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Problem :

Make a program code of Euler Method and classical Runge-Kutta for

$$\begin{cases} y' = 5(2-y)y \quad (0 \le t \le 1) \\ y(0) = 0.04 \end{cases}$$

with C language and draw a graph to compare the numerical solution and exact solution for $N = 4, 8, 16, 32, 64, 128$ and $h = 1/N$. The exact solution is

$$y(t) = \frac{2}{1 + 49e^{-10t}}$$

C Code :

```c
#include<stdio.h>
#include<stdlib.h>
#include<math.h>

int main(int argc, char *argv[])
{
    int i;
    double y, yy, h, t, er, k1, k2, k3, k4;

    FILE *euler;
    FILE *rk;
    char filename1[30];
    char filename2[30];

    for(int dev=4;dev<=128;dev=dev*2)
    {
        h = 1./dev;

        sprintf(filename1, "euler_%.3d.txt",dev);
        sprintf(filename2, "rk_%.3d.txt",dev);

        euler = fopen(filename1,"w");
        rk = fopen(filename2, "w");

        y = 0.04; //initial condition

        fprintf(euler,"%.3f %.3f %.3f %.3f \n", 0., y, y, 0.);
        fprintf(rk,"%.3f %.3f %.3f %.3f \n", 0., y, y, 0.);

        //Using Euler
        for(i=1;i<=dev;i++)
        {
            t=i*h;
            y = 5*h*(2-y)*y + y;
            yy = 2./(1+49*exp(-10*t));
            er = fabs(yy-y);
            fprintf(euler,"%.3f %.3f %.3f %.3f \n", t, y, yy, er);
        }

        //Using RK
        y = 0.04; //initial condition
        for(i=1;i<=dev;i++)
        {
            t=i*h;
            k1 = 5*(2-y)*y;
            k2 = 5*(2-(y+(k1*h/2.)))*(y+(k1*h/2.));
            k3 = 5*(2-(y+(k2*h/2.)))*(y+(k2*h/2.));
            k4 = 5*(2-(y+(k3*h)))*(y+(k3*h));
            y = h*(k1 + 2*k2 + 2*k3 + k4)/6. + y;
```

```
50              yy = 2./(1+49*exp(-10*t));
51              er = fabs(yy-y);
52              fprintf(rk,"%.3f %.3f %.3f %.3f \n", t, y, yy, er);
53          }
54
55          //plot
56          FILE *pipe = popen("gnuplot", "w");
57          fprintf(pipe, "reset \n");
58          fprintf(pipe, "set terminal png \n");
59          fprintf(pipe, "set output 'plot%.3d.png'\n", dev);
60          fprintf(pipe, "set title 'Euler and Runge-Kutta for devider %.3d' \n", dev);
61          fprintf(pipe, "set xrange [0:1] \n");
62          fprintf(pipe, "set xlabel 't' \n");
63          fprintf(pipe, "set yrange [0:4] \n");
64          fprintf(pipe, "set ylabel 'y' \n");
65          fprintf(pipe, "plot '%s' using 1:2 w linespoint title 'Euler y numeric',
               '%s' using 1:2 w linespoint title 'RK y numeric', '%s' using 1:3 w
               linespoint title 'Exact y' , '%s' using 1:4 w linespoint title 'Euler
               differences' , '%s' using 1:4 w linespoint title 'RK differences'\n",
               filename1, filename2, filename1, filename1, filename2);
66          fclose(pipe);
67      }
68
69      fclose(euler);
70      fclose(rk);
71  }
```

Euler.c

GNUPLOT :

