

Problems

Problem 5.1. (2 points)

Show the details of the proof that the bilinear form a and the functional F defined by

$$a(u, \varphi) = \int_{\Omega} \nabla u \nabla \varphi \, dx, \quad u, \varphi \in H_0^1(\Omega)$$

$$F(\varphi) = \int_{\Omega} f \varphi \, dx, \quad \varphi \in H_0^1(\Omega)$$

satisfy the assumptions of Lax-Milgram lemma. In particular, explain

- why is H_0^1 a Hilbert space: what is its inner product? Show that this inner product satisfies the axioms of inner product.
- why is a a bilinear form?
- why is a coercive? We have to show that $a(u, u) \geq \beta \|u\|_{H^1(\Omega)}^2$ for some $\beta > 0$.
(Hint: use Friedrichs' inequality)
- why is F a bounded linear functional for $f \in L^2(\Omega)$?
(Hint: use an estimate similar as in the case of showing the continuity of a in the lecture)