Topics in Computational Science Report
Derivation of Position Function on Chaos Process

Alifian Mahardhika Maulana

June 13, 2018

Problem 1. *For $\alpha \in \mathbb{R}^+$ and $x(t) : \mathbb{R}^+ \to \mathbb{R}$, Let:*

$$\frac{dx}{dt} = \alpha(1-x)x \tag{1}$$

Proof the following:

$$x(t) = \frac{1}{1 + Ae^{-\alpha t}}, \quad with \ A = \frac{1 - x(0)}{x(0)}$$

Proof.

$$\frac{dx}{dt} = \alpha(1-x)x$$

$$\frac{dx}{(1-x)x} = \alpha dt$$

$$\left(\frac{1}{x} + \frac{1}{(1-x)}\right)dx = \alpha dt$$

taking integral on the both side, we obtain:

$$\int \frac{1}{x}dx + \int \frac{1}{(1-x)}dx = \int \alpha dt$$

$$\ln(x) - \ln(1-x) = \alpha t + C$$

$$\ln\left(\frac{x}{(1-x)}\right) = \alpha t + C$$

multiplied by $e(\exp)$ on the both side, we obtain:

$$\exp\left(\ln\left(\frac{x}{(1-x)}\right)\right) = \exp(\alpha t + C)$$

$$\frac{x}{(1-x)} = e^{\alpha t + C}$$

let $e^{\alpha t + C} = e^{\alpha t}e^C = Ce^{\alpha t}$, thus:

$$\frac{x}{(1-x)} = Ce^{\alpha t} \tag{2}$$

for $t = 0$, we get:

$$\frac{x(0)}{(1-x(0))} = C$$

insert C to equation (2), hence the equation becomes:

$$\frac{x}{(1-x)} = \frac{x(0)}{(1-x(0))}e^{\alpha t}$$

$$\frac{(1-x)}{x} = \frac{(1-x(0))}{x(0)}e^{-\alpha t}$$

$$\frac{1}{x} - 1 = \frac{(1-x(0))}{x(0)}e^{-\alpha t}$$

let $A = \frac{(1-x(0))}{x(0)}$ thus the equation becomes:

$$\frac{1}{x} - 1 = Ae^{-\alpha t}$$

$$\frac{1}{x} = 1 + Ae^{-\alpha t}$$

$$\therefore x = \frac{1}{1 + Ae^{-\alpha t}}$$

$\square$