)} = \frac{12 \times 5890 \times 10^{-8}}{(1.60 - 1)} = \mathbf{1.178 \times 10^{-3}\ cm}$$

**Example 8.24**. *When a thin sheet of transparent material of thickness $6.3 \times 10^{-4}$ cm is introduced in the path of one of the interfering beams, the central fringe shifts to a position occupied by the sixth bright fringe. If $\lambda = 5460$ Å, find the refractive index of the sheet.*

Here,

$$t = 6.3 \times 10^{-4} \text{ cm}$$

$$\lambda = 5460 \text{ Å} = 5460 \times 10^{-8} \text{ cm}$$

$$n = 6, \quad \mu = ?$$

$$(\mu - 1)\, t = n\lambda$$

$$\mu = \frac{n\lambda}{t} + 1 = \frac{6 \times 5460 \times 10^{-8}}{6.3 \times 10^{-4}} + 1 = \mathbf{1.52}$$

**Example 8.25**. *On placing a thin sheet of mica of thickness $12 \times 10^{-5}$ cm in the path of one of the interfering beams in a biprism arrangement, it is found that the central bright band shifts a distance equal to the width of a bright fringe. Calculate the refractive index of mica.* *(Gorakhpur 1966)*
$[\lambda = 6 \times 10^{-5}$ cm$]$

Here,

$$t = 12 \times 10^{-5} \text{ cm}, \quad \lambda = 6 \times 10^{-5} \text{ cm}$$

$$n = 1, \quad \mu = ?$$

$$(\mu - 1)\, t = n\lambda$$

$$\mu = \frac{n\lambda}{t} + 1 = \frac{1 \times 6 \times 10^{-5}}{12 \times 10^{-5}} + 1 = \mathbf{1.50}$$

**Example 8.26**. *Fresnel's fringes are produced with homogeneous light of wavelength $6 \times 10^{-5}$ cm. A thin glass film ($\mu = 1.50$) is interposed in the path of one of the interfering beams. The central bright band is shifted to the position previously occupied by the 5 th bright band. Find the thickness of the film.* *[Delhi 1988]*

Here,

$$\lambda = 6 \times 10^{-5} \text{ cm}$$

$$\mu = 1.50 \quad \text{and} \quad n = 5$$

$$t = ?$$

$$(\mu - 1)\, t = n\lambda$$

or

$$t = \frac{n\lambda}{(\mu - 1)}$$

$$= \frac{5 \times 6 \times 10^{-5}}{(1.5 - 1)}$$

$$= \mathbf{6 \times 10^{-4} \text{ cm}}$$

of $V$ and $R$. Interference occurs between the beams from $V R$ and those from $V' R'$. The violet fringes are produced by $V$ and $V'$ while the red *fringes are produced by* $R$ and $R'$.

Suppose, $VV' = d_1$ and $RR' = d_2$

If $\frac{\lambda}{d_1} = \frac{\lambda}{d_2}$, the fringe width $\beta$ will be the same and interference fringes due to different colours will overlap and white achromatic fringes are produced in the field of view. The white and dark fringes are seen through the eyepiece or can be produced on the screen.

Instead of a diffraction grating, a prism of small angle can also be used.

## 8.15 INTERFERENCE IN THIN FILMS

Newton and Hooke observed and developed the interference phenomenon due to multiple reflections from the surface of thin transparent materials. Everyone is familiar with the beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of a soap bubble. Hooke observed such colours in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings when a convex lens was placed on a plane glass-plate. Young was able to explain the phenomenon on the basis of interference *between light reflected from the top and the bottom surface of a thin film.* It has been observed that interference in the case of thin films takes place due to (1) reflected light and (2) transmitted light.

## 8.16 INTERFERENCE DUE TO REFLECTED LIGHT (THIN FILMS)

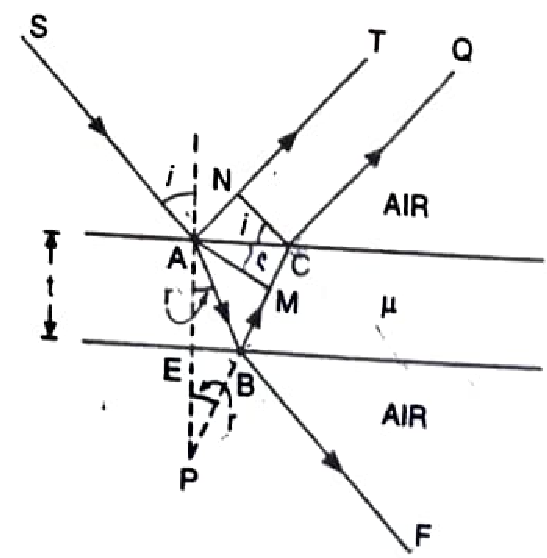



Fig. 8. 15

Consider a transparent film of thickness $t$ and refractive index $\mu$. A ray $SA$ incident on the upper surface of the film is partly reflected along $AT$ and partly refracted along $AB$. At $B$ part of it is reflected along $BC$ and finally emerges out along $CQ$. The difference in path between the two rays $AT$ and $CQ$ can be calculated. Draw $CN$ normal to $AT$ and $AM$ normal to $BC$. The angle of incidence is $i$ and the angle of refraction is $r$. Also produce $CB$ to meet $AE$ produced at $P$. Here $\angle APC = r$ (Fig. 8.15).

The optical path difference

$$x = \mu (AB + BC) - AN$$

Here,
$$\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$$

$$\therefore \quad AN = \mu \,.\, CM$$

$$x = \mu (AB + BC) - \mu \,.\, CM$$

$$\therefore \quad x = \mu (AB + BC - CM) = \mu (PC - CM)$$

$$= \mu.PM$$

In the $\Delta\, APM$,

$$\cos r = \frac{PM}{AP}$$

or
$$PM = AP.\cos r = (AE + EP)\cos r$$

$$= 2t \cos r$$

$$(\because \quad AE = EP = t)$$

$$\therefore \quad x = \mu.PM = 2\,\mu t \cos r \qquad ...(i)$$

This equation (i), in the case of reflected light does not represent the correct path difference but only the apparent. It has been established on the basis of electromagnetic theory that, when light is reflected from the surface of an optically denser medium (air-medium interface) a phase change $\pi$ equivalent to a path difference $\frac{\lambda}{2}$ occurs.

Therefore, the correct path difference in this case,

$$x = 2\,\mu t \cos r - \frac{\lambda}{2} \qquad ...(ii)$$

(1) If the path difference $x = n\,\lambda$ where $n = 0, 1, 2, 3, 4$ etc., constructive interference takes place and the film appears bright.

$$\therefore \; 2\,\mu\, t \cos r - \frac{\lambda}{2} = n\lambda$$

or
$$2\,\mu t \cos r = (2n+1)\frac{\lambda}{2} \qquad ...(iii)$$

(2) If the path difference $x = (2n+1)\frac{\lambda}{2}$ where $n = 0, 1, 2$ ...etc., destructive interference takes place and the film appears dark.

$$\therefore \quad 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or
$$2\mu t \cos r = (n+1)\,\lambda \qquad ...(iv)$$

Here $n$ is an integer only, therefore $(n + 1)$ can also be taken as $n$.

$$\therefore \qquad 2\mu t \cos r = n\lambda \qquad \ldots(v)$$

where $n = 0, 1, 2, 3, 4, \ldots$ etc.

It should be remembered that the interference pattern will not be perfect because the intensities of the rays $AT$ and $CQ$ will not be the same and their amplitudes are different. The amplitudes will depend on the amount of light reflected and transmitted through the films. It has been found that for normal incidence, about 4% of the incident light is reflected and 96% is transmitted. Therefore, the intensity never vanishes completely and perfectly dark fringes will not be observed for the rays $AT$ and $CQ$ alone. But in the case of multiple reflection, the intensity of the minima will be zero.

Consider reflected rays 1, 2, 3 etc. as shown in Fig. 8.16. The amplitude of the incident ray is $a$. Let $r$ be the reflection coefficient, $t$ the transmission coefficient from rarer to denser medium and $t'$ the transmission coefficient from denser to rarer medium.

The amplitudes of the reflected rays are: $ar$, $atrt'$, $atr^3t'$, $atr^5t' \ldots$ etc. The ray 1 is reflected at the surface of a denser medium. It undergoes a phase change $\pi$. The rays 2, 3, 4 etc. are all in phase but out of phase with ray 1 by $\pi$.

The resultant amplitude of 2, 3, 4 etc. is given by

$$A = atrt' + atr^3t' + atr^5t' + \ldots$$

$$A = att'r\,[1 + r^2 + r^4 + \ldots]$$

As $r$ is less than 1, the terms inside the brackets form a geometric series.

$$A = att'r\left[\frac{1}{1-r^2}\right] = \left[\frac{att'r}{1-r^2}\right]$$

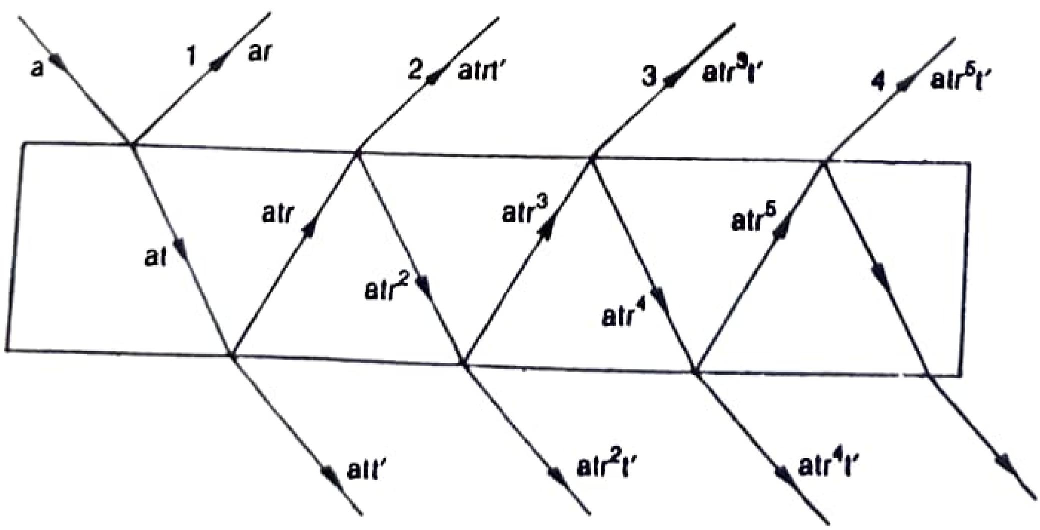



Fig. 8.16

According to the principle of reversibility.,

$$tt' = 1 - r^2$$

$$\therefore \qquad A = \frac{a(1-r^2)\,r}{(1-r^2)} = ar$$

Thus, the resultant amplitude of 2, 3, 4,...etc. is equal in magnitude of the amplitude of ray 1 but out of phase with it. Therefore the minima of the reflected system will be of zero intensity.

