Problems

Problem 3.1. (1 point)

Show that the space $C([0,1])$ of continuous functions on the interval $[0,1]$ equipped with the L^2-norm is not a Banach space.

Hint: *The L^2-norm is the norm*

$$\|f\|_{L^2(0,1)} = \Big(\int_0^1 f^2(x)\,dx \Big)^{1/2}.$$

It is enough to find a sequence of functions which is a Cauchy sequence in this L^2-norm and which converges to a discontinuous function.

Problem 3.2. (1 point)

Show using Hölder's inequality that the following relation holds:

$$p \le q \qquad \Rightarrow \qquad L^q(0,1) \subset L^p(0,1) \qquad\qquad \text{for any } \; p,q > 0.$$