1 09-04-2018

We will learn about : Basics of functions of several variables. In this lecture:

1.1 A sequence in the Euclidean space and its application

Using these notation :

- $\mathbb{N}$: set of natural number ($\mathbb{N} = \{1, 2, 3, \dots\}$)
- $\mathbb{Z}$: set of integers ($\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$)
- $\mathbb{Q}$: set of rational number ($\mathbb{Q} = \{0, \pm 1, \pm 2, \frac{2}{3}, \dots\}$)
- $\mathbb{R}$: set of real number
- $\mathbb{C}$: set of complex number

Definition 1. *A sequence $(x_n)_{n=1}^{\infty}$ is an assignment of (real) number $x_n \in \mathbb{R}$ to natural number $n \in \mathbb{N}$ ($x_n \in \mathbb{R}$).*
Example : $x_n = \frac{1}{n}$. $x_1 = 1, x_2 = \frac{1}{2}, \dots$

Definition 2. *A subsequence of a sequence $(x_n)_{n=1}^{\infty}$ is a sequence $(y_j)_{j=1}^{\infty}$ defined by $y_j = x_{n_j}$ for some sequence $(n_j)_{j=1}^{\infty}$ in $\mathbb{N}$ such that $n_j < n_{j+1}$ ($j = 1, 2, \dots$).*
Example : sequence $(x_n)_{n=1}^{\infty} = 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{100}$, takes $n_1 = 1, n_2 = 3, n_3 = 5, n_4 = 100$
subsequence $(x_{n_j})_{j=1}^{\infty} = x_{n_1}, x_{n_2}, x_{n_3}, x_{n_4} = 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{100}$.

Definition 3. *Let $(x_n)_{n=1}^{\infty}$ be a sequence converges to $\alpha \in \mathbb{R}$ if for any $\epsilon > 0$, there is an $N \in \mathbb{N}$ such that $n > N$, $|x_n - \alpha| < \epsilon$.*
In the mathematical symbol $\forall \epsilon > 0, \exists N \in \mathbb{N}$ such that $n > N$, $|x_n - \alpha| < \epsilon$ for $n > N$.
In this case we write, $\lim\limits_{n \to \infty}$ or $x_n \to \alpha$ ($n \to \infty$)

Example 1.

Theorem 1. *$(x_n)_{n=1}^{\infty}, (y_n)_{n=1}^{\infty}$ is sequence. Suppose $x_n \to \alpha$ and $y_n \to \beta$ as $n \to \infty$.*

1. $x_n \pm y_n \to \alpha \pm \beta$, $(n \to \infty)$
2. $x_n \cdot y_n \to \alpha \cdot \beta$, $(n \to \infty)$
3. *if* $\beta \neq 0$, $\frac{x_n}{y_n} \to \frac{\alpha}{\beta}$, $(n \to \infty)$

Remark 1. *On 3, $\frac{x_n}{y_n}$ is not defined for all $n \in \mathbb{N}$ because $y_n = 0$ possibly for some $n \in \mathbb{N}$. But, since $y_n \to \beta \neq 0$, y_n eventually is not 0. Hence $\frac{x_n}{y_n}$ is defined eventually.*

Theorem 2. $(x_n)_{n=1}^{\infty}$