Assignment 5
Topics of Mathematical Science

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1. Suppose h : continuous on $[a, b]$ (complex valued), proof that:

$$\Rightarrow \Big| \int_a^b h(t)dt \Big| \leq \int_a^b |h(t)|dt \tag{1}$$

Answer:

Approximating left hand side of (1) as Riemann sum, we have:

$$\Big| \int_a^b h(t)dt \Big| \approxeq \Big| \sum_{k=1}^{n} h(t_k) \triangle t \Big| \tag{2}$$

By triangle inequality,

$$\Big| \sum_{k=1}^{n} h(t_k) \triangle t \Big| \leq \sum_{k=1}^{n} |h(t_k)| \triangle t \tag{3}$$

we know that $\sum_{k=1}^{n} |h(t_k)| \triangle t$ is Riemann sum of $\int_a^b |h(t)|dt$. Hence,

$$\begin{gathered} \Big| \int_a^b h(t)dt \Big| \approxeq \Big| \sum_{k=1}^{n} h(t_k) \triangle t \Big| \leq \sum_{k=1}^{n} |h(t_k)| \triangle t \approxeq \int_a^b |h(t)|dt \\ \therefore \Big| \int_a^b h(t)dt \Big| \leq \int_a^b |h(t)|dt \end{gathered} \tag{4}$$