תרגיל מס.1

עפיף חלומה 302323001

2 בדצמבר 2009

1 שאלה 2

$$\begin{aligned} X &\sim \operatorname{Pois}(5) \\ P(X=\alpha) &= \frac{\lambda^\alpha e^{-5}}{\alpha!} \end{aligned}$$

בהינתן שנכנסו X זבבות לחדר ההסתברות שY מהם יתפסו היא בדיוק $P(Y=\beta|X) = \binom{X}{\beta} p^\beta (1-p)^{X-\beta}$ כלומר

$$\begin{aligned} P(Y=\beta) &= \sum_{\alpha=0}^{\infty} P(Y=\beta|X)\cdot P(X=\alpha) \\ &= \sum_{\alpha=0}^{\infty} P(Y=\beta)\cdot P(X=\alpha) \\ &= \sum_{\alpha=0}^{\infty} \binom{\alpha}{\beta}\left(\frac{1}{2}\right)^\beta\left(1-\frac{1}{2}\right)^{\alpha-\beta}\cdot\frac{5^\alpha e^{-5}}{\alpha!} \end{aligned}$$

1.1 א

$$\begin{aligned} P(Y=0) &= \sum_{\alpha=0}^{\infty} \binom{\alpha}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{\alpha}\cdot\frac{5^\alpha e^{-5}}{\alpha!} \\ &= \sum_{\alpha=0}^{\infty} \frac{5^\alpha e^{-5}}{2^\alpha \alpha!} \end{aligned}$$

1.2 ב

$$P(Y=1) = \sum_{\alpha=0}^{\infty} \binom{\alpha}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{\alpha-1}\cdot\frac{5^\alpha e^{-5}}{\alpha!}$$

$$
\begin{aligned}
&= \sum_{\alpha=0}^{\infty}\left(\frac{1}{2}\right)^{\alpha}\cdot\frac{5^{\alpha}e^{-5}}{\alpha!} \\
&= \sum_{\alpha=0}^{\infty}\frac{5^{\alpha}e^{-5}}{2^{\alpha}\alpha!}
\end{aligned}
$$

1.3 ג

$$
\begin{aligned}
P\left(Y=k\right) &= \sum_{\alpha=0}^{\infty}\binom{\alpha}{k}\left(\frac{1}{2}\right)^{k}\left(\frac{1}{2}\right)^{\alpha-k}\cdot\frac{5^{\alpha}e^{-5}}{\alpha!} \\
&= \sum_{\alpha=0}^{\infty}\binom{\alpha}{k}\frac{5^{\alpha}e^{-5}}{2^{\alpha}\alpha!}
\end{aligned}
$$

1.4 ד

1.4.1 א

$$
\begin{aligned}
P\left(Y=0\right) &= \sum_{\alpha=0}^{\infty}\binom{\alpha}{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{\alpha}\cdot\frac{5^{\alpha}e^{-5}}{\alpha!} \\
&= \sum_{\alpha=0}^{\infty}\frac{3^{\alpha}5^{\alpha}e^{-5}}{4^{\alpha}\alpha!}
\end{aligned}
$$

1.4.2 ב

$$
\begin{aligned}
P\left(Y=1\right) &= \sum_{\alpha=0}^{\infty}\binom{\alpha}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{\alpha-1}\cdot\frac{5^{\alpha}e^{-5}}{\alpha!} \\
&= \sum_{\alpha=0}^{\infty}\frac{3^{\alpha-1}5^{\alpha}e^{-5}}{4^{\alpha}\alpha!}
\end{aligned}
$$

1.4.3 ג

$$
\begin{aligned}
P\left(Y=k\right) &= \sum_{\alpha=0}^{\infty}\binom{\alpha}{k}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{\alpha-k}\cdot\frac{5^{\alpha}e^{-5}}{\alpha!} \\
&= \sum_{\alpha=0}^{\infty}\binom{\alpha}{k}\frac{3^{\alpha-k}5^{\alpha}e^{-5}}{4^{\alpha}\alpha!}
\end{aligned}
$$

2 שאלה 3

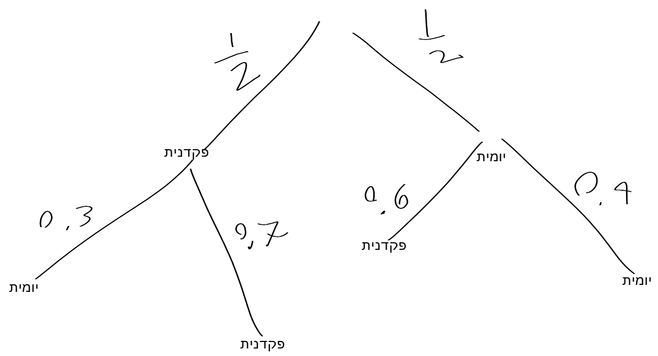
X הוא מספר הנסיונות של הפיכח:

$$\begin{aligned} P\left(x=k\right) &= \left(\sum_{i=1}^{k-1}\left(\frac{n-i-1}{n-i}\right)\right)\cdot\frac{1}{n-k+1} \\ E\left(x\right) &= \sum_{j=1}^{n} j\cdot\left(\sum_{i=1}^{j-1}\left(\frac{n-i-1}{n-i}\right)\right)\cdot\frac{1}{n-j+1} \end{aligned}$$

Y הוא השיכור:

$$\begin{aligned} P\left(X=k\right) &= \left(\frac{n-1}{n}\right)^{k-1}\left(\frac{1}{n}\right)^{1} \\ &= \frac{(n-1)^{k-1}}{n^k} \\ E\left(x\right) &= \sum_{i=1}^{n} i\cdot\frac{(n-1)^{i-1}}{n^i} \end{aligned}$$

3 שאלה 4



איור 1: עץ הסתברות

$$\begin{aligned} P\left(X=0\right) &= \frac{1}{2}\cdot 0.7 \\ P\left(X=1\right) &= \frac{1}{2}\cdot 0.6+\frac{1}{2}\cdot 0.3 \\ P\left(x=2\right) &= \frac{1}{2}\cdot 0.4 \end{aligned}$$

ב 3.1

$$P = \frac{1}{2} \cdot 0.3 + \frac{1}{2} \cdot 0.4$$