An application of analytic functions to axisymmetric flow problems

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An extension of analytic functions for the solution of axisymmetric flow problems is presented. Axisymmetric flow problems have been transformed to two-dimensional ones by using an integral transformation that can be solved by the well-known methods of the theory of analytic functions. Here a plane problem on the flow has been solved approximately by means of conformal mapping. As an example we have investigated flow problems of an ellipsoid, a gas bubble motion, and an axisymmetric cavitational flow. © *1997 by Elsevier Science Inc.*

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1. Introduction

A plane problem of the motion of bodies in an incompressible, inviscid fluid can be investigated efficiently by using the theory of analytic functions. Numerous problems of plane flows on the basis of the analytic functions have been investigated.[1,2] However, the methods of the theory of analytic functions cannot be used for the solution of axisymmetric flow problems directly. Not long ago, G. N. Polozhii[3] considered an extension of analytic functions of a special kind and gave integral transformations that depended on ordinary analytic functions. The velocity potential and the streamfunction of an axisymmetric flow are the same kind of functions. Therefore Polozhii's transformations allow transformation of an axisymmetric flow problem into a two-dimensional plane flow one, though with more complicated boundary conditions. Polozhii's transformations have been used previously,[4,5] where other transformations also have been considered.

A number of problems of axisymmetric flow with free boundary are investigated below by means of Polozhii's transformation. The examples considered show that these techniques are very useful both from the theoretical point of view and for numerical computations.

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2. The main equations of axisymmetric flow problems

An axisymmetric irrotational flow of an inviscid fluid is described by a velocity potential ϕ and a streamfunction ψ which satisfy two equations in the fluid domain using cylindrical coordinates (x, y):

$$\phi_x = \psi_y/y,\ \phi_y = -\psi_x/y \tag{1}$$

These equations are different from Cauchy-Riemann's conditions in the coefficient $1/y$. Polozhii[3] has considered a p-analytic function $F(z) = U(x, y) + iV(x, y)$, which satisfies two equations: $U_x = V_y/p$ and $U_y = -V_x/p$. For the characteristic function $p = x^k$ he gave an integral transformation that defines the dependence of a p-analytic function $F(z)$ on an ordinary analytic function $f(z) = u(x, y) + iv(x, y)$. For the complex potential $w = \phi + i\psi$ with $p = y$, Polozhii's transformation can be represented as

$$w(z) = \phi + i\psi = -\frac{1}{2}\int_\Gamma f(\zeta)(i + \zeta - x)\frac{d\zeta}{g(\zeta)}, \tag{2}$$

$$g(\zeta) = \sqrt{(\zeta - z)(\zeta - \bar{z})}$$

where the line Γ joints the points $\bar{z} = x - iy$ and $z = x + iy$.

The functions $v(x, y)$ and $\psi(x, y)$ on the real x axis are assumed to be equal to zero, i.e., $v(x, 0) = 0$, $\psi(x, 0) = 0$.

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If the analytic function for $z \to \infty$ satisfies the condition $f(z) \sim z^{-1-\varepsilon}$ where $\varepsilon = \text{const} > 0$, then the line Γ can be arbitrarily deformed and the complex integral transformation (2) can be written as two real transformations:

$$\phi = \operatorname{Im} \int_z^{x+i\infty} f(\zeta)\frac{d\zeta}{g(\zeta)},\qquad \psi = -\operatorname{Im} \int_z^{x+i\infty} f(\zeta)\frac{(\zeta - x)\,d\zeta}{g(\zeta)} \tag{3}$$

The function $g(\zeta)$ is not uniquely defined; therefore a single-valued branch cut must be chosen for it, for instance, $\operatorname{Im} g(\zeta) > 0$ on the interval $(z, x + i\infty)$.

It can be shown after substituting the new variable $\zeta = z + i\eta$ into the integrals (3) and then differentiation, that the functions (3) satisfy equations (1). The partial derivatives are written in the same form as

$$\phi_x = \operatorname{Im} \int_z^{x+i\infty} f'(\zeta)\frac{d\zeta}{g(\zeta)},\qquad \psi_x = -\operatorname{Im} \int_z^{x+i\infty} f'(\zeta)\frac{(\zeta - x)\,d\zeta}{g(\zeta)} \tag{4}$$

where the $'$ represents derivative, e.g., $f'(z) = df(z)/dz$.

From the first integral (3) as $y \to 0$ the potential value on the real x-axis is given by $\phi(x,0) = \pm\pi f(x,0)/2$, where the plus and minus signs correspond to the points to the left and right of the body, respectively.

The integrals (3) can be considered as the potential and the streamfunction due to the perturbations caused by the body. Therefore the potential and the streamfunction for a flow over a body can be written as follows:

$$\Phi = \phi + V_\infty x, \qquad \Psi = \psi + V_\infty y^2/2 \tag{5}$$

where V_∞ is the free stream velocity.

If the analytic function $f(z)$ is found, then the flow problem can be solved completely. For some axisymmetric flow problems, the analytic function $f(z)$ can be found exactly, but for general problems it must be determined numerically.

3. Numerical investigation of flow problems

A numerical investigation can be carried out by employing the well-known methods of the theory of analytic functions. Here one can use power series as an approximation. In order to do so the flow domain in the z-plane must be conformally mapped onto the outer domain of the unity circle in the τ plane. The mapping function $z(\tau)$ and the analytic function $f[z(\tau)]$ can be expressed as

$$z = z_0(\tau) + z_1(\tau),\ z_1(\tau) = \sum_{n=-1}^{N} \frac{a_n}{\tau^n} \tag{6}$$

$$f = f_0(\tau) + f_1(\tau),\ f_1(\tau) = \sum_{n=1}^{M} \frac{b_n}{\tau^{n+1}} \tag{7}$$

The functions $z_0(\tau)$ and $f_0(\tau)$ should have the same singularities as the functions $z(\tau)$ and $f(\tau)$, since they improve the convergence of the coefficients of the series $z_1(\tau)$ and $f_1(\tau)$ and reduce the error of the calculations.[5]

The integrals (3) and (4) can be transformed into integrals along the boundary of the body from the stagnation point x_0 to the point z on the body, i.e., along the circular line $\tau = e^{i\sigma}$ in the transformed domain. Furthermore, after using the kinematic and dynamic conditions at separate points on the circle, a system of equations for the unknown parameters and coefficients of the series (6) and (7) can be generated.

Some problems illustrating this technique are presented in the next sections.

4. Flow problems for fixed axisymmetric bodies

If the surface of the body is known, the functions $z(\tau)$ and $f(\tau)$ can be calculated separately. The kinematic condition is given by

$$\Psi(\sigma) = 0 \text{ or } \Phi_x(\sigma)y'(\sigma) + \Psi_x(\sigma)x'(\sigma)/y(\sigma) = 0 \tag{8}$$

where $x'(\sigma) + iy'(\sigma) = dz/d\sigma$. Moreover the derivative of the potential (Φ_x) must be zero at the stagnation point, i.e.,

$$V_\infty \frac{dz}{d\tau} + \frac{\pi}{2}\frac{df}{d\tau} = 0 \text{ for } \tau = -1 \tag{9}$$

On smooth boundaries, both functions $z(\tau)$ and $f(\tau)$ have no singularities therefore, $z_0(\tau) = 0$, $f_0(\tau) = 0$.

As an example, we consider flow around an ellipsoid. The ellipsoid meridian and the mapping function are given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } z(\tau) = \frac{1}{2}\left[(a+b)\tau + \frac{a-b}{\tau}\right] \tag{10}$$

For comparison, *Tables 1* and *2* show the results of numerical calculations for the velocity and potential on the ellipsoid by the previously given approaches and by exact analytic formulas.[7]

5. Gas bubble flow

Most problems of gas bubbles have been investigated under the assumption of plane flows.[6,8–10] The first theoretical results for a plane bubble in a channel were obtained by Joukowsky.[8] An exact result for the special case of a bubble was obtained by McLeod.[9] Kiselev[10] gave a method for constructing a solution for the small deformation of the bubble surface. The steady-state problem has been investigated in detail by Zhitnikov and Terentiev.[6] An axisymmetric bubble problem has been considered for the time-dependent deformation of the boundary using a numerical method only.[11,12] The problem of steady bubble flow is considered below.

The location of the bubble shape is not known *a priori*. Thus all coefficients of series (6) and (7) are unknown. On the bubble surface kinematic and dynamic conditions must be satisfied. The kinematic boundary conditions are given by equations (8) and (9), while the dynamic one is given by Laplace equation as

$$\frac{L}{\pi}\left(\frac{\cos\theta}{y}-\frac{d\theta}{ds}\right)=We\left(\mu-1+\frac{\Phi_x^2+\Phi_y^2}{V_\infty^2}\right) \qquad (11)$$

where $We=\rho V_\infty^2 L/2T$ is Weber number, $\mu=2(P_0-P_\infty)/\rho V_\infty^2$ is the pressure number, P_0 and P_∞ are the pressure inside the bubble and at the infinity, respectively, ρ is the fluid density, T is the surface tension (the same in all directions and at any point on the surface), L is the length of the bubble meridian, θ is the angle of inclination of the tangent, y is the distance of the surface point from the x-axis, s is the arc length of the generator of the axisymmetric body, whose positive direction is clockwise.

Unbounded flow domain. In this case it also must be assumed that $z_0(\tau)=0$, $f_0(\tau)=0$.

Figure 1 shows the contours of the axisymmetric bubble with $\mu=5$, 1, 0.681, and 0 (lines 1–4, respectively) and for the plane bubble,[6] with $\mu=0.546$, 1, and 5 (lines 5–7). The values of $\mu_0=0.681$ for the axisymmetric bubble and $\mu_0=0.546$ for the plane bubble correspond to the touching of the bubble surface at two stagnation points. The contour of the bubbles self-intersect for all numbers $\mu<\mu_0$. Such a process is impossible. It can be concluded that the bubble transforms itself for $\mu=\mu_0$ into other bubbles or into a toroidal bubble.

Table 1. Numerical and exact data for flow around an ellipsoid with $a=1$, $b=0.8$, the number of coefficients is $M=5$

	Velocity		Potential	
x	V, numerical	V, exact	Φ, numerical	Φ, exact
−0.9511	0.5198	0.5197	−1.3136	−1.3136
−0.8090	0.9294	0.9286	−1.1173	−1.1174
−0.5879	1.1941	1.1941	−0.8116	−0.8118
−0.3090	1.3361	1.3368	−0.4267	−0.4268
0.0000	1.3812	1.3812	0.0000	0.0000
0.3090	1.3369	1.3368	0.4268	0.4268
0.5879	1.1940	1.1941	0.8118	0.8118
0.8090	0.9278	0.9286	1.1173	1.1174
0.9511	0.5181	0.5197	1.3131	1.3136

Table 2. Numerical and exact data for flow around an ellipsoid with $a=1$, $b=0.5$, the number of coefficients is $M=10$

	Velocity		Potential	
x	V, numerical	V exact	Φ, numerical	Φ, exact
−0.9511	0.6598	0.6593	−1.1508	−1.1508
−0.8090	0.9966	0.9968	−0.9789	−0.9789
−0.5879	1.1368	1.1373	−0.7112	−0.7112
−0.3090	1.1957	1.1944	−0.3739	−0.3739
0.0000	1.2077	1.2100	0.0000	0.0000
0.3090	1.1979	1.1944	0.3739	0.3739
0.5879	1.1325	1.1373	0.7112	0.7112
0.8090	1.0046	0.9968	1.9793	0.9789
0.9511	0.6244	0.6593	1.1503	1.1508

The convergence of the series for z_1 and f_1 has been examined numerically. *Table 3* shows a comparison of values obtained for different numbers N and M of terms kept in the series for axisymmetric bubbles. The numerical results coincide for all numbers $N, M>20$.

A bubble in a tube. A solution of this problem can be obtained by conformally mapping the flow domain in the z-plane onto the half-ring of the τ-plane. Functions (6) and (7) can be expressed as

$$z_0=\frac{2}{\pi}h\ell n\frac{1+\tau}{1-\tau},\ z_1=\sum_{n=1}^{N}a_{2n-1}(\tau^{2n-1}+\tau^{1-2n}) \qquad (12)$$

$$f_0=A\frac{\tau}{\tau^2-1},\ f_1=\sum_{n=1}^{M}b_{-2n}\tau^{-2n}+\sum_{n=1}^{M}b_{2n+1}\tau^{2n+1} \qquad (13)$$

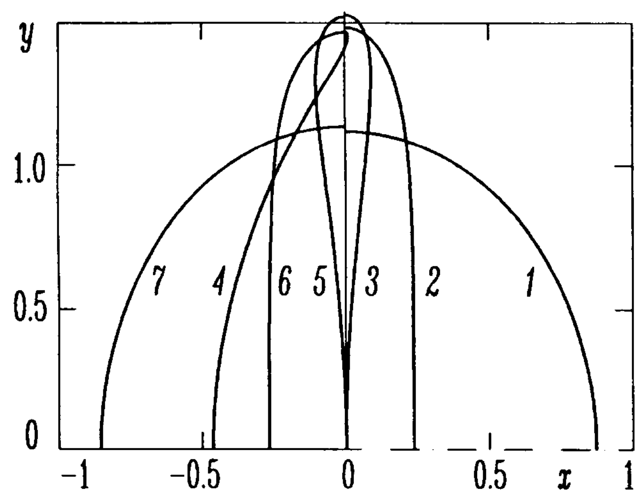

Figure 1. Calculated bubble shapes for axisymmetric flow (lines 1–4) and for plane flow (lines 5–7).

Table 3. Numerical data for different numbers (N, M) of terms kept in the series for $\mu = \mu_0$

N, M	μ_0	We	V_{max}
5	0.681358	0.914399	4.575548
10	0.681435	0.913266	4.582812
20	0.681436	0.913264	4.582844
30	0.681436	0.913264	4.582835
40	0.681436	0.913264	4.582842

Symmetric flow with respect to x = 0. In this case Polozhii's transformation (3) takes the form

$$\Phi(z) = \mathrm{Im}\left(\int_{x_L+i0}^{z} f(\zeta)\frac{d\zeta}{g(\zeta)} - \int_{x_L+i0}^{-\bar{z}} f(\zeta)\frac{d\zeta}{g_1(\zeta)}\right) \tag{14}$$

$$\Psi(z) = -\mathrm{Im}\left(\int_{x_L+i0}^{z} f(\zeta)\frac{(\zeta - x)\,d\zeta}{g(\zeta)} + \int_{x_L+i0}^{-\bar{z}} f(\zeta)\frac{(\zeta + x)\,d\zeta}{g_1(\zeta)}\right) \tag{15}$$

where $g_1(\zeta) = \sqrt{(\zeta + z)(\zeta + \bar{z})}$, and x_L is some arbitrary point on the x-axis.

Boundary conditions (8), (9), and (11) are satisfied on the bubble shape, and the condition $\Psi = 0.5V_\infty h^2$ is fulfilled on the surface of the cylindrical tube, where h is the tube radius.

The Weber number, We, depends on both the pressure number μ and the ratio h/L (h is the radius of the tube or half width of the channel). The numerical investigation shows that the dependence of $We(\mu)$ for the fixed ratio h/L is a two-valued function for $h/L < H \simeq 0.505$ and $\mu > \mu_0$ (μ_0 depends on the ratio h/L), i.e., two kinds of flows exist for the same value of μ. But for $h/L > H$ the function $We(\mu)$ is single valued and the bubble has only one shape.

Figure 2 shows the bubble shapes for different Weber numbers and two values of the ratio h/L: (a) $h/L = 0.5 < H$ and (b) $h/L = 0.557 > H$. The curves on the right of the y-axis are the bubbles for axisymmetric flows, whereas those on the left correspond to plane flow. Bubble shapes 1–6 in *Figure 2(a)* correspond to $\mu = 14, 5.5, 5, 7, 8.5$ and 9, respectively, and curves 1–3 in *Figure 2(b)* correspond to $\mu = 3, 1, 0$. *Figure 2(a)* shows two kinds of bubble shapes (1) for the planar flow.

6. Axisymmetric cavity flow

Cavitation is a hydrodynamic phenomenon connected with the appearance of ruptures on continuous liquids that significantly influence the flow field and the resultant forces on a submerged body. Therefore it is very important in shipbuilding to make a study of cavitating flows. Until now most cavity problems, particularly planar problems, have been widely investigated. A review of this subject can be found in the works of Birkhoff and Zarantonello,[2] Ivanov,[17] and Terentiev.[13]

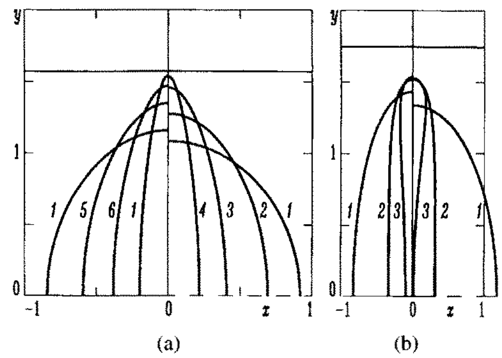

Figure 2. Bubble shapes for different Weber's number and two ratios of h/L: (a) $h/L = 0.5$; (b) $h/L = 0.557$.

It must be noted that axisymmetric flow problems have been studied numerically only,[14–19] although the problem of cavity flow on slender bodies has been investigated using analytic methods.[20]

A boundary of cavity flow has a wet surface on the solid body and an unknown surface on the cavity. Kinematic boundary conditions must be satisfied on both the surface of the solid body and on the cavity. The dynamic condition on the cavity surface is given by

$$\Phi_x^2(\tau) + \Phi_y^2(\tau) = V_\infty^2(1 + K) \tag{16}$$

where $K = 2(P_0 - P_\infty)/\rho V_\infty^2$ is the cavitation number.

The cavity flow presents several difficult problems connected with the cavity detachment from the body, the shape of the aft portion of the cavity, etc. It is assumed that the detachment line is fixed and that the shape of the aft portion of the axisymmetric cavity is the same as that of a flat one. It is known[13, 19] that the function at the aft point has the singularity $\ln(dw/dz) \sim w^{-1/2}$, i.e., $dw/dz \sim \exp(w^{-1/2})$. Therefore the function $z_0(\tau)$ is given by

$$z_0(\tau) = a\int_{-1}^{\tau}\left(e^{(2c/t-1)}\frac{t^2 - 1}{t^2} - \frac{2c}{t}\right)dt \tag{17}$$

where a and c are unknown parameters that must be determined from additional conditions. Such conditions can be given by $z_1(-1) = 0$, $z_1'(1) = 2ac$. Then the center of the fast convergent spiral (the endpoint on the cavity boundary) is obtained from

$$x_c = 2ae^{-c}\{2 - c[e^{-c}\overline{Ei}(c) - e^{c}Ei(-c)]\} + \sum_{n=-1}^{N} a_n \tag{18}$$

$$y_c = 2\pi ac(1 - e^{-2c}) \tag{19}$$

where Ei is the exponential integral.

Collocating the boundary conditions (8) and (16) at separate nodes and satisfying the above-mentioned conditions leads to a system of nonlinear equations. The numerical solution of this system presents several difficulties, namely, it is quite sensitive to the initial values of the unknown parameters.

