תרגיל מס.1

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14 במרץ 2010

1 שאלה 1

$$O_\alpha \circ X(t) = X(\alpha t)$$

1.1 ליניאריות

$$\begin{aligned}
O_\alpha \circ (\alpha_1 x_1 + \alpha_2 x_2)(t) &\stackrel{?}{=} \alpha_1 (O_\alpha \circ x_1)(t) + \alpha_2 (O_\alpha \circ x_2)(t) \\
(\alpha_1 x_1 + \alpha_2 x_2)(\alpha t) &\stackrel{?}{=} \alpha_1 x_1(\alpha t) + \alpha_2 x_2(\alpha t) \\
\alpha_1 x_1(\alpha t) + \alpha_2 x_2(\alpha t) &\stackrel{\checkmark}{=} \alpha_1 x_1(\alpha t) + \alpha_2 x_2(\alpha t)
\end{aligned}$$

1.2 אינוריציה בזמן

$$\begin{aligned}
(O_\alpha \circ Sh_\tau \circ f)(t) &\stackrel{?}{=} (Sh_\tau \circ O_\alpha \circ f)(t) \\
\tau &= 2 \\
\alpha &= -2 \\
f(t) &= \begin{cases} 1 & 0.5 < t < 1.5 \\ 0 & otherwise \end{cases} \\
(O_\alpha \circ Sh_\tau \circ f)(t) &= \begin{cases} 1 & -3 < t < -1 \\ 0 & otherwise \end{cases} \\
(Sh_\tau \circ O_\alpha \circ f)(t) &= \begin{cases} 1 & -1 < t < 1 \\ 0 & otherwise \end{cases} \\
(O_\alpha \circ Sh_\tau \circ f)(t) &\neq (Sh_\tau \circ O_\alpha \circ f)(t)
\end{aligned}$$

2 שאלה 2

2.1 $Sh_\tau \circ x(t) = x(t-\tau)$

2.1.1 ליניאריות:

$$\begin{aligned}
Sh_\tau \circ (\alpha_1 x_1 + \alpha_2 x_2)(t) &\overset{?}{=} \alpha_1 \cdot Sh_\tau \circ x_1(t) + \alpha_2 \cdot Sh_\tau \circ x_2(t) \\
(\alpha_1 x_1 + \alpha_2 x_2)(t-\tau) &\overset{?}{=} \alpha_1 \cdot x_1(t-\tau) + \alpha_2 \cdot x_2(t-\tau) \\
\alpha_1 x_1(t-\tau) + \alpha_2 x_2(t-\tau) &\overset{\checkmark}{=} \alpha_1 \cdot x_1(t-\tau) + \alpha_2 \cdot x_2(t-\tau)
\end{aligned}$$

2.1.2 אינווריציה בזמן

$$\begin{aligned}
Sh_\alpha \circ Sh_\tau \circ x(t) &\overset{?}{=} Sh_\tau \circ Sh_\alpha \circ x(t) \\
x(t-\tau-\alpha) &\overset{?}{=} x(t-\alpha-\tau) \\
x(t-\tau-\alpha) &\overset{\checkmark}{=} x(t-\tau-\alpha)
\end{aligned}$$

2.2 $O \circ x(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

2.2.1 ליניאריות

$$\begin{aligned}
O \circ (\alpha_1 x_1 + \alpha_2 x_2) &\overset{?}{=} \alpha_2 O \circ x_1 + \alpha_2 O \circ x_2 \\
\int_{-\infty}^{t} (\alpha_1 x_1(\tau) + \alpha_2 x_2(\tau))\, d\tau &\overset{?}{=} \alpha_2 \int_{-\infty}^{t} x_1 d\tau + \alpha_2 \int_{-\infty}^{t} x_2(\tau)\, d\tau \\
\int_{-\infty}^{t} \alpha_1 x_1(\tau)\, d\tau + \int_{-\infty}^{t} \alpha_2 x_2(\tau)\, d\tau &\overset{?}{=} \alpha_2 \int_{-\infty}^{t} x_1 d\tau + \alpha_2 \int_{-\infty}^{t} x_2(\tau)\, d\tau \\
\alpha_1 \int_{-\infty}^{t} x_1(\tau)\, d\tau + \alpha_2 \int_{-\infty}^{t} x_2(\tau)\, d\tau &\overset{\checkmark}{=} \alpha_2 \int_{-\infty}^{t} x_1 d\tau + \alpha_2 \int_{-\infty}^{t} x_2(\tau)\, d\tau
\end{aligned}$$

