C. Kleinwort, 09.01.14

Telescope alignment

Beam telescope

Set of detector planes at fixed Z-positions measuring X(Z) (and Y(Z)).

Alignment

The alignment of the detector planes is performed by minimizing a global Chi2, e.g. with Millepede-II.

Weak modes

Systematic geometry distortions which don't or only very weakly change the global Chi2. Alignment will give 'random' answers.

Rigid body alignment

For each detector plane (up to) 3 offsets and 3 rotations are determined.

Without bending in a magnetic field the measurement expectations depend linearly on Z (with track parameters (p_0,p_1)):

$$m_i = p_0 + p_1 \cdot z_i$$

Offsets in measurement direction

Offsets Δm in the measurement directions linear in Z can be absorbed by the (local) track fit by changed track parameters:

$$\Delta m_i = a + b \cdot z_i \Rightarrow m_i + \Delta m_i = p_0 + p_1 \cdot z_i + a + b \cdot z_i = (p_0 + a) + (p_1 + b) \cdot z_i$$

Therefore the Chi2 of the track fit will not change. Two offsets (like first and last layer) have to be fixed to control this weak mode. Offsets nonlinear in Z would change the Chi2 and can be determined without fixing additional parameters. In case the track model allows a (constant) non-zero curvature offsets parabolic in Z would be again a weak mode requiring the fixing of a third offset.

Offsets in Z-direction

Offsets Δz_i change the measurement prediction according to the track slope:

$$\Delta m_i = \frac{\partial m_i}{\partial z_i} \Delta z_i = p_1 \cdot \Delta z_i$$

For offsets linear in Z the change Δm_i is linear in Z too and the track Chi2 will not change resulting in a weak mode requiring the fixing of the offsets of two measurement planes.

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Rotations in MZ plane

For a small rotation β (around the intersection of measurement direction and Z-axis in that plane) the Z position is slightly modified:

$$\Delta z_i = \beta \cdot m_i$$

This changes the measurement prediction according to the track slope:

$$\Delta m_i = \frac{\partial m_i}{\partial z_i} \Delta z_i = \beta \cdot p_1 \cdot m_i = \beta \cdot p_1 \cdot \left(p_0 + p_1 \cdot z_i \right)$$

For constant β the change Δm_i is linear in Z and the track Chi2 will not change resulting in a weak mode requiring the fixing of the rotation of one measurement plane. For $\beta = \beta_0 + \beta_1 \cdot Z$ there will be a term $\beta_1 \cdot p_1^2 \cdot z_i^2$ quadratic in Z in Δm_i. This will change the Chi2 and β_1 is not a weak mