MATH-2203

Wednesday
Date- 03.05.2023

Course teacher - Prof. Dr. Humaira Farzana
Dept A&S (Mathematics)

Scalar রাশি $\leftarrow$ $\boxed{W = \overline{F} \cdot \overline{r}}$ $\downarrow$ workdone

$\boxed{\overline{M} = \overline{F} \times \overline{r}}$ $\downarrow$ Moment

$|\overline{M}| \longrightarrow$ torque (Scalar)

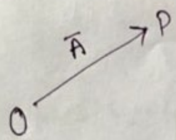
velocity $\rightarrow$ vector
Speed $\rightarrow$ Scalar

$\overline{v} = \overline{\omega} \times \overline{r}$

$\overline{v}$ $\rightarrow$ linear velocity
$\overline{\omega}$ $\rightarrow$ angular velocity
$\overline{r}$ $\rightarrow$ displacement vector

Scalar & Vectors

vectors $\longrightarrow$ $\overline{A}$ or $\overrightarrow{A}$ or $\mathbf{A}$



$\overline{A} = \overline{OP} = \overline{a}$

$-\overline{A} = \overline{PO}$

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Unit vector = $\frac{\bar{A}}{|\bar{A}|} = \hat{a}$ $\boxed{\wedge \rightarrow \text{Cap}}$

Any vector

Any vector, $\bar{A} = \hat{a}\,|\bar{A}|$

Magnitude of $\bar{A} \longrightarrow |\bar{A}| = a$

$$\begin{array}{ccc} x & y & z \\ \uparrow & \uparrow & \uparrow \\ \end{array}$$
$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

row vector

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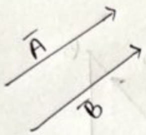


* Parallel vectors →* magnitude same or opposite হতে পারে

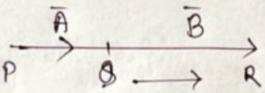
$\bar{A} = k\bar{B}$ (for Parallel vector)

$\bar{A} = \frac{1}{2}\bar{B}$

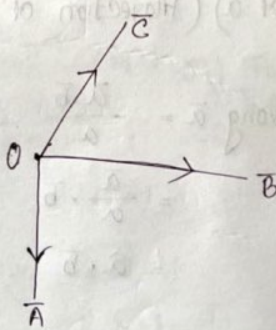
$\bar{A} = -\frac{1}{2}\bar{B}$

$\bar{A} = 2\bar{B}$

* equal vectors →* both magnitude and direction same



Co-linear vectors (lie on the same line)



Co-initial vectors (O is the initial point)

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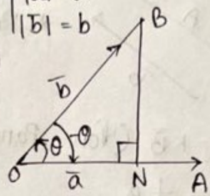
Scalar or Dot product of two products $\bar{A}$ & $\bar{B}$

$$\bar{a}\cdot\bar{b} = ab\cos\theta$$

$|\bar{a}| = a$
$|\bar{b}| = b$



Symmetry: $\bar{b}\cdot\bar{a} = ba\cos(-\theta) = ab\cos\theta$
$= \bar{a}\cdot\bar{b}$

$$\boxed{\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}}$$

Geometrical interpretation,

$\overline{OA} = \bar{a}$
$\overline{OB} = \bar{b}$

$$\bar{a}\cdot\bar{b} = (OA)(OB)\cos\theta$$

$$\Rightarrow \bar{a}\cdot\bar{b} = OA\, OB\, \frac{ON}{OB}$$

$$\Rightarrow \bar{a}\cdot\bar{b} = (OA)(ON)$$

$\Rightarrow \bar{a}\cdot\bar{b}$ = (length of $\bar{a}$) (Projection of $\bar{b}$ along $\bar{a}$)

$\Rightarrow$ Projection of $\bar{b}$ along $\bar{a}$ $= \dfrac{\bar{a}\cdot\bar{b}}{a}$

$$= \frac{\bar{a}}{a}\cdot\bar{b}$$

$$= \hat{a}\cdot\bar{b}$$

$\therefore$ Projection of $\bar{b}$ along $\bar{a}$ $= \bar{b}\cdot\hat{a}$ → Unit vector (along)

Similar → Projection of $\bar{a}$ along $\bar{b}$ $= \bar{a}\cdot\hat{b}$

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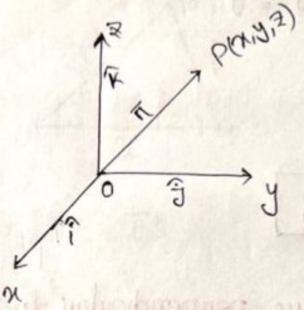
Position vector with respect to the origin -



$$\boxed{\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

↓

Position vector

* x, y, z এর Unit vector কে $\hat{i}, \hat{j}, \hat{k}$ দিয়ে প্রকাশ করা হয়।

* Position vector must be Passing through the origin.

* Any vector origin দিয়ে Pass হবে না।

$O(0,0,0)$, $P(1,2,3)$

$$\bar{r} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Useful result :- (If $\theta = 0°$) Parallel

$$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0°$$
$$= 1.1.1 = 1$$

Similarly, $\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

$$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}}$$

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If $\theta = 90^\circ$ (Perpendicular)

$$\hat{i} \cdot \hat{j} = 1 \cdot 1 \cos 90^\circ = 0$$

$$\hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{i} = 0$$

$$\boxed{\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0}$$

* if any two vectors are perpendicular then there dot product always 0.

* if any two vectors are parallel then there dot product is 1.

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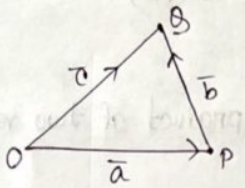
MATH 2203

Wednesday

Date- 10.05.2023

Triangle law:-

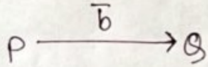


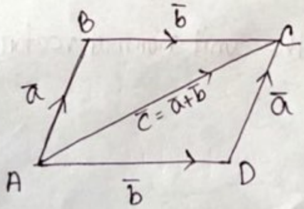
* One vectors initial is another vectors terminal point.

$$\overline{OP} + \overline{PQ} = \overline{OQ}$$

$$\Rightarrow \bar{a} + \bar{b} = \bar{c}$$

Sum or Resultant of two vectors:-





Given, $\overline{AB} = \bar{a}$

$\overline{PQ} = \bar{b}$

Draw $\overline{BC} = \overline{PQ} = \bar{b}$ from the terminal point A, then $\overline{AC} = \bar{c}$ directed from initial point of $\bar{a}$ to the

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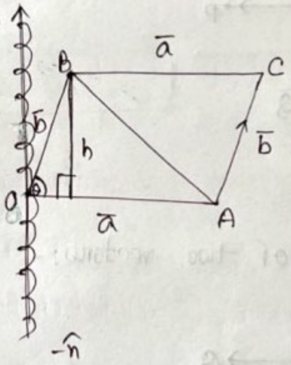
terminal point of $\bar{b}$ & joining the initial of $\bar{a}$ to the terminal of $\bar{b}$ to get $\bar{c}$ ($\overline{AC}$)

vector product or cross product of two vectors :-



$$\bar{c} = \bar{a} \times \bar{b}$$

$$\hat{n} = \frac{\bar{c}}{|\bar{c}|}$$ * $\hat{n}$ is the unit normal vector.

$$\bar{a} \times \bar{b} = \hat{n}\,(ab \sin\theta)$$

magnitude :- $|\bar{a} \times \bar{b}| = ab \sin\theta$

$\therefore |\hat{n}| = 1$

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$$\bar{a}\times\bar{b} = \hat{n}\,(b\sin\theta)\,a$$
$$= \hat{n}\,(ah)$$

$\sin\theta = \frac{h}{b}$

$\Rightarrow h = b\sin\theta$

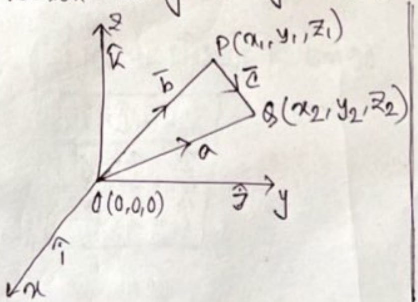
= vector area of the parallelogram OACB.

$|\bar{a}\times\bar{b}| = |\hat{n}\,(ah)| = |\hat{n}|\,|ah| = ah$ = area of the parallelogram OACB.

$= 2$ (area of ΔOAB)

$\Rightarrow$ area of the ΔOAB formed by the two adjacent sides $\bar{a}, \bar{b}$ = $\boxed{\frac{1}{2}|\bar{a}\times\bar{b}|}$

\# Position vector using triangle law :-



$O(0,0,0)$, $P(x,y,z)$

$$\overline{OP} = \bar{P} - \bar{O}$$
$$= (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k}$$
$$= x\hat{i} + y\hat{j} + z\hat{k}$$

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$$\overline{OP} + \overline{PQ} = \overline{OQ}$$

$$\bar{b} + \bar{c} = \bar{a}$$

$$\bar{c} = \bar{a} - \bar{b} = \overline{PQ}$$

$$\overline{PQ} = \overline{OQ} - \overline{OP}$$

if two vectors are given

$$= \boxed{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$

* $A(1, 2, 3)$, $B(-1, 0, 5)$

$$\overline{AB} = \bar{B} - \bar{A}$$

$$= (-1-1)\hat{i} + (0-2)\hat{j} + (5-3)\hat{k}$$

$$= -2\hat{i} - 2\hat{j} + 2\hat{k}$$

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Thrusday
Date- 11.05.2023

$\bar{a} \times \bar{b} = \hat{n}\ (ab\sin\theta)$

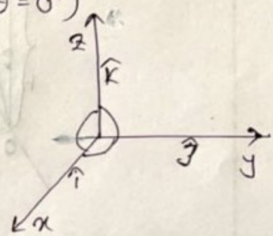
$\bar{b} \times \bar{a} = \hat{n}\ ab\sin(-\theta) = -\hat{n}\ ab\sin\theta = -(\bar{a} \times \bar{b})$

$\boxed{\bar{b} \times \bar{a} = -(\bar{a} \times \bar{b})}$ useful properties

(1) For parallel unit vectors $(\theta = 0°)$

$\hat{i} = \hat{j} \quad (\theta = 0)$

$\boxed{\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0}$

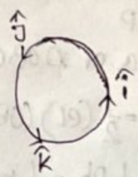


(2) For perpendicular unit vectors $(\theta = 90°)$

$\hat{i} \times \hat{j} = |\hat{i}|\,|\hat{j}|\ \overset{\rightarrow \hat{n}}{\hat{k}}\ \sin 90°$

$$\boxed{\begin{array}{c} \hat{i} \times \hat{j} = \hat{k} \\ \hline \hat{k} \times \hat{i} = \hat{j} \\ \hat{j} \times \hat{k} = \hat{i} \end{array}}$$

→ Anti clock wise

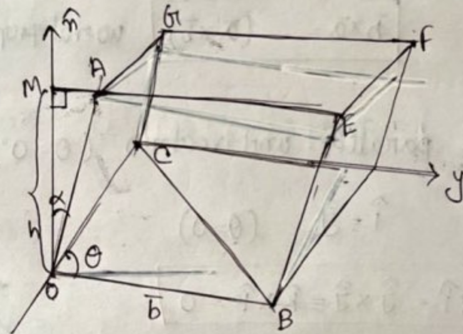


* clockwise হলে sign negative

Scalar triple product of 3 vectors $\bar{a}, \bar{b}, \bar{c}$

Let $\bar{a}, \bar{b}, \bar{c}$ be the co-terminous edges

$\bar{a}, \hat{n}$ lie on the same side of $\bar{b}, \bar{c}$



now $\bar{a}.(\bar{b}\times\bar{c}) = \bar{a}(bc\sin\theta)\hat{n} = \bar{a}.\hat{n}(bc\sin\theta)$

$\rightarrow a\cos\alpha\,(bc\sin\theta)$

$= |\bar{a}||\hat{n}|\,\cos\alpha(bc\sin\theta) = h(bc\sin\theta)$

$= 2\left(\frac{1}{2}bc\sin\theta\right)$

$= 2(\text{area of the tringle } \Delta OBC)\times h$

= volume of the parallelopiped formed by vectors $\bar{a}, \bar{b}, \bar{c}$ as its co-terminous edges.

CL = P
area of ΔOBC

$= \frac{1}{2}(CL)(OB)$

$= \frac{1}{2}Pb = \frac{1}{2}bc\sin\theta$

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* If $\bar{a}\cdot(\bar{b}\times\bar{c})=0$ then $\bar{a},\bar{b},\bar{c}$ are coplanar that means they are on the same plane.

Condition for co-planarity of 3 vectors.