2.2.2 אינווריציה בזמן

$$\begin{aligned}
O \circ Sh_\lambda \circ x &\overset{?}{=} Sh_\lambda \circ O \circ x \\
O \circ x(\tau-\lambda) &\overset{?}{=} Sh_\lambda \circ \int_{-\infty}^{t} x(\tau)\, d\tau \\
\int_{-\infty}^{t} x(\tau-\lambda) \cdot 1 d\tau &\overset{\checkmark}{=} \int_{-\infty}^{t} x(\tau-\lambda)\, d\tau
\end{aligned}$$

2.3 $O\circ x(t)=\begin{cases}\int_0^t x(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}$

2.3.1 ליניאריות

$$\begin{aligned}
O\circ(\alpha_1x_1+\alpha_2x_2) &\overset{?}{=} \alpha_1O\circ x_1+\alpha_2O\circ x_2\\
\begin{cases}\int_0^t(\alpha_1x_1(\tau)+\alpha_2x_2(\tau))\,d\tau & t\geq 0\\ 0 & t<0\end{cases} &\overset{?}{=} \alpha_1\cdot\begin{cases}\int_0^t x_1(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}+\alpha_2\cdot\begin{cases}\int_0^t x_2(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}\\
\begin{cases}\int_0^t \alpha_1x_1(\tau)\,d\tau+\int_0^t\alpha_2x_2(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases} &\overset{?}{=} \begin{cases}\alpha_1\cdot\int_0^t x_1(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}+\begin{cases}\alpha_2\cdot\int_0^t x_2(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}\\
\begin{cases}\alpha_1\int_0^t x_1(\tau)\,d\tau+\alpha_2\int_0^t x_2(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases} &\overset{\checkmark}{=} \begin{cases}\alpha_1\cdot\int_0^t x_1(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}+\begin{cases}\alpha_2\cdot\int_0^t x_2(\tau)\,d\tau & t\geq 0\\ 0 & t<0\end{cases}
\end{aligned}$$

2.3.2 אינוורינט

דוגמה נגדית: $x=1$

$$\begin{aligned}
O\circ Sh_\tau\circ x &= \begin{cases}t & t\geq 0\\ 0 & t<0\end{cases}\\
Sh_\tau\circ O\circ x &= \begin{cases}t-\tau & t-\tau\geq 0\\ 0 & t<0\end{cases}
\end{aligned}$$

2.4 $O\circ x(t)=x\left(t^2\right)$

2.4.1 ליניאריות

$$\begin{aligned}
O\circ(\alpha_1x_1+\alpha_2x_2)(t) &\overset{?}{=} \alpha_1O\circ x_1(t)+\alpha_2O\circ x_2(t)\\
(\alpha_1x_1+\alpha_2x_2)\left(t^2\right) &\overset{?}{=} \alpha_1x_1\left(t^2\right)+\alpha_2x_2\left(t^2\right)\\
\alpha_1x_1\left(t^2\right)+\alpha_2x_2\left(t^2\right) &\overset{\checkmark}{=} \alpha_1x_1\left(t^2\right)+\alpha_2x_2\left(t^2\right)
\end{aligned}$$

2.4.2 אינוורינט זמן

$x(t)=t$

$$\begin{aligned}
Sh_\tau\circ O\circ x(t) &= Sh_\tau\circ x\left(t^2\right)\\
&= x\left(t^2-\tau\right)\\
O\circ Sh_\tau\circ x(t) &= O\circ x(t-\tau)\\
&= x\left((t-\tau)^2\right)\\
&= x\left(t^2-2\tau t+\tau^2\right)
\end{aligned}$$

2.5 $O \circ x = x \cdot \frac{\partial x}{\partial t}$

2.5.1 ליניאריות

$x_1 = t^2, x_2 = \cos(t)$

$$\begin{aligned}
O \circ (x_1 + x_2) &\stackrel{?}{=} O \circ x_1 + O \circ x_2 \\
(x_1 + x_2) \frac{\partial (x_1 + x_2)}{\partial t} &\stackrel{?}{=} x_1 \frac{\partial x_1}{\partial t} + x_2 \frac{\partial x_2}{\partial t} \\
\left(t^2 + \cos(t)\right) \cdot (2t - \sin(t)) &\stackrel{?}{=} t^2 \cdot 2t + \cos(t) \cdot (-\sin(t)) \\
2t^3 - t^2 \sin(t) + 2t \cos(t) - \cos(t) \sin(t) &\stackrel{X}{=} 2t^3 - \sin(t) \cos(t)
\end{aligned}$$

