

Assignment 1

Topics of Mathematical Science

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Chain Rule

1. Let $f(x) = x^2, g(y) = y^3 + 2y$, then $(g \circ f) = x^6 + 2x^2$

$$\therefore \frac{d(g \circ f)}{dx} = 6x^5 + 4x$$

2. Let $f(x) = x^3 + 2x, g(y) = y^2 + 3y$, then $(g \circ f) = x^6 + 4x^4 + 13x^2 + 6$

$$\therefore \frac{d(g \circ f)}{dx} = 6x^5 + 16x^3 + 26x$$

3. Let $\gamma(t) = (t^2, t^3 + t), f(x, y) = x^3y$ then $(f \circ \gamma) = t^9 + t^7$

$$\therefore \frac{d(f \circ \gamma)}{dt} = 9t^8 + 7t^6$$

4. Let $\gamma(t) = (t, t^2 + t), f(x, y) = xe^y$ then $(f \circ \gamma) = te^{t^2+t}$

$$\therefore \frac{d(f \circ \gamma)}{dt} = e^{t^2+t} + te^{t^2+t}(2t + 1) = e^{t^2+t}(2t^2 + t + 1)$$

5. Let $f(x, y) = (ax+by, cx+dy), g(u, v) = (pu+qv, ru+sv)$ with (a, b, c, d, p, q, r, s) are constant then $(g \circ f) = (p(ax + by) + q(cx + dy), r(ax + by) + s(cx + dy))$

$$\therefore J(g \circ f) = \begin{pmatrix} pa + qc & pb + qd \\ ra + sc & rb + sd \end{pmatrix}$$

$$Jac(f) = \det J(f) = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

6. Let $f(x, y) = (x^2y, x^5y^2), g(u, v) = (u^2, v^3)$ then $(g \circ f) = ((x^4y^2), (x^{15}y^6))$

$$\therefore J(g \circ f) = \begin{pmatrix} 4x^3y^2 & x^42y \\ 15x^{14} & x^{15}6y^5 \end{pmatrix}$$

$$Jac(f) = \det J(f) = \det \begin{pmatrix} 2xy & x^2 \\ 5x^4y^2 & x^52y \end{pmatrix} = 4x^6y^2 - 5x^6y^2 = -x^6y^2$$

7. Let $f(r, \theta) = (r \cos \theta, r \sin \theta), g(x, y) = (x^2, y)$ then $(g \circ f) = ((r \cos \theta)^2, (r \sin \theta))$

$$\therefore J(g \circ f) = \begin{pmatrix} 2r \cos^2 \theta & -2r^2 \cos \theta \sin \theta \\ \sin \theta & r \cos \theta \end{pmatrix}$$

$$Jac(f) = \det J(f) = \det \begin{pmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{pmatrix} = r \cos^2 \theta + r \sin^2 \theta = r(\cos^2 \theta + \sin^2 \theta) = r$$