Proposition

A Proposition or a statement or logical sentence is a declarative sentence which is either true or false.

Example1: The following statements are all propositions:

- Soekarno is the first president of Indonesia
- It rained Yesterday.

Example2: The following statements are not propositions:

- Please report at 11 a.m. sharp
- What is your name?
- $x^2=13$

Propositional Variables

The lower case letters starting from P onwards are used to represent propositions

Example: p: India is in Asia
q: 2 + 2 = 4

Compound Statements

Statements or propositional variables can be combined by means of logical connectives (operators) to form a single statement called compound statements.

The five logical connectives are:

Symbol	Connective	Name
~	Not	Negation
∧	And	Conjunction
∨	Or	Disjunction
→	Implies or if...then	Implication or conditional
↔	If and only if	Equivalence or biconditional

Basic Logical Operations

1. **Negation:** It means the opposite of the original statement. If p is a statement, then the negation of p is denoted by ~p and read as 'it is not the case that p.' So, if p is true then ~ p is false and vice versa.

Example: If statement p is Paris is in France, then ~ p is 'Paris is not in France'.

p	~ p
T	F
F	T

2. Conjunction: It means Anding of two statements. If p, q are two statements, then "p and q" is a compound statement, denoted by $p \land q$ and referred as the conjunction of p and q.
The conjunction of p and q is true only when both p and q are true. Otherwise, it is false.

p	q	$p \land q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction: It means Oring of two statements. If p, q are two statements, then "p or q" is a compound statement, denoted by p V q and referred to as the disjunction of p and q.
The disjunction of p and q is true whenever at least one of the two statements is true, and it is false only when both p and q are false.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

4. Implication / if-then (⟶): An implication $p \longrightarrow q$ is the proposition "if p, then q." It is false if p is true and q is false. The rest cases are true.

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	F

5. If and Only If (↔): p ↔ q is bi-conditional logical connective which is true when p and q are same, i.e., both are false or both are true.

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Derived Connectors

1. NAND: It means negation after ANDing of two statements. Assume p and q be two propositions. Nanding of pand q to be a proposition which is false when both p and q are true, otherwise true. It is denoted by p ↑ q.

p	q	p ∨ q
T	T	F
T	F	T
F	T	T
F	F	T

2. NOR or Joint Denial: It means negation after ORing of two statements. Assume p and q be two propositions. NORing of p and q to be a proposition which is true when both p and q are false, otherwise false. It is denoted by p ↑ q.

p	q	p ↓ q
T	T	F
T	F	F
F	T	F
F	F	T

3. XOR: Assume p and q be two propositions. XORing of p and q is true if p is true or q is true but not both and vice-versa. It is denoted by **p ⊕ q**.

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Example1: Prove that $X \oplus Y \cong (X \wedge \sim Y) \vee (\sim X \wedge Y)$.

Solution: Construct the truth table for both the propositions.

X	Y	$X \oplus Y$	$\sim Y$	$\sim X$	$X \wedge \sim Y$	$\sim X \wedge Y$	$(X \wedge \sim Y) \vee (\sim X \wedge Y)$
T	T	F	F	F	F	F	F
T	F	T	T	F	T	F	T
F	T	T	F	T	F	T	T
F	F	F	T	T	F	F	F

As the truth table for both the proposition is the same.

$X \oplus Y \cong (X \wedge \sim Y) \vee (\sim X \wedge Y)$. Hence Proved.

Problem: Show that $(p \oplus q) \vee (p \downarrow q)$ is equivalent to $p \uparrow q$.

Solution: Construct the truth table for both the propositions.

p	q	$p \oplus q$	$(p \downarrow q)$	$(p \oplus q) \vee (p \downarrow q)$	$p \uparrow q$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	T	T	T

? Exercises for Section 2.1

1. Suppose that Daisy says, “If it does not rain, then I will play golf.” Later in the day you come to know that it did rain but Daisy still played golf. Was Daisy’s statement true or false? Support your conclusion.
2. Suppose that P and Q are statements for which $P \to Q$ is true and for which $\urcorner Q$ is true. What conclusion (if any) can be made about the truth value of each of the following statements?

   (a) P
   (b) $P \wedge Q$
   (c) $P \vee Q$
3. Suppose that P and Q are statements for which $P \to Q$ is false. What conclusion (if any) can be made about the truth value of each of the following statements?

   (a) $\urcorner P \to Q$
   (b) $Q \to P$
   (c) $P\ veeQ$
4. Suppose that P and Q are statements for which Q is false and $\urcorner P \to Q$ is true (and it is not known if R is true or false). What conclusion (if any) can be made about the truth value of each of the following statements?

   (a) $\urcorner Q \to P$
   (b) P
   (c) $P \wedge R$
   (d) $R \to \urcorner P$
5. Construct a truth table for each of the following statements:

   (a) $P \to Q$
   (b) $Q \to P$
   (c) $\urcorner P \to \urcorner Q$
   (d) $\urcorner Q \to \urcorner P$

   Do any of these statements have the same truth table?
6. Construct a truth table for each of the following statements:

   (a) $P \vee \urcorner Q$
   (b) $\urcorner(P \vee Q)$
   (c) $\urcorner P \vee \urcorner Q$
   (d) $\urcorner P \wedge \urcorner Q$

   Do any of these statements have the same truth table?
7. Construct truth table for $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$. What do you observe.