* HKDas Book → example (15, 16, 17, 23, 24, 22, 26-31)

exercise - 5.3

vector triple product (Page - 377)

$$\bar{a}\times(\bar{b}\times\bar{c})$$

$\bar{a}=\hat{i}a_1+\hat{j}a_2+\hat{k}a_3$

$\bar{b}=\hat{i}b_1+\hat{j}b_2+\hat{k}b_3$

$\bar{c}=\hat{i}c_1+\hat{j}c_2+\hat{k}c_3$

$$\bar{b}\times\bar{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$=\hat{i}($

$$\boxed{* \ \bar{a}\times(\bar{b}\times\bar{c})=\bar{b}(\bar{a}\cdot\bar{c})-\bar{c}(\bar{a}\cdot\bar{b})}$$

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Example- 15 Find the projection of the vector

$$\underbrace{\hat{i}-2\hat{j}+\hat{k}}_{a} \text{ on } \underbrace{4\hat{i}-4\hat{j}+7\hat{k}}_{b}$$

Projection of $\bar{a}$ on $\bar{b}$ = $a \cdot \hat{b}$

$$= (\hat{i}-2\hat{j}+\hat{k}) \cdot \left(\frac{4\hat{i}-4\hat{j}+7\hat{k}}{\sqrt{4^2+4^2+7^2}}\right)$$

$$= \frac{1}{9}(4+8+7)$$

$$= \frac{19}{9} \text{ (Ans:)}$$

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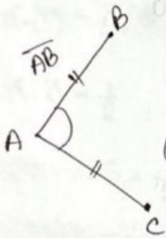
Monday

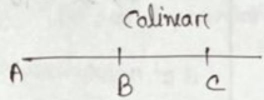
Course - MATH 2203 Date- 15.05.2023

Ex-29 (Book)

① Determine λ & μ by using vectors such that the points $A(-1,3,2)$, $B(-4,2,-2)$, $C(5,\lambda,\mu)$ lie on the straight line.



non colinear (not on the same line)



* when $\theta = 0$, they are parallel

Take, $\overline{AB} \times \overline{AC} = 0$

$$\overline{AB} = \overline{B} - \overline{A} = (-4+1)\hat{i} + (2-3)\hat{j} + (-2-2)\hat{k}$$
$$= -3\hat{i} - \hat{j} - 4\hat{k}$$

$$\overline{AC} = \overline{C} - \overline{A} = (5+1)\hat{i} + (\lambda-3)\hat{j} + (\mu-2)\hat{k}$$
$$= 6\hat{i} + (\lambda-3)\hat{j} + (\mu-2)\hat{k}$$

$\overline{AB} \times \overline{AC} = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & -4 \\ 6 & \lambda-3 & \mu-2 \end{vmatrix} = 0$$

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$= \hat{i}(-\mu+2+4\lambda-12) + \hat{j}(-3\mu+6+24) + \hat{k}(-3\lambda+9+6) = 0$

$= \hat{i}(-\mu+4\lambda-10) - \hat{j}(-3\mu+30) + \hat{k}(-3\lambda+15) = 0$

$-3\mu+30=0$	$-3\lambda+15=0$
$\Rightarrow \mu = 10$	$\Rightarrow \lambda = 5$

Ex-28

Given, $\bar{A}, \bar{B}$

$$\bar{A} = 2\hat{i}-3\hat{j}+\hat{k}, \quad \bar{B} = 7\hat{i}-5\hat{j}+\hat{k}$$

Find a unit vector perpendicular to both vectors.

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 7 & -5 & 1 \end{vmatrix}$$

$$= \hat{i}(-3+5) - \hat{j}(2-7) + \hat{k}(-10+21)$$

$$= 2\hat{i}+5\hat{j}+11\hat{k}$$

$$\hat{n} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|} = \frac{2\hat{i}+5\hat{j}+11\hat{k}}{\sqrt{2^2+(-5)^2+(11)^2}} = \frac{2\hat{i}+5\hat{j}+11\hat{k}}{5\sqrt{6}}$$

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Ex-31

$\bar{a} \times (\bar{b} \times \bar{c}) = \frac{1}{2}\bar{b}$ find the angle by which $\bar{a}$ makes with $\bar{b}$ & $\bar{c}$, $\bar{b}, \bar{c}$ being non parallel unit vector.

$\bar{b}.(\bar{a}.\bar{c}) - \bar{c}(\bar{a}.\bar{b}) = \frac{1}{2}\bar{b} - \bar{c}.0$

$\Rightarrow \bar{a}.\bar{c} = \frac{1}{2}$

$\Rightarrow |\bar{a}||\bar{c}| \cos\theta = \frac{1}{2}$

$\Rightarrow 1.1 \cdot \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

$\bar{a}.\bar{b} = 0$ (Dot product perpendicular so 0)

$\bar{a}$ is perpendicular to $\bar{b}$

$\theta = \frac{\pi}{2}$

Ex-42

$\bar{\alpha} = 5\bar{a} + 6\bar{b} + 7\bar{c}$

$\bar{\beta} = 7\bar{a} - 8\bar{b} + 9\bar{c}$

$\bar{\gamma} = 3\bar{a} + 20\bar{b} + 5\bar{c}$

Show that the vectors are co-planar. $\bar{a}, \bar{\beta}$ & $\bar{\gamma}$ are three non-colinear vectors.

not parallel

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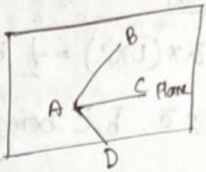
Condition

$\bar{\alpha}\cdot(\bar{\beta}\times\bar{\gamma}) = 0 \Rightarrow \bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are coplanar vectors



$\bar{a} = \hat{i}a_1 + \hat{j}a_2 + \hat{k}a_3$

$\bar{b} = \hat{i}b_1 + \hat{j}b_2 + \hat{k}b_3$

$\bar{c} = \hat{i}c_1 + \hat{j}c_2 + \hat{k}c_3$

$$\bar{a}\cdot(\bar{b}\times\bar{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\bar{\alpha}\cdot(\bar{\beta}\times\bar{\gamma}) = \begin{vmatrix} 5 & 6 & 7 \\ 7 & -8 & 9 \\ 3 & 20 & 5 \end{vmatrix} = 0$$

$$= 5(-40 - 180) - 6(35 - 27) + 7(140 + 24)$$

$$= 0$$

As $\bar{\alpha}\cdot(\bar{\beta}\times\bar{\gamma}) = 0$ then $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are coplanar vectors.

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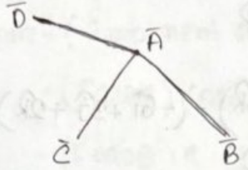
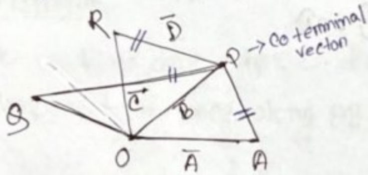
Wednesday

Course - MATH 2203 Date - 17.05.2023



$$\left.\begin{aligned} \bar{a} &= \bar{A} \\ \bar{b} &= \bar{B} \\ \bar{c} &= \bar{C} \\ \bar{d} &= \bar{D} \end{aligned}\right\} \text{4 Points given}$$

* 4 Position vectors are given prove that they are coplanar.



$\overline{PA}$, $\overline{PQ}$, $\overline{PR}$ show that $\overline{PA} \cdot (\overline{PQ} \times \overline{PR}) = 0$

$\underbrace{\bar{B}-\bar{A}}_{\bar{\alpha}}$ $\underbrace{\bar{B}-\bar{C}}_{\bar{\beta}}$ $\underbrace{\bar{B}-\bar{D}}_{\bar{\gamma}}$

$\bar{\alpha} \cdot (\bar{\beta} \times \bar{\gamma}) = 0$

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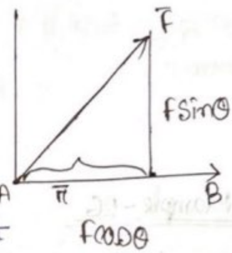
Workdone as a scalar product

workdone = (Component of $\bar{F}$ along $\bar{AB}$) (distance)

$= (F\cos\theta)(AB)$

$= (F\cos\theta)\, r$

$= Fr\cos\theta$



$(F\cos\theta).r = \bar{F}.\bar{r}$

both are magnitude

Work done = Force . displacement

Component of force along AB =

$$F\cos\theta = \frac{\bar{F}.\bar{r}}{r} = \bar{F}.\frac{\bar{r}}{r} = \bar{F}.\hat{r}$$

Example -25

$\bar{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$

$\bar{Q} = -\hat{i} + 2\hat{j} - \hat{k}$, act on a particle and displaces it from A to B the position vector of $\bar{A}$ & $\bar{B}$ are $4\hat{i} - 3\hat{j} - 2\hat{k}$ and $6\hat{i} + \hat{j} - 3\hat{k}$ resp. find the work done.

Total force, $\bar{F} = \bar{P} + \bar{Q}$

$= \hat{i} - 3\hat{j} + 5\hat{k}$

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Displacement vector, $\overline{AB} = \bar{r} = \bar{B} - \bar{A}$

$$= 2\hat{i} + 4\hat{j} - \hat{k}$$

workdone $= \bar{F}\cdot\bar{r} = (\hat{i} - 3\hat{j} + 5\hat{k})\cdot(2\hat{i} + 4\hat{j} - \hat{k})$

$$= 2 - 12 - 5$$

$$= -15$$

$$= 15 \text{ (Ans:)}$$

Example - 26

Magnitude of forces = 3kg & 5kg along the 2 directions

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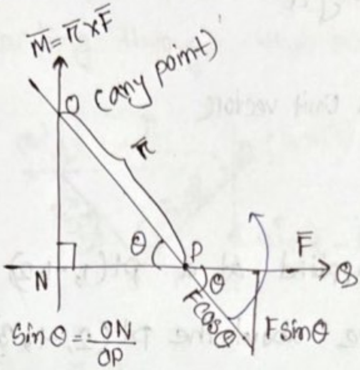
Thursday

Course - MATH2203 Date- 18.05.2023

Moment



$\sin\theta = \frac{ON}{OP}$

* moment Create করতে must rotate করবে

* r to F ঘুরলে Positive moment

Moment of a force -

Let a force $\overline{F}=\overline{PQ}$ acts at a point P.

Moment of a force about O.

$= (\text{Product of force})(\text{Per p. dist}^n)$

$= (PQ)(ON)\,\hat{n}$

$= (PQ)(OP\sin\theta)\,\hat{n}$

$= \overline{OP}\times\overline{PQ}$

$$\boxed{M = \overline{r}\times\overline{F}}$$

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* 5kg force acting along $\bar{d}$

$$\bar{F} = 5\left(\frac{\bar{d}}{|d|}\right) \rightarrow \text{Unit vector}$$

* Example - 34

$\bar{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ applied at a pt$(1,-1,2)$. Find the torque of the force about the pt$(2,-1,3)$.

$P(1,-1,2)$, $O(2,-1,3)$

$$\bar{r} = \overline{OP} = \bar{P} - \bar{O}$$
$$= -\hat{i} - \hat{k}$$

$$\bar{M} = \bar{r} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= 2\hat{i} - 7\hat{j} - 2\hat{k}$$

torque $= |M| = \sqrt{57}$

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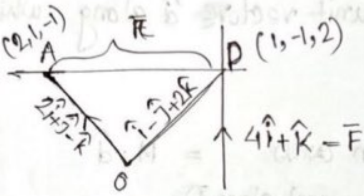
* Find the torque about the point $2\hat{i}+\hat{j}-\hat{k}$ of a force $4\hat{i}+\hat{k}$ acting through the point $\hat{i}-\hat{j}+2\hat{k}$.



$$\bar{r}=\overline{AP}=\bar{P}-\bar{A}$$
$$=-\hat{i}-2\hat{j}+3\hat{k}$$

$$\bar{M}=\bar{r}\times\bar{F}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -2 & 3\\ 4 & 0 & 1\end{vmatrix}$$

$$=\hat{i}(-2-0)-\hat{j}(-1-12)+\hat{k}(0+8)$$
$$=-2\hat{i}+13\hat{j}+8\hat{k}$$

$$|\bar{M}|=\sqrt{(-2)^2+(13)^2+(8)^2}$$

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Formula -

Moment about an axis PQ = dot product of the moment vector and the unit vectore $\hat{d}$ along which we want to find moment.

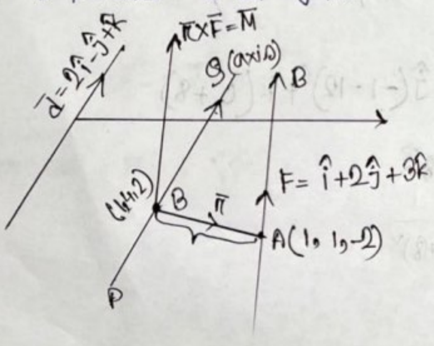
$$\text{Moment about an axis} = \bar{M} \cdot \hat{d}$$

(Projection of the moment along $\bar{d}$)

(i) Find $\bar{M}$

(ii) along any direction $\bar{d} = \bar{M} \cdot \hat{d}$

H.W Find the moment of the force $\hat{i}+2\hat{j}+3\hat{k}$ passing through the point $(1,1,-2)$ about the axis through $(1,4,2)$ which is parallel to $2\hat{i}-\hat{j}+\hat{k}$.



