Helix Derivatives

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1 Track based alignment

Each hit measurement, y_i, is assumed to be described by a (non-)linear track model $f(x_i, \mathbf{q})$ which depends on a small number of parameter $\mathbf{q}$. Typically these are the five track parameters (see above) for tracks in a uniform magnetic field,

$$y_i = f(x_i, \mathbf{q}) + \epsilon_i. \tag{1}$$

The coordinate x_i is the coordiante of the hit y_i and ϵ is the uncertainty on y_i. The local-fit function f is linearized, if needed, by expressing it as a linear function of the local parameter corrections $\mathbf{\Delta q}$ at some reference value $\mathbf{q}_k$,

$$f(x_i, \mathbf{q}_k + \mathbf{\Delta q}) = f(x_i, \mathbf{q}_k) + \frac{\partial f}{\partial q_1}\Delta \mathrm{q}_1 + \frac{\partial f}{\partial q_2}\Delta \mathrm{q}_2 + ... \tag{2}$$

The local fit relies on minimizing the measured residual z_i for each hit,

$$z_i = y_i - f(x_i, \mathbf{q}_k). \tag{3}$$

By solving for $\mathbf{\Delta q}$ for each iteration k and updating with $q_{k+1} = q_k + \mathbf{\Delta q}$ convergence and optimal $\mathbf{\Delta q}$ can be obtained. These local parameter corrections $\mathbf{\Delta q}$ is used in the global fit. The derivatives of f w.r.t. the local parameters $\mathbf{q}$ is calculated in Sec. 3.

Describe the addition of global parameters here.

2 Helical Track Equations

Equations for trajectories in the XY plane

Point on helix (x, y) satisfies,

$$\mathrm{R}^2 = (x - x_c)^2 + (y - y_c)^2 \tag{4}$$

and the coordinate of the centre of the circle can be written,

$$x_c = x + \mathrm{R}\sin\phi \tag{5}$$

$$y_c = y - \mathrm{R}\cos\phi. \tag{6}$$

Equation for trajectory in XZ plane

$$z = z_0 + \mathrm{s} \times \mathrm{slope} \tag{7}$$

3 Local Parameters

Many of these are deliberately not fully simplified but adhere to the way they are implemented in the software.

Local derivative of d_0

Using 21,

$$\begin{aligned}
\frac{\partial x}{\partial d_0} &= -\sin(\phi_0) + \mathrm{sign}(\mathrm{R}) \frac{\partial y_c}{\partial d_0} && (8) \\
&= -\sin(\phi_0) + \mathrm{sign}(\mathrm{R}) \left(\frac{\mathrm{sign}(\mathrm{R})^2 \sin(\phi_0)(x - x_c)}{\sqrt{\mathrm{R}^2 - \mathrm{sign}(\mathrm{R})^2 \left(\mathrm{R}^2 - (x - x_c)^2\right)}} \right) && (9)
\end{aligned}$$

Local derivative of z_0

Local derivative of slope

Local derivative of ϕ_0

Local derivative of R

4 Global Transformations

4.1 Translation

Using Eq. 4, 5, 6,we can write,

$$\begin{aligned}
x &= x_c + \mathrm{sign}(\mathrm{R}) \times \mathrm{R} \times \sin\phi && (10) \\
y &= y_c + \mathrm{sign}(\mathrm{R}) \times \mathrm{R} \times \cos\phi && (11) \\
z &= z_0 - \mathrm{R} \times \mathrm{slope} \times \Delta\phi. && (12)
\end{aligned}$$