2.5.2 אינווריציה בזמן

$$\begin{aligned}
O \circ Sh_\tau \circ x(t) &\stackrel{?}{=} Sh_\tau \circ O \circ x \\
O \circ x(t - \tau) &\stackrel{?}{=} Sh_\tau \circ x(t) \cdot \frac{\partial x(t)}{\partial t} \\
x(t - \tau) \frac{\partial x(t - \tau)}{\partial t} &\stackrel{\checkmark}{=} x(t - \tau) \cdot \frac{\partial x(t - \tau)}{\partial t}
\end{aligned}$$

3 שאלה 3

$$\begin{aligned}
f * g &= \int_{-\infty}^{\infty} f(\tau) g(t - \tau) \, d\tau \\
&\underbrace{=}_{\tau = t - \alpha} \int_{\infty}^{-\infty} f(t - \alpha) g(\alpha) (-d\alpha) \\
&= -\int_{\infty}^{-\infty} f(t - \alpha) g(\alpha) \, d\alpha \\
&= \int_{-\infty}^{\infty} f(t - \alpha) g(\alpha) \, d\alpha \\
&= g * f
\end{aligned}$$

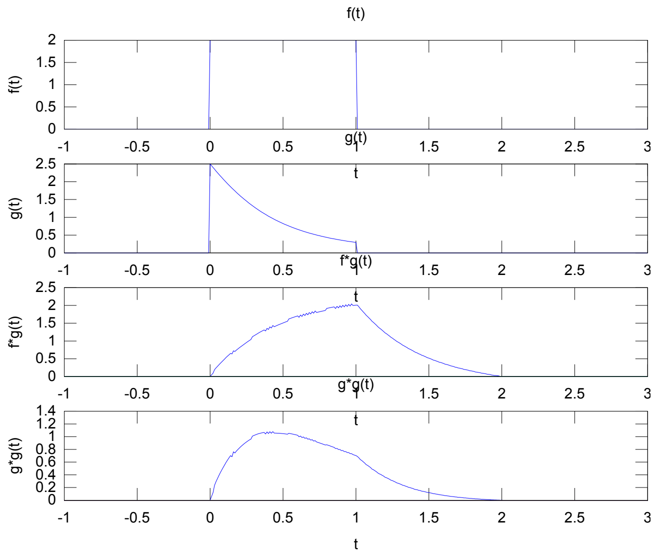
4 שאלה 4

$$
\begin{aligned}
f*f &= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{\tau^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{(t-\tau)^2}{2}}d\tau \\
&= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{\tau^2}{2}-\frac{(t-\tau)^2}{2}}d\tau \\
&= \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{4}}\int_{-\infty}^{\infty}\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\tau^2+t\tau}d\tau \\
&= \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\int_{-\infty}^{\infty}\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\tau^2+t\tau}d\tau \\
&\underbrace{=}_{\frac{\alpha}{\sqrt{2}}=\tau-t/2} \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}e^{\frac{t^2}{4}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\left(\tau-\frac{t}{2}\right)^2} \\
&= \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{4}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{\alpha^2}{2}}\cdot\frac{1}{\sqrt{2}}d\alpha \\
&= \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{4}}\cdot\frac{1}{\sqrt{2}}\cdot 1 \\
&= \frac{1}{\sqrt{2}\sqrt{2\pi}}e^{-\frac{t^2}{2\cdot(\sqrt{2})^2}} \\
\mu &= 0 \\
\sigma^2 &= 2
\end{aligned}
$$

5 שאלה 5

לחןעיפסט 1 תוכנית המתלב

```
#!/bin/usr/env octave
DELTA=0.01;
t=-1:DELTA:3;
f = inline('2*(t>0 & t<=1)');
g = inline('5 ./ ( (t+1).^4 + 1 ) .* (t>0 & t<=1)');
t=-1:DELTA:3;
fg=zeros(length(t));
for i=1:length(t)
fg(i)=sum(f(t).*g(t(i)-t).*DELTA );
end
gg=zeros(length(t));
for i=1:length(t)
gg(i)=sum(g(t).*g(t(i)-t).*DELTA );
end
subplot(4,1,1); plot(t,f(t)); title('f(t)'); xlabel('t'); ylabel('f(t)');
subplot(4,1,2); plot(t,g(t)); title('g(t)'); xlabel('t'); ylabel('g(t)');
subplot(4,1,3); plot(t,fg ); title('f*g(t)'); xlabel('t'); ylabel('f*g(t)');
subplot(4,1,4); plot(t,gg ); title('g*g(t)'); xlabel('t'); ylabel('g*g(t)');
print -dsvg hw1.svg
```



איור 1: תוצאת הרצת התוכנית