תרגיל מס.8

עפיף חלומה 302323001

22 בדצמבר 2009

1 שאלה 1

1.1 א

$$
\begin{aligned}
g(x,y) &= \left[rect\left(\frac{x}{a/2}\right)\cdot rect\left(\frac{y}{b/2}\right)\right] * \left(\delta(x-d,y)\cdot\delta(x+d,y)\right)\\
\mathcal{F}\left(rect\left(\frac{x}{a/2}\right)\cdot rect\left(\frac{y}{b/2}\right)\cdot\right) &= |ab|\, sinc\left(a/2k_x, b/2k_y\right)\\
\mathcal{F}\left(\delta(x-d,y)+\delta(x+d,y)\right) &= 2\cos(k_x d)\\
a(k_x,k_y) &= 2|ab|\cos(k_x d)\, sinc\left(\frac{a}{2}k_x, \frac{b}{2}k_y\right)
\end{aligned}
$$

1.2 ב

$$
\begin{aligned}
\mathcal{F}\left(\frac{1}{2}\left(1+m\cos(2\pi f_0 x)\right)\right) &= \mathcal{F}\left(\frac{1}{2}+\frac{m}{4}e^{i2\pi f_0 x}+\frac{m}{4}e^{-i2\pi f_0 x}\right)\\
&= \frac{1}{2}\delta(k_x,k_y)+\frac{m}{4}\delta(k_x-2\pi f_0)+\frac{m}{4}\delta(k_x+2\pi f_0)\\
\mathcal{F}\left(rect\left(\frac{x}{l}\right)rect\left(\frac{y}{l}\right)\right) &= 4l^2 sinc(lk_x)\, sinc(lk_y)\\
\mathcal{F}(g(x,y)) &= \left(4l^2 sinc(lk_x)\, sinc(lk_y)\right) * \left(\frac{1}{2}\delta(k_x,k_y)+\frac{m}{4}\delta(k_x-2\pi f_0)+\frac{m}{4}\delta(kx+2\pi f_0)\right)\\
&= 2l^2 sinc(lk_y)\left[sinc(lk_x)+\frac{m}{2}sinc\left(l(k_x-2\pi f_0)\right)+\frac{m}{2}sinc(lk_x+2\pi f_0)\right]
\end{aligned}
$$

2 שאלה 2

$$
\begin{aligned}
\vec{S} &= \frac{1}{\mu_0}\vec{E}\times\vec{B}\\
I &= \frac{n\varepsilon_0 c}{2}|E|^2
\end{aligned}
$$

$$
\begin{aligned}
\vec{E} &= E_0 \cos\left(\omega t - \vec{k}\vec{r}\right) \\
\vec{B} &= B_0 \cos\left(\omega t - \vec{k}\vec{r}\right)(-\cos\theta\hat{x} + \sin\theta\hat{z}) \\
|B_0| &= \frac{n}{c} E_0 \\
S &= \frac{1}{\mu_0}\frac{n}{c} E_0^2 \cos\left(\omega t - \vec{k}\vec{r}\right) \\
\langle |S| \rangle &= \frac{1}{T}\int_0^T \frac{1}{\mu_0} E_0^2 \frac{n}{c} \cos^2\left(\omega t - \vec{k}\vec{r}\right) dt \\
&= \frac{1}{\mu_0}\frac{n}{c}\frac{1}{2} = \frac{nE_0^2}{2\mu_0 c} \\
c^2 &= \frac{1}{\mu_0 \varepsilon_0} \\
\langle S \rangle &= \frac{nE_0^2}{2c}\varepsilon_0 c^2 = \frac{1}{2} c n \varepsilon_0 \left|E_0\right|^2 \\
I &= \langle S \rangle
\end{aligned}
$$