The following numerical scheme was used:

(i) solve the plane problem for a given cavitation number and a detachment point;
(ii) solve the axisymmetric flow problem for the calculated plane shape of the cavity;
(iii) calculate the new cavitation number using the velocity at the detachment point;
(iv) solve the total flow problem for the given cavitation number, which does not differ much from the calculated value, and for initial values of the unknown parameters equal to those found above.

Furthermore the flow problem has been calculated step by step with respect to the detachment point at a fixed cavitation number (K). The preceding data was taken as initial values for the next calculation.

Some calculated results for a circular cylinder and sphere are depicted in *Figures 3–5*. Calculated cavity shapes for plane and axisymmetric flows are shown in *Figures 3* and *4*, respectively. The angles of the location of the detachment points are the same, $\alpha = 60°$. The cavitation number for plane flow is $K = 1$ and for axisymmetric flow $K = 0.3$. The plane cavity shape coincides with the results obtained by Terentiev[13] (□ on *Figure 3*). An exact proof of a convergence of series (6) and (7) is quite difficult, but the numerical results for different N (see *Figure 3*) provide sufficient accuracy.

Figure 5 shows the dependence of the drag coefficient $C_D = 2D/\rho\pi R^2 V_\infty^2$, where R is the radius of the sphere on the detachment angle α for fixed cavitation numbers. It is to be noted that the function $C_D(\alpha)$ has a maximum at two points as for a plane flow (dotted lines) where $C_D = D/\rho R V_\infty^2$.[13] The symbols on the curves correspond to results for the sphere obtained numerically by Brennen,[14] (□), Kojouro,[15] (△), and Ivanov,[17] (○) for the case of smooth separation.[2]

7. Conclusions

The present paper shows an application of generalized analytic functions of complex variables to the investigation of axisymmetric flow problems with free surfaces. As an illustration of its efficiency several problems have been analyzed, e.g., a flow about axisymmetric bubbles and a cavitation flow past a sphere. This method can be used also for the investigation of nonaxisymmetric flow problems,[21] but it must be further refined, particularly regarding the free surfaces.

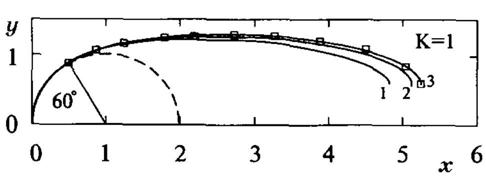

Figure 3. The plane cavity shapes (1, N=5; 2, N=10; 3, N=28).

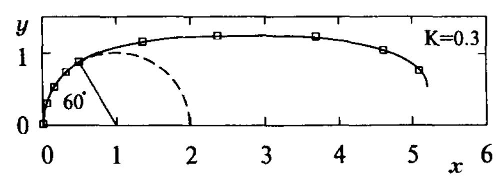

Figure 4. The axisymmetric cavity shape (N=M=9, □-collocation points).

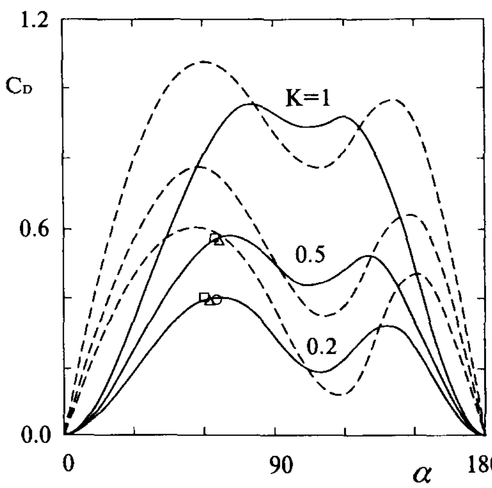

Figure 5. Drag coefficient as a function of detachment angle α (—, axisymmetric flow; - - -, plane flow).

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