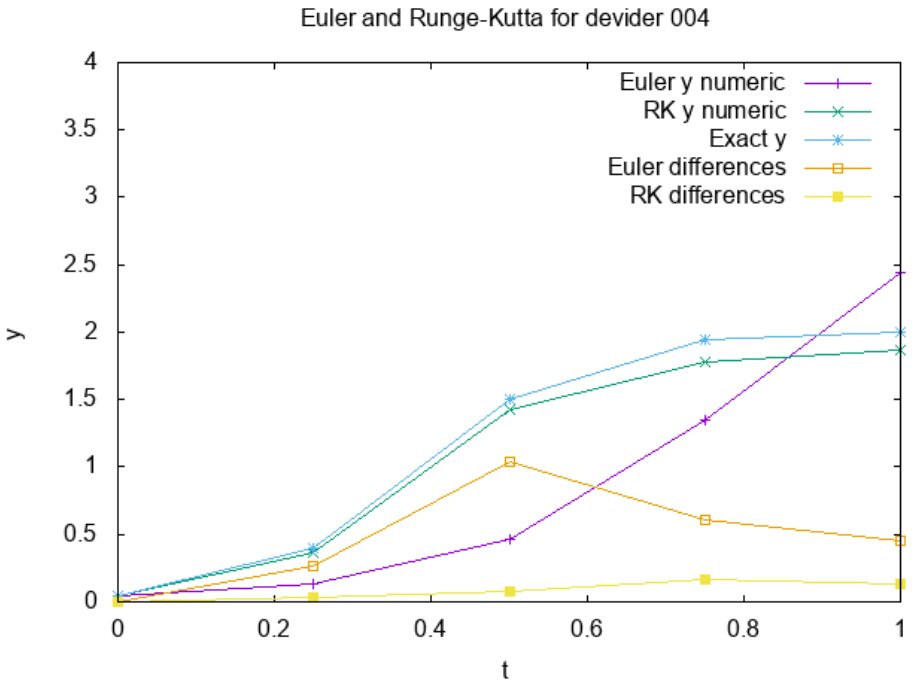

Figure 1:

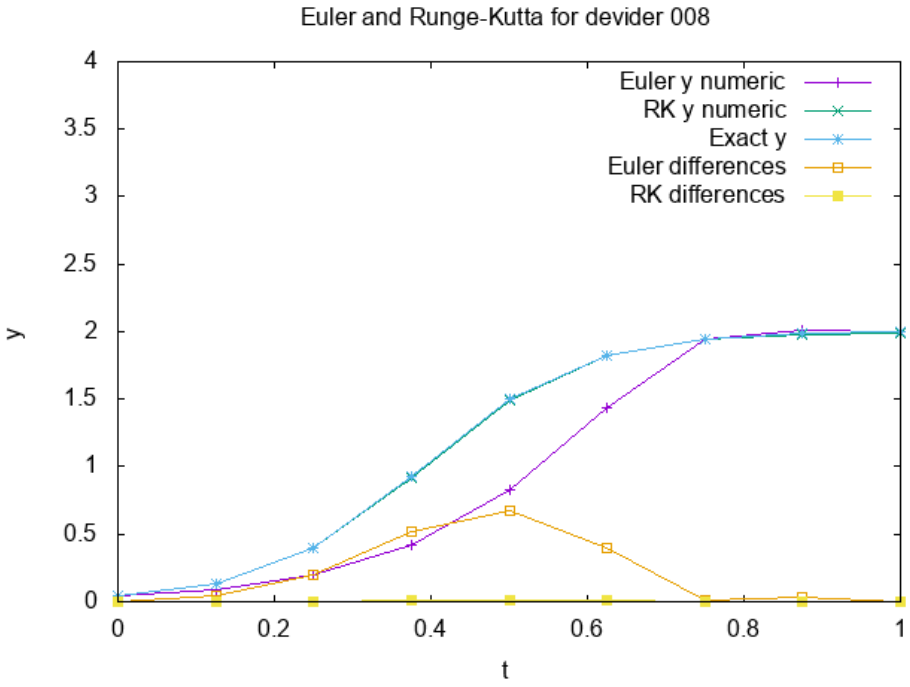

Figure 2:

