תרגיל מס.6

עפיף חלומה 302323001

31 בדצמבר 2009

1 שאלה 1

$$
\begin{aligned}
I_{N1} &= \frac{E_s}{R_2} \\
X_{C\|R_2} &= X_C \parallel R_2 \\
&= \frac{R_2 \cdot \frac{1}{i\omega C}}{R_2 + \frac{1}{i\omega C}} \\
&= \frac{R_2}{R_2 \cdot i\omega C + 1}
\end{aligned}
$$

נעבור לתבינין

$$
\begin{aligned}
V_{T1} &= I_{N1} \cdot X_{C\|R_2} \\
&= \frac{R_s}{R_2 i\omega C + 1} \\
X_{T1} &= X_{C\|R_2} + X_L \\
&= \frac{R_2}{R_2 \cdot i\omega C + 1} + i\omega L \\
&= \frac{R_2\left(1 - \omega^2 CL\right) + i\omega L}{R_2 i\omega C + 1}
\end{aligned}
$$

נעבור לנורטון

$$
\begin{aligned}
I_{N2} &= \frac{V_{T1}}{X_{C\|R_2+L}} \\
&= \frac{E_s}{R_2 j\omega C + 1} \cdot \frac{R_2 i\omega C + 1}{R_2\left(1 - \omega^2 CL\right) + i\omega L} \\
&= \frac{E_s}{R_2\left(1 - \omega^2 CL\right) + i\omega L} \\
X_{N2} &= X_{C\|R_2+L} \parallel X_{R_1} \\
&= \frac{\left(\frac{R_2\left(1-\omega^2 CL\right)+i\omega L}{R_2 i\omega C+1} \cdot R_1\right)}{\left(\frac{R_2\left(1-\omega^2 CL\right)+i\omega L}{R_2 i\omega C+1} + R_1\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{R_1R_2\left(1-\omega^2CL\right)+i\omega LR_1}{R_2\left(1-\omega^2CL\right)+i\omega L+R_2R_1\cdot i\omega C+R_1} \\
&= \frac{R_1R_2\left(1-\omega^2CL\right)+i\omega LR_1}{R_2\left(1-\omega^2CL\right)+R_1+i\omega\left(L+R_2R_1\cdot C\right)}
\end{aligned}
$$

2 שאלה 2

2.1 א

$$
\begin{aligned}
Z &= R+i\omega L \\
I &= \frac{V}{Z} \\
I &= \frac{1\cdot e^{i\cdot 0}}{\left(R^2+\omega^2L^2\right)e^{i\tan^{-1}\left(\frac{\omega L}{R}\right)}} \\
|I|\,e^{i\theta} &= \frac{1}{\left(R^2+\omega^2L^2\right)}\cdot e^{-i\tan^{-1}\left(\frac{\omega L}{R}\right)} \\
|I| &= \frac{1}{\left(R^2+\omega^2L^2\right)} \\
\theta &= -\tan^{-1}\left(\frac{\omega L}{R}\right) \\
-\tan^{-1}\left(\frac{\omega L}{R}\right) &= -78.69^\circ \\
\frac{\omega L}{R} &= 5 \\
\omega &= \frac{5R}{L}
\end{aligned}
$$

2.2 ב

$$
\begin{aligned}
V_L &= I\cdot X_L \\
&= \frac{1}{\left(R^2+(\omega)^2L^2\right)}e^{i(-78.69^\circ)}\cdot\omega Le^{i\cdot(90^\circ)} \\
&= \frac{1}{\left(R^2+\left(\frac{5R}{L}\right)^2L^2\right)}e^{i(-78.69^\circ)}\cdot\left(\frac{5R}{L}\right)Le^{i\cdot(90^\circ)} \\
&= \frac{5}{26R}e^{i(-78.69^\circ)}\cdot e^{i\cdot(90^\circ)} \\
&= \frac{5}{26R}e^{i11.4^\circ}
\end{aligned}
$$

