

תרגיל מס. 6.

עפיף חלומה 302323001

16 בדצמבר 2009

שאלה 1

א 1.1

$$\begin{aligned}h(t) &= e^{-\alpha t} u(t) \\x(t) &= \sin(\omega t) u(t)\end{aligned}$$

$$\begin{aligned}(h * x)(t) &= \int_{-\infty}^t h(\tau) \cdot x(t - \tau) d\tau \\&= \begin{cases} t < 0 & 0 \\ t > 0 & \int_{-\infty}^t h(\tau) \cdot x(t - \tau) d\tau \end{cases} \\&= \begin{cases} t < 0 & 0 \\ t > 0 & \int_{-\infty}^t e^{-\alpha \tau} \cdot \sin(\omega(t - \tau)) d\tau \end{cases} \\&= \begin{cases} t < 0 & 0 \\ t > 0 & \left[\frac{e^{-\alpha k} (\alpha \sin(\omega(\tau - t)) + \omega \cos(\tau - t))}{\alpha^2 + \omega^2} \right]_0^t \end{cases} \\&= \begin{cases} t < 0 & 0 \\ t > 0 & \frac{\omega e^{-\alpha t} + \alpha \sin(\omega t) - \omega \cos(\omega t)}{\alpha^2 + \omega^2} \end{cases}\end{aligned}$$

ב 1.2

$$\begin{aligned}h(t) &= 2 \sin(\omega t) u(t) \\x(t) &= 2 \cos(\omega t) u(t)\end{aligned}$$

$$\begin{aligned}
(h * x)(t) &= \begin{cases} t < 0 & 0 \\ t \geq 0 & \int_0^t h(t-\tau) x(\tau) d\tau \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & 4 \int_0^t \sin(\omega(t-\tau)) \cos(\omega\tau) d\tau \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & 4 \cdot \frac{2\tau\omega \sin(\omega t) + \cos(\omega(2\tau-t))}{4\omega} \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & \frac{2\tau\omega \sin(\omega t) + \cos(\omega(2\tau-t))}{\omega} \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & 2t \sin(\omega t) \end{cases}
\end{aligned}$$

1.3

$$\begin{aligned}
h(t) &= 8r(t) \\
x(t) &= 3u(t) - 2u(t-6)
\end{aligned}$$

$$\begin{aligned}
(h * x)(t) &= \begin{cases} t < 0 & 0 \\ t \geq 0 & h(t-\tau) x(\tau) \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & \int_0^t (8r(t-\tau)) (3u(\tau) - 2u(\tau-6)) d\tau \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & \int_0^t (8r(t-\tau)) (3u(\tau) - 2u(\tau-6)) d\tau \end{cases} \\
&= \begin{cases} t < 0 & 0 \\ t \geq 0 & [(\tau-6)u(\tau-6)(\tau-2t+6) - \frac{3}{2}\tau u(\tau)(\tau-2t)] \cdot 8r(t) \end{cases} \\
&= \begin{cases} t \geq 0 & 8u[-6+k](36-k^2 + (-36+t^2)u[6-t] + (k^2-t^2)u[k-t]) + \\ & -12u[k](-k^2 + (k^2-t^2)u[k-t] + t^2u[-t]) \\ t < 0 & 0 \end{cases} \\
&= \begin{cases} 12t^2 & 0 < t \leq 6 \\ 4(72+t^2) & t > 6 \\ 0 & \text{Else} \end{cases}
\end{aligned}$$

שאלה 2

א 2.1

$$\begin{aligned}x(t) &= (1-t)u(t)u(1-t) \\h(t) &= (1-t)u(t)u(1-t)\end{aligned}$$

בתחום $t < 0$ מקבלים $y = 0$
בתחום $0 < t < 1$ יש חפיפה מצד שמאל אזי:

$$\begin{aligned}h * x &= \int_0^t (1-\tau)(t-\tau) d\tau \\&= \frac{t^2}{2} - \frac{t^3}{6}\end{aligned}$$

בתחום $2 > t > 1$

$$\int_{t-1}^1 (1-\tau)(t-\tau) d\tau = \frac{2}{3} - \frac{t^2}{2} + \frac{t^3}{6}$$

בתחום $t > 2$ מקבלים $y = 0$.