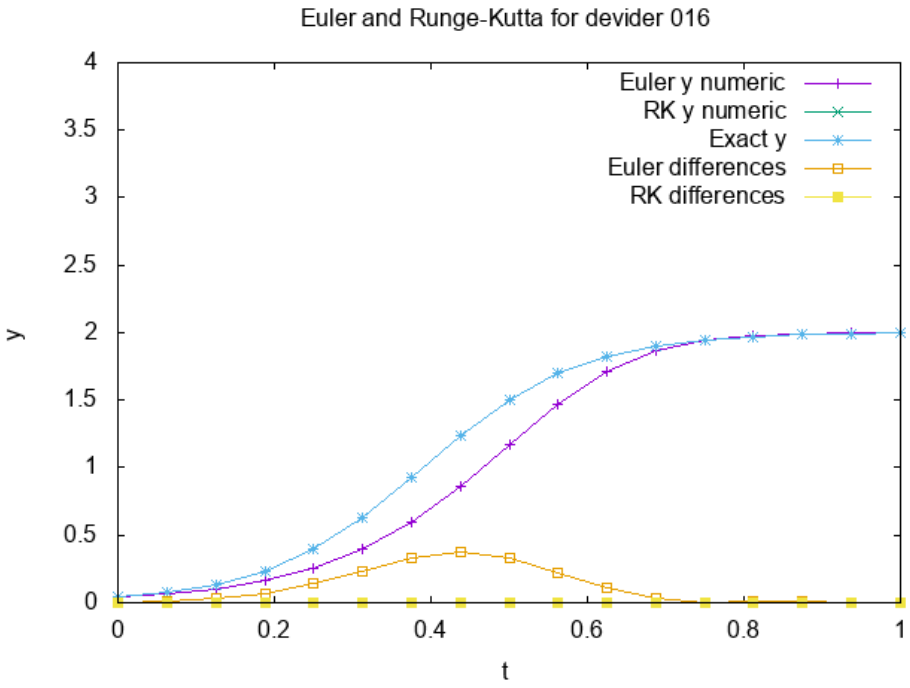

Figure 3:

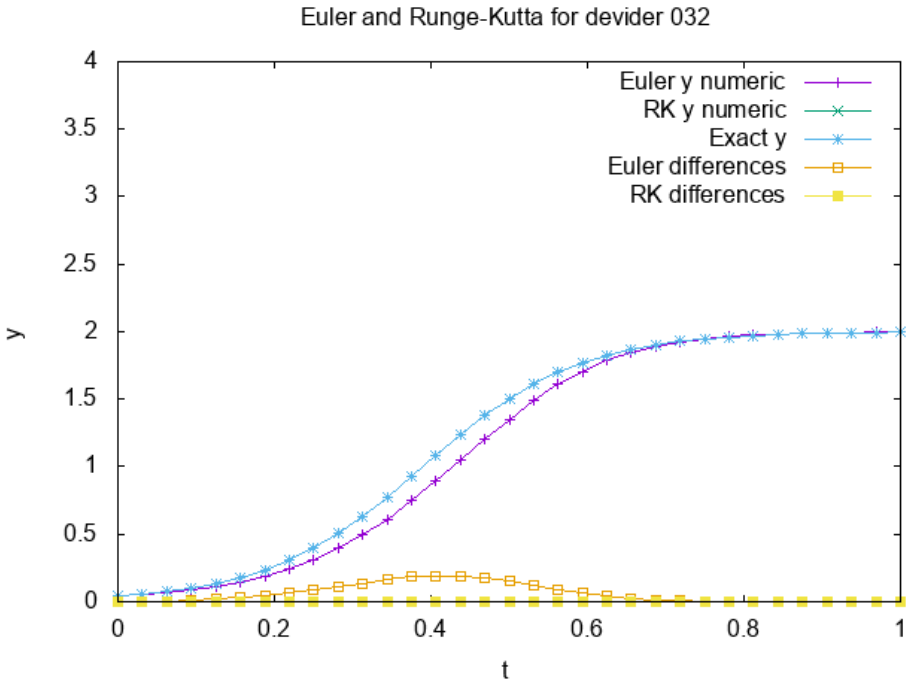

Figure 4:

