Problems

Problem 1.1. (2 points)

Decide if the following identities are correct or not and give the reason. Work in 3-dimensional space. Here, f is a scalar function and $\boldsymbol{u}, \boldsymbol{v}$ are 3-d vector functions.
("$\nabla\cdot$" is the same as "div", while "$\nabla\times$" is the same as "rot")

(A) $\nabla \cdot (f\boldsymbol{v}) = f\nabla \cdot \boldsymbol{v} + \boldsymbol{v} \cdot \nabla f$

(B) $\nabla \cdot (\boldsymbol{u} \times \boldsymbol{v}) = \boldsymbol{v} \cdot (\nabla \times \boldsymbol{u}) - \boldsymbol{u} \cdot (\nabla \times \boldsymbol{v})$

(C) $\nabla \times (f\boldsymbol{u}) = \nabla f \times \boldsymbol{u} + f\nabla \times \boldsymbol{u}$

(D) $\nabla \cdot (\nabla \times \boldsymbol{u}) = 0$

(E) $\nabla \times (\nabla \times \boldsymbol{u}) = \nabla(\nabla \cdot \boldsymbol{u}) - \Delta \boldsymbol{u}$

Nonlinear partial differential equations • April, 18
Deadline: May, 2

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