```
yc[R_, d0_, phi0_] := -(R - d0) * Cos[phi0];
xc[R_, d0_, phi0_] := (R - d0) * Sin[phi0];
y0[d0_, phi0_] := d0 * Cos[phi0];
x0[d0_, phi0_] := -d0 * Sin[phi0];
yint[R_, d0_, phi0_, xint_] :=
  yc[R, d0, phi0] + Sign[R] * Sqrt[R * R - (xint - xc[R, d0, phi0]) ^ 2];

yintPos[R_, d0_, phi0_, xint_] := yc[R, d0, phi0] + Sqrt[R * R - (xint - xc[R, d0, phi0]) ^ 2];
yintNeg[R_, d0_, phi0_, xint_] := yc[R, d0, phi0] - Sqrt[R * R - (xint - xc[R, d0, phi0]) ^ 2];

(*phi1[R_,d0_,phi0_,xint_]:=
 ArcTan[y0[d0,phi0]-yc[R,d0,phi0],x0[d0,phi0]-xc[R,d0,phi0]];
phi2[R_,d0_,phi0_,xint_]:=
 ArcTan[yint[R,d0,phi0,xint]-yc[R,d0,phi0],xint-xc[R,d0,phi0]];*)
phi1[R_, d0_, phi0_, xint_] :=
  ArcTan[x0[d0, phi0] - xc[R, d0, phi0], y0[d0, phi0] - yc[R, d0, phi0]];
phi2[R_, d0_, phi0_, xint_] := ArcTan[xint - xc[R, d0, phi0],
   yint[R, d0, phi0, xint] - yc[R, d0, phi0]];
phi2Pos[R_, d0_, phi0_, xint_] := ArcTan[xint - xc[R, d0, phi0],
   yintPos[R, d0, phi0, xint] - yc[R, d0, phi0]];
phi2Neg[R_, d0_, phi0_, xint_] := ArcTan[xint - xc[R, d0, phi0],
   yintNeg[R, d0, phi0, xint] - yc[R, d0, phi0]];
tmpdphi[R_, d0_, phi0_, xint_] := phi2[R, d0, phi0, xint] - phi1[R, d0, phi0, xint];
dphiPos[R_, d0_, phi0_, xint_] := phi2Pos[R, d0, phi0, xint] - phi1[R, d0, phi0, xint];
dphiNeg[R_, d0_, phi0_, xint_] := phi2Neg[R, d0, phi0, xint] - phi1[R, d0, phi0, xint];
dphi[R_, d0_, phi0_, xint_] := If[tmpdphi[R, d0, phi0, xint] > Pi,
   tmpdphi[R, d0, phi0, xint] - 2 * Pi, If[tmpdphi[R, d0, phi0, xint] > Pi,
    tmpdphi[R, d0, phi0, xint] + 2 * Pi, tmpdphi[R, d0, phi0, xint]]];
s[R_, d0_, phi0_, xint_] := -R * dphi[R, d0, phi0, xint];
sPos[R_, d0_, phi0_, xint_] := -R * dphiPos[R, d0, phi0, xint];
sNeg[R_, d0_, phi0_, xint_] := -R * dphiNeg[R, d0, phi0, xint];
pos = Simplify[D[sPos[R, d0, phi0, xint], R]]
```

$$\left(-\text{xint}-\text{d0}\,\text{Sin}[\text{phi0}]+\text{ArcTan}[-R\,\text{Sin}[\text{phi0}],R\,\text{Cos}[\text{phi0}]]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}-\text{ArcTan}\left[\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}],\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)\Big/\left(\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)$$

$$\left(-\text{xint}-\text{d0}\,\text{Sin}[\text{phi0}]+\text{ArcTan}[-R\,\text{Sin}[\text{phi0}],R\,\text{Cos}[\text{phi0}]]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}-\text{ArcTan}\left[\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}],\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)\Big/\left(\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)$$

$$\left(-\text{xint}-\text{d0}\,\text{Sin}[\text{phi0}]+\text{ArcTan}[-R\,\text{Sin}[\text{phi0}],R\,\text{Cos}[\text{phi0}]]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}-\text{ArcTan}\left[\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}],\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)\Big/\left(\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)$$


```
neg = Simplify[D[sNeg[R, d0, phi0, xint], R]]
```

$$\left(\text{xint}+\text{d0}\,\text{Sin}[\text{phi0}]+\text{ArcTan}[-R\,\text{Sin}[\text{phi0}],R\,\text{Cos}[\text{phi0}]]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}-\text{ArcTan}\left[\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}],-\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right]\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)\Big/\left(\sqrt{R^2-(\text{xint}+(\text{d0}-R)\,\text{Sin}[\text{phi0}])^2}\right)$$

$$\left(\mathbf{xint + d0\,Sin[phi0] + ArcTan[-R\,Sin[phi0],\,R\,Cos[phi0]]\,\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2} - ArcTan\left[xint + (d0 - R)\,Sin[phi0],\,-\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}\,\right]\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right) \Big/ \left(\mathbf{\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)$$

$$\left(\mathrm{xint + d0\,Sin[phi0] + ArcTan[-R\,Sin[phi0],\,R\,Cos[phi0]]\,\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2} - ArcTan\left[xint + (d0 - R)\,Sin[phi0],\,-\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}\,\right]\sqrt{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right) \Big/ \left(\sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)$$

```
Simplify[D[sPos[R, d0, phi0, xint], phi0]]
```