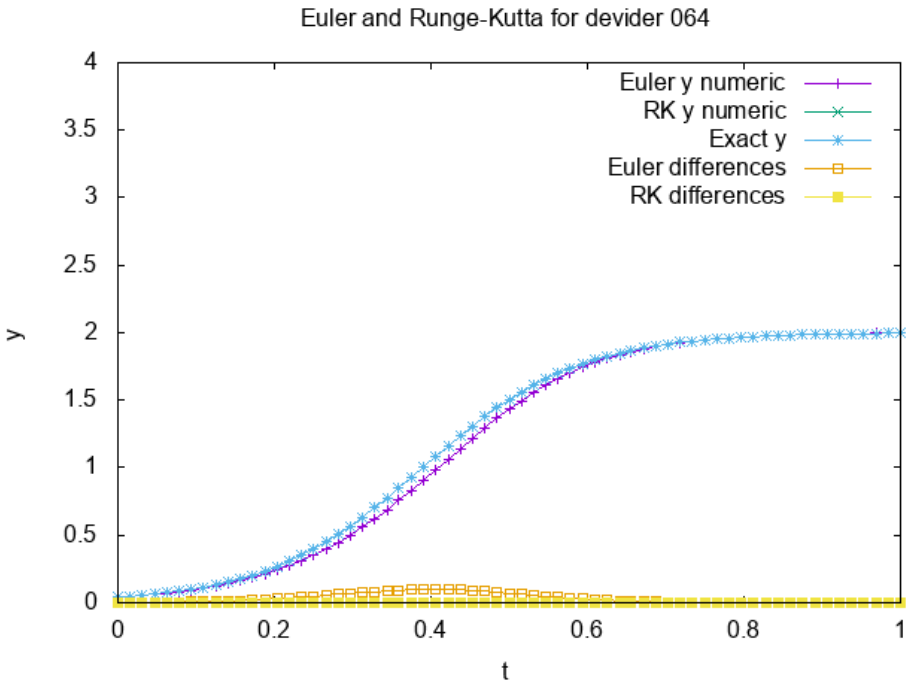

Figure 5:

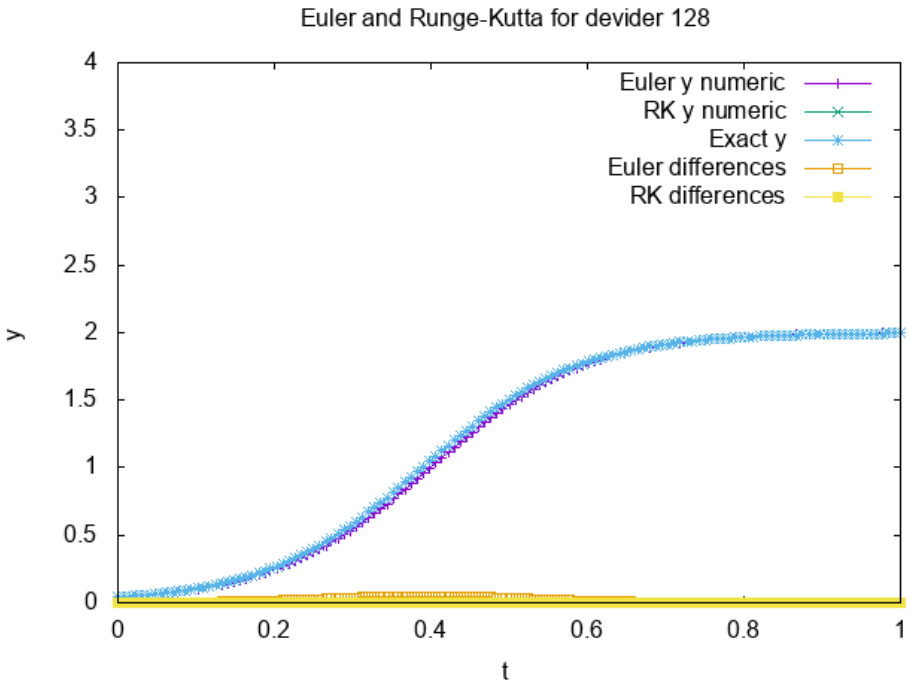
Euler and Runge-Kutta for devider 128
Euler y numeric
RK y numeric
Exact y
Euler differences
RK differences
y
t
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.2
0.4
0.6
0.8
1

Figure 6: