

## תרגיל 5

עפ"י חלומה 302323001

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### 1 שאלה 1

נסמן:  $\frac{\partial}{\partial x}\phi(x, y) = \beta(x, y)$  ו  $\frac{\partial}{\partial x}\phi(x, y) = \alpha(x, y)$

$$\begin{aligned}\frac{\partial}{\partial x}\phi(X(x, y), Y(x, y)) &= \alpha(X(x, y), Y(x, y)) \cdot \left( \frac{\partial X(x, y)}{\partial x} + \frac{\partial Y(x, y)}{\partial x} \right) \\ \frac{\partial^2}{\partial x^2}\phi(X(x, y), Y(x, y)) &= \beta(X(x, y), Y(x, y)) \cdot \left( \frac{\partial X(x, y)}{\partial x} + \frac{\partial Y(x, y)}{\partial x} \right) \cdot \frac{\partial X(x, y)}{\partial x} \\ &\quad + \alpha(X(x, y), Y(x, y)) \cdot \left( \frac{\partial^2 X(x, y)}{\partial x^2} + \frac{\partial^2 Y(x, y)}{\partial x^2} \right) \\ \frac{\partial^2}{\partial y^2}\phi(X(x, y), Y(x, y)) &= \beta(X(x, y), Y(x, y)) \cdot \left( \frac{\partial X(x, y)}{\partial y} + \frac{\partial Y(x, y)}{\partial y} \right) \cdot \frac{\partial X(x, y)}{\partial y} \\ &\quad + \alpha(X(x, y), Y(x, y)) \cdot \left( \frac{\partial^2 X(x, y)}{\partial y^2} + \frac{\partial^2 Y(x, y)}{\partial y^2} \right)\end{aligned}$$

$$\begin{aligned}X &= \frac{x^2 + y^2 - 1}{(x-1)^2 + y^2} \\ Y &= \frac{-2y}{(x-1)^2 + y^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial x}X &= \frac{2x \cdot ((x-1)^2 + y^2) - (x^2 + y^2 - 1) \cdot (2x - 2)}{((x-1)^2 + y^2)^2} \\ &= \frac{2(y-x+1)(y+x-1)}{(y^2 + x^2 - 2x + 1)^2} \\ \frac{\partial^2}{\partial x^2}X &= -\frac{4(x-1)(3y^2 - x^2 + 2x - 1)}{(y^2 + x^2 - 2x + 1)^3}\end{aligned}$$