Note that,

$$\begin{aligned}
\phi &= \mathrm{atan2}\left(\sin\phi_0 - \frac{x - x_0}{\mathrm{R}}, \cos\phi_0 + \frac{y - y_0}{\mathrm{R}}\right) \\
\frac{\partial \phi}{\partial x} &= -\frac{1}{R} \times \frac{\cos\phi_0 + \frac{y - y_0}{\mathrm{R}}}{\left(\cos\phi_0 + \frac{y - y_0}{\mathrm{R}}\right)^2 + \left(\sin\phi_0 - \frac{x - x_0}{\mathrm{R}}\right)^2} \\
\frac{\partial \phi}{\partial y} &= -\frac{1}{R} \times \frac{\sin\phi_0 - \frac{x - x_0}{\mathrm{R}}}{\left(\cos\phi_0 + \frac{y - y_0}{\mathrm{R}}\right)^2 + \left(\sin\phi_0 - \frac{x - x_0}{\mathrm{R}}\right)^2} \\
\frac{\partial \phi}{\partial z} &= -\frac{1}{\mathrm{R} \times \mathrm{slope}}. && (13)
\end{aligned}$$

4.1.1 Translation in x

Using Eq. 4, 5, 6,

$$\begin{aligned} \frac{\partial x}{\partial x} &= 1 \\ \frac{\partial y}{\partial x} &= -\mathrm{sign}\left(\mathrm{R}\right) \times \mathrm{R} \sin\phi \frac{\partial \phi}{\partial x} \\ \frac{\partial z}{\partial x} &= -\mathrm{R} \times \mathrm{slope} \frac{\partial \phi}{\partial x}, \end{aligned} \tag{14}$$

with $\frac{\partial \phi}{\partial x}$ given byEq. 13.

4.1.2 Translation in y

Using Eq. 4, 5, 6,

$$\begin{aligned} \frac{\partial x}{\partial y} &= \mathrm{sign}\left(\mathrm{R}\right) \times \mathrm{R} \cos\phi \frac{\partial \phi}{\partial y} \\ \frac{\partial y}{\partial y} &= 1 \\ \frac{\partial z}{\partial y} &= -\mathrm{R} \times \mathrm{slope} \frac{\partial \phi}{\partial y}, \end{aligned} \tag{15}$$

with $\frac{\partial \phi}{\partial y}$ given byEq. 13.

4.1.3 Translation in z

Using Eq. 4, 5, 6,

$$\begin{aligned} \frac{\partial x}{\partial z} &= \mathrm{sign}\left(\mathrm{R}\right) \times \mathrm{R} \cos\phi \frac{\partial \phi}{\partial z} \\ \frac{\partial y}{\partial z} &= -\mathrm{sign}\left(\mathrm{R}\right) \times \mathrm{R} \times \sin\phi \frac{\partial \phi}{\partial z} \\ \frac{\partial z}{\partial z} &= 1, \end{aligned} \tag{16}$$

with $\frac{\partial \phi}{\partial z}$ given byEq. 13.

4.2 Rotation

4.2.1 Rotation around x - α

4.2.2 Rotation around y - β

4.2.3 Rotation around z - γ

Appendix

Practical Formulas

$$y = y_c + \mathrm{sign}\left(()\, \mathrm{R}\right) \times \sqrt{\left(\mathrm{R}^2 - (x - x_c)^2\right)} \tag{17}$$

$$x = x_c + \mathrm{sign}\left(()\, \mathrm{R}\right) \times \sqrt{\left(\mathrm{R}^2 - (y - y_c)^2\right)} \tag{18}$$

$$\Delta\phi = \mathrm{atan2}(x - x_c, y - y_c) - \mathrm{atan2}(x_0 - x_c, y_0 - y_c) \tag{19}$$

Arc length, s, can be written as:

$$\mathrm{s} = -\Delta\phi \times \mathrm{R}. \tag{20}$$

$$\frac{\partial y_c}{\partial d_0} = \frac{\mathrm{sign}\left(\mathrm{R}\right)^2 \sin\left(\phi_0\right)\left(x - x_c\right)}{\sqrt{\mathrm{R}^2 - \mathrm{sign}\left(\mathrm{R}\right)^2 \left(\mathrm{R}^2 - (x - x_c)^2\right)}} \tag{21}$$