## 8.17 INTERFERENCE DUE TO TRANSMITTED LIGHT (THIN FILMS)

Consider a thin transparent film of thickness $t$ and refractive index $\mu$. A ray $SA$ after refraction goes along $AB$. At $B$ it is partly reflected along $BC$ and partly refracted along $BR$. The ray $BC$ after reflection at $C$, finally emerges along $DQ$. Here at $B$ and $C$ reflection takes place at the rarer medium (medium-air interface). Therefore, no phase change occurs. Draw $BM$ normal to $CD$ and $DN$ normal to $BR$. The optical path difference between $DQ$ and $BR$ is given by,

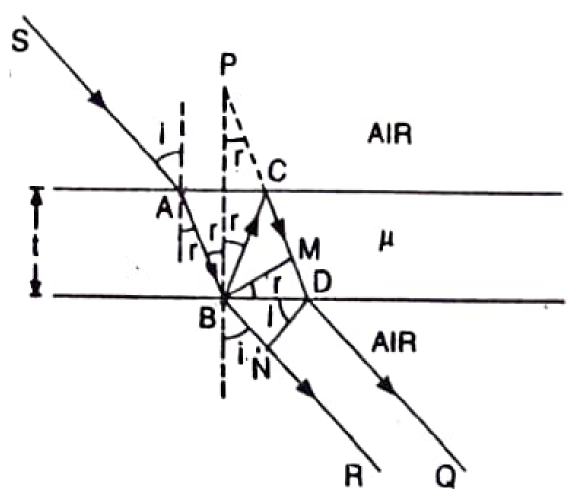



Fig. 8.17.

$$x = \mu(BC + CD) - BN$$

Also $$\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD} \quad \text{or} \quad BN = \mu.MD$$

In Fig. 8.17,

$$\angle BPC = r \text{ and } CP = BC = CD$$

$$\therefore \qquad BC + CD = PD$$

$$\therefore \qquad x = \mu(PD) - \mu(MD) = \mu(PD - MD) = \mu\, PM$$

In the $\Delta BPM$, $\cos r = \dfrac{PM}{BP}$ or $PM = BP.\cos r$

But, $$BP = 2\,t$$

$$\therefore \qquad PM = 2\,t\cos r$$

$$\therefore \qquad x = \mu.PM = 2\,\mu\,t\cos r \qquad ...(i)$$

(*i*) For bright fringes, the path difference $x = n\lambda$

$$\therefore \quad 2\mu t \cos r = n\lambda \qquad \text{...(ii)}$$

where $n = 0, 1, 2, 3 \ldots$ etc.

(*ii*) For dark fringes, the path difference $x = (2n+1)\dfrac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos r = \frac{(2n+1)\lambda}{2}$$

where $n = 0, 1, 2, 3, \ldots$ etc.

In the case of transmitted light, the interference fringes obtained are less distinct because the difference in amplitude between *BR* and *DQ* is very large. However, when the angle of incidence is nearly 45°, the fringes are more distinct.

## 8.18 INTENSITIES OF MAXIMA AND MINIMA IN THE INTERFERENCE PATTERN OF REFLECTED AND TRANSMITTED BEAMS IN THIN FILMS

The intensity of the transmitted beam is given by (vide theory of Fabry-Perot Interferometer)

$$I_t = \frac{I_0}{1 + \dfrac{4r^2}{(1-r^2)^2}\sin^2\dfrac{\delta}{2}}$$

Here $\delta$ is the phase difference, $r^2$ is the reflection coefficient and $I_0$ is the maximum intensity.

For values of $\delta = \pi, 3\pi, 5\pi$ etc.

$$\sin^2\frac{\delta}{2} = 1$$

For $r^2 = 0.04$ [*i.e.* Reflectance of 4% ]

$$I_t = \frac{I_0}{1 + \dfrac{4 \times 0.04}{(1-0.04)^2}}$$

$$I_t = 0.8521\ I_0$$

Taking $I_0 = 1$

$$I_t = 85.21\%$$

and $I_r = 100 - 85.21 = 14.79\%$

(1) In the reflected system, the intensity of the interference maxima will be 14.79% of the incident intensity and the intensity of the minima will be zero (Fig. 8.18).

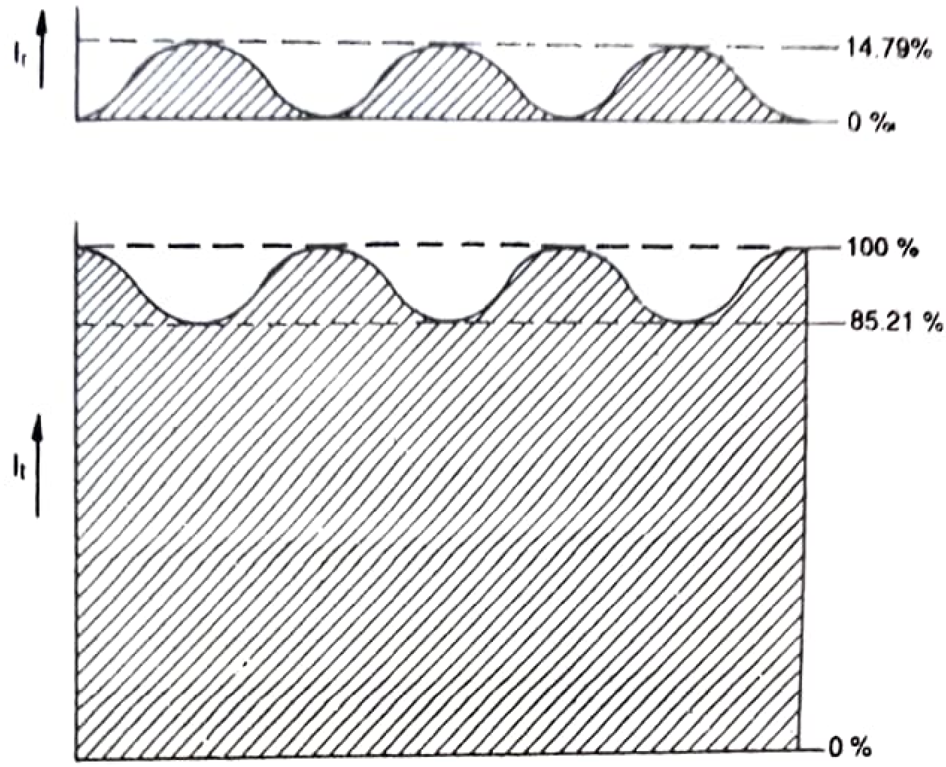



Fig. 8.18

(2) In the transmitted system, the intensity of the maxima will be 100% and intensity of the minima will be 85.21%. It means the visibility of the fringes is much higher in the reflected system than in the transmitted system. Thus the fringes are more sharp in reflected light.

## 8.19 COLOURS OF THIN FILMS

When white light is incident on a thin film, the light which comes from any point from it will not include the colour whose wavelength satisfies the equation $2\mu t \cos r = n\lambda$, in the reflected system. Therefore, the film will appear coloured and the colour will depend upon the thickness and the angle of inclination. If $r$ and $t$ are constant, the colour will be uniform. In the case of oil on water, different colours are seen because $r$ and $t$ vary. This is clear from the following solved example.

**Example 8.29**. *A parallel beam of light* ($\lambda = 5890 \times 10^{-8}$ cm) *is incident on a thin glass plate* ($\mu = 1.5$) *such that the angle of refraction into the plate is* $60°$. *Calculate the smallest thickness of the glass plate which will appear dark by reflection.* (*Punjab 1973*)

Here $2\mu t \cos r = n\lambda$

$$\mu = 1.5, \quad r = 60^\circ, \quad \cos 60^\circ = 0.5$$

$$n = 1, \quad \lambda = 5890 \times 10^{-8} \text{ cm}$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$t = \mathbf{3.926 \times 10^{-5} \text{ cm}}$$

**Example 8.30.** *A soap film* $5 \times 10^{-5}$ cm *thick is viewed at an angle of* $35^\circ$ *to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light* ($\mu = 1.33$) *(Bombay)*

Let $i$ be the angle of incidence and $r$ the angle of refraction.

Then $\mu = \dfrac{\sin i}{\sin r}$ or $1.33 = \dfrac{\sin 35^\circ}{\sin r}$

or $r = 25.55^\circ$ and $\cos r = 0.90$

Applying the relation and taking

$$2\mu t \cos r = n\lambda$$

$$t = 5 \times 10^{-5} \text{ cm}$$

(*i*) For the first order, $n = 1$

$$\therefore \quad \lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90$$

$$= 12.0 \times 10^{-5} \text{ cm}$$

which lies in the infra-red (invisible) region.

(*ii*) For the second order, $n = 2$

$$\lambda_2 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90$$

$$\lambda_2 = 6.0 \times 10^{-5} \text{ cm}$$

which lies in the visible region.

(*iii*) Similarly, taking $n = 3$

$$\lambda_3 = 4.0 \times 10^{-5} \text{ cm}$$

which also lies in the visible region.

(*iv*) If

$$n = 4$$

$$\lambda_4 = 3.0 \times 10^{-5} \text{ cm}$$

which lies in the ultra-violet (invisible) region.

Hence, absent wavelengths in the reflected light are $\mathbf{6.0 \times 10^{-5}}$ **cm** and $\mathbf{4.0 \times 10^{-5}}$ **cm.**

**Example 8.31**. *A soap film of refractive index* $\frac{4}{3}$ *and of thickness* $1.5 \times 10^{-4}$ cm *is illuminated by white light incident at an angle of* $60°$. *The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of* $5 \times 10^{-5}$ cm. *Calculate the order of interference of the dark band.* [*Delhi* (*Hons*)]

Here, $$2\,\mu t \cos r = n\lambda$$

$$\mu = \frac{4}{3} \quad ; \quad \lambda = 5 \times \times 10^{-5} \text{ cm}$$

$$t = 1.5 \times 10^{-4} \text{ cm} \quad ; \quad i = 60°$$

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \frac{4}{3} = \frac{\sin 60}{\sin r}$$

$$\sin r = \frac{0.866}{4/3} = 0.6495$$

$$r = 40.5° \quad \text{and} \quad \cos r = 0.7604$$

$\therefore$ $$n = \frac{2\mu t \cos r}{\lambda}$$

$$n = \frac{2 \times 4/3 \times 1.5 \times 10^{-4} \times 0.7604}{5 \times 10^{-5}}$$

$$n = 6.0832$$

Hence, the order, **n = 6**.

**Example 8.32**. *A beam of parallel rays is incident at an angle of* $30°$ *with the normal on a plane parallel film of thickness* $4 \times 10^{-5}$ cm *and refractive index 1.50. Show that the reflected light whose wavelength is* $7.539 \times 10^{-5}$ cm, *will be strengthened by reinforcement.*

(*Bombay, 1986*)

Here $$t = 4 \times 10^{-5} \text{ cm}, \quad \lambda = 7.539 \times 10^{-5} \text{ cm}; \quad \mu = 1.50$$

For the film to appear bright by reflection,

$$2\,\mu t \cos r = \frac{n\lambda}{2}$$

Here, $n$ must be odd.

$$n = \frac{4\,\mu t \cos r}{\lambda}$$

Here, $$i = 30°, \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin 30}{\sin r}$$

$$\therefore \qquad \sin r = \frac{0.5}{\mu} = \frac{0.5}{1.5} = 0.33$$

$$\therefore \qquad r = 19.4^\circ \quad \text{and} \quad \cos r = 0.9432$$

$$n = \frac{4 \times 1.5 \times 4 \times 10^{-5} \times 0.9432}{7.539 \times 10^{-5}}$$

$$n = 3.002$$

or

$$n = \mathbf{3}$$

As $n$ is odd, the film appears bright by reflected light.

**Example 8.33.** *A parallel beam of light ($\lambda$ = 5890 Å) is incident on a thin glass plate ($\mu$ = 1.5) such that the angle of refraction is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.*

*(Lucknow, 1990 ; Kanpur 1990)*

Here,

$$2\,\mu t \cos r = n\,\lambda$$

$$\mu = 1.5, \qquad r = 60^\circ \quad ; \quad \cos 60^\circ = 0.5$$

For minimum thickness,

$$n = 1$$

$$\lambda = 5890 \text{ Å}$$

$$\lambda = 5890 \times 10^{-10} \text{ m}$$

$$t = \frac{n\lambda}{2\,\mu \cos r}$$

$$t = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.5 \times 0.5}$$

$$t = 3926 \times 10^{-10} \text{ m}$$

$$t = \mathbf{3.926 \times 10^{-7} \text{ m}}$$

$$t = \mathbf{3.926 \times 10^{-4} \text{ mm}}$$

**Example 8.34.** *A soap film of refractive index 1.33 is illuminated with light of different wavelengths at an angle of 45°. There is complete destructive interference for $\lambda$ = 5890 Å. Find the thickness of the film.*

*(I.A.S, 1991)*

Here

$$\mu = 1.33$$

$$r = 45^\circ$$

$$\cos 45 = 0.707$$

$$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

$$n = 1 \; ; \; t = ?$$

$$2 \mu t \cos r = n \lambda$$

$$t = \frac{n \lambda}{2 \mu \cos r}$$

$$t = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.33 \times 0.707}$$

$$t = \mathbf{3.132 \times 10^{-7} \text{ m}}$$

$$t = \mathbf{3.132 \times 10^{-4} \text{ mm}}$$

**Example 8.35**. *A thin film of soap solution is illuminated by white at an angle of incidence,* $i = \sin^{-1}\left(\frac{4}{5}\right)$. *In reflected light,two dark consecutive overlapping fringes are observed corresponding to wavelengths* $6.1 \times 10^{-7}$ m *and* $6.0 \times 10^{-7}$ m. $\mu$ *for the soap solution is* $\frac{4}{3}$. *Calculate the thickness of the film.* *(Kanpur,1991)*

Here $$n \lambda_1 = (n+1) \lambda_2$$

$$n (6.1 \times 10^{-7}) = (n+1) \times 6 \times 10^{-7}$$

$$n = \mathbf{60}$$

$$\sin i = \frac{4}{5}$$

$$\mu = \frac{4}{3} = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = 0.6$$

$$\cos r = \left[1 - \sin^2 r\right]^{1/2} = 0.8$$

Also $$2 \mu t \cos r = n \lambda_1$$

$$t = \frac{n \lambda_1}{2 \mu \cos r} = \frac{60 \times 6.1 \times 10^{-7}}{2 \times (4/3)\, 0.8}$$

$$t = \mathbf{1.72 \times 10^{-5} \text{ m}}$$

$$t = \mathbf{1.72 \times 10^{-2} \text{ mm}}$$

## 8.20 NECESSITY OF A BROAD SOURCE

Interference fringes obtained in the case of Fresnel's biprism, inclined mirrors and Lloyd's single mirror were produced by two coherent sources. The source used is narrow. These fringes can be obtained on the screen or can be viewed with an eyepiece. In the case of interference in thin films, the narrow source limits the visibility of the film.

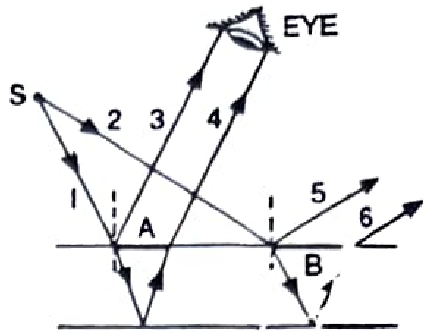



Fig. 8.19.

Consider a thin film and a narrow source of light at $S$ (Fig. 8.19). The ray 1 produces interference fringes because 3 and 4 reach the eye whereas the ray 2 meets the surface at some different angle and is reflected along 5 and 6. Here, 5 and 6 do not reach the eye. Similarly we can take other rays incident at different angles on the film surface which do not reach the eye. Therefore, the portion A of the film is visible and not the rest.

If an extended source of light is used (Fig. 8.20), the ray 1 after reflection from the upper and the lower surface of the film emerges as

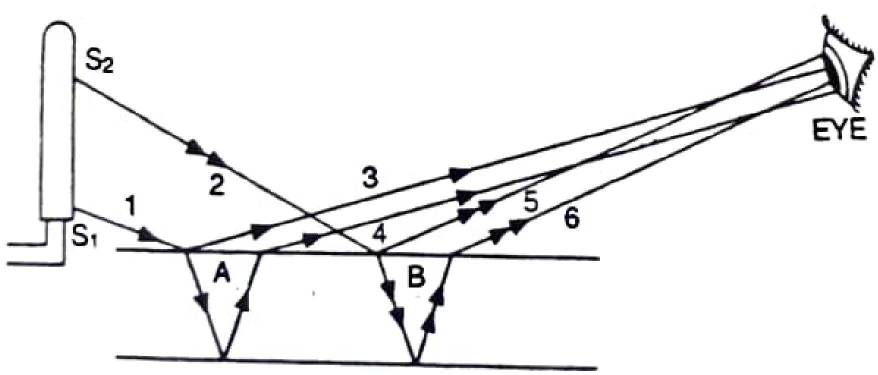



Fig. 8.20

3 and 4 which reach the eye. Also ray 2 from some other point of the source after reflection from the upper and the lower surfaces of the film emerges as 5 and 6 which also reach the eye. Therefore, in the case of such a source of light, the rays incident at different angles on the film are accommodated by the eye and the field of view is large. Due to this reason, to observe interference phenomenon in thin films, a broad source of light is required. With a broad source of light, rays of light are incident at different angles and the reflected parallel beams reach the eye or the microscope objective. Each such ray of light has its origin at a different point on the source.

## 8.21 FRINGES PRODUCED BY A WEDGE SHAPED THIN FILM

Consider two plane surfaces $OA$ and $OB$ inclined at an angle $\theta$ and enclosing a wedge shaped air film. The thickness of the air film increases from $O$ to $A$ (Fig. 8.21). When the air film is viewed with reflected monochromatic light, a system of equidistant interference fringes are observed which are parallel to the line of intersection of the two surfaces. The interfering rays do no enter the eye parallel to each other but they appear to diverge from a point near the film. The effect is best observed when the angle of incidence is small.

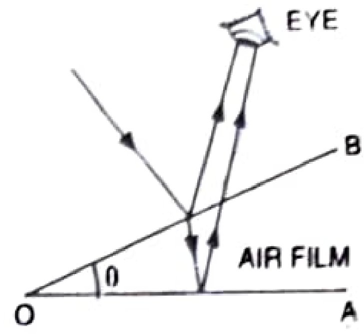



Fig. 8.21.

Suppose the $n$ th bright fringe occurs at $P_n$ (Fig. 8.22). The thickness of the air film at $P_n = P_n Q_n$. As the angle of incidence is small, $\cos r = 1$

Applying the relation for a bright fringe,

$$2\,\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

Here, for air $\mu = 1$ and $\cos r = 1$

and $\quad t = P_n Q_n$

$$\therefore \quad 2\,P_n Q_n = (2n+1)\frac{\lambda}{2} \quad ...(i)$$

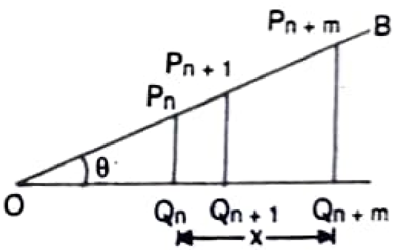



Fig 8.22

The next bright fringe $(n+1)$ will occur at $P_{n+1}$, such that

$$2\,P_{n+1} Q_{n+1} = [2(n+1)+1]\frac{\lambda}{2}$$

or

$$2\,P_{n+1} Q_{n+1} = (2n+3)\frac{\lambda}{2} \quad ...(ii)$$

Subtracting $(i)$ from $(ii)$

$$P_{n+1}Q_{n+1} - P_n Q_n = \frac{\lambda}{2} \quad ...(iii)$$

Thus the next bright fringe will occur at the point where the thickness of the air film increases by $\frac{\lambda}{2}$. Suppose the $(n+m)$ th bright fringe is at $P_{n+m}$. Then, there will be $m$ bright fringes between $P_n$ and $P_{n+m}$ such that

$$P_{n+m} Q_{n+m} - P_n Q_n = \frac{m\lambda}{2} \qquad ...(iv)$$

If the distance $\qquad Q_n Q_{n+m} = x$

$$\theta = \frac{P_{n+m} Q_{n+m} - P_n Q_n}{Q_n Q_{n+m}} = \frac{m\frac{\lambda}{2}}{x} = \frac{m\lambda}{2x} \qquad ...(v)$$

or

$$x = \frac{m\lambda}{2\theta} \qquad ...(vi)$$

Therefore, the angle of inclination between $OA$ and $OB$ can be known. Here, $x$ is the distance corresponding to $m$ fringes. The fringe width

$$\beta = \frac{x}{m} = \frac{\lambda}{2\theta} \qquad ...(vii)$$

## 8.22 TESTING THE PLANENESS OF SURFACES

If the two surfaces $OA$ and $OB$ are perfectly plane, the air-film gradually varies in thickness from $O$ to $A$. The fringes are of equal thickness because each fringe is the locus of the points at which the thickness of the film has a constant value (Fig. 8.23).

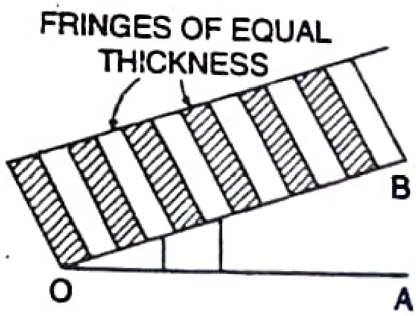



Fig. 8.23.

This is an important application of the phenomenon of interference. If the fringes are not of equal thickness it means the surfaces are not plane. The standard method is to take an optically plane surface $OA$ and the surface to be tested $OB$. The fringes are observed in the field of view and if they are of equal thickness the surface $OB$ is plane. If not, the surface $OB$ is not plane. The surface $OB$ is polished and the process is repeated. When the fringes observed are of equal width, it means that the surface $OB$ is plane.

**Example 8.36.** *Two glass plates enclose a wedge shaped air film, touching at one edge and are separated by a wire of 0.05* mm *diameter at a distance of 15* cm *from the edge. Calculate the fringe width. Monochromatic light of* $\lambda$ = 6,000 Å *from a broad source falls normally on the film.*

(*Rajasthan 1989*)

$$x = 15 \text{ cm}, \quad \lambda = 6000\,\text{Å} = 6000 \times 10^{-8} \text{ cm}$$

$$AB = 0.005 \text{ cm}$$

Fringe width, $$\beta = \frac{\lambda}{2\,\theta}$$

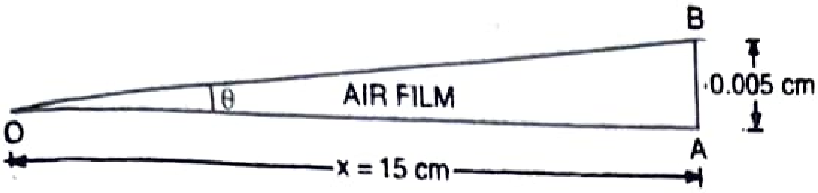



Fig. 8.24

$$\theta = \frac{AB}{OA} = \frac{0.005}{15}$$

$$\beta = \frac{\lambda}{2\,\theta} = \frac{6000 \times 10^{-8} \times 15}{2 \times 0.005}$$

$$= \mathbf{0.09\ cm}$$

**Example 8.37**. *Light of wavelength 6000 Å falls normally on a thin wedge shaped film of refractive index 1.4, forming fringes that are 2* mm *apart. Find the angle of the wedge.* [*Delhi (Hons.) 1986*]

$$\beta = \frac{\lambda}{2\,\theta\,\mu} \qquad \theta = \frac{\lambda}{2\,\mu\,\beta},$$

Here $$\lambda = 6000\text{ Å} = 6000 \times 10^{-8}\ \text{cm}$$

$$\mu = 1.4\ ;\ \beta = 2\ \text{mm} = 0.2\ \text{cm}$$

$\therefore$ $$\theta = \frac{6000 \times 10^{-8}}{2 \times 1.4 \times 0.2}$$

$$\theta = \mathbf{1.07 \times 10^{-4}\ radian}$$

Example 8.38. *A glass wedge of angle 0.01 radian is illuminated by monochromatic light of wavelenth 6000 Å falling normally on it. At what distance from the edge of the wedge, will the 10 th fringe be observed by reflected light.* (*Punjab 1984*)

Here $$\theta = 0.01\ \text{radian},\ n = 10,$$

$$\lambda = 6000 \times 10^{-8}\ \text{cm}$$

$$2\,t = n\,\lambda$$

But $$\theta = \frac{t}{x}$$

or $$t = \theta\,x$$

$\therefore$ $$2\,\theta\,x = n\,\lambda$$

or
$$x = \frac{n\lambda}{2\theta} = \frac{10 \times 6000 \times 10^{-8}}{2 \times 0.01}$$
$$= \mathbf{0.30\ cm}$$

**Example 8.39**. *Interference fringes are produced with monochromatic light falling normally on a wedge-shaped film of cellophane whose refractive index is 1.40. The angle of the wedge is 10 seconds of an arc and the distance between the successive fringes is 0.5 cm. Calculate the wavelength of light used.*

*(Punjab 1985)*

Here
$$\theta = 10 \text{ seconds of an arc} = \frac{10 \times \pi}{60 \times 60 \times 180} \text{ radian}$$
$$\beta = 0.50 \text{ cm}, \quad \mu = 1.40 \quad \beta = \frac{\lambda}{2\theta\mu}$$
$$\lambda = 2\theta\mu\beta = \frac{2 \times 10 \times 22 \times 1.40 \times 0.50}{60 \times 60 \times 180 \times 7}$$
$$= \mathbf{6790 \times 10^{-8}\ cm = 6790\ Å}$$

**Example 8.40**. *A square piece of cellophane film with index of refraction 1.5 has a wedge shaped section so that its thickness at the two opposite sides are $t_1$ and $t_2$. If with a light of $\lambda$ = 6000 Å, the number of fringes appearing in the film is 10, calculate the difference $t_2 - t_1$.*

*(Agra 1987)*

Here
$$x = 10\beta, \qquad \beta = \frac{x}{10}$$
$$\theta = \frac{t_2 - t_1}{x} \qquad \lambda = 6000 \times 10^{-8} \text{ cm}$$
$$\mu = 1.50$$

But
$$\beta = \frac{\lambda}{2\theta\mu}$$
$$\lambda = 2\theta\mu\beta$$
$$6000 \times 10^{-8} = \frac{2 \times (t_2 - t_1) \times 1.5 \times x}{x \times 10}$$
$$t_2 - t_1 = \frac{6000 \times 10^{-8} \times 10}{2 \times 1.5}$$
$$= \mathbf{2 \times 10^{-4}\ cm}$$

**Example 8.41**. *Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and wavelength of light is 5893 Å. Calculate the angle of the wedge in seconds of an arc.*

Here $\mu = 1.52$ ; $\lambda = 5893 \times 10^{-8}$ cm

$$\beta = 1 \text{ mm} = 0.1 \text{ cm}$$

$$\beta = \frac{\lambda}{2\mu\theta} \quad \theta = \frac{\lambda}{2\mu\beta}$$

or
$$\theta = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \text{ radian}$$

$$\theta = \frac{5893 \times 10^{-8} \times 180 \times 7 \times 60 \times 60}{0.304 \times 22}$$

$$\theta = \mathbf{39.96 \text{ seconds of an arc}}$$

**Example 8.42**. *Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophane of refractive index 1.40. If the angle of the wedge is 20 seconds of an arc and the distance between successive fringes is 0.25* cm, *calculate the wavelength of light.* *(Delhi (sub) 1989)*

