Applied Analysis Notes

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1 Finite Element Method

1.1 Strong Form

Find $u : \Omega \to \mathbb{R}$ s.t.

$$\begin{cases} -\triangle u = f \ in \ \Omega \\ u = g \ on \ \partial\Omega \end{cases} \tag{1}$$

where $f : \Omega \to \mathbb{R}, g : \partial\Omega \to \mathbb{R}$ are given functions

1.2 Weak Form

Find $u \in V(g)$ s.t.

$$a(u,v) = (f,v), \forall v \in V,$$

where

$$\begin{aligned} (u,v) &\equiv \int_\Omega u(x)v(x)dx, \\ a(u,v) &\equiv (\nabla u, \nabla v) = \int_\Omega \nabla u \cdot \nabla v dx, \\ V(g) &\equiv v \in H^1(\Omega); v = g \ on \ \partial\Omega, \\ V &\equiv V(0) \end{aligned}$$

1.3 FEM

$$\begin{aligned} Find u_h \in V_h(g_h) s.t. \\ a(u_h, v_h) = (f, v_h), \forall v_h \in V_h \\ where \\ V_h \subset V, dim V_h < +\infty \\ g_h \ g, approximation of g \\ u_h \ u, approximation solution of u \\ h : meshsize \end{aligned}$$

1.4 1-Dimension Case

Let us define basis functions $\varphi_{i i}^{4}, \varphi_i : (0,1) \to \mathbb{R}$

$$\varphi_i : piecewiselinear$$

$$\varphi_i(x_j) = \delta_{ij} = \begin{cases} 1(i = j) \\ 0(i! = j) \end{cases}$$

$$X_h = <\varphi_0, \cdots, \varphi_4> = \sum_{i=0}^{4} c_i\varphi_i(x); c_i \in \mathbb{R}, i = 0, \cdots, 4$$

$$\begin{aligned} V_h(g) &= v_h \in X_h; v_h(x_0) = g_0, v_h(x_1) = g_1 \\ &= g_0\varphi_0(x) + \sum_{i=1}^{3} c_i\varphi_i(x) + g_1\varphi_4(x); c_i \in \mathbb{R}, i = \end{aligned}$$

$$dimX_h = 5, dimV_h(g) = 3$$

$$V_h = V_h(0) = \sum_{i=1}^{3} c_i\varphi_i(x)c_i \in \mathbb{R}, i = 1, 2, 3$$

Rewrite FEM

$$u_h(x) = g_0\varphi_0(x) + \sum_{i=1}^{3} c_i\varphi_i(x) + g_1\varphi_4(x) \begin{cases} Findc_{i i=1}^{3} \subset \mathbb{R}s.t. \\ a(u_h, v_h), \forall v_h \in V_h \end{cases} \quad \begin{cases} Findc_{i i=1}^{3} \\ a(u_h, \varphi_i) = (f, \varphi_i), i = 1, 2, 3, \end{cases} \quad pro$$