Course- MATH 2203

Monday

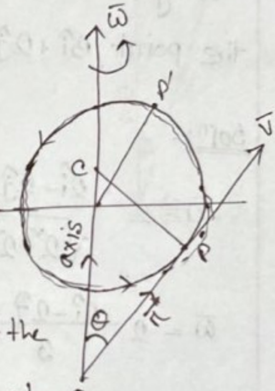
Date- 22.05.2023

Quiz - 01

31 May (Wed)

Class time Syllabus up to this week

Angular velocity $\bar{\omega}$



Let a rigid body is rotating about this axis OC with the angular velocity $\bar{\omega}$. magnitude is ω rad/sec. its direction is parallel to the axis of rotation. Let P be any point on the body. Let the velocity of P be $\bar{v}$. Let $\hat{n}$ is the unit vector perpendicular to $\bar{\omega}$ & $\bar{r}$.

$$\bar{\omega} \times \bar{r} = (\omega r \sin\theta)\hat{n} = (\omega CP)\hat{n} = (\text{Speed of P})\ \hat{n}$$

angular velocity

displacement

Scalar

$\bar{v}$ = velocity of P perpendicular to $\bar{\omega}$ and $\bar{r}$ and its direction is towards the tangent at P.

linear velocity

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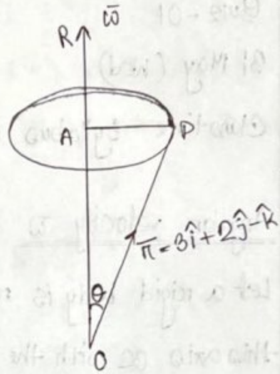
Exp-36

magnitude of angular velocity

Given, $\bar{\omega} = 2$ rad/sec

$\overline{OR} = 2\hat{i} - 2\hat{j} + \hat{k}$ → $\bar{\omega}$ is in the direction

O(origin). Find the velocity of the point $3\hat{i} + 2\hat{j} - \hat{k}$



Solⁿ:-

$$\bar{\omega} = 2 \frac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{2^2 + 2^2 + 1^2}}$$

$$\bar{\omega} = 2 \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$$

$$\bar{v} = \bar{\omega} \times \bar{r} = \frac{2}{3} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= \frac{10}{3} (\hat{j} + 2\hat{k}) \quad \text{(Ans.)}$$

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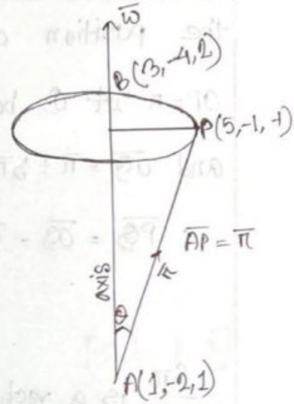
Ex-37

Given, $\omega = 2.5$ rad/sec rotating about an axis AB



Find $\overline{AP} = \bar{P} - \bar{A}$

$$\overline{AP} = \bar{r} = (5-1)\hat{i} + (-1+2)\hat{j} + (-1-1)\hat{k}$$

$$\overline{AB} = \bar{B} - \bar{A}$$

$$= (3-1)\hat{i} + (-4+2)\hat{j} + (2-1)\hat{k}$$

$$= 2\hat{i} - 2\hat{j} + \hat{k}$$

$\bar{\omega}$ is in the direction $\overline{AB}$

$$\bar{\omega} = \underbrace{2.5}_{\text{magnitude}} \underbrace{\left(\frac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{2^2 + 2^2 + 1^2}} \right)}_{\text{Unit vector}}$$

$v = \bar{\omega} \times \bar{r}$

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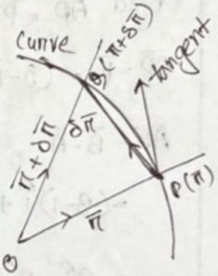
Wednesday

Date-24.05.2023

Differentiation of vectors - Let O be the origin and P be the position of a moving particle at time t, let $OP = r$ let Q be the position vector at time $t+\delta t$ and $\overline{OQ} = \overline{r} + \delta \overline{r}$

$$\overline{PQ} = \overline{OQ} - \overline{OP} = (\overline{r} + \delta \overline{r}) - \overline{r}$$
$$= \delta \overline{r}$$



$\frac{\delta \overline{r}}{\delta t}$ is a vector,

As $\delta t \to 0$, $Q \to P$ and the chord becomes the tangent at P.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$
$$= \tan\theta = m$$

We define

$$\frac{d\overline{r}}{dt} = \lim_{\delta t \to 0} \frac{\delta \overline{r}}{\delta t},$$ then $\frac{d\overline{r}}{dt}$ is a vector in the direction of the tangent at P. $\frac{d\overline{r}}{dt}$ gives the velocity of the particle at P, along the tangent to the curve.

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$\frac{d^2\bar{r}}{dt^2}$ gives the acceleration of the particle at P.

Formula

1) $\frac{d}{dt}(\bar{F}+\bar{G}) = \frac{d\bar{F}}{dt} + \frac{d\bar{G}}{dt}$

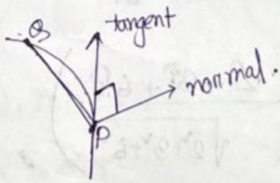
2) $\frac{d}{dt}(\bar{F}\phi) = \frac{d\bar{F}}{dt}\phi + \bar{F}\frac{d\phi}{dt}$

3) $\frac{d}{dt}(\bar{F}\cdot\bar{G}) = \bar{F}\cdot\frac{d\bar{G}}{dt} + \frac{d\bar{F}}{dt}\cdot\bar{G}$ $\quad [a\cdot b = b\cdot a]$

*** 4) $\frac{d}{dt}(\bar{F}\times\bar{G}) = \bar{F}\times\frac{d\bar{G}}{dt} + \frac{d\bar{F}}{dt}\times\bar{G}$ $\quad [\bar{a}\times\bar{b} = -\bar{b}\times\bar{a}]$

sequence maintain

* normal vector is perpendicular to the tangent.



Ex-52

Curve, $x = t^3+1$, $y = t^2$, $z = 2t+5$

Component of velocity at $t=1$ $dt = 2$

in the direction $2\hat{i}+3\hat{j}+6\hat{k}$

$x^2+y^2=a^2$

$\boxed{\begin{matrix} x = a\cos t \\ y = a\sin t \end{matrix}}$

↓

Parametric of this circle

$\bar{v} = \dfrac{d\bar{r}}{dt}$

$\boxed{\bar{r} = x\hat{i}+y\hat{j}+z\hat{k}}$

$= (t^3+1)\hat{i}+(t^2)\hat{j}+(2t+5)\hat{k}$

$\bar{v} = 3\hat{i}+2\hat{j}+2\hat{k}$

$\dfrac{d\bar{r}}{dt} = 3t^2\hat{i}+2t\hat{j}+2\hat{k}$

at, $t=1$

$\dfrac{d\bar{r}}{dt} = 3\hat{i}+2\hat{j}+2\hat{k}$

Component of velocity $\bar{v}$ along

$2\hat{i}+3\hat{j}+6\hat{k} = \bar{a}\ (\delta+y)$

$= \bar{v}\cdot\hat{a}$

Ans: $(3\hat{i}+2\hat{j}+2\hat{k})\cdot\left(\dfrac{2\hat{i}+3\hat{j}+6\hat{k}}{\sqrt{2^2+3^2+6^2}}\right)$

another part

$acc^n = \dfrac{d^2\bar{r}}{dt^2} = \dfrac{d}{dt}\left(\dfrac{d\bar{r}}{dt}\right) = \dfrac{d}{dt}\left(3t^2\hat{i}+2t\hat{j}+2\hat{k}\right)$

$= 6t\hat{i}+2\hat{j}+0$

at $t=1$ $\dfrac{d^2\bar{r}}{dt^2} = 6\hat{i}+2\hat{j}$

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Component of accl^n along $2\hat{i}+3\hat{j}+6\hat{k} = (6\hat{i}+2\hat{j})\cdot\left(\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}\right)$

Formula for tangent vector $\bar{T} = \frac{d\bar{r}}{dt}$

Formula for normal vector $\bar{N} = \frac{d\hat{T}}{dt}$ where $\hat{T}$ is the unit tangent vector

Exp-54 Unit tangent vector, $\hat{T} = \frac{1}{5}\left(-3\ \text{unit} + 3\cos t\,\hat{j} + 4\hat{k}\right)$

$$\frac{d\hat{T}}{dt} = \bar{N}$$

$\hat{N} \longrightarrow$ Unit

Ex-(15-43), 52, 54

Upto angular velocity

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Thursday

Course - MATH 2203 Date-24.05.2023

5.33 **field** :-

Scalar pt. function

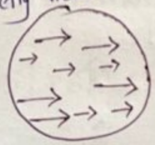
Vector pt. function

$T(x,y,z) = xy + yz + zx$

Scalar pt function

* যে region এ temperature সবগুলো define করা যাবে সেটা temperature field.

velocity field



velocity field $\bar{v}(x,y,z) = \hat{i}\,xy + \hat{j}\,x^2 z + \hat{k}\,xyz$

Gradient, Divergence, Curl

Gradient of a scalar function $\phi(x,y,z)$

$\nabla\phi$ or Grad ϕ

nabla / Grad $= \nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{xy} + \hat{k}\frac{\partial}{\partial z}$

is a vector operator

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$$\boxed{\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}}$$

rate of change বের করা যায়

Divergence of a vector function $\bar{v}(x,y,z)$

$$\text{div}\,\bar{v} = \nabla\cdot\bar{v} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}v_1 + \hat{j}v_2 + \hat{k}v_3\right)$$

$$= \boxed{\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}}$$

$\boxed{\nabla\cdot\bar{v} > 0}$ $\boxed{\nabla\cdot\bar{v} < 0}$ $\boxed{\nabla\cdot\bar{v} = 0}$

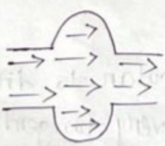
Source

Sink

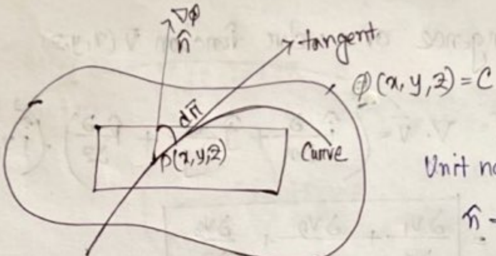
Steady flow

* flow বাড়লে Positive divergence

* flow কমলে negative divergence.



$$\text{Curl } \bar{v} = \nabla \times \bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$



Unit normal vector,

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

Normal to a surface $\phi(x,y,z) = C$

$\nabla\phi$ is a normal to the surface $\phi(x,y,z) = C$ — $\boxed{d\phi = 0}$

Proof:- $d\phi = \nabla\phi \cdot d\bar{r}$

$$= \left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right) \cdot (\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

$0 = \nabla\phi \cdot d\bar{r}$

$\nabla\phi \cdot d\bar{r} = 0 \Rightarrow \nabla\phi$ is perpendicular to $d\bar{r}$ i.e it is perpendicular to every tangent vector to the surface.

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Course-MATH 2203

Monday

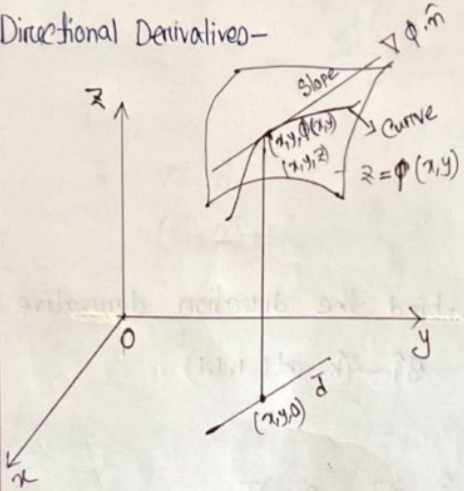
Date-29.05.2023

Quiz 01 $\longrightarrow$ 05.06.2023

Topic - first to derivatives

Quiz 02 $\longrightarrow$ 19.06.2023

Directional Derivatives-



$\nabla\phi$ = normal to the surface $\phi(x,y,z)=c$

$$\boxed{\frac{\nabla\phi}{|\nabla\phi|}=\hat{n}} \rightarrow \text{Unit normal vector}$$

* Directional derivative is the component of $\nabla\phi$ in the direction $\vec{d}$ at (x,y,z) is called directional derivative $=$

$$\boxed{\nabla\phi.\hat{n}}$$

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$\nabla\phi \cdot \hat{n}$

$= |\nabla\phi| \cos\theta \longrightarrow$ when, $\theta = 0$ | when, $\theta = \pi/2$

$= D_J(a, b, f(a,b))$ $\quad D_n\phi \longrightarrow$ maximum | $D_u\phi \rightarrow$ minimum

if $\theta = \pi$

$-|\nabla\phi|$

direction এর জন্য negative

Ex-57

$T = xy + yz + zx$. Find the direction derivative of T in the direction $3\hat{i} - 4\hat{k}$ at $(1,1,1)$

$$\nabla T = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}(xy + yz + zx) + \hat{j}\frac{\partial}{\partial y}(xy + yz + zx) + \hat{k}\frac{\partial}{\partial z}(xy + yz + zx)$$

$$= \hat{i}(y + 0 + z) + \hat{j}(x + z + 0) + \hat{k}(0 + y + x)$$

মান বসিয়ে $\nabla T/_{(1,1,1)} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

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Ex 55, 56

Directional Derivative of ∇T in the direction $D\hat{d} = \nabla T \cdot \hat{n}$

$$\hat{n} = \frac{3\hat{i} - 4\hat{k}}{\sqrt{3^2+4^2}}$$

$$= \frac{3\hat{i} - 4\hat{k}}{5}$$

$$\bar{d} = \nabla T \cdot \hat{n}$$

$$= (2\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \frac{3\hat{i} - 4\hat{k}}{5}$$

$$= -\frac{2}{5}$$

Exp-60

$\bar{V}^2 = ?$

$\bar{V} = xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$

Find the direct derivative of $\bar{V}^2$ at $(2,0,3)$ in the direction of the outward normal to the sphere $x^2+y^2+z^2=14$ at $(3,2,1)$

Soln :-

$$\bar{V}^2 = \bar{V} \cdot \bar{V}$$

$$= x^2y^4 + z^2y^4 + x^2z^4 \rightarrow \text{Scalar}$$

Course - MATH 2203 Wednesday Date - 31.05.2023

Exp-60

Find the directional derivative of $\bar{v}^2$ where

$\bar{v} = xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$ at $(2,0,3)$ in the direction of the outward normal to the sphere $x^2+y^2+z^2=14$ at $(3,2,1)$

Soln:-

$$\bar{v}^2 = \bar{v}.\bar{v}$$

Scalar

$$= x^2y^4 + z^2y^4 + x^2z^4$$

Now, $\nabla \bar{v}^2 = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^2y^4 + z^2y^4 + x^2z^4\right)$

$$= (2xy^4 + 2xz^4)\hat{i} + (4x^2y^3 + 4z^2y^3)\hat{j} + (2zy^4 + 4x^2z^3)\hat{k}$$

$$\nabla\bar{v}^2/_{(2,0,3)} = 324\hat{i} + 432\hat{k} = 108(3\hat{i} + 4\hat{k})$$

$\nabla\phi$ is the normal vector,

$$\phi(x,y,z) = 0$$

$$x^2+y^2+z^2-14=0$$

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find $\nabla(x^2+y^2+z^2-14)$

Ex-55, 56, 57, 59, 60, 61, 63, 71, 72

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla\phi\Big/_{(3,2,1)} = 6\hat{i} + 4\hat{j} + 2\hat{k}$$

Outward normal to $x^2+y^2+z^2-14=0$ is $6\hat{i}+4\hat{j}+2\hat{k} = \bar{d}$ (say)

Directional derivative $= \nabla\bar{v} \cdot \hat{d}$

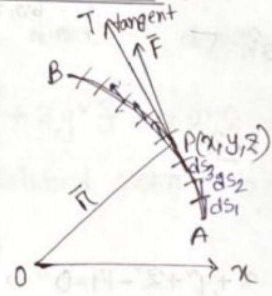
$$= 108(3\hat{i}+4\hat{k}) \cdot \left(\frac{6\hat{i}+4\hat{j}+2\hat{k}}{2\sqrt{14}}\right)$$

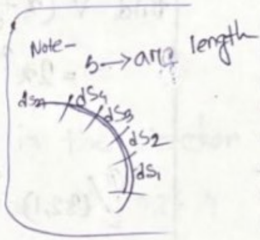
$$= \frac{1404}{\sqrt{14}} \text{ (Ans.)}$$

Thursday

Course - MATH 2203 | Date - 01.06.2023

Integration of vectors (Line integral)





Curve AB and $\bar{F}$ is a vector function Line integral of $\bar{F}(x,y,z)$ along a curve AB is defined as the integral component of $\bar{F}$ along the tangent to the curve AB.

