

Assignment 5

Topics of Mathematical Science

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1. Suppose h : continuous on $[a, b]$ (complex valued), proof that:

$$\Rightarrow \left| \int_a^b h(t) dt \right| \leq \int_a^b |h(t)| dt \quad (1)$$

Answer:

Approximating left hand side of (1) as Riemann sum, we have:

$$\left| \int_a^b h(t) dt \right| \cong \left| \sum_{k=1}^n h(t_k) \Delta t \right| \quad (2)$$

By triangle inequality,

$$\left| \sum_{k=1}^n h(t_k) \Delta t \right| \leq \sum_{k=1}^n |h(t_k)| \Delta t \quad (3)$$

we know that $\sum_{k=1}^n |h(t_k)| \Delta t$ is Riemann sum of $\int_a^b |h(t)| dt$. Hence,

$$\begin{aligned} \left| \int_a^b h(t) dt \right| &\cong \left| \sum_{k=1}^n h(t_k) \Delta t \right| \leq \sum_{k=1}^n |h(t_k)| \Delta t \cong \int_a^b |h(t)| dt \\ \therefore \left| \int_a^b h(t) dt \right| &\leq \int_a^b |h(t)| dt \end{aligned} \quad (4)$$