$$\left(\mathrm{R}\left(\mathrm{d0\,Cos[phi0] - R\,Cos[phi0]} + \sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)\right) \Big/ \left(\sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)$$

```
Simplify[D[sNeg[R, d0, phi0, xint], phi0]]
```

$$\left(\mathrm{R}\left(\mathrm{-d0\,Cos[phi0] + R\,Cos[phi0]} + \sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)\right) \Big/ \left(\sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}\right)$$

```
Simplify[D[sPos[R, d0, phi0, xint], d0]]
```

$$\frac{\mathrm{R\,Sin[phi0]}}{\sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}}$$

```
Simplify[D[sNeg[R, d0, phi0, xint], d0]]
```

$$-\frac{\mathrm{R\,Sin[phi0]}}{\sqrt{\mathrm{R^2 - (xint + (d0 - R)\,Sin[phi0])^2}}}$$

```
(*phis[R_,d0_,phi0_,xint_]:=phi0-s[R,d0,phi0,xint]/R;*)

(*y[R_,d0_,phi0_,xint_]:=yc[R,d0,phi0]+R/Sin[phis[R,d0,phi0,xint]];*)

(*z[R_,d0_,phi0_,xint_,z0_,slope_]:=z0+s[R,d0,phi0,xint]*slope;*)

(*D[y[R_,d0_,phi0_,xint_],R_];
D[y[R_,d0_,phi0_,xint_],d0_];
D[y[R_,d0_,phi0_,xint_],phi0_];*)

(*D[z[R_,d0_,phi0_,xint_,z0_,slope_],R_];
D[z[R_,d0_,phi0_,xint_,z0_,slope_],d0_];
D[z[R_,d0_,phi0_,xint_,z0_,slope_],phi0_];
D[z[R_,d0_,phi0_,xint_,z0_,slope_],z0_];
D[z[R_,d0_,phi0_,xint_,z0_,slope_],slope_];*)

(*phis[R_,phi0_,stmp_]:=phi0-stmp/R;*)
phis[R_, d0_, phi0_, xint_] := phi0 - stmp[R, d0, phi0, xint] / R
y[R_, d0_, phi0_, xint_] := yc[R, d0, phi0] + R / Sin[phis[R, d0, phi0, xint]];
z[R_, d0_, phi0_, xint_, z0_, slope_] := z0 + stmp[R, d0, phi0, xint] * slope;
```

```
dydr = D[y[R, d0, phi0, xint], R]
```

$$-\mathrm{Cos}[\mathrm{phi0}] + \mathrm{Csc}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] - R\,\mathrm{Cot}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \mathrm{Csc}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \left(\frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R^2} - \frac{\mathrm{stmp}^{(1,0,0,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right)$$

```
D[y[R, d0, phi0, xint], d0]
```

$$\mathrm{Cos}[\mathrm{phi0}] + \mathrm{Cot}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \mathrm{Csc}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \mathrm{stmp}^{(0,1,0,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]$$

```
D[y[R, d0, phi0, xint], phi0]
```

$$-(d0 - R)\,\mathrm{Sin}[\mathrm{phi0}] - R\,\mathrm{Cot}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \mathrm{Csc}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \left(1 - \frac{\mathrm{stmp}^{(0,0,1,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right)$$

```
(-(d0 - R))*Sin[phi0] - R*Cot[phi0 - stmp[R, d0, phi0, xint]/R]*Csc[phi0 - stmp[R, d0, phi(
   (1 - Derivative[0, 0, 1, 0][stmp][R, d0, phi0, xint]/R)
```

$$(-d0 + R)\,\mathrm{Sin}[\mathrm{phi0}] - R\,\mathrm{Cot}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \mathrm{Csc}\left[\mathrm{phi0} - \frac{\mathrm{stmp}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right] \left(1 - \frac{\mathrm{stmp}^{(0,0,1,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]}{R}\right)$$

```
D[z[R, d0, phi0, xint, z0, slope], R]
```

$\mathrm{slope}\ \mathrm{stmp}^{(1,0,0,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]$

```
D[z[R, d0, phi0, xint, z0, slope], d0]
```

$\mathrm{slope}\ \mathrm{stmp}^{(0,1,0,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]$

```
D[z[R, d0, phi0, xint, z0, slope], phi0]
```

$\mathrm{slope}\ \mathrm{stmp}^{(0,0,1,0)}[R, d0, \mathrm{phi0}, \mathrm{xint}]$

```
D[z[R, d0, phi0, xint, z0, slope], z0]
```

1

```
D[z[R, d0, phi0, xint, z0, slope], slope]
```

stmp[R, d0, phi0, xint]

```
myd0 = 0.48454;
myz0 = 0.05180;
myslope = 1.61182;
myphi0 = 6.24804;
myR = -4759.26979;
myxint = 700;
s[myR, myd0, myphi0, myxint]
```

701.136

```
xc[myR, myd0, myphi0]
```

167.249

```
yc[myR, myd0, myphi0]
```

4756.82

```
dphi[myR, myd0, myphi0, myxint]
```

0.14732

```
x0[myd0, myphi0]
y0[myd0, myphi0]
```

0.0170258

0.484241

```
yint[myR, myd0, myphi0, myxint]
```

27.4573

```
phi1[myR, myd0, myphi0, myxint]
```

-1.60594

```
phi2[myR, myd0, myphi0, myxint]
```

-1.45862