Here, $\theta = 20$ seconds of an arc

or
$$\theta = \frac{20 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

$$\beta = 0.25 \text{ cm} \; ; \; \mu = 1.40$$

$$\beta = \frac{\lambda}{2\theta\mu} \quad \text{or} \quad \lambda = 2\theta\mu\beta$$

$$\lambda = \frac{2 \times 20 \times 22 \times 1.40 \times 0.25}{60 \times 60 \times 180 \times 7}$$

$$= 6790 \times 10^{-8} \text{ cm}$$

$$\lambda = \mathbf{6790 \text{ Å}}$$

**Example 8.43**. *A vertical rectangular soap film of total length 12* cm *is illuminated with light of wavelength* $6 \times 10^{-5}$ cm. *Just before the film breaks, there are 12 dark and 11 bright interference bands between the upper and the lower ends. If the refractive index of soap solution is 1.33, find the angle of the wedge so formed and the thickness of the film at the base just before the film breaks.* *[Delhi (Hons.) 1985]*

(*i*) Here $\beta = \dfrac{x}{m} = \dfrac{\lambda}{2\mu\theta}$ ; $x = 1.2$ cm and $m = 11$

$$\lambda = 6 \times 10^{-5} \text{ cm} \; ; \; \mu = 1.33$$

$$\theta = \frac{\lambda m}{2 \mu x} = \frac{6 \times 10^{-5} \times 11}{2 \times 1.33 \times 12}$$

$$= \mathbf{2.0625 \times 10^{-5} \text{ radian.}}$$

(ii)

$$t = x\theta$$

$$x = 12 \text{ cm}$$

$$\theta = 2.0625 \times 10^{-5} \text{ radian.}$$

$$\therefore \quad t = \mathbf{2.475 \times 10^{-4} \text{ cm.}}$$

**Example 8.44**. *A beam of monochromatic light of wavelength $5.82 \times 10^{-7}$ m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per* cm *of the wedge length.* (*IAS, 1987*)

$$\theta = 20 \text{ seconds of an arc}$$

$$= \frac{20 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

$$\lambda = 5.82 \times 10^{-7} \text{ m} \; ; \; \mu = 1.5$$

$$\beta = \frac{\lambda}{2\theta\mu} = \frac{5.82 \times 10^{-7} \times 60 \times 60 \times 180}{2 \times 20 \times \pi \times 1.5}$$

$$= 2 \times 10^{-3} \text{ m} = 0.2 \text{ cm}$$

Number of fringes per cm $= \frac{1}{0.2} =$ **5 per cm**

**Example 8.45**. *Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds, of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light (blue) of wavelength 4800 Å there are 18 bands per* cm. (*Delhi, 1992*)

Fringe width,

$$\beta = \frac{1}{18} \text{ cm} = \frac{1}{1800} \text{ m}$$

$$\beta = \frac{\lambda}{2\theta} \quad \therefore \quad \theta = \frac{\lambda}{2\beta}$$

Here

$$\lambda = 4800 \text{ Å} = 4.8 \times 10^{-7} \text{ m}$$

$$\theta = \frac{4.8 \times 10^{-7} \times 1800}{2 \times 1}$$

$$= 4.32 \times 10^{-4} \text{ radian}$$

$$\theta = \frac{4.32 \times 10^{-4} \times 180 \times 60 \times 60}{3.14} \text{ sec. of an arc}$$

$$= \mathbf{89 \text{ seconds of an arc}}$$

## 8.23 NEWTONS'S RINGS

When a plano-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre

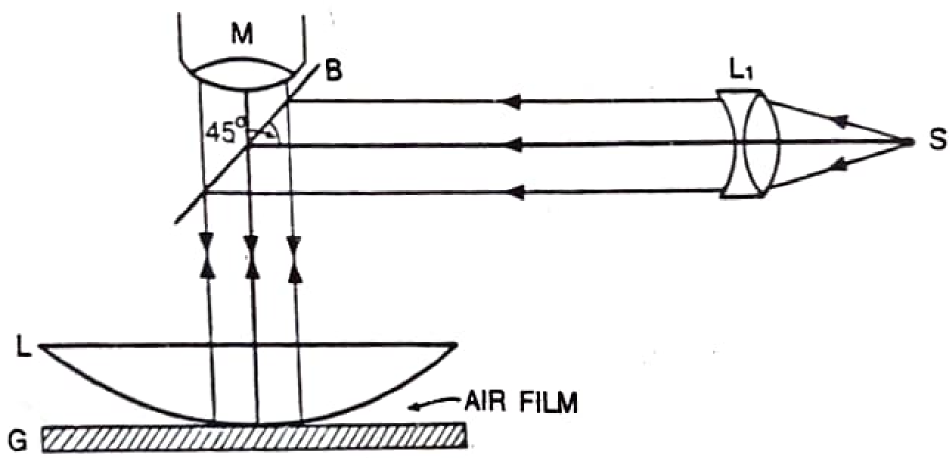



Fig. 8.25

outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are coloured. With monochromatic light, bright and dark circular fringes are produced in the air film.

$S$ is a source of monochromatic light at the focus of the lens $L_1$ (Fig. 8.25). A horizontal beam of light falls on the glass plate $B$ at 45°. The glass plate $B$ reflects a part of the incident light towards the air film enclosed by the lens $L$ and the plane glass plate $G$. The reflected beam from the air film is viewed with a microscope. Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate $G$.

**Theory.** (*i*) **Newton's rings by reflected light**. Suppose the radius of curvature of the lens is $R$ and the air film is of thickness $t$ at a distance of $OQ = r$, from the point of contact $O$.

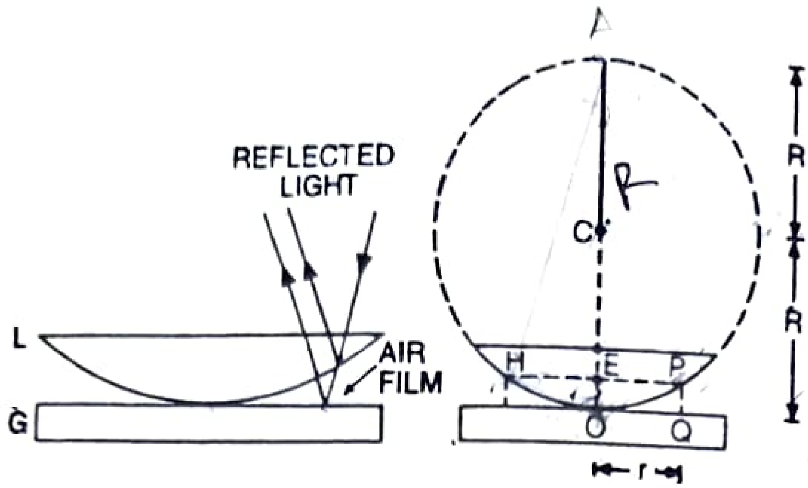



Fig. 8.26

Here, interference is due to reflected light. Therefore, for the bright rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \qquad ...(i)$$

where

$$n = 1, 2, 3 \ldots \text{etc.}$$

Here, θ is small, therefore $\cos\theta = 1$

For air,

$$\mu = 1$$

$$2t = (2n-1)\frac{\lambda}{2} \qquad ...(ii)$$

For the dark rings,

$$2\mu t \cos\theta = n\lambda$$

or

$$2t = n\lambda$$

where

$$n = 0, 1, 2, 3 \ldots \text{etc.} \qquad ...(iii)$$

In Fig. 8.26,

$$EP \times HE = OE \times (2R - OE)$$ 2t

But

$$EP = HE = r, \quad OE = PQ = t$$

and

$$2R - t = 2R \quad \text{(approximately)}$$

$$r^2 = 2R.t$$

10.02×07.000

or
$$t = \frac{r^2}{2R}$$

Substituting the value of $t$ in equations (*ii*) and (*iii*).

For bright rings

$$r^2 = \frac{(2n+1)\lambda R}{2} \quad ...(iv)$$

$$r = \sqrt{\frac{(2n+1)\lambda R}{2}} \quad ...(v)$$

For dark rings,

$$r^2 = n\lambda R \quad ...(vi)$$

$$r = \sqrt{n\lambda R} \quad ...(vii)$$

When $n = 0$, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately dark and bright rings are produced (Fig. 8.27).

**Result**. The radius of the dark ring is proportional to (*i*) $\sqrt{n}$ (*ii*) $\sqrt{\lambda}$ and (*iii*) $\sqrt{R}$. Similarly the radius of the bright ring is proportional to

(*i*) $\sqrt{\frac{2n-1}{2}}$ (*ii*) $\sqrt{\lambda}$ and (*iii*) $\sqrt{R}$.

If $D$ is the diameter of the dark ring,

$$D = 2r = 2\sqrt{n\lambda R} \quad ...(viii)$$

For the central dark ring

$$n = 0$$

$$D = 2\sqrt{n\lambda R} = 0$$

*This corresponds to the centre of the Newton's rings. While counting the order of the dark rings 1, 2, 3, etc. the central ring is not counted.*

Therefore for the first dark ring,

$$n = 1$$

$$D_1 = 2\sqrt{\lambda R}$$

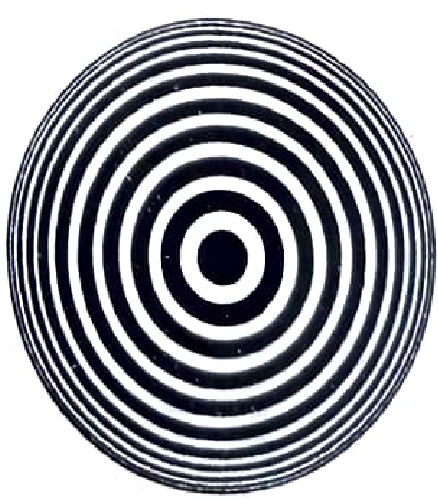


Fig. 8.27.

