Problems

Problem 6.1. (2 points)

Let $I = (-1, 1)$. Define the operator $F : L^2(I) \to L^2(I)$ by

$$F(u)(x) = \int_I \frac{u(y)}{2 + x^2 + y^2}\, dy$$

Answer the following questions.

(i) Is the operator F linear?

(ii) Is the operator F bounded? What is its norm?

(iii) Is F a strict contraction?

(Hint: Use Cauchy-Schwarz inequality for (ii).)