3 שאלה 3

3.1 א

$$
\begin{aligned}
Z &= R + X_L \parallel X_C \\
&= R + \left(\frac{1}{X_L} + \frac{1}{X_c}\right)^{-1} \\
&= R + \left(\frac{1}{i\omega L} + i\omega C\right)^{-1} \\
&= R + \left(\frac{1-\omega^2 LC}{i\omega L}\right)^{-1} \\
&= R + \overbrace{\frac{i\omega L}{1-\omega^2 LC}}^{X_L \parallel X_C} \\
&= \frac{i\omega L + \left(1-\omega^2 LC\right) R}{1-\omega^2 LC} \\
V &= V_s \cdot \frac{X_L \parallel X_C}{R + X_L \parallel X_C} \\
&= V_s \frac{\left(\frac{i\omega L}{1-\omega^2 LC}\right)}{\left(\frac{i\omega L + (1-\omega^2 LC) R}{1-\omega^2 LC}\right)} \\
&= \left(\frac{i\omega L}{1-\omega^2 LC}\right)\left(\frac{1-\omega^2 LC}{i\omega L + (1-\omega^2 LC) R}\right) \\
&= \frac{i\omega L}{i\omega L + (1-\omega^2 LC) R} \\
|V| &= \frac{\omega L}{\sqrt{\omega^2 L^2 + (1-\omega^2 LC)^2 R^2}} \\
&= \frac{\omega}{\sqrt{4 - 3\omega^2 + \omega^4}} \\
\angle V &= \tan^{-1}\left(\frac{2}{\omega} - \omega\right) \\
V(t) &= \frac{\omega}{\sqrt{4 - 3\omega^2 + \omega^4}} \cos\left(\omega t + \angle V\right)
\end{aligned}
$$

3.2 ב

$$H_{max} = \frac{H(j\omega)}{H(j\omega_0)} = \frac{1}{\sqrt{2}}$$

כדי למצא את ω_0 צריך הפאזור בין המקור לזרם הנכנס יהיה 0.
כאן זה קורה כאשר אדמיטנס מקבל מינימום כלומר

$$1 - \omega_0^2 LC = 0$$

$$
\begin{aligned}
H\left(j\omega_0\right) &= \frac{V\left(j\omega_0\right)}{V_S\left(j\omega_0\right)} = \frac{1+0j}{1+0j} = 1 = H_{max} \\
H\left(j\omega\right) &= \frac{1}{\sqrt{2}} = \frac{|V|}{|V_s|} = |V| \\
\frac{1}{2} &= \frac{\omega^2}{4+\omega^4-3\omega^2} \\
\omega^4-5\omega^2+4 &= 0 \\
\omega_{1,2}^2 &= \frac{5\pm 3}{2} \\
\omega_{1,2}^2 &= 4 \\
\omega_{3,4}^2 &= 1 \\
\omega_{1,2} &= \pm 2 \\
\omega_{3,4} &= \pm 1
\end{aligned}
$$

$$
\omega > 0 \Rightarrow \omega_1 = 2, \omega_2 = 1 \Rightarrow \Delta\omega = 1
$$

3.3 ג

$$
H = \frac{|V|}{\left(\underbrace{|V_s|}_{1}\right)} = |V|
$$

$$
\begin{aligned}
\frac{\partial |V|}{\partial \omega} &= \frac{\omega\left(-6\omega+4\omega^4\right)}{2\left(4-3\omega^2+\omega^4\right)^{3/2}} + \frac{1}{\left(4-3\omega^2+\omega^4\right)^{3/2}} \\
0 &= \frac{4-\omega^4}{\left(4-3\omega^2+\omega^4\right)^{3/2}} \\
\omega &= \sqrt{2}
\end{aligned}
$$

4 שאלה 4

$$
\begin{aligned}
Z &= 1+i\omega+\left(\frac{1}{i\omega} \parallel 1\right) \\
&= 1+i\omega+\frac{1\cdot\frac{1}{i\omega}}{1+\frac{1}{i\omega}}
\end{aligned}
$$

$$
\begin{aligned}
&= 1+i\omega+\frac{1}{1+i\omega} \\
&= \frac{1+(1+i\omega)^2}{1+i\omega} \\
&= \frac{2+2i\omega-\omega^2}{1+i\omega}\cdot\frac{1-i\omega}{1-i\omega} \\
&= \frac{2-2i\omega+2\omega^2-\omega^2+i\omega^3}{1+\omega^2} \\
&= \frac{2+\omega^2+i\omega^3}{1+\omega^2}
\end{aligned}
$$

$$
\begin{aligned}
|Z| &= \frac{\sqrt{(2+\omega^2)^2+\omega^6}}{1+\omega^2} \\
\angle Z &= \tan^{-1}\left(\frac{\omega^3}{2+\omega^2}\right)
\end{aligned}
$$

4.1 ב

$$
\begin{aligned}
\omega &= 2 \\
|V| &= 10 \\
\angle V &= 0 \\
|Z| &= 2 \\
\angle Z &= 53.13^\circ
\end{aligned}
$$

אזי

$$
\begin{aligned}
I &= \frac{V}{Z} \\
&= \frac{10e^0}{2e^{i53.13^\circ}} \\
&= 5e^{-i53.13^\circ}
\end{aligned}
$$

4.2 ג

מכיוון שהכל ליניארי אנחנו פשוט מאוד מחשבים כל רכיב בנפרד ומסכמים:

$$
\begin{aligned}
V &= 1 \\
\omega &= 0 \\
|V| &= 0 \\
\angle V &= 0
\end{aligned}
$$

$$
\begin{aligned}
|Z| &= \frac{2}{1} \\
\angle Z &= \tan^{-1}\left(\frac{8}{6}\right) = 0
\end{aligned}
$$

$$
V = IZ = 1 \cdot 2 = 2\angle 0
$$

$$
\begin{aligned}
V &= \cos t \\
\omega &= 1 \\
|V| &= 1
\end{aligned}
$$

$$
\begin{aligned}
|Z| &= \frac{\sqrt{10}}{2} \\
\angle Z &= \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ
\end{aligned}
$$

$$
\begin{aligned}
V &= IZ = 1\angle 0 \cdot \frac{\sqrt{10}}{2}\angle 18.43 \\
&= \frac{\sqrt{10}}{2}\angle 18.43
\end{aligned}
$$

$$
\begin{aligned}
\omega &= 2 \\
|I| &= 1 \\
|Z| &= 2 \\
\angle Z &= 53.1^\circ \\
V &= |I|\angle I \cdot 2\angle 53.1^\circ \\
&= 2\cos\left(2t + 53.1\right)
\end{aligned}
$$

אזי

$$
V = 2 + \frac{\sqrt{10}}{2}\cos\left(t + 18.43^\circ\right) + 2\cos\left(2t - 53.13^\circ\right)
$$

5 שאלה 5

5.1 א

נרשום את המתחים בצורה יותר נוחה:

$$\begin{aligned} V_a &= 10\angle 60^\circ \\ V_b &= 5\angle -30^\circ \end{aligned}$$

נשתמש במחלק מתח כדי למצא את Z_n:

$$\begin{aligned} V_a &= V_b + V_n \\ V_b &= V_a \cdot \frac{Z_c}{Z_c + Z_n} \\ 5\angle -30^\circ &= (10\angle 60^\circ) \cdot \frac{-10j}{-10j + Z_n} \\ (5\angle -30^\circ)(10\angle -90^\circ + Z_n) &= (10\angle 60^\circ) \cdot (10\angle -90^\circ) \\ (50\angle -120^\circ) + Z_n (5\angle -30^\circ) &= (10\angle 60^\circ)(10\angle -90^\circ) \\ Z_n &= \frac{(10\angle 60^\circ)(10\angle -90^\circ) - (50\angle -120^\circ)}{(5\angle -30^\circ)} \\ &= \frac{(100\angle -30^\circ)}{(5\angle -30^\circ)} - \frac{(50\angle -120^\circ)}{(5\angle -30^\circ)} \\ &= 20\angle 0^\circ - 10\angle -90^\circ \\ &= \sqrt{500}\angle 26.5^\circ \end{aligned}$$

5.2 ב

$$\begin{aligned} I &= I_c \\ &= \frac{5\angle -30^\circ}{10\angle -90} \\ &= 0.5\angle 60^\circ \\ V_n &= V_a - V_b \\ &= 10\angle 60 - 5\angle -30 \\ &= 0.6698 + 11.16i \\ &= 11.18\angle 86.56^\circ \\ P_{av} &= \frac{1}{2}|V||I|\cos(\angle V - \angle I) \\ &= \frac{1}{2}(11.18)(0.5)\cos(86.56^\circ - 60^\circ) \\ &= 2.5\mathrm{W} \end{aligned}$$