Component of $\bar{F}$ along a tangent PT at $P = \bar{F} \cdot \frac{d\bar{r}}{ds}$

($\frac{d\bar{r}}{ds}$ is a unit tangent vector along tangent PT)

A থেকে B তে যাব - that's why sum

Line integral $= \sum \bar{F} \cdot \frac{d\bar{r}}{ds}$ from A to B along the curve AB

Scalar

Line integral $= \int_C \left(\bar{F} \cdot \frac{d\bar{r}}{ds}\right) ds = \int_C \bar{F} \cdot d\bar{r}$

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$C : AB$

$\bar{F} \rightarrow$ given

$\int_C \bar{F} \cdot d\bar{r}$

$d\bar{r} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$

$\bar{r} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$

line integral along the closed curve

Note –

$\oint_c \rightarrow$ closed curve

$\int_c \rightarrow$ not closed

1) work

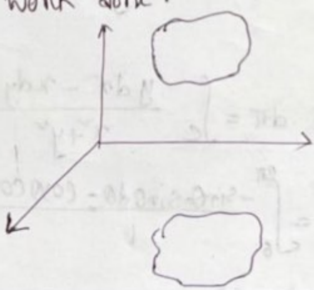
if

Total work done $= \int_c \bar{F} \cdot d\bar{r}$

Circulation : If $\bar{F}$ represents the velocity of a liquid then $\boxed{\oint_c \bar{F} \cdot d\bar{r}}$ is called the circulation of $\bar{F}$ round the curve c.

Exp-89

$\bar{F} = 2x^2y\,\hat{i} + 3xy\,\hat{j}$ displaces a particle in the xy plane from $(0,0)$ to $(1,1)$ along a curve $y = 4x^2$; Find the work done.



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Line integral $= \int_C \vec{F} \cdot d\vec{r}$

$$= \int_{(0,0)}^{(1,1)} (2x^2 y\, dx + 3xy\, dy)$$

along $y = 4x^2$

$dy = 8x\,dx$

$$= \int_0^1 2x^2 \cdot 4x^2 dx + 3x \cdot 4x^2 \cdot 8x\,dx$$

$$= \int_0^1 8x^4 dx + \int_0^1 96\, x^4 dx$$

$$= \int_0^1 104\, x^4 \cdot dx$$

$$= \frac{104}{5}$$

Exp-90

Compute $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F} = \frac{\hat{i}y - \hat{j}x}{x^2+y^2}$ and the circle $x^2+y^2=1$

Line integral $\int \vec{F} \cdot d\vec{r} = \int_C \frac{y\,dx - x\,dy}{x^2+y^2}$

$$= \int_0^{2\pi} \frac{-\sin\theta \sin\theta\, d\theta - \cos\theta \cos\theta\, d\theta}{1}$$

Put, $r = 1$

$x = \cos\theta$

$y = \sin\theta$

$dx = -\sin\theta\, d\theta$

$dy = \cos\theta\, d\theta$

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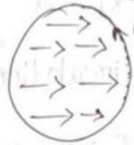
$$= -\int (\sin^2\theta + \cos^2\theta)\, d\theta$$

$$= -\int_0^{2\pi} d\theta$$

$$= -2\pi$$

$\oint_c \vec{F} \cdot d\vec{r}$

$C: x^2+y^2=1$



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Course - MATH 2203 Sunday Date - 04.06.2023

Irrotational field Properties -

(i) $\nabla \times \bar{F} = 0$

close curve (ii) Circulation $\oint_C \bar{F}\cdot d\bar{r}$ along every closed path is zero.

(iii) If $\nabla \times \bar{F} = 0$ then $\bar{F}$ is called irrotational and

work done হবে না

$\boxed{\bar{F} = \nabla\phi}$, $\bar{F}$ is called conservative vector field. $(\nabla \times \bar{F} = 0)$

Again since $\nabla \times \bar{F} = 0 \Rightarrow \nabla \times \nabla\phi = 0 \Rightarrow \boxed{\nabla^2\phi = 0}$

Thus the vector $\bar{F}$ can always be expressed as a gradient of scalar function ϕ.

When $\oint F\cdot dr = 0$ as stated in (ii) the field is conservative, i.e, no work is done in displacing a particle from a point A to another point B in the mechanical energy is conserved.

** $\boxed{\text{Every irrotational field is conservative.}}$

$\boxed{\int_C F\cdot d\bar{r} \longrightarrow \text{line integral}}$

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$$\boxed{\nabla \times \bar{F} = 0 \Rightarrow \bar{F} = \nabla \phi}$$

$\bar{F}$ conservative + irrotational

* Conservative হলে must irrotational and irrotational হলে must হবে Conservative

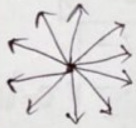
(iv) when div $\bar{V} = 0$ or $\boxed{\nabla \cdot \bar{V} = 0}$

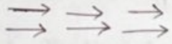
the field $\bar{F}$ is called $\boxed{\text{solenoidal}}$

$\boxed{\text{flux is defined as circulation of per unit area}}$

It measures the outward flux of a vector field from an infinitesimal volume around a given point.

$\nabla \cdot \bar{V} > 0$





$\nabla \cdot \bar{V} = 0$

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Ex - 41

Conservative field line material is indep of path

If $\boxed{\bar{F} = \nabla\phi}$ Show that the work done in moving a Particle from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ is independent of path joining the two points.

Sol^n :-

$$\text{Work done} = \int_A^B \bar{F}\cdot d\bar{r} = \int_A^B \nabla\phi \cdot d\bar{r}$$

$$= \int_A^B \left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right)\left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$= \int_A^B \left(\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial x}dz\right)$$

$$= \int_A^B d\phi = \left[\phi\right]_A^B$$

$$= \phi(B) - \phi(A)$$

$$= \phi(x_2, y_2, z_2) - \phi(x_1, y_1, z_1)$$

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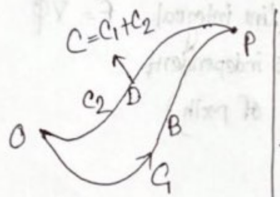
* Circulation : $\oint_C \bar{F}\cdot d\bar{r}$

$\int_{A}^{B} \bar{F}\cdot d\bar{r} \rightarrow$ line integral

If $\oint_C \bar{F}\cdot d\bar{r} = 0$

then the field is conservative ($\bar{F} = \nabla\phi$)



$\int_{C_1} = \int_{OBP}$, $\int_{C_2} = \int_{ODP}$

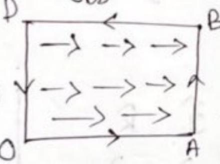
$\int_{OBP} = -\int_{PDO} = \int_{ODP} = \int_{C_2}$

$\int_{C_1} = \int_{C_2}$

* $\bar{F}$ is not conservative since $\oint_C \bar{F}\cdot d\bar{r} \neq 0$

$\nabla\times\bar{F} \neq 0 \quad \bar{F} \neq \nabla\phi$

$\int_{BD} \bar{F}\cdot d\bar{r} = -$ve (Parallel but not in same direction)



$\int_{DO} \bar{F}\cdot d\bar{r} = 0$

$\int_{AB} \bar{F}\cdot d\bar{r} = 0$ (Perpendicular dot product is 0)

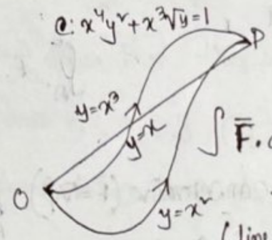
$\int_{OA} \bar{F}\cdot d\bar{r} \neq 0$

(Same direction & parallel)

$\boxed{\text{So, } \oint_C \bar{F}\cdot d\bar{r} = 0}$

$\bar{F}$ conservative so, $\bar{F} = \nabla\phi$

* field conservative হলে we can choose any path.



$\int \bar{F}\cdot d\bar{r}=0$ | $\bar{F}$ is Conservative | $\nabla\times\bar{F}=0$

$\downarrow$ (line integral is independent of path) | $\Downarrow$ $\bar{F}=\nabla\phi$ | $\Downarrow$ irrotational

$y=x$ $\rightarrow$ Convential

(*) Solenoidal vector field : $\nabla\cdot\bar{V}=0$ (div $\bar{V}=0$)

Example

$$\bar{F}=(y^2\cos x+z^3)\hat{i}+(2y\sin x-4)\hat{j}+(3xz^2+2)\hat{k}$$

$\boxed{\text{Show that}}$ $\bar{F}$ is irrotational, also find a scalar Potential show function ϕ such that $\bar{F}=\nabla\phi$

$$\nabla\times F=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ y^2\cos x+z^3 & 2y\sin x-4 & 3xz^2+2\end{vmatrix}=0$$

$$=\hat{i}\left[\frac{\partial}{\partial y}(3xz^2+2)-\frac{\partial}{\partial z}(2y\sin x-4)\right]-$$

$\hat{j}$ -

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$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz = \nabla \phi \cdot d\bar{r} = \bar{F} \cdot d\bar{r}$$

$$\left(i \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right) \left(\hat{i}\, dx + \hat{j}\, dy + \hat{k}\, dz \right)$$

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Thursday

Course - MATH-2023 Date - 08.06.23

last math →

$d\phi = \nabla\phi \cdot d\vec{r} = \vec{F} \cdot d\vec{r} = (y^2\cos x + z^3)dx + (2y\sin x - 4)dy + (3xz^2 + 2)dz$

↓

$\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz = D0$

Comparing the co-effs of dx, dy, dz on both side

$\frac{\partial\phi}{\partial x} = y^2\cos x + z^3$ — (1)

$\frac{\partial\phi}{\partial y} = 2y\sin x - 4$ — (2)

$\frac{\partial\phi}{\partial z} = 3xz^2 + 2$ — (3)

Int (1) with respect to x partially -

$\phi = y^2\sin x + xz^3 + f(, z)$ — (4)

Int (2) w.r. to y -

$\phi = y^2\sin x - 4y + f(x, z)$ — (5)

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Int (3) w.r.to z -

$$\phi = 3x\frac{z^3}{3} + 2z + f(x,y)$$

$$= xz^3 + 2z + f(x,y) \quad \text{---} (6)$$

$$(4)-(5) \Rightarrow \quad xz^3 + f(y,z) = -4y + f(x,z) \quad \text{---} (8)$$

$$xz^3 = f(x,z)$$

Substitute $f(x,z) = xz^3$ in (5)

$$\phi = y^2 \sin x - 4y + xz^3 \quad (\text{Ans:})$$

verify $\nabla\phi = \bar{F}$

$$\hat{i}\frac{\partial}{\partial x}(y^2 \sin x - 4y + xz^3) + \hat{j}\frac{\partial}{\partial y}(\text{DO}) + \hat{k}\frac{\partial}{\partial z}(\text{DO})$$

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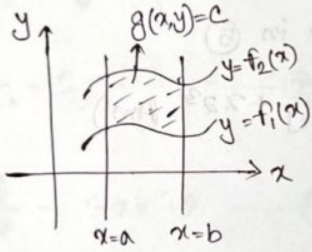
Green's theorem for a plane :

Statement - If $\phi(x,y)$, $\psi(x,y)$, $\frac{\partial \phi}{\partial y}$, $\frac{\partial \psi}{\partial x}$ be cont^s in region R bounded by a simple closed curve c, then

$$\boxed{\oint_c \phi\, dx + \psi\, dy = \int_R\int \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dy\, dx}$$



* যার level constant তাকে আগে use করতে হবে।

$$g(x,y)\ \overset{①}{dy}\ \overset{②}{dx}$$

① verify Green's theorem for,

$\int_c \left[(xy+y^2)\,dx + x^2\,dy\right]$ where c is bounded by

$y=x$ & $y=x^2$

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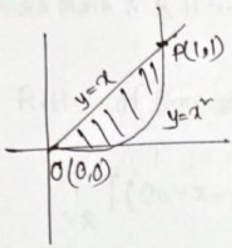

$$\left.\begin{array}{l} y = x \\ y = x^2 \end{array}\right\}$$

$$x(1-x) = 0$$
$$x = 0, 1$$
$$y = 0, 1$$

along $y = x^2$, $dy = 2x\,dx$

along $y = x$, $dy = dx$

$$\phi = xy + y^2 \quad , \quad \psi = x^2$$

$$\frac{\partial \phi}{\partial y} = x + 2y \quad ; \quad \frac{\partial \psi}{\partial x} = 2x$$

L.H.S of Green's theorem,

$$\oint_C \phi\,dx + \psi\,dy = \int_C (xy + y^2)\,dx + x^2\,dy$$

1) along $y = x^2$

$$\int_0^1 (x \cdot x^2 + x^4)\,dx + x^2 \cdot 2x\,dx = \frac{19}{20}$$

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2) Along $y = x$

$$\int_1^0 2x^2 \cdot dx + x^2 dx = -1$$

$$L.H.S = -1 + \frac{19}{20} = -\frac{1}{20}$$

Quiz-02

irrotation, Conservative vector field, line integral, Solenoidal, Green's theorem

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Monday

MATH-2203 Date-12.06.2023

Previous Math's R.H.S $\rightarrow$ $\boxed{\text{Ex-96, 97, 98}}$

R.H.S of Green's theorem -

$$\int_R\int (2x - x - 2y)\,dy\,dx = \int_0^1\int_{y=x^2}^{y=x} (x-2y)\,dy\,dx$$

$$= \int_0^1 \left(\int_{y=x^2}^{y=x} [x-2y]\,dy\right) dx$$

$$= \int_0^1 \left[(xy - y^2) \right]_{y=x^2}^{y=x} dx$$

$$= \int_0^1 \left[(x^2 - x^2) - (x^3 - x^4) \right] dx$$

$$= \int_0^1 (x^4 - x^3)\,dx$$

$$= \left[\frac{x^5}{5} - \frac{x^4}{4} \right]_0^1$$

$$\text{R.H.S} = -\frac{1}{20}$$

L.H.S = R.H.S [verified]

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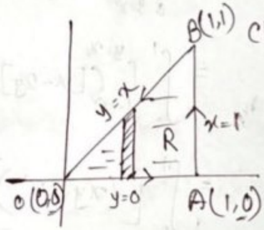
Ex-97

Apply green's theorem $\oint_C (x^2 y\,dx + x^2 dy)$

c: vertices of a tringle $(0,0), (1,0), (1,1)$



Closed curve triangle C

For OA : $y = 0$
$dy = 0$

For AB : $x = 1$
$dx = 0$

y varies from 0 to 1 .

