Networks, Games, and Collective Behaviour
Homework 2.

Q1:-

$$\frac{\Sigma d_i}{n_1} = \frac{\Sigma d_j}{n_2} = k$$

The total no. of edges (friendships) in the network is distributed among both groups. The average degree for each group is assumed to be the same, k.

Let
E_{11} be the no. of edges within G_1
E_{22} be the no. of edges within G_2
E_{12} be the no. of edges between the two groups

Total no. of friendships in the network $= E_{11} + E_{22} + E_{12}$

~~Homophily metrics:~~

$$h_1 = \frac{2E_{11}}{2E_{11} + E_{12}}, \quad h_2 = \frac{2E_{22}}{2E_{22} + E_{12}}$$

Since $n_1 > n_2$, & the average degree is the same, there are more total edges involving group 1 than group 2. However because homophily is between same-group friendships, a larger group tends to have more within-group friendships due to more possible connections within itself

Since n_1 is larger, it is more likely that individuals from group 1 will find friends within their own group rather than across groups. However, individuals in the smaller group i.e. group 2 have higher chance of forming cross group friendships since they have fewer same group options.

This means $\dfrac{E_{11}}{E_{12}} > \dfrac{E_{22}}{E_{12}}$

which leads to: $h_1 > h_2$

When both groups have the same average degree and $0 < h_1, h_2 < 1$, the larger group, G_1 has higher homophily metric than the smaller group, G_2 i.e, $h_1 > h_2$. This happen because individuals in the smaller group have fewer in group friendship options, leading to more cross-group friendships.