

# תרגיל מס. 1.

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שאלה 1

א 1.1

$$\begin{aligned} f(x) &= \frac{\partial}{\partial x} F(x) \\ &= \begin{cases} 0 & x < 0 \\ nx^{n-1} & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases} \end{aligned}$$

ב 1.2

$$\begin{aligned} E[x] &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^1 nx^n \\ &= \left[ n \cdot \frac{x^{n+1}}{n+1} \right]_0^1 \\ &= \frac{n}{n+1} \end{aligned}$$

$$\begin{aligned} E[x^2] &= \int_0^1 nx^{n+1} \\ &= \left[ \frac{n}{n+2} x^{n+2} \right]_0^1 \\ &= \frac{n}{n+2} \end{aligned}$$

$$\begin{aligned} \text{var}[x] &= E[x^2] - E^2[x] \\ &= \frac{n}{n+2} - \frac{n^2}{(n+1)^2} \end{aligned}$$

ג 1.3

$$\begin{aligned} P(X > 0.5) &= \int_{0.5}^{\infty} f(x) dx \\ &= F(x) \Big|_{0.5}^{\infty} \\ &= 1 - 0.5^n \end{aligned}$$

$$\begin{aligned} P(X > 0.5 | X < 0.7) &= \frac{P(0.5 < X < 0.7)}{P(X < 0.7)} \\ &= \frac{F(0.7) - F(0.5)}{F(0.7)} \\ &= \frac{0.7^n - 0.5^n}{0.7^n} \end{aligned}$$

שאלה 2

א 2.1

דורשים רציפות

$$\begin{aligned} ax|_5 &= a(10 - x)|_5 \\ 5a &= 5a \end{aligned}$$

זה מתקיים עבור כל  $x$   
דורשים שכלל ההסתברות תהיה 1:

$$\begin{aligned} \int_{-\infty}^{\infty} f(x) dx &= 1 \\ \int_0^5 ax + \int_5^{10} a(10 - x) &= 1 \\ \left[ a \frac{x^2}{2} \right]_0^5 + \left[ 10ax - \frac{x^2}{2}a \right]_5^{10} &= 1 \\ \frac{25a}{2} + 100a - 50a - 50a + \frac{25}{2}a &= 1 \\ 25a &= 1 \\ a &= \frac{1}{25} \end{aligned}$$

ב 2.2

$$\begin{aligned}
 F(x) &= \int_0^x f(x) dx \\
 &= \begin{cases} 0 & x < 0 \\ \int_0^x f(x) & 0 < x < 5 \\ \int_0^x f(x) & 5 < x < 10 \\ 1 & x > 10 \end{cases} \\
 &= \begin{cases} 0 & x < 0 \\ a \frac{x^2}{2} & 0 < x < 5 \\ \int_0^5 ax dx + \int_5^x a(10-x) dx & 5 < x < 10 \\ 1 & x > 10 \end{cases} \\
 &= \begin{cases} 0 & x < 0 \\ a \frac{x^2}{2} & 0 < x < 5 \\ 25a - \frac{ax^2}{2} + 10ax - \frac{75a}{2} & 5 < x < 10 \\ 1 & x > 10 \end{cases}
 \end{aligned}$$

ג 2.3

$$P(X < 3) = a \frac{x^2}{2} = \frac{9a}{2}$$

$$\begin{aligned}
 P(X < 6 | X > 2) &= \frac{P(2 < X < 6)}{P(X > 2)} \\
 &= \frac{F(6) - F(2)}{1 - P(X < 2)} \\
 &= \frac{F(6) - F(2)}{1 - F(2)}
 \end{aligned}$$

ד 2.4

$$\begin{aligned}
 P(x < t | x > 6) &= \frac{P(x < t) \cap P(x > 6)}{P(x > 6)} \\
 &= \frac{P(x < t) \cap P(x > 6)}{1 - P(X < 6)} \\
 &= \frac{1}{1 - F(6)} \cdot (P(x < t) \cap P(x > 6)) \\
 &= \frac{1}{1 - F(6)} \cdot (P(x < t) \cap P(x > 6))
 \end{aligned}$$

$$(P(x < t) \cap P(x > 6)) = \begin{cases} 0 & x < 6 \\ F(t) - F(6) & x > 6 \end{cases}$$

### שאלה 3

$$f(x) = \begin{cases} ae^{-x/5} & x > 0 \\ 0 & otherwise \end{cases}$$

אנ

$$\begin{aligned} \int_0^\infty ae^{-\frac{x}{5}} dx &= 1 \\ -5ae^{-\frac{x}{5}} \Big|_0^\infty &= 1 \\ -5a \cdot 0 + 5a \cdot 1 &= 1 \\ 5a &= 1 \\ a &= 1/5 \end{aligned}$$

ב 3.1

$$\int_0^\infty xae^{-\frac{x}{5}} dx =$$

### שאלה 4

$$f(x) = \begin{cases} \frac{3+ax^2}{b} & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

נתון:

$$\begin{aligned} E(X) &= \frac{5}{8} \\ \int_{-\infty}^\infty xf(x) dx &= \frac{5}{8} \\ \int_0^1 \frac{3x+ax^3}{b} dx &= \frac{5}{8} \\ \left[ \frac{3}{2b}x^2 + \frac{a}{4b}x^4 \right]_0^1 &= \frac{5}{8} \\ \frac{3}{2b} + \frac{a}{4b} &= \frac{5}{8} \end{aligned}$$

יודעים גם כי

$$\begin{aligned}\int_0^1 f(x) &= 1 \\ \int_0^1 \frac{3+ax^2}{b} &= 1 \\ \left[ \frac{3x + \frac{ax^3}{3}}{b} \right]_0^1 &= 1 \\ \left[ \frac{3}{b}x + \frac{a}{3b}x^3 \right]_0^1 &= 1 \\ \frac{3}{b} + \frac{a}{3b} &= 1\end{aligned}$$

$$\begin{aligned}a &= 4b \left( \frac{5}{8} - \frac{3}{2b} \right) \\ &= \frac{5b}{2} - 6b \\ \frac{3}{b} + \frac{a}{3b} &= 1 \\ \frac{3}{b} + \frac{\frac{5b}{2} - 6b}{3b} &= 1 \\ \frac{3}{b} + \frac{5b}{6} - 2 &= 1 \\ \frac{3}{b} + \frac{5b}{6} - 3 &= 0 \\ 5b^2 - 18b + 18 &= 0 \\ b &= \frac{18 \pm \sqrt{18^2 - 4 \cdot 5 \cdot 18}}{10} \\ &= \frac{18 \pm \sqrt{-32}}{10}\end{aligned}$$

כנראה יש לי טעות חישוב, אבל היה לי מבחן בתחילת השבוע ואין זמן לתקן כרגע

ג 4.1

$$\begin{aligned}p &= P(x > 0.7) \\ &= 1 - P(x < 0.7)\end{aligned}$$

נגדיר  $Y = \text{Geom}(p)$  ואז נחשב את  $P_Y(y > 3)$