Discussion 6 Worksheet

Spring 2019

1 Rules

$$\text{Id}\ \frac{A(x) = v}{A; x \longrightarrow v} \qquad \text{Int}\ \frac{}{A; n \longrightarrow n}$$

$$\text{Bool-True}\ \frac{}{A; \textbf{true} \longrightarrow \textbf{true}} \qquad \text{Bool-False}\ \frac{}{A; \textbf{false} \longrightarrow \textbf{false}}$$

$$\text{Let}\ \frac{A; e1 \longrightarrow v1 \quad A, x : v1; e2 \longrightarrow v2}{A; \text{let } x = e1 \text{ in } e2 \longrightarrow v2}$$

$$\text{Add}\ \frac{A; e1 \longrightarrow v1 \quad A; e2 \longrightarrow v2 \quad v3 \text{ is } v1 + v2}{A; e1 + e2 \longrightarrow v3} \qquad \text{Mul}\ \frac{A; e1 \longrightarrow v1 \quad A; e2 \longrightarrow v2 \quad v3 \text{ is } v1 * v2}{A; e1 * e2 \longrightarrow v3}$$

$$\text{If-True}\ \frac{A; e1 \longrightarrow \textbf{true} \quad A; e2 \longrightarrow v2}{A; \text{if } e1 \text{ then } e2 \text{ else } e3 \longrightarrow v2} \qquad \text{If-False}\ \frac{A; e1 \longrightarrow \textbf{false} \quad A; e3 \longrightarrow v3}{A; \text{if } e1 \text{ then } e2 \text{ else } e3 \longrightarrow v3}$$

$$\text{Not-True}\ \frac{A; e1 \longrightarrow \textbf{true}}{A; \text{not } e1 \longrightarrow \textbf{false}} \qquad \text{Not-False}\ \frac{A; e1 \longrightarrow \textbf{false}}{A; \text{not } e1 \longrightarrow \textbf{true}}$$

2 Derivations

Given the rules above, prove the following.

$$\frac{}{\bullet; 1 + 2 * 3 \longrightarrow 7}$$

$$\frac{}{y : 3;\ \text{let } x = 1 \text{ in } x + y \longrightarrow 4}$$

$$\frac{}{\bullet;\ \text{let } x = \textbf{true} \text{ in if not } x \text{ then } 4 \text{ else } 6 \longrightarrow 6}$$

3 Solutions

$$\dfrac{\dfrac{}{\bullet; 1 \longrightarrow 1} \quad \dfrac{\dfrac{}{\bullet; 2 \longrightarrow 2} \quad \dfrac{}{\bullet; 3 \longrightarrow 3}\ 6 \text{ is } 2 * 3}{\bullet; 2 * 3 \longrightarrow 6}}{\bullet; 1 + 2 * 3 \longrightarrow 7}\ 7 \text{ is } 1 + 6$$

$$\dfrac{\dfrac{}{y : 3; 1 \longrightarrow 1} \quad \dfrac{\dfrac{A(x) = 1}{y : 3, x : 1; x \longrightarrow 1} \quad \dfrac{A(y) = 3}{y : 3, x : 1; y \longrightarrow 3}\ 4 \text{ is } 1 + 3}{y : 3, x : 1; x + y \longrightarrow 4}}{y : 3;\ \text{let } x = 1 \text{ in } x + y \longrightarrow 4}$$

$$\dfrac{\dfrac{}{\bullet; \mathbf{true} \longrightarrow \mathbf{true}} \quad \dfrac{\dfrac{\dfrac{\dfrac{A(x) = \mathbf{true}}{x : \mathbf{true}; x \longrightarrow \mathbf{true}}}{x : \mathbf{true}; \text{not } x \longrightarrow \mathbf{false}} \quad \dfrac{}{x : \mathbf{true}; 6 \longrightarrow 6}}{x : \mathbf{true}; \text{if not } x \text{ then 4 else 6} \longrightarrow 6}}{\bullet;\ \text{let } x = \mathbf{true} \text{ in if not } x \text{ then 4 else 6} \longrightarrow 6}$$