For the second dark ring, $n = 2$,

$$D_2 = 2\sqrt{2\lambda R}$$

and for the $n$ th dark ring,

$$D_n = 2\sqrt{n\lambda R}$$

Take the case of 16 th and 9 th rings,

$$D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R},$$

$$D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$$

The difference in diameters between the 16 th and the 9 th rings,

$$D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Similarly the difference in the diameters between the fourth and first rings,

$$D_4 - D_1 = 2\sqrt{4\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Therefore, the fringe width decreases with the order of the fringe and the fringes got closer with increase in their order.

For bright rings,

$$r^2 = \frac{(2n-1)\lambda R}{2} \quad ...(ix)$$

or

$$D^2 = 2(2n-1)\lambda R \quad ...(x)$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}} \quad ...(xi)$$

*In equation (ix), substituting $n = 1, 2, 3$ (number of the ring) the radii of the first, second, third etc., bright rings can be obtained directly.*

**(ii) Newton's rings by transmitted light.** In the case of transmitted light (Fig. 8.28), the interference fringes are produced such that for bright rings,

$$2\mu t\cos\theta = n\lambda \quad ....(xii)$$

and for dark rings

$$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2} \quad ...(xiii)$$

Here, for air

$$\mu = 1,$$

and

$$\cos\theta = 1$$

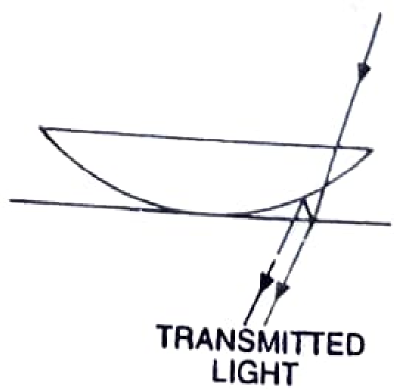



Fig. 8.28

For bright rings,

$$2t = n\lambda$$

and for dark rings $$2t = (2n-1)\frac{\lambda}{2}$$

Taking the value of $t = \frac{r^2}{2R}$, where $r$ is the radius of the ring and $R$ the radius of curvature of the lower surface of the lens, the radius for the bright and dark rings can be calculated.

For bright rings,

$$r^2 = n\lambda R \quad \text{...(xiv)}$$

For dark rings,

$$r^2 = \frac{(2n-1)\lambda R}{2} \quad \text{...(xv)}$$

where $n = 1, 2, 3 \ldots$ etc.

When, $n = 0$, for bright rings

$$r = 0.$$

Therefore, in the case of Newton's rings due to transmitted light, the central ring is bright (Fig. 8.29) *i.e.*, just opposite to the ring pattern due to reflected light.

Fig. 8.29.

**Example 8.46**. *A thin equiconvex lens of focal length 4 metres and reflective index 1.50 rests on and in contact with an optical flat, and using light of wavelength 5460 Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5 th bright ring ?*

The diameter of the $n$ th bright ring is given by

$$D_n = \sqrt{2(2n-1)\lambda R}$$

Here $n = 5, \quad \lambda = 5460 \times 10^{-8}$ cm

$$f = 400 \text{ cm}, \quad \mu = 1.50$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here $R_1 = R, \quad R_2 = -R$

$\therefore$ $$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{400} = (1.50 - 1)\left(\frac{2}{R}\right)$$

$$R = 400 \text{ cm}$$

$$\therefore \quad D_n = \sqrt{2 \times (2 \times 5 - 1) \times 5460 \times 10^{-8} \times 400}$$

$$D_n = \mathbf{0.627 \text{ cm}}$$

## 8.24 DETERMINATION OF THE WAVELENGTH OF SODIUM LIGHT USING NEWTONS'S RINGS

The arrangement used is shown in Fig. 8.25. $S$ is a source of sodium light. A parallel beam of light from the lens $L_1$ is reflected by the glass plate $B$ inclined at an angle of 45° to the horizontal. $L$ is a plano-convex lens of large focal length. Newton's rings are viewed through $B$ by the travelling microscope $M$ focussed on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the $n$ th dark ring.

Suppose, the diameter of the $n$ th ring $= D_n$

$$r_n^2 = n\lambda R$$

But,
$$r_n = \frac{D_n}{2}$$

$$\therefore \quad \frac{(D_n)^2}{4} = n\lambda R$$

or
$$D_n^2 = 4n\lambda R \qquad ...(i)$$

Measure the diameter of the $n+m$ th dark ring.

Let it be $D_{n+m}$

$$\therefore \quad \frac{(D_{n+m})^2}{4} = (n+m)\lambda R$$

or
$$(D_{n+m})^2 = 4(n+m)\lambda R \qquad ...(ii)$$

Subtracting ($i$) from ($ii$)

$$(D_{n+m})^2 - (D_n^2) = 4m\lambda R$$

or
$$\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR} \qquad ...(iii)$$

Hence, $\lambda$ can be calculated. Suppose the diameters of the 5 th ring and the 15 th ring are determined. Then, $m = 15 - 5 = 10$.

$$\therefore \quad \lambda = \frac{(D_{15})^2 - (D_5)^2}{4 \times 10R} \qquad ...(iv)$$

The radius of curvature of the lower surface of the lens is determined with the help of a spherometer but more accurately it is determined by

Boy's method. Hence the wavelength of a given monochromatic source of light can be determined.

**Example 8.47.** *A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8 th dark ring in the transmitted system is 0.72 cm. Calculate the wavelength of light used.*

[*Delhi B.Sc.(Hons) 1986*]

For the transmitted system,

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Here $n = 8, \quad D = 0.72 \text{ cm}, \quad r = 0.36 \text{ cm}$

$R = 300 \text{ cm}, \quad \lambda = ?$

$$\lambda = \frac{2r^2}{(2n-1)R} = \frac{2\times(0.36)^2}{(2\times 8-1)\,300}$$

$$= 5760\times 10^{-8} \text{ cm}$$

or $\lambda = \mathbf{5760\ Å}$

**Example 8.48.** *In a Newton's rings experiment the diameter of the 15 th ring was found to be 0.590 cm and that of the 5 th ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of light used.*

Here $D_5 = 0.336 \text{ cm} \quad D_{15} = 0.590 \text{ cm}.$

$R = 100 \text{ cm} \; ; \; m = 10,$

$$\lambda = \frac{(D_{n+m})^2-(D_n)^2}{4mR} = \frac{D_{15}^2 - D_5^2}{4\times 10\times R}$$

$$\lambda = \frac{(0.590)^2-(0.336)^2}{4\times 10\times 100} = 5880\times 10^{-8} \text{ cm}$$

$$\lambda = \mathbf{5880\ Å}$$

**Example 8.49.** *In a Newton's rings experiment, the diameter of the 5 th ring was 0.336 cm and the diameter of the 15 th ring = 0.590 cm. Find the radius of curvature of the plano-convex lens, if the wavelength of light used is 5890 Å.*

Here $D_5 = 0.336 \text{ cm.}, \; D_{15} = 0.590 \text{ cm},$

and $m = 10, \; \lambda = 5890\times 10^{-8} \text{ cm}, \; R = ?$

$$R = \frac{(D_{n+m})^2-(D_n)^2}{4m\lambda} = \frac{D_{15}^2 - D_5^2}{4\times 10\times \lambda}$$

$$R = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

$$= \mathbf{99.82\ cm}$$

**Example 8.50.** *In a Newton's rings experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength* $5890 \times 10^{-8}$ cm, *the diameter of the third dark ring is* 3.2 mm. *The light is falling at such an angle that it passes through the air film at an angle of zero degree to the normal.*

*[Rajasthan, 1987]*

For dark rings

$$\boxed{r^2 = n\lambda R} \; ; \quad R = \frac{r^2}{n\lambda}$$

Here $$r = \frac{3.2}{2}\ \text{mm} = 1.6\ \text{mm} = 0.16\ \text{cm}$$

$$n = 3\ ;\ \lambda = 5890 \times 10^{-8}\ \text{cm}$$

$$\therefore \quad R = \frac{(0.16)^2}{3 \times 5890 \times 10^{-8}}$$

$$R = \mathbf{144.9\ cm}$$

## 8.25 REFRACTIVE INDEX OF A LIQUID USING NEWTON'S RINGS

The experiment is performed when there is an air film between the plano-convex lens and the optically plane glass plate. These are kept in a metal container $C$. The diameter of the $n$ th and the $(n+m)$ th dark rings are determined with the help of a travelling microscope (Fig. 8.30).

For air, $$(D_{n+m})^2 = 4(n+m)\lambda R\ ; \quad D_n^2 = 4n\lambda R$$

$$D_{n+m}^2 - D_n^2 = 4m\lambda R \qquad ...(i)$$

The liquid is poured in the container $C$ without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the $n$ th ring and the $(n+m)$ th ring are determined.

For the liquid, $2\mu t\cos\theta = n\lambda$ for dark rings

or $$2\mu t = n\lambda. \text{ But, } t = \frac{r^2}{2R}$$

or $$\frac{2\mu r^2}{2R} = n\lambda$$

or $$r^2 = \frac{n\lambda R}{\mu}. \text{ But } r = \frac{D}{2};\ D^2 = \frac{4\,n\lambda R}{\mu}$$

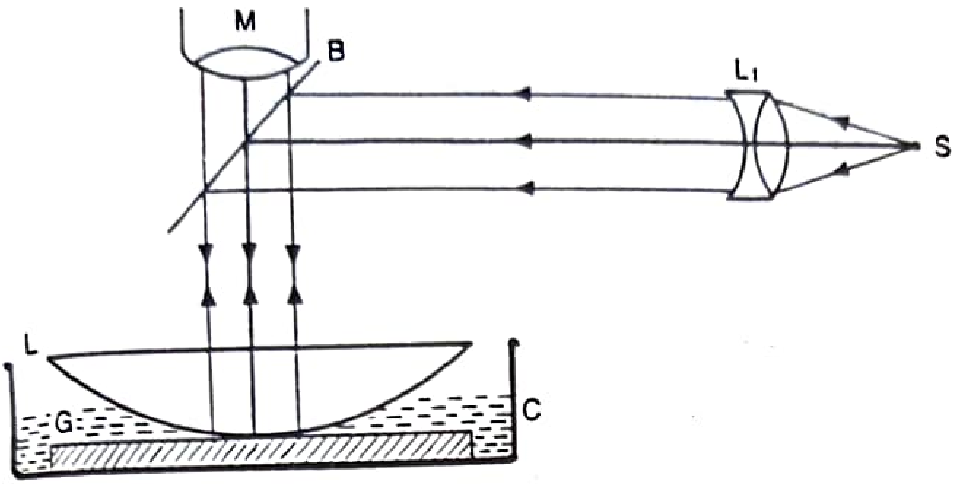



Fig. 8.30

If $D_n'$ is the diameter of the $n$ th ring and $D'_{n+m}$ is the diameter of the $(n+m)$ th ring

then, $$(D'_{n+m})^2 = \frac{4(n+m)\lambda R}{\mu}\ ;\quad (D_n')^2 = \frac{4n\lambda R}{\mu}$$

or $$(D'_{n+m})^2 - (D'_n)^2 = \frac{4m\lambda R}{\mu} \quad ...(ii)$$

or $$\mu = \frac{4m\lambda R}{(D'_{n+m})^2 - (D'^2_n)} \quad ...(iii)$$

If $m, \lambda, R, D'_{n+m}$ and $D'_n$ are known $\mu$ can be calculated. If $\lambda$ is not known, then divide $(iii)$ By $(i)$

$$\mu = \frac{(D_{n+m})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2} \quad ...(iv)$$

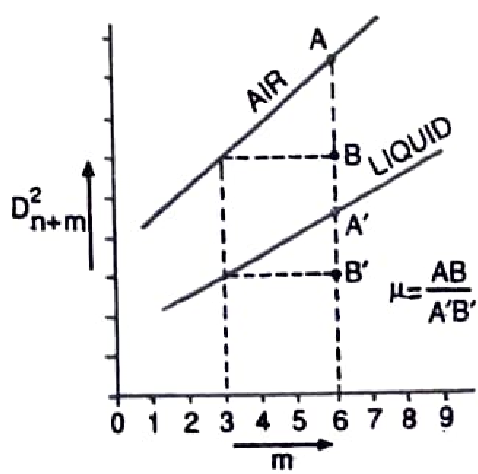



Fig. 8.31

**Graphical method.** The diameters of the dark rings are determined for various orders, varying from the $n$ th ring to the $(n+m)$ th ring, first with air as the medium and then with the liquid. A graph is plotted between $D^2_{n+m}$ along the $y$-axis and $m$ along the $x$-axis, where

$m = 0, 1, 2, 3 \ldots$ etc. The ratio of the slopes of the two lines (air and liquid), gives the refractive index of the liquid.

$$\mu = \frac{AB}{A'B'}$$

**Example 8.51**. *In a Newton's rings experiment the diameter of the 10 th ring changes from 1.40 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.*

*(Nagpur 1985)*

For liquid medium $D_1^2 = \dfrac{4n\lambda R}{\mu}$ ...(*i*)

For air medium $D_2^2 = 4n\lambda R$ ...(*ii*)

Dividing (*ii*) by (*i*)

$$\mu = \left(\frac{D_2}{D_1}\right)^2$$

Here $D_1 = 1.27$ cm, $D_2 = 1.40$ cm

$$\therefore \quad \mu = \left(\frac{1.40}{1.27}\right)^2 = \mathbf{1.215}$$

**Example 8.52.** *In a Newton's rings arrangement, if a drop of water ($\mu$ = 4/3) be placed in between the lens and the plate, the diameter of the 10 th ring is found to be 0.6 cm. Obtain the radius of curvaiure of the face of the lens in contact with the plate. The wavelength of light used is 6000 Å.*

*(Delhi 1983)*

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad \text{or} \quad R = \frac{\mu D_n^2}{4n\lambda}$$

Here $\mu = \dfrac{4}{3}$, $D_n = 0.6$ cm

$$n = 10, \; \lambda = 6000 \text{ Å} = 6 \times 10^{-5} \text{ cm}$$

$$R = ?$$

$$R = \frac{4 \times (0.6)^2}{3 \times 4 \times 10 \times 6 \times 10^{-5}}$$

$$= \mathbf{200 \text{ cm}}$$

**Example 8.53**. *Newton's rings are formed by reflected light of wave-length 5895 Å with a liquid between the plane and curved surfaces. If the diameter of the 5 th bright ring is 3* mm *and the radius of curvature of the curved surface is 100* cm, *calculate the reflective index of the liquid.*
(*Gorakhpur 1986*)

Here, for the $n$ th bright ring,

$$\mu = \frac{(2n-1)\ \lambda R}{2r^2}$$

Here $n = 5$, $\lambda = 5895 \times 10^{-8}$ cm, $R = 100$ cm, $r = \frac{3}{2}$ mm $= 0.15$ cm

$\mu = ?$

$$\mu = \frac{(2 \times 5 - 1) \times 5895 \times 10^{-8} \times 100}{2\,(0.15)^2}$$

$$\mu = \mathbf{1.179}$$

**Example 8.54**. *In a Newton's rings experiment the diameter of the 15 th ring was found to be 0.590* cm *and that of the 5 th ring was 0.336* cm. *If the radius of the plano-convex lens is 100 cm, calculate the wave-length of light used.* (*Delhi ; 1984*)

Here $D_5 = 0.336 \text{ cm} = 3.36 \times 10^{-3}$ m

$$D_{15} = 0.590 \text{ cm} = 5.90 \times 10^{-3} \text{ m}$$

$$R = 100 \text{ cm} = 1 \text{ m}, \quad \lambda = ?$$

$$\lambda = \frac{(D_{n+m})^2 - D_n^2}{4m\,R} = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times R}$$

$$\lambda = \frac{(5.9 \times 10^{-3})^2 - (3.36 \times 10^{-3})^2}{4 \times 10 \times 1}$$

$$= \mathbf{5.880 \times 10^{-7}\ m}$$

$$\lambda = \mathbf{5880\ Å}$$

**Example 8.55.** *In a Newton's rings experiment the diameter of the 12 th ring changes from 1.50* cm *to 1.35* cm *when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.*
(*Delhi 1990*)

For liquid medium

$$D_1^2 = \frac{4\,n\,\lambda\,R}{\mu} \qquad ...(i)$$

For air medium

$$D_2^2 = 4\,n\,\lambda\,R \qquad ...(ii)$$

Dividing (*ii*) by (*i*)

$$\mu = \left(\frac{D_2}{D_1}\right)^2$$

Here

$$D_1 = 1.35 \text{ cm}$$
$$D_2 = 1.50 \text{ cm}$$

$$\mu = \left(\frac{1.50}{1.35}\right)^2$$

$$\mu = \mathbf{1.235}$$

**Example 8.56.** *Newton's rings are observed in reflected light of* $\lambda = 5.9 \times 10^{-5}$ cm. *The diameter of the 10 th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the air film.*

*(Delhi, 1991)*

(*i*) Here.

$$r^2 = n\lambda R$$

$$\lambda = 5.9 \times 10^{-5} \text{ cm} = 5.9 \times 10^{-7} \text{ m}$$

$$n = 10$$

$\therefore$

$$R = \frac{(2.5 \times 10^{-3})^2}{10 \times 5.9 \times 10^{-7}}$$

$$R = \mathbf{1.059 \ m}$$

(*ii*) Thickness of the air film $= t$

$$2t = n\lambda$$

$$t = \frac{n\lambda}{2}$$

$$= \frac{10 \times 5.9 \times 10^{-7}}{2}$$

$$t = \mathbf{2.95 \times 10^{-6} \ m}$$

## 8.26 NEWTON'S RINGS FORMED BY TWO CURVED SURFACES

Consider two curved surfaces of radii of curvature $R_1$ and $R_2$ in contact at the point $O$. A thin air film is enclosed between the two surfaces (Fig. 8.32). The dark and bright rings are formed and can be viewed with a travelling microscope. Suppose the radius of the $n$ th dark ring $= r$. The thickness of the air film at $P$, is

$$PQ = PT - QT$$

From geometry,

$$PT = \frac{r^2}{2R_1}$$

and $$QT = \frac{r^2}{2R_2}$$

$$\therefore \quad PQ = \frac{r^2}{2R_1} - \frac{r^2}{2R_2}$$

But $$PQ = t$$

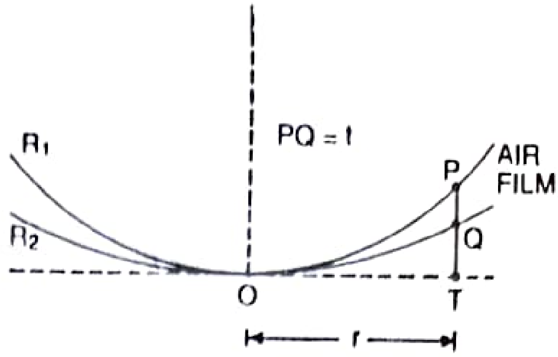



Fig. 8.32.

For reflected light,

$$2\mu t \cos\theta = n\lambda, \text{ for dark rings.}$$

Here, for air $$\mu = 1$$

$$\cos\theta = 1$$

$$2t = n\lambda$$

or $$2\left(\frac{r^2}{2R_1} - \frac{r^2}{2R_2}\right) = n\lambda$$

$$r^2\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = n\lambda \quad ...(i)$$

where $$n = 0, 1, 2, 3 \ldots \text{etc.}$$

For bright rings,

$$2\mu t \cos\theta = \frac{(2n+1)\lambda}{2}$$

Taking $$\mu = 1$$

and $$\cos\theta = 1$$

$$2t = \frac{(2n+1)\lambda}{2}$$

or $$r^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(2n+1)\lambda}{2} \quad ...(ii)$$

where $n = 0, 1, 2, 3 \ldots$ etc.

For the 10 th bright ring, the value of $n = 10 - 1 = 9$

$\therefore$ For $n$ th bright ring,

$$r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left[\frac{[2(n-1)+1]\lambda}{2}\right] = \frac{(2n-1)\lambda}{2} \quad ...(iii)$$

**Special Case.** When the lower surface as seen from above is convex (Fig. 8.33).

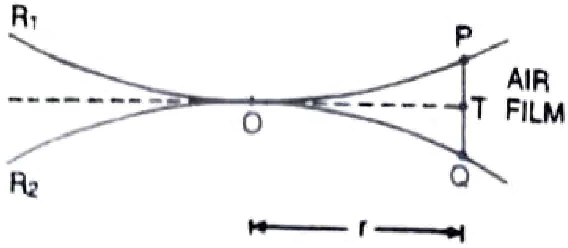



Fig. 8.33.

$$PQ = PT + QT$$

$$= \frac{r^2}{2R_1} + \frac{r^2}{2R_2}$$

For dark rings,

$$2PQ = n\lambda$$

$$r^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = n\lambda \quad ...(iii)$$

For bright rings,

$$2PQ = (2n+1)\frac{\lambda}{2}$$

$$r^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (2n+1)\frac{\lambda}{2}$$

When $n = 0, 1, 2, 3 \ldots$ etc.

For the first bright ring, $n = 0$

$$\therefore \quad r_1^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\lambda}{2}$$

For the 10 th bright ring, $n = 9$

$$r_{10}^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = [2(9)+1]\frac{\lambda}{2}$$

For the $n$ th bright ring

$$r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = [2(n-1)+1]\frac{\lambda}{2}$$

$$r_n^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{(2n-1)\lambda}{2} \quad ...(iv)$$

**Example 8.57.** *A convex surface of radius 300 cm of a planoconvex lens rests on a concave spherical surface of radius 400 cm and Newton's rings are viewed with reflected light of wavelength $6 \times 10^{-5}$ cm. Calculate the diameter of the 13 th bright ring.*

Here, $R_1 = 300$ cm, $R_2 = 400$ cm, $n = 13$, $\lambda = 6 \times 10^{-5}$ cm

For the bright ring,

$$r_n^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(2n-1)\lambda}{2}$$

$$r_{13}^2\left(\frac{1}{300}-\frac{1}{400}\right)=\frac{25}{2}\times 6\times 10^{-5}$$

or $$r_{13} = 0.95 \text{ cm}$$

$\therefore$ Diameter of the 13 th bright ring $D_{13} = 2r_{13} = 1.90$ cm

**Example 8.58**. *If in the example 8.57 the plano-convex lens rests on a convex spherical surface (the other data remaining the same), calculate the diameter of the 11 th bright ring.*

Here, $R_1 = 300$ cm, $R_2 = 400$ cm, $n = 11$, $\lambda = 6\times 10^{-5}$ cm

For the bright ring,

$$r_n^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{(2n-1)\lambda}{2}$$

$$r_{11}^2\left(\frac{1}{300}+\frac{1}{400}\right)=\frac{21\times 6\times 10^{-5}}{2}$$

$$r_{11} = 0.33 \text{ cm}$$

$\therefore$ Diameter of the 11th bright ring

$$= 2\,r_{11} = \mathbf{0.66\ cm}$$

## 8.27 NEWTON'S RINGS WITH BRIGHT CENTRE DUE TO REFLECTED LIGHT

The rings formed by reflected light have a dark centre when there is an air film between the lens and the plane glass plate. At the centre, the two surfaces are just in contact but the two interfering rays are reflected under different conditions due to which a path difference of half a wavelength occurs (since one of the rays undergoes a phase change of $\pi$, when reflected from the glass plate).

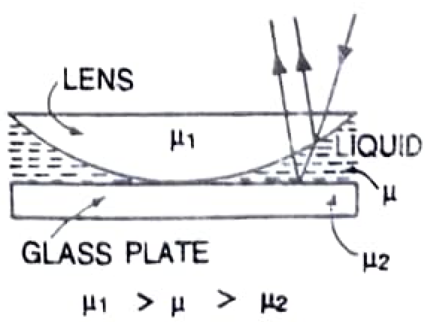


Fig. 8.34.

Consider a transparent liquid of refractive index $\mu$ trapped between the two surfaces in contact (Fig. 8.34). The refractive index of the material of the lens is $\mu_1$ and that of the glass plate is $\mu_2$ such that $\mu_1 > \mu > \mu_2$. This is possible if a little oil of sassafaras is placed between a convex lens of crown glass and a plate of flint glass. The reflections in both the cases will be from denser to rarer medium and the two interfering rays are reflected under the same conditions. Therefore, in this case the central spot will be **bright.**

The diameter of the $n$ th bright ring,

$$D_n = 2\sqrt{\frac{n\lambda R}{\mu}}$$

The central spot will also be bright, if $\mu_1 < \mu < \mu_2$, because a path difference of $\frac{\lambda}{2}$ takes place at both the upper and the lower glass-liquid surfaces. Here again the two interfering beams are reflected under similar conditions. In this case also the central spot is bright due to reflected light.

## 8.28 NEWTON'S RINGS WITH WHITE LIGHT

With monochromatic light, Newton's rings are alternately dark and bright. The diameter of the ring depends upon the wavelength of light used. When white light is used, the diameter of the rings of the different colours will be different and coloured rings are observed. Only the first few rings are clear and after that due to overlapping of the rings of different colours, the rings cannot be viewed.

**Example 8.59.** *Light containing two wavelengths $\lambda_1$ and $\lambda_2$ falls normally on a plano-convex lens of radius of curvature R resting on a glass plate. If the n th dark ring due to $\lambda_1$, coincides with the (n + 1) th dark ring due to $\lambda_2$, prove that the radius of the n th dark ring of $\lambda_1$ is*

$$= \sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1 - \lambda_2}}$$

*(Mysore 1971)*

The radius of the $n$ th dark ring due to $\lambda_1$

$$= \sqrt{n\lambda_1 R} \quad \text{...(i)}$$

The radius of the $(n + 1)$ th dark ring due to $\lambda_2$

$$= \sqrt{(n+1)\lambda_2 R} \quad \text{...(ii)}$$

As (*i*) and (*ii*) are equal

$$r = \sqrt{n\lambda_1 R} = \sqrt{(n+1)\lambda_2 R} \quad \text{...(iii)}$$

$\therefore$

$$n\lambda_1 R = (n+1)\lambda_2 R$$

or

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Substituting the value of $n$ in equation (*iii*)

$$r = \sqrt{n\lambda_1 R} = \sqrt{\frac{\lambda_1 \lambda_2 R}{\lambda_1 - \lambda_2}}$$

**Example 8.60.** *Newton's rings arrangement is used with a source emitting two wavelengths $\lambda_1$ and $\lambda_2$. It is found that the n th dark ring due to $\lambda_1$ coincides with (n + 1) th dark ring due do $\lambda_2$. Find the diameter of the n th dark ring for wavelength $\lambda_1$ given $\lambda_1 = 6 \times 10^{-5}$ cm, $\lambda_2 = 5.9 \times 10^{-5}$ cm and radius of curvature of the lens is 90 cm.*

(*Gorakhpur 1967*)

$$d = 2r = 2 \times \sqrt{\frac{\lambda_1 \lambda_2 R}{\lambda_1 - \lambda_2}}$$

$$d = 2 \times \sqrt{\frac{6 \times 10^{-5} \times 5.9 \times 10^{-5} \times 90}{(6 - 5.9) \times 10^{-5}}}$$

$$d = \mathbf{1.129 \text{ cm}}$$

## 8.29 HAIDINGER'S FRINGES

In the relation $2\,\mu t \cos r = n\lambda$, if $t$ is large, a very small change in $r$ will change the path difference by one wavelength. In this case the ray must pass through a plate as a parallel beam and must be received by the eye or the telescope focussed for infinity. The interference patterns are known as fringes of equal inclination. These are different from Newton's rings. These fringes of equal inclination were first observed by Haidinger and afterwards studied by Lummer and Mascart. From an extended source $S$, light rays fall on the plate. The rays striking

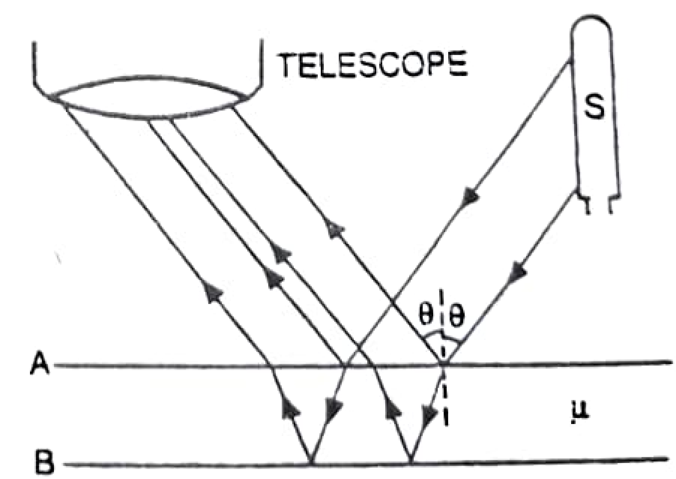



Fig. 8.35.

at the same angle and refracted at the same angle form a parallel beam and are viewed through the telescope focussed for infinity (Fig. 8.35). The pattern is a series of concentric circles whose centre is the principal focus