Report Topics in Computational Science

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Problem : For $\alpha \in \mathbb{R}^+$ and $x(t) : \mathbb{R}^+ \to \mathbb{R}$, solve

$$\frac{dx}{dt} = \alpha(1-x)x$$

Solution : $x = \dfrac{1}{1 + Ae^{-\alpha t}}$ where $A = \dfrac{1 - x(0)}{x(0)}$

Answer :

$$\begin{aligned}
& & \frac{dx}{dt} &= \alpha(1-x)x \\
&\Leftrightarrow & \frac{dx}{(1-x)x} &= \alpha\ dt \\
&\Leftrightarrow & \Big(\frac{1}{1-x} + \frac{1}{x}\Big)dx &= \alpha\ dt \\
&\Leftrightarrow & \int \frac{1}{1-x}\ dx + \int \frac{1}{x}\ dx &= \int \alpha\ dt && \text{(integrate both side)} \\
&\Leftrightarrow & -\ln(1-x) + ln(x) &= \alpha t + c && \text{(for any constant c)} \\
&\Leftrightarrow & e^{-\ln(1-x)+ln(x)} &= e^{\alpha t + c} && \text{(power by eksponential)} \\
&\Leftrightarrow & e^{-\ln(1-x)}e^{ln(x)} &= e^{\alpha t + c} \\
&\Leftrightarrow & \frac{x}{1-x} &= Ce^{\alpha t} && \text{(for any constant } C = e^c \text{)} \\
&\Leftrightarrow & \frac{1-x}{x} &= Ae^{-\alpha t} && \text{(for any constant } A = 1/C \text{)} \\
&\Leftrightarrow & \frac{1}{x} - 1 &= Ae^{-\alpha t} \\
&\Leftrightarrow & \frac{1}{x} &= 1 + Ae^{-\alpha t} \\
&\Leftrightarrow & x &= \frac{1}{1 + Ae^{-\alpha t}}
\end{aligned}$$

For $t = 0$ we obtain value of constant A

$$x(0) = \frac{1}{1+A} \Leftrightarrow A = \frac{1 - x(0)}{x(0)}$$

such that the solution of the problem is

$$x = \frac{1}{1 + Ae^{-\alpha t}}$$

where

$$A = \frac{1 - x(0)}{x(0)}$$