For BO : $y = x$
$dy = dx$

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$$\int_{BO} x^2 y\, dx + x^2\, dy = \int_1^0 (x^3\, dx + x^2\, dx)$$

$$= \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_1^0$$

$$= -\frac{7}{12}$$

$$\text{Total line integral} = \left(0 + 1 + \left(-\frac{7}{12}\right)\right)$$

$$= \frac{5}{12}$$

use / Apply Green's th^m –

$$\oint_C (\underbrace{x^2 y}_{\phi}\, dx + \underbrace{x^2}_{\psi}\, dy) \qquad \begin{array}{|l} \phi = x^2 y \\ \psi = x^2 \end{array}$$

$$\text{R.H.S of Green's} = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dy\, dx$$

$$= \int\int_y (2x - x^2)\, dy\, dx$$

$$= \int_0^1 \int_{y=0}^{y=x} \underbrace{(2x - x^2)}_{\text{Const}}\, dy\, dx$$

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$$= \int_0^1 (2x - x^2) \left[y \right]_{y=0}^{y=x} dx$$

$$= \int_0^1 x(2x - x^2)\, dx$$

$$= \int_0^1 (2x^2 - x^3)\, dx$$

$$= \left[2 \cdot \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{5}{12} \text{ (Ans:)}$$

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Wednesday

Date-14.06.2023

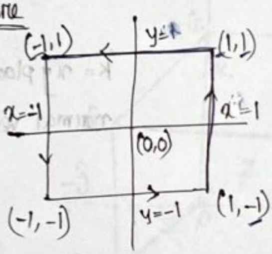
Course -MATH 2203

FLUX: Flow rate accross the entire surface per unit time

Exp-98

$C \begin{cases} x : \pm 1 \\ y : \pm 1 \end{cases}$ bounded by line (closed curve c)

Square

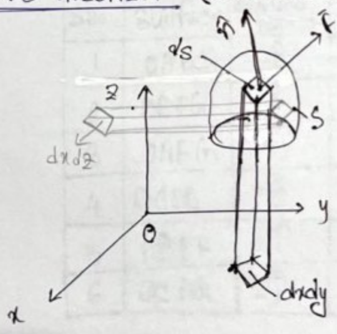


R.H.S

$$\iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dy\, dx$$

$$\int_{x=-1}^{1}\int_{y=-1}^{1} (\quad) \, dy\, dx$$

STOKE'S THEOREM (Surface Integral)



* Surface integral always double integral

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Component of $\bar{F}$ along the normal $\hat{n}$

$$\iint_S \bar{F} \cdot \hat{n} \, ds = \text{Surface integral (flux)}$$

$\hat{n} \rightarrow$ unit vector

If $\iint_S \bar{F} \cdot \hat{n} \, ds = 0$, then $\bar{F}$ is solenoidal vector field.

- Projection on xy plane

$$ds = \frac{dy \, dx}{|\hat{n} \cdot \hat{k}|}$$

$\hat{k} =$ xy plane এর perpen. normal vector

extra - Projection on xz plane

$$ds = \frac{dx \, dz}{|\hat{n} \cdot \hat{j}|}$$

extra - Projection on yz plane

$$ds = \frac{dy \, dz}{|\hat{n} \cdot \hat{i}|}$$

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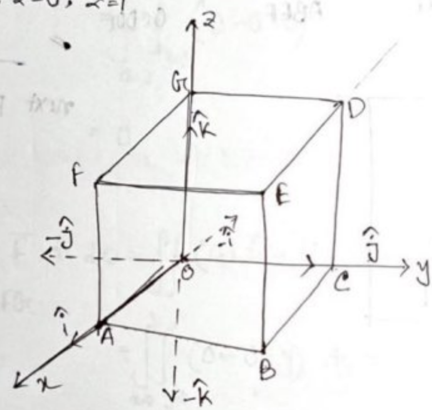
Ex-94

Show that $\iint_S F.\hat{n}\, ds = \frac{3}{2}$, $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$,

where S is bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$



S.No	Surface	outward normal ($\hat{n}$)	ds	Eqn of Surface
1	OABC	$-\hat{k}$	$dy\,dx$	$z=0$
2	DEFG	$\hat{k}$	$dy\,dx$	$z=1$
3	OAFG	$-\hat{j}$	$dx\,dz$	$y=0$
4	BCDE	$\hat{j}$	$dx\,dz$	$y=1$
5	ABEF	$\hat{i}$	$dy\,dz$	$x=1$
6	OCDG	$-\hat{i}$	$dy\,dz$	$x=0$

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$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iint_{OABC} DO + \iint_{DEFG} DO + \iint_{OAFG} DO +$$

$$\iint_{BCDE} DO + \iint_{ABEF} DO + \iint_{OCDG} DO \longrightarrow ① \text{ eqn}$$

next page

$$\boxed{\begin{aligned} &\iint_{OABC} \vec{F}\cdot\hat{n}\,ds \\ &\iint_{OABC} \vec{F}\cdot\hat{n}\,\frac{dy\,dx}{|\hat{n}\cdot\hat{k}|} \end{aligned}}$$

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Monday

Course - MATH 2203 Date- 19.06.2023

Ex-94

বাকি অংশ

$F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$

$$\iint_{OABC} F.\hat{n}\, ds = \iint_{OABC} (4xz\hat{i} - y^2\hat{j} + yz\hat{k}).(-\hat{k})\, dy\, dx$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} (0-0+0) \qquad [z=0]$$

$$= 0$$

$$ds = \frac{dx\,dy}{|\hat{n}.\hat{k}|}$$

$$ds = \frac{dx\,dy}{|\hat{k}.\hat{k}|} \to 1$$

$$ds = dx\,dy$$

$$\iint_{DEFG} \overline{F}.\hat{n}\, ds = \iint (4xz\hat{i} - y^2\hat{j} + yz\hat{k}).\hat{k}\, dy\, dx$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} (0-0+y)\, dy\, dx \qquad [z=1]$$

$$= \int_0^1 \left[\frac{y^2}{2}\right]_0^1 dx$$

$$= \int_0^1 \frac{1}{2}\, dx$$

$$= \frac{1}{2}$$

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$$\iint_{OAFG} \bar{F}\cdot\hat{n}\, ds = \iint_{OAFG} (4xz\hat{i} - y^2\hat{j} + yz\hat{k})\cdot(-\hat{j})\, dx\, dz$$

$$= \int_{x=0}^{1}\int_{z=0}^{1} (0 + y^2 + 0)\, dx\, dz \qquad [y=0]$$

$$= 0$$

$$\iint_{BCDE} \bar{F}\cdot\hat{n}\, ds = \iint_{BCDE} (4xz\hat{i} - y^2\hat{j} + yz\hat{k})\cdot(\hat{j})\, dx\, dz$$

$$= \int_{x=0}^{1}\int_{z=0}^{1} (0 - y^2 + 0)\, dx\, dz \qquad [y=1]$$

$$= -\int_{x=0}^{1} [z]_0^1\, dx$$

$$= -\int_{x=0}^{1} dx$$

$$= -[x]_0^1$$

$$= -1$$

$$\iint_{ABEF} \bar{F}\cdot\hat{n}\, ds = \iint_{ABEF} (4xz\hat{i} - y^2\hat{j} + yz\hat{k})\cdot(\hat{i})\, dy\, dz$$

$$= \int_{y=0}^{1}\int_{z=0}^{1} 4xz\, dy\, dz$$

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$$= \int_{y=0}^{1} \int_{z=0}^{1} 4z \, dy \, dz \qquad [x=1]$$

$$= 4\int_{y=0}^{1} \left[\frac{z^2}{2}\right]_0^1 dy$$

$$= 4 \times \frac{1}{2} \int_{y=0}^{1} dy$$

$$= 2\left[y\right]_0^1$$

$$= 2$$

$$\iint_{OCDG} \bar{F}.\hat{n}\, ds = \iint_{OCDG} (4xz\hat{i} - y^2\hat{j} + yz\hat{k}) . (-\hat{i})\, dy\, dz$$

$$= \int_{y=0}^{1} \int_{z=0}^{1} -4xz \, dy\, dz \qquad [x=0]$$

$$= 0$$

Putting these values in eq^n (1) we get,

$$\iint_S F.\hat{n}\, ds = 0 + \frac{1}{2} + 0 - 1 + 2 - 0$$

$$= \frac{3}{2}$$

[Proved]

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STOKE'S THEOREM:- (relation between line integral & Surface integral)

Statement:- Surface integral of the component of curl $\bar{F}$ along the normal to the Surface S, taken over the surface S, bounded by Curve C is equal to the line integral of the vector point function $\bar{F}$ taken along the closed curve C.

Mathematically, $$\oint_C \bar{F}\cdot d\bar{r} = \iint_S \underbrace{(\nabla\times\bar{F})}_{\text{Curl of the Surface}}\cdot\hat{n}\, ds$$

Ex-99

Use stokes theorem to evaluate,

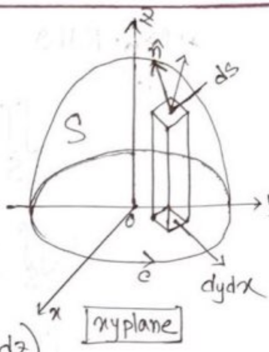
$$\int_C \left[(2x-y)\,dx - yz^2\,dy - y^2 z\,dz\right] \quad , \quad \text{where } C: x^2+y^2=1$$

corresponding to the surface of the sphere of unit radius.

$\boxed{x^2+y^2+z^2=1}$ Sphere

$\underline{\text{Sol}^n}$:- By stoke's theorem,

$$\oint_C F \cdot d\bar{r} = \iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, ds$$



Given, $\oint_C F \cdot d\bar{r}$ form ⇒ আছে

$$\int_C \left[(2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}\right] \cdot \left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(-y^2z) - \frac{\partial}{\partial z}(-yz^2)\right] - \hat{j}\left[\frac{\partial}{\partial x}(-y^2z) - \frac{\partial}{\partial z}(2x-y)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(-yz^2) - \frac{\partial}{\partial y}(2x-y)\right]$$

$$= \hat{i}\left[-2yz + 2yz\right] - \hat{j}\left[0-0\right] + \hat{k}\left[0+1\right]$$

$$= \hat{k}$$

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Now, R.H.S of Stoke's theorem -

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds$$

$$ds = \frac{dy\, dx}{|\hat{n} \cdot \hat{k}|}$$

$$= \iint_S \hat{k} \cdot \hat{n} \frac{dy\, dx}{|\hat{n} \cdot \hat{k}|}$$

$$= \iint_S dy\, dx$$

$C: x^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-x^2}$

$$= \int_{x=-1}^{x=1} \int_{y=\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} dy\, dx$$

$$= 2\int_0^1 2\int_0^{\sqrt{1-x^2}} dy\, dx$$

$$= 4\int_{x=0}^{1} [y]_0^{\sqrt{1-x^2}}\, dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$= 4\int_{x=0}^{1} \sqrt{1-x^2}\, dx$$

$$= 4\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 2\int_0^{\pi/2} (1+\cos 2\theta)\, d\theta$$

$$= \pi$$

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general form of Plane

$ax+by+cz+d=0$

Ex-103

Use Stoke's to evaluate,

$$\int_C [(x+2y)\,dx + (x-z)\,dy + (y-z)\,dz]$$

where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$, $(0,0,6)$ oriented anticlockwise direction.

$\triangle ABC, \triangle OBC$

$\triangle OAC, \triangle OAB$

Solⁿ :-

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y & x-z & y-z \end{vmatrix}$$

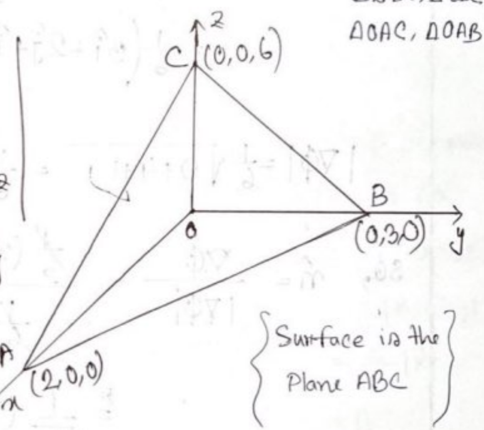
$$= \hat{i}\left\{\frac{\partial}{\partial y}(y-z) - \frac{\partial}{\partial z}(x-z)\right\}$$

$$- \hat{j}\left\{\frac{\partial}{\partial x}(y-z) - \frac{\partial}{\partial z}(x+2y)\right\}$$

$$+ \hat{k}\left\{\frac{\partial}{\partial x}(x-z) - \frac{\partial}{\partial y}(x+2y)\right\}$$

$$= \hat{i}(1+1) - \hat{j}(0-0) + \hat{k}(1-2)$$

$$= 2\hat{i} - \hat{k}$$



{Surface is the Plane ABC}

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So, S is the surface of the plane.

*surface এর gradient তার normal vector

$$S : \phi : \frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$$

Unit normal to $\boxed{\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}}$

Now, $\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{6} - 1\right)$

$$= \frac{\hat{i}}{2} + \frac{\hat{j}}{3} + \frac{\hat{k}}{6}$$

$$= \frac{1}{6}\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$$

$$|\nabla\phi| = \frac{1}{6}\sqrt{9+4+1} = \frac{1}{6} \times \sqrt{14}$$

So, $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\cancel{\frac{1}{6}}(3\hat{i} + 2\hat{j} + \hat{k})}{\cancel{\frac{1}{6}} \times \sqrt{14}}$

$$= \frac{1}{\sqrt{14}}\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$$

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Wednesday

Course - MATH 2203 Date - 21.06.2023

$$\iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, ds = \iint_S (2\hat{i} - \hat{k}) \cdot \frac{1}{\sqrt{14}}(3\hat{i} + 2\hat{j} + \hat{k})\, ds$$

$$= \iint_S \frac{5}{\sqrt{14}}\, ds$$

$$= \iint_S \frac{5}{\sqrt{14}} \frac{dx\,dy}{|\hat{n} \cdot \hat{k}|}$$

$$= \iint_S \frac{5}{\sqrt{14}} \frac{dx\,dy}{\underbrace{\frac{1}{\sqrt{14}}(3\hat{i}+2\hat{j}+\hat{k})}_{\hat{n}} \cdot \hat{k}}$$

$$= 5 \iint_S dx\,dy$$

Area of triangle ΔOAB formed by the Side $\overline{OA}, \overline{OB}$

$$= 5 \times 3$$

$$= 15$$

$$\Delta OAB = \frac{1}{2} |\overline{OA} \times \overline{OB}|$$

$$= \frac{1}{2} |2i \times 3j|$$

$$= \frac{6}{2} |\hat{k}|$$

$$= 3$$

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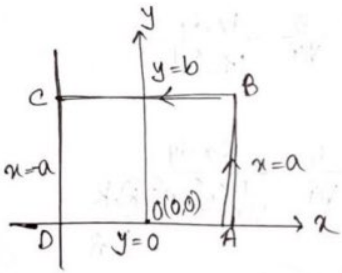
Exp-106

Verify the stoke's th^m for $F = (x^2+y^2)\hat{i} - 2xy\hat{j}$ around the rectangle bounded by the lines $x = \pm a$, $y=0$, $y=b$



verify → both side

Sol^n

Let ABCD be the given rectangle,

$$\oint_C \bar{F}\cdot d\bar{r} = \int_{AB} \bar{F}\cdot d\bar{r} + \int_{BC} \bar{F}\cdot d\bar{r} + \int_{CD} \bar{F}\cdot d\bar{r} + \int_{DA} \bar{F}\cdot d\bar{r}$$

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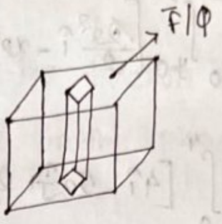
Wednesday

Course - MATH 2203 Date- 12.07.2023

Volume Integral

$\bar{F} \to$ vector point function/

Scalar pt. function



$\boxed{dv = dx\,dy\,dz}$

Volume integral $= \iiint_V \phi \, dv$ or $\iiint_V \bar{F}\, dv$

Here,
ϕ = Scalar
dv = Scalar
$\bar{F}$ = vector

Example - 95

$\bar{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, find $\iiint_V \bar{F} dv$, where v is the region bounded by the surfaces $x=0$; $y=0$; $x=2$, $y=4$, $\boxed{z = x^2,\ z = 2}$

$$\iiint (2z\hat{i} - x\hat{j} + y\hat{k})\, dx\, dy\, dz \quad (1)$$

$$= \int_{x=0}^{x=2}\int_{y=0}^{y=4}\left[\int_{z=2}^{z=x^2}(2z\hat{i} - x\hat{j} + y\hat{k})\, dz\right] dy\, dx$$

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$$= \int_{x=0}^{x=2}\int_{y=0}^{y=4}\left[\frac{2z^2}{2}\hat{i} - xz\hat{j} + yz\hat{k}\right]_{z=2}^{z=x^2} dy\, dx$$

$$= \int_0^2\int_0^4 \left[4\hat{i} - 2x\hat{j} + 2y\hat{k} + x^4\hat{i} + x^3\hat{j} - x^2 y\hat{k}\right] dy\, dx$$

$$= \int_0^2 \left[4y\hat{i} - 2xy\hat{j} + y^2\hat{k} + x^4 y\hat{i} + x^3 y\hat{j} - x^2\frac{y^2}{2}\hat{k}\right]_{y=0}^{y=4} dx$$

$$= \int_0^2 \left(16\hat{i} - 8x\hat{j} + 16\hat{k} - 4x^4\hat{i} + 4x^3\hat{j} - 8x^2\hat{k}\right) dx$$

$$= \left[16x\hat{i} - 4x^2\hat{j} + 16x\hat{k} - \frac{4x^5}{5}\hat{i} + x^4\hat{j} - \frac{8x^3}{3}\hat{k}\right]_0^2$$

$$= 32 - 16\hat{j} + 32\hat{k} - \frac{128}{5}\hat{i} + 16\hat{j} - \frac{64}{3}\hat{k}$$

$$= \frac{32\hat{i}}{5} + \frac{32\hat{k}}{3}$$

$$= \frac{32}{15}(3\hat{i} + 5\hat{k})$$

(Ans:)

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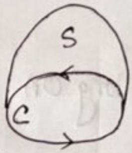
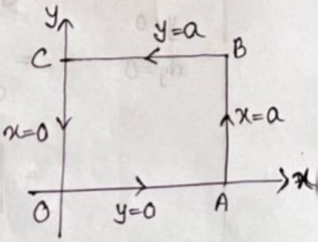
Example - 106

Verify Stoke's theorem for the function $\bar{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z=0$, $y=0$, $x=a$, $y=a$

Stoke's theorem -



$$\oint_C \bar{F} \cdot d\bar{r} = \iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, ds$$



Here,

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & -xy & 0 \end{vmatrix}$$

$$= (0-0)\hat{i} - (0-0)\hat{j} + (-y-0)\hat{k}$$

$$= -y\hat{k}$$

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Thursday

Date - 13.06.2023

Course - MATH 2203

Previous Math → Ans

$$\bar{F}\cdot d\bar{r} = (x^2\hat{i} - xy\hat{j})\cdot(\hat{i}\,dx + \hat{j}\,dy)$$

$$= x^2dx - xy\,dy$$

$$\oint_C \bar{F}\cdot d\bar{r} = \int_{OA}\bar{F}\cdot d\bar{r} + \int_{AB}\bar{F}\cdot d\bar{r} + \int_{BC}\bar{F}\cdot d\bar{r} + \int_{CO}\bar{F}\cdot d\bar{r}$$

Along OA:

$y=0$

$dy=0$

$$\int_{OA} x^2dx - xy\,dy$$

$$= \int_{x=0}^{a} x^2dx - x\times 0$$

$$= \left[\frac{x^3}{3}\right]_0^a$$

$$= \frac{a^3}{3}$$

along AB:

$x=a$

$dx=0$

$$\int_{AB}(x^2dx - xy\,dy)$$

$$= \int_{y=0}^{a} 0 - ay\,dy$$

$$= \left[-a\frac{y^2}{2}\right]_0^a$$

$$= -\frac{a^3}{2}$$

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Along BC:

$y = a$

$dy = 0$

$$\int_{BC} x^2 dx - xy\, dy$$

$$= \int_a^0 x^2 dx - 0$$

$$= \left[\frac{x^3}{3}\right]_a^0$$

$$= -\frac{a^3}{3}$$

Along CO:

$x = 0$

$dx = 0$

$$\int_{CO} (x^2 dx - xy\, dy)$$

$$= \int_0^0 0 - 0$$

$$= 0$$

Total line integral $= \cancel{\frac{a^3}{3}} - \frac{a^3}{2} - \cancel{\frac{a^3}{3}} + 0$

$$= -\frac{a^3}{2}$$

$$= \text{L.H.S}$$

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$$R.H.S = \iint_S -y\hat{k}\cdot\hat{k}\frac{dy\,dx}{|\hat{n}\cdot\hat{k}|}$$

$$= \iint_S -y\,dy\,dx$$

$$= \int_{x=0}^{x=a}\int_{y=0}^{y=a} -y\,dy\,dx$$

$$= \int_0^a \left[-\frac{y^2}{2}\right]_0^a dx$$

$$= \int_0^a -\frac{a^2}{2}\,dx$$

$$= \left[-\frac{a^2 x}{2}\right]_0^a$$

$$= -\frac{a^3}{2}$$

$L.H.S = R.H.S$ [Proved]

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Gauss Divergence Theorem (relation bet^n Surface integral and volume integral)

Statement: from book (Reading)

Mathematically

$$\iint_S \underbrace{\bar{F} \cdot \hat{n} \, ds}_{\text{flux accross the Surface}} = \iiint_V (\text{div } \bar{F}) \, dv$$

$$\boxed{\text{div } \bar{F} = \lim_{\Delta v \to 0} \frac{\iint_S \bar{F} \cdot \hat{n} \, ds}{\Delta v}}$$

Outward flux per Unit volume

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Monday

Date-17.07.2023

Course-MATH 2203

Ex-110

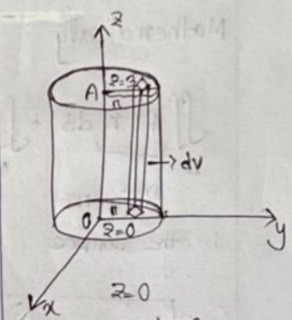
Use Divergence th^m of find $\iint_S \bar{F}\cdot d\bar{s}$, where $\bar{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, S is the surface bounding the region $x^2+y^2=4$, $z=0$, $z=3$

In 3D Circle : Sphere . plane

Circle $\boxed{x^2+y^2+z^2=a^2,\ z=K}$

$x^2+y^2+z^2=a^2 \downarrow$ Sphere, $z=K \downarrow$ Plane

In 3D $x^2+y^2=r^2$, $z=a$ and $z=b$



z=0

xy plane

$$\iint_S \bar{F}\cdot d\bar{s} = \iiint_V \text{div}\ \bar{F} dv$$

Divergence th^m:-

$$\iint_S F\cdot\hat{n}\ ds = \iiint_V \text{div}\ \bar{F} dv \qquad [d\bar{s} = \hat{n}\, ds]$$

$$\text{div}\,\bar{F} = \nabla\cdot\bar{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}\right)$$

$$= 4 - 4y + 2z$$

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Now,

$$\iiint_V (4-4y+2z)\,dx\,dy\,dz \qquad \boxed{dv = dx\,dy\,dz}$$

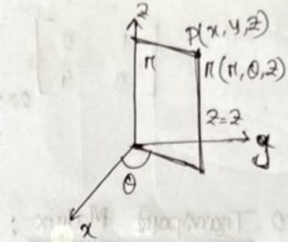
$$= \int_{x=-2}^{x=2}\int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}}\int_{z=0} 2(2-2y+z)\,dz\,dy\,dx$$

$x = r\cos\theta$
$y = r\sin\theta$

$$\boxed{dy\,dx = r\,dr\,d\theta}$$

$r^2 = 4$
$r = 2$

Cylindrical Co-ordinate System(3D)



$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

Cylindrical co-ordinate system:-

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2} 2(2-2r\sin\theta+z)\,dz\,(r\,dr\,d\theta)$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\left[2\left(2z-2r\sin\theta z+\frac{z^2}{2}\right)\right]_{z=0}^{3} r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2}(21-12r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi}\left[21\frac{r^2}{2}-12\frac{r^3}{3}\sin\theta\right]_{r=0}^{2} d\theta$$

$$= \int_{\theta=0}^{2\pi}\left(\frac{21r^2}{2}-4r^3\sin\theta\right)_0^2 d\theta$$

$$= \int_0^{2\pi} d\theta\,(42-32\sin\theta)$$

$$= (42\theta+32\cos\theta)_0^{2\pi}$$

$$= 84\pi + 32 - 32$$

$$= 84\pi$$

(Ans:)

Matrix

Classification :-

1. **Square matrix** : row = Column

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \quad \begin{bmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{bmatrix}_{3\times 3} \quad \text{etc}$$

2. **Transpose Matrix** : A^T, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

3. **Diagonal Matrix** :

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

4. **Symmetric matrix** : (Square Matrix)

$$\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$$

$a_{ii} \neq 0$ $a_{12} = a_{21}$

$i = j$ $a_{13} = a_{31}$

$a_{ij} = a_{ji}$ $a_{32} = a_{23}$

$A^T = A$

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5. Skew Symmetric Matrix :

$$\begin{pmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{pmatrix}$$

$a_{ii} = 0$

$i = j$

$a_{ij} = -a_{ji}$

$A^T = -A$

$a_{12} = a_{21}$

$a_{13} = a_{31}$

$a_{32} = a_{23}$

6. Identity Matrix :

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Thursday

Course-MATH 2203 Date-20.07.2023

* Row Matrix : $\begin{bmatrix} 1 & 2 & 3 & 5 & 8 \end{bmatrix}_{(1\times5)}$

* Column : $\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}_{(3\times1)}$

* Upper triangular matrix :-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

→ Diagonal এর নিচের elements গুলো zero.

* Complex Matrix :-

$$A = \begin{bmatrix} 1 & -2i \\ 2+3i & 4i \end{bmatrix}$$

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* Conjugate : $\bar{A} = \begin{bmatrix} 1 & -2i \\ 2-3i & -4i \end{bmatrix}$

* Hermitian Matrix (Square matrix)

$$\bar{A}^T = A$$

* Skew - Hermitian:-

$$\bar{A}^T = -A$$

* Orthogonal Matrix:-

$$AA^T = A^T A = I$$

* Null - Matrix: -

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Unitary Matrix: (Square)

$$A\bar{A}^T = \bar{A}^T A = I$$

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Inverse of a matrix-

$AB = BA = I$ (Two square matrix A and B are said to be invertable)

$A = B^{-1}I = B^{-1}$

Or $B = A^{-1}$

$AB = I$

$$\boxed{AA^{-1} = I = A^{-1}A}$$

Gauss Jordan Method (elementary row transformation)

Find A^{-1}, $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Sol^n

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - R_1 = R_2'$

$R_3 + 2R_1 = R_3'$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

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$R_2 + R_3 = R_3'$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$$

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Wednesday

Date- 26.07.2023

Course- MATH 2203

Quiz-03

02.08.2023 → Wednesday

Syllabus up to this week

Echelon form/ row- reduced echelon form

An $(m \times n)$ matrix is said to be row echelon form if

a)

b)

c)

* zero row গুলোকে নিচে দিতে হবে
* যদি zero row গুলো non zero এর আগে থাকে then interchange

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

#. Row - echelon Form-

first non zero entry

non-zero entry Backward always

second non zero entry

3rd row non zero entry

$$\begin{pmatrix} 3 & 2 & 0 & 7 & 9 \\ 0 & 4 & 5 & 10 & 0 \\ 0 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

→ non zero entry নিচের সব column zero হবে।

Stair case

→ non zero entry এর বাম দিকের সব zero

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Row-reduced echelon form -

$$\begin{pmatrix} 1 & 0 & 5 & 0 & 4 \\ 0 & -1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{pmatrix}$$

* non-zero entry এর উপরে নিচে both

Normal form - (Canonical form)

(i) I_r (ii) $[I_r, O]$ (iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ (iv) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

Rank: The number of non-zero rows in a matrix is called rank. (lowest number of rank is 1)

$r \to$ rank , $I \to$ Identity matrix

i) $I_r \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2} = I_2$; Rank = 2

ii) $[I_r, O] \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [I_3, 0]$; Rank = 3

O column থাকবে

I_3 0

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(iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$ ∴ Rank = 2

0 row থাকবে

(iv) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

0 column & row থাকবে

∴ Rank = 2

Example-

Reduce to echelon & row reduced echelon form-

$$\begin{bmatrix} 2 & 3 & 5 & -3 & -2 \\ 1 & 1 & -2 & 2 & -1 \\ 5 & 6 & -1 & 3 & -5 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 1 & -2 & 2 & -1 \\ 2 & 3 & 5 & -3 & -2 \\ 5 & 6 & -1 & 3 & -5 \end{bmatrix}$$

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$R_2 - 2R_1 = R_2'$

$R_3 - 5R_1 = R_3'$

$$\begin{bmatrix} 1 & 1 & -2 & 2 & -1 \\ 0 & 1 & 9 & -7 & 0 \\ 0 & 1 & 9 & -7 & 0 \end{bmatrix}$$

$R_2 - R_3 = R_3'$

$$\begin{bmatrix} 1 & 1 & -2 & 2 & -1 \\ 0 & 1 & 9 & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

echelon form

$R_1 - R_2 = R_1$

$$\begin{bmatrix} 1 & 0 & 11 & -9 & -1 \\ 0 & 1 & 9 & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{row echelon reduced form}$$

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Consistency of a system of linear equations-

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n &= b_1 \\ a_{21}x + a_{22}x_2 + \cdots\cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots + a_{mn}x_n &= b_n \end{aligned}\right\} (A)$$

(A) can be written as,

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

↑ Co-efficient matrix

↑ solutions matrix

Augmented matrix, $C = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_n \end{array}\right]$

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A. Consistent equations : Rank(A) = Rank(C)

$\Rightarrow$ System has solutions

Unique Sol^n | Infinite Sol^{n}s

i) Unique Sol^n : If Rank(A) = Rank(C) = no of unknowns = n

ii) Infinite Sol^{n}s : If Rank(A) = Rank(C) = $r < n$

Rank < unknown

B. ~~Co~~ Inconsistence system

\# RANK(A) $\neq$ RANK C

$\Rightarrow$ no solution.

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① Test for consistency & solve.

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

Soln

$$C = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

for soln always use row transformation

$5R_2 - 3R_1 = R_2'$

$5R_3 - 7R_1 = R_3'$

$$\sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

$\frac{1}{11} R_2$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

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$R_2 + R_3 = R_3'$

obtained from co-eff matrix (A)

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$$

from 2^{nd} row,

$11y - z = 3$ — (i)

from 1^{st} row,

$5x + 3y + 7z = 4$ — (ii)

Rank (C) = 2 = no. of non-zero rows

Rank (A) = 2 no. of non-zero rows

$R(A) = R(C) = 2$

$\Rightarrow$ System consistent and has solutions.

Rank (A) = Rank (C) = 2 < 3 (no of unknowns) so

sol^n will be infinite.

Unknown - eq^n

$3 - 2 = 1$

free variable = 1

$z = K$

$y = \dfrac{3 + K}{11}$

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Ex-48

Determine for what values of λ & μ the following System has i) No solution ii) Unique Sol^n iii) Infinite no of sol^{n}s.

$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

Sol^n:-

$$C=\left[\begin{array}{ccc|c}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{array}\right]$$

R_2-R_1, R_3-R_1

$$\left[\begin{array}{ccc|c}1&1&1&6\\0&1&2&4\\0&1&\lambda-1&\mu-6\end{array}\right]$$

R_3-R_2

$$\left[\begin{array}{ccc|c}\multicolumn{3}{c|}{\overbrace{\hspace{6em}}^{A}}&\\1&1&1&6\\0&1&2&4\\0&0&\underset{z=K}{\lambda-3}&\mu-10\end{array}\right]$$

$\mu-10 \rightarrow$ 0 or non zero

Note-

$\lambda=3$

$0x+0y+0z = 0$ or K

$0+0+0=5$ $\quad K\neq 0$

$0=5$

inconsistent

(i) For no solⁿ -

$R(A) \neq R(C)$

$\lambda = 3, \ \mu \neq 10$

$0+0+0 =$ any non zero value

$$\underbrace{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & k \end{array}\right]}_{A} \quad k \neq 0$$

* inconsistence system & no solution

$R(A) = 2, \ R(C) = 3$

(ii) Unique solⁿ -

$\boxed{R(A) = R(C)} = n$ no of unknowns

→ has solution

→ Consistent

unique solution এ ৩ টা value non zero হতে হবে

For $\lambda \neq 3$, μ may have any value

$R(A) = 3 = R(C)$

⇒ Unique solⁿ

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(iii) Infinite sol^n -

$R(A) = R(C) = 2$

$\lambda = 3, \mu = 10$

Quiz-03

Ex - 94, 95, 99, 102, 103, 104, 105, 106, 110, 111, 116

→ vectors

Matrix → Pdf → given to CR

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Thursday

Course – MATH 2203 Date – 03.08.2023

Quadratic form expressed in Matrice's -

Quadratic form $= X^T A X$

Where, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

must be symmetrix

$$X^T A X = \underset{(1\times 3)}{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}} \underset{(3\times 3)}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

row×column

$$= \underset{(1\times 3)}{\begin{bmatrix} [a_{11}x_1 + a_{21}x_2 + a_{31}x_3] & [a_{12}x_1 + a_{22}x_2 + a_{32}x_3] & [a_{13}x_1 + a_{23}x_2 + a_{33}x_3] \end{bmatrix}} \xrightarrow{\text{Multiply}} \underset{(3\times 1)}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$$

$$= [a_{11}x_1^2 + a_{21}x_1x_2 + a_{31}x_1x_3 + a_{12}x_1x_2 + a_{22}x_2^2 + a_{32}x_2x_3 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{33}x_3^2]$$

$$= [a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12}+a_{21})x_1x_2 + (a_{23}+a_{32})x_2x_3 + (a_{31}+a_{13})x_1x_3]$$

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$x^T A x = [a_{11} x_1^2 + a_{22} x_2^2 + a_{33} x_3^2 + 2 a_{12} x_1 x_2 + 2 a_{23} x_2 x_3 + 2 a_{13} x_1 x_3]$

→ matrix polynomial

as, $a_{12} = a_{21}$

$a_{13} = a_{31}$

$a_{23} = a_{32}$

Symmetric matrix

* Polynomial বের করতে $\boxed{x^T A x}$.
* Symmetric না হলে Polynomial বের করা যায় না।

In general,

$a_{ij} = \frac{1}{2}$ co-efficient of x_{ij}, $i \neq j$

→ Polynomial দেয়া থাকলে co-efficient বের করতে হবে।

<u>Example</u> —

Write down the matrix of the quadratic form

$x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1 x_2 + 8 x_1 x_3 + 5 x_2 x_3$

→ $2a_{12} x_1 x_2$
$= 2(-2) x_1 x_2$

Here, $a_{11} = 1$ $\quad a_{12} = -2 = a_{21}$

$a_{22} = 2$ $\quad a_{13} = 4 = a_{31}$

$a_{33} = -7$ $\quad a_{23} = \frac{5}{2} = a_{32}$

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$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & -5/2 \\ 4 & -5/2 & -7 \end{bmatrix}$$

Ans: $X^T A X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & -5/2 \\ 4 & -5/2 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

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Monday

Date- 07.08.2023

Course-MATH 2203

Vector Space (the set of all vectors)

$$V = \{\bar{v}_1, \bar{v}_2, \bar{v}_3 \cdots\cdots \bar{v}_n\}$$

Property for all vectors :

(i) $u, v \in V$, $u+v = v+u$

(ii) $u+(v+\omega) = (u+v)+\omega$

(iii) $0 \in V$ st, $0+v = v+0$ for all $v \in V$

(iv) for each $v \in V$, $v+(-v) = (-v)+v = 0$

(v) $u, v \in V$, $k(u+v) = ku+kv$

$(k+\alpha)v = kv+\alpha v$

(vi) $(k\alpha)v = k(\alpha v)$, $\alpha, k \in F$, $v \in V$

Linearly dependent & independent vectors -

$x_1, x_2, x_3, \ldots\ldots x_n$ are linearly dependent vectors

if (a) all the vectors (row/col) are of the same order

(b) n scalars $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_n$ exist such that, their linear combination.

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Combination vector সুধু আগে scalar গুন

$$\boxed{\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \cdots\cdots + \lambda_n x_n = 0} \rightarrow \text{linear Combination}$$

where at least one is non zero (not all the scalars are zero)

$\rightarrow (\lambda_1 = \lambda_2 = \lambda_3 \cdots\cdots \lambda_n = 0)$ [Scalar all zero]

$$\underset{x_1}{\begin{pmatrix} 1 \\ 2 \end{pmatrix}_{(1\times 2)}} \quad \underset{x_2}{\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}_{(1\times 3)}}$$

not same size

Test for linear dependency –

$$x_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \quad x_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Put, $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$$\lambda_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 0$$

$$2\lambda_1 + 3\lambda_2 + \lambda_3 = 0 \quad \text{—— (1)}$$

$$\lambda_1 + 2\lambda_2 + 2\lambda_3 = 0 \quad \text{—— (2)}$$

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Now, eq^n (1) $- 2\times 2 \Rightarrow$

$$2\lambda_1 + 3\lambda_2 + \lambda_3 = 0$$
$$2\lambda_1 + 4\lambda_2 + 4\lambda_3 = 0$$
$$(-)\quad(-)\quad(-)$$

$$-\lambda_2 - 3\lambda_3 = 0$$
$$\Rightarrow \lambda_2 = -3\lambda_3 \quad \text{--- (3)}$$

$\lambda_2 = -3\lambda_3$ eq^n (1) এ বসাই,

$$2\lambda_1 + 3(-3\lambda_3) + \lambda_3 = 0$$
$$\Rightarrow 2\lambda_1 - 8\lambda_3 = 0$$
$$\Rightarrow \lambda_1 - 4\lambda_3 = 0 \quad \text{--- (4)}$$

$$\lambda_2 + 3\lambda_3 = 0 \quad \text{--- (5)}$$

Let, $\lambda_3 = 1$
$\lambda_2 = -3$
$\lambda_1 = 4$

Let, $\lambda_3 = 2$
$\lambda_2 = -6$
$\lambda_1 = 8$

$$\left.\begin{array}{l}\lambda_3 = K \\ \lambda_2 = -3K \\ \lambda_1 = 4K\end{array}\right\} \text{linearly dependent}$$

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Thursday

Date- 10.08.2023

Example - 53

Test for linear dependency -

$x_1 = (1\ 2\ 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$

$$\boxed{\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0} \text{ --- (A)}$$

$$= \lambda_1 (1\ 2\ 4) + \lambda_2 (2\ -1\ 3) + \lambda_3 (0\ 1\ 2) + \lambda_4 (-3\ 7\ 2) = 0$$

$$\left.\begin{aligned} \lambda_1 + 2\lambda_2 + 0\lambda_3 - 3\lambda_4 &= 0 \\ 2\lambda_1 - \lambda_2 + \lambda_3 + 7\lambda_4 &= 0 \\ 4\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 &= 0 \end{aligned}\right\}$$

In matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 2 & -1 & 1 & 7 & 0 \\ 4 & 3 & 2 & 2 & 0 \end{array}\right]$$

$R_2 - 2R_1$

$R_3 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & -5 & 1 & 13 & 0 \\ 0 & -5 & 2 & 14 & 0 \end{array}\right]$$

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$R_3 - R_2 \Rightarrow$

$$
\begin{array}{cccccc}
\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & & \\
\end{array}
$$

$$
\left[\begin{array}{cccc|c}
1 & 2 & 0 & -3 & 0 \\
0 & -5 & 1 & 13 & 0 \\
0 & 0 & 1 & 1 & 0
\end{array}\right]
$$

$$
\left.\begin{aligned}
\lambda_1 + 2\lambda_2 + 0 - 3\lambda_4 &= 0 \quad (1)\\
-5\lambda_2 + \lambda_3 + 13\lambda_4 &= 0 \quad (2)\\
\lambda_3 + \lambda_4 &= 0 \quad (3)
\end{aligned}\right\}
$$

In (3) Put $\lambda_4 = \boxed{K}$ > arbitary (যেকোনো real number)

$$\lambda_3 = -K$$

In (2) eqn,

$$-5\lambda_2 - K + 13K = 0$$

$$\Rightarrow \lambda_2 = \frac{12K}{5}$$

(1) eqn, $\lambda_1 = -\frac{2K}{5}$

Substitute $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ in (A)

$$-\frac{9K}{5}x_1 + \frac{12K}{5}x_2 - Kx_3 + Kx_4 = 0$$

$$\Rightarrow 9x_1 - 12x_2 + 5x_3 - 5x_4 = 0$$

[vectors are linearly dependent]

(Ans:)

Ex-55

Test for linear dependency,

$$x_1 = (2\ 2\ 1)^T,\ x_2 = (1\ 3\ 1)^T,\ x_3 = (1\ 2\ 2)^T$$

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$$

$$\lambda_1\begin{pmatrix}2\\2\\1\end{pmatrix} + \lambda_2\begin{pmatrix}1\\3\\1\end{pmatrix} + \lambda_3\begin{pmatrix}1\\2\\2\end{pmatrix} = 0$$

$$2\lambda_1 + \lambda_2 + \lambda_3 = 0$$

$$2\lambda_1 + 3\lambda_2 + 2\lambda_3 = 0$$

$$\lambda_1 + \lambda_2 + 2\lambda_3 = 0$$

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$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 2 & 3 & 2 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right]$$

$R_2 - R_1$

$R_3 - \frac{1}{2}R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & 0 \end{array}\right]$$

For linearly independent vectors,

$|A| \neq 0$

$R_3 - \frac{1}{4}R_2$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & \frac{5}{4} & 0 \end{array}\right]$$

$$2\lambda_1 + \lambda_2 + \lambda_3 = 0$$

$$2\lambda_2 + \lambda_3 = 0$$

$$\frac{5}{4}\lambda_3 = 0$$

Putting values,

$\lambda_3 = 0$

$2\lambda_2 + \lambda_3 = 0$

$\Rightarrow \lambda_2 = 0$

$2\lambda_1 + \lambda_2 + \lambda_3 = 0$

$\Rightarrow 2\lambda_1 + 0 + 0 = 0$

$\Rightarrow \lambda_1 = 0$

As al the values are 0 So this eqn is independent

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x nonzero vector [Eigenvalue & eigenvector]

$$Ax = \lambda x$$

$$Ax - \lambda x = 0$$

$$[A - \lambda I]\, x = 0$$

$|A - \lambda I| = 0 \rightarrow$ characteristic eqn.

$\lambda \rightarrow$ eigenvalue, $x \rightarrow$ eigen vector

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Monday

Date-14.08.2023

Course - MATH 2203

Eigen value & Eigen vector

$A\bar{v} = \lambda \bar{v}$ $\quad v \rightarrow$ eigen vector (non zero vector in $\mathbb{R}^n$)

$A\bar{v} - \lambda\bar{v} = 0$ $\quad$ Corresponding to λ.

[eigen means Proper or own value]

$\boxed{(A-\lambda)\bar{v} = 0}$ $\quad \lambda \rightarrow$ eigenvalue (scalar)

$\rightarrow$ Characteristics matrix equation

$\Rightarrow \boxed{|A - \lambda I| = 0} \rightarrow$ Characteristics equation

The set of all vectors is called eigenspace of λ

$A = \begin{bmatrix} \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \end{bmatrix}_{(n\times n)}$

$\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$

$|A| = \lambda_1 \times \lambda_2 \times \lambda_3 \times \ldots\ldots \times \lambda_4$

KA

(i) $K\lambda_1, K\lambda_2, \ldots\ldots K\lambda_n$

(ii) A^m are $\lambda_1^m, \lambda_2^m \ldots\ldots \lambda_n^m$

(iii) A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots\ldots, \frac{1}{\lambda_n}$

Example -

① $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, find the eigenvalues of A.

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Solution -

The characteristic eq^n of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(3-\lambda)(-2-\lambda) = 0$$

$$\Rightarrow \lambda = 1, 3, -2$$

CALEY - HAMILTON THEOREM

Statement - Every square matrix satisfies its own Characteristic equation. $\boxed{A - \lambda I = 0}$

Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}_{(2\times 2)}$ we know characteristic eq^n is now the characteristic Polynomial of A is,

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix}$$

$$= \boxed{\lambda^2 - 3\lambda - 4} \rightarrow (1)$$

From caley Hamilton A Satisfies its characteristic polynomial.

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Put $\lambda = A$ in (1)

$A^2 - 3A - 4I$

$= \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(2) Using caley Hamilton theorem find the inverse of

$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Sol^n:-

Characteristic eq^n of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 3 & 1-\lambda & 0 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda^3 + 3\lambda^2 + 5\lambda - 1 = 0$ —— (1)

Put $\lambda = A$ in (1) $-A^3 + 3A^2 + 5A - 1 = 0$ —— (2)

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$5 = 5I$
$AA^{-1} = I$

Multiply both sides of ② by A^{-1} (or divided by A)

$$-A^2 + 3A + 5I - A^{-1} = 0$$

$$A^{-1} = -A^2 + 3A + 5I$$

$$= -\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{bmatrix} \text{ (Ans:)}$$

Example -

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & -2-\lambda & 0 \\ 0 & -5 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 4) = 0$$

$$\Rightarrow (\lambda - 1)(\lambda + 2)(\lambda - 2) = 0$$

$\lambda = 1, -2, 2 \rightarrow$ eigen values

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Now by def^n $\bar{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is an eigen vector of A

we have,

$$(A - \lambda I)\bar{v} = 0 \quad \text{for } \lambda = 1$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & -5 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2v_2 - v_3 = 0 \qquad -5v_2 + v_3 = 0$

$-3v_2 = 0$

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$y + 2z = 0$; here x arbitrary

$y = 0$

$z = 0$

Let, $x = k$

$y = 0$

$z = 0$

Eigen vector - $\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$

→ Particular Eigen, $k = 1$

so, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\lambda = 2, 2$

$v_1 = \begin{bmatrix} \ \end{bmatrix}, v_2 = \begin{bmatrix} \ \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$C_1 v_1 + C_2 v_2 + C_3 v_3 = 0$

$\Rightarrow C_1 = C_2 = C_3 = 0$ → independent Eigen vector

Fourier Series

If $f(x)$ is periodic in $(-l, l)$ / $(-\pi, \pi)$ / $(0, 2\pi)$ then the fourier series is -

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega x + b_n \sin n\omega x)}$$

a_0, a_n, b_n → Coefficients

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For $(-l, l)$

where $\boxed{a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\, dx}$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{\begin{aligned} a_n &= \frac{1}{l}\int_{-l}^{l} f(x) \cos n\omega x\, dx \\ b_n &= \frac{1}{l}\int_{-l}^{l} f(x) \sin n\omega x \end{aligned}}$$

$m^2 + \mu = 0$

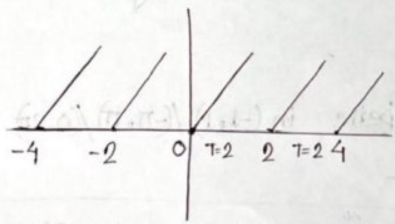
$m = \pm i\sqrt{\mu}$

$\sqrt{\mu} = \omega \quad [m = \pm i\omega]$

$$\frac{d^2 x}{dt^2} = -\mu x$$

$$x(t) = A\cos\underbrace{\sqrt{\mu}}_{\omega} t + B \sin\sqrt{\mu}\, t$$

$$= A\cos\omega t + B\sin\omega t$$



$$f(x) = f(x+T) = f(x+2T)$$

$$= f(x+3T) = \ldots\ldots\ldots = f(x+nT)$$

Period = least value of T

$f(x) = \sin(x+2\pi) = \sin(x+4\pi)$

$\sin(x+6\pi) = \ldots\ldots = \sin(x+2n\pi)$

$\boxed{T=2\pi} \rightarrow$ least

* Same for $\cos x$

Period -

$(-l,l) \Rightarrow T = l-(-l) = 2l$

$(-\pi,\pi) \Rightarrow T = \pi-(-\pi) = 2\pi$

$(0,2\pi) \Rightarrow T = 2\pi$

$(-2,2) \Rightarrow T = 2-(-2) = 4$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos n\frac{\pi}{l}x\,dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin n\frac{\pi}{l}x\,dx$$

for $(-\pi,\pi)$

$T = 2\pi$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$\boxed{\omega = 1}$

$$\Rightarrow f(x) = \frac{a_0}{2}\sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

for $(0, 2\pi)$,

$$a_0 = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\, dx$$

$$a_1 = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\sin nx\, dx$$

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Monday

Date- 21.08.2023

Course - MATH 2203

$f(x)$ is periodic

For even function : $f(-x) = f(x)$

For odd function : $f(-x) = -f(x)$

→ For $f(x)$ even

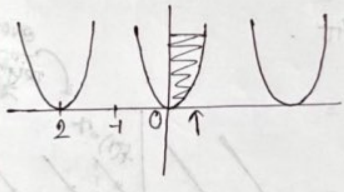
Find a_0, a_n and $b_n = 0$ where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$;

$$a_n = \frac{2}{l}\int_0^l f(x)\cos n\omega x\,dx$$

$$\left[\omega = \frac{2\pi}{2l} = \frac{\pi}{l}\right]$$

For $f(x)$ odd,

$a_n = 0$ and find b_n, where $b_n = \frac{2}{l}\int_0^l f(x)\sin n\omega x\,dx$



$$\int_{-1}^{1} \underset{\uparrow \text{even}}{f(x)}\,dx = 2\int_0^1 f(x)\,dx$$

For $(-\pi,\pi)$, $(0,2\pi)$

$a_0 = \frac{1}{\pi}\int_0^\pi f(x)\,dx$

$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$

For $(-\pi,\pi)$, $(0,2\pi)$

$b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

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* $\int_{-a}^{a} f(x)\,dx = 0$

$f(-x) = -f(x)$

$\Rightarrow f(x)$ is odd

* $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

if $f(-x) = f(x)$

$\Rightarrow f(x)$ is even

* $f(x) = x^3$

$f(-x) = (-x)^3$

$\Rightarrow -f(x) = -x^3$

$\uparrow$
odd

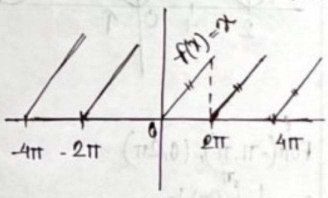
Example-1

$f(x) = x$, $0 < x < 2\pi$; find the Fourier Series & Sketch the graph from -4π to 4π.

Here, $T = 2\pi$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

even/odd কোনটাই না ~~এর~~ function



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Fourier Series -

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= 2\pi$$

useful formula -

$\sin n\pi = 0$

$\cos n\pi = (-1)^n$

$\cos 2n\pi = 1$

$$a_n = \frac{1}{\pi}\int \underbrace{x}_{u} \underbrace{\cos nx}_{v}\, dx = \frac{1}{\pi}\left[\frac{x \sin nx}{n}\right]_0^{2\pi} - \frac{1}{\pi}\int_0^{2\pi} \frac{\sin nx}{n}\, dx$$

$$= 0 + \frac{1}{\pi n^2}\left[\cos nx\right]$$

$$= \frac{1}{\pi n^2}\left[\cos 2n\pi - 1\right]$$

$$= \frac{1}{\pi n^2}(1-1) = 0$$

$$\boxed{\therefore a_n = 0}$$

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[Exp-8, 9]

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n}\right]_0^{2\pi} + \int_0^{2\pi} \frac{\cos nx}{n}\, dx$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\cos 2n\pi}{n} - 0\right] + \frac{1}{\pi}\left[\frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= -\frac{1}{\pi}\left[-\frac{2\pi}{n}\right] + \frac{1}{\pi}[0-0]$$

$$= -\frac{2}{n}$$

$$\boxed{\therefore b_n = -\frac{2}{n}}$$

$$\boxed{* \int uv\, dx = u\int v\, dx - \int\left\{\frac{du}{dx}\int v\, dx\right\} dx} \rightarrow \text{সূত্র}$$

for
$n = 1, 2, 3$
$b_1, b_2, b_3, b_4 \ldots$

মান বের করতে হবে

$b_n = -\frac{2}{n}$
$b_1 = -2$
$b_2 = -1$
$b_3 = -\frac{2}{3}$
$b_4 = -\frac{1}{2}$

F series -

$$x = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \pi + (b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \ldots\ldots) \quad [a_n = 0]$$

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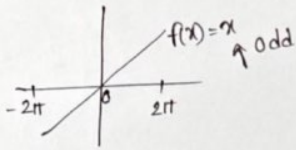
* $-\pi < x < \pi$

$f(x) = x$



[even-odd করতে হলে function must positive - negative হতে হবে]

Example-8

$f(x) = x^2 \quad ; \; -\pi < x < \pi$

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$b_n = 0$ [Since $f(x)$ is even]

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$

Example-9

$f(x) = x^3 \quad ; \; -\pi < x < \pi$

$\boxed{a_n = a_0 = 0}$

$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

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