Helix Derivatives

P. Hansson Adrian[1], M. Graham[1]

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1 Introduction

Give short overview of the helix equations and describe the conventions used.

1.1 Equations for trajectories in the XY plane

Point on helix (x, y) satisfies,

$$\mathrm{R}^2 = (x - x_c)^2 + (y - y_c)^2 \tag{1}$$

1.2 Equation for trajectory in XZ plane

$$z = z_0 + \mathrm{s} \times \mathrm{slope} \tag{2}$$

2 Local Transformations

3 Global Transformations

3.1 Translation

3.1.1 Translation in x

Using 1 and 2,

$$\begin{aligned}
\frac{\partial x}{\partial x} &= 1 && (3)\\
\frac{\partial y}{\partial x} &= \frac{\partial}{\partial x}\left(y_c + \mathrm{sign}(\mathrm{R}) \times \sqrt{\left(\mathrm{R}^2 - (x - x_c)^2\right)}\right) && (4)\\
&= -\mathrm{sign}(\mathrm{R}) \times \frac{x - x_c}{y - y_c} && (5)\\
\frac{\partial z}{\partial x} &= \frac{\partial}{\partial x}\left(z_0 - \mathrm{R} \times \Delta\phi \times \mathrm{slope}\right) && (6)\\
&= -\mathrm{R} \times \mathrm{slope} \times \frac{\partial \phi}{\partial x} && (7)
\end{aligned}$$

where (using 19),

$$\begin{aligned}\frac{\partial\phi}{\partial x} &= \frac{\partial}{\partial x}\left(\operatorname{atan2}(x-x_c, y-y_c)\right) && (8)\\ &= \frac{y-y_c}{(y-y_c)^2+(x-x_c)^2} && (9)\end{aligned}$$

and thus,

$$\frac{\partial z}{\partial x} = -\mathrm{R}\times\mathrm{slope}\times\frac{y-y_c}{(y-y_c)^2+(x-x_c)^2} \tag{10}$$

3.1.2 Translation in z

Trivially,

$$\frac{\partial z}{\partial z} = 1. \tag{11}$$

From 2 and 20,

$$\operatorname{atan2}(x-x_c, y-y_c) = -\frac{z-z_0}{\mathrm{R}\times\mathrm{slope}} + \operatorname{atan2}(x_0-x_c, y_0-y_c) \tag{12}$$

$$\begin{aligned} y &= y_c + \frac{x-x_c}{\mathrm{R}\times\mathrm{slope}}\times(1+\frac{1}{\tan(\Delta\phi+\operatorname{atan2}(x_0-x_c,y_0-y_c))}) && (13)\\ x &= x_c + (x+x_c)\times\frac{1}{\tan\left(\Delta\phi+\operatorname{atan2}(x_0-x_c,y_0-y_c)\right)}, && (14)\end{aligned}$$

Derivatives then become,

$$\frac{\partial y}{\partial z} = \frac{(x-x_c)}{\mathrm{R}\times\mathrm{slope}}\times\left(1+\frac{1}{\tan^2\left(\Delta\phi+\operatorname{atan2}(x_0-x_c,y_0-y_c)\right)}\right), \tag{15}$$

and

$$\frac{\partial x}{\partial z} = -\frac{(y-y_c)}{\mathrm{R}\times\mathrm{slope}}\times\left(1+\tan^2\left(\Delta\phi+\operatorname{atan2}(x_0-x_c,y_0-y_c)\right)\right). \tag{16}$$

Appendix

Practical Formulas

$$y = y_c + \mathrm{sign}(\mathrm{R}) \times \sqrt{\left(\mathrm{R}^2 - (x - x_c)^2\right)} \tag{17}$$

$$x = x_c + \mathrm{sign}(\mathrm{R}) \times \sqrt{\left(\mathrm{R}^2 - (y - y_c)^2\right)} \tag{18}$$

$$\Delta\phi = \mathrm{atan2}(x - x_c, y - y_c) - \mathrm{atan2}(x_0 - x_c, y_0 - y_c) \tag{19}$$

Arc length, s, can be written as:

$$\mathrm{s} = -\Delta\phi \times \mathrm{R}. \tag{20}$$