ב 2.2

קודם נפתור עבור $h = u(t) - u(1-t)$
עבור $t < -1$ מקבלים $y = 0$
עבור $-1 < t < 0$ מקבלים:

$$\begin{aligned}\int_{-1}^t 1 \cdot (-2\tau) d\tau &= \left[-2\frac{1}{2}\tau^2 \right]_{-1}^t \\&= -t^2 + 1\end{aligned}$$

עבור $0 < t < 1$:

$$\begin{aligned}\int_{t-1}^0 1 \cdot (-2\tau) + \int_0^t -\tau d\tau &= [-\tau^2]_{t-1}^0 + \left[-\frac{1}{2}\tau^2 \right]_0^t \\&= (t-1)^2 + \frac{1}{2}t^2\end{aligned}$$

עבור $1 < t < 2$:

$$\begin{aligned}\int_{t-1}^2 \tau d\tau &= \left[\frac{1}{2} \tau^2 \right]_{t-1}^2 \\ &= 2 - \frac{1}{2} (t-1)^2\end{aligned}$$

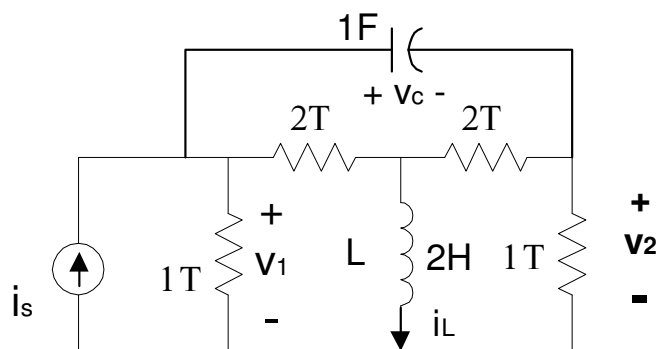
עבור $t > 2$ מקבלים $y = 0$.

$$y(t) = \begin{cases} t < -1 & 0 \\ -1 < t < 0 & -t^2 + 1 \\ 0 < t < 1 & (t-1)^2 + \frac{1}{2}t^2 \\ 1 < t < 2 & 2 - \frac{1}{2}(t-1)^2 \\ t > 2 & 0 \end{cases}$$

אזי התשובה הסופית:

$$\begin{aligned}y_{final} &= y(t) + y(t-2) \\ &= \begin{cases} t < -1 & 0 \\ -1 < t < 0 & -t^2 + 1 \\ 0 < t < 1 & (t-1)^2 + \frac{1}{2}t^2 \\ 1 < t < 2 & 2 - \frac{1}{2}(t-1)^2 \\ t > 2 & 0 \end{cases} + \begin{cases} t < 1 & 0 \\ 1 < t < 2 & -(t-2)^2 + 1 \\ 2 < t < 3 & (t-3)^2 + \frac{1}{2}(t-2)^2 \\ 3 < t < 4 & 2 - \frac{1}{2}(t-3)^2 \\ t > 4 & 0 \end{cases} \\ &= \begin{cases} t < -1 & 0 \\ -1 < t < 0 & -t^2 + 1 \\ 0 < t < 1 & (t-1)^2 + \frac{1}{2}t^2 \\ 1 < t < 2 & 2 - \frac{1}{2}(t-1)^2 + -(t-2)^2 + 1 \\ 2 < t < 3 & (t-3)^2 + \frac{1}{2}(t-2)^2 \\ 3 < t < 4 & 2 - \frac{1}{2}(t-3)^2 \\ t > 4 & 0 \end{cases}\end{aligned}$$

שאלה 3 3



איור 1: מעגל מסובך

$$\begin{pmatrix} 1 + \frac{1}{2} + s & -\frac{1}{2} & -s \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{2} + \frac{1}{2s} & -\frac{1}{2} \\ -s & -\frac{1}{2} & \frac{1}{2} + \frac{1}{1} + s \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} I_s \\ 0 \\ 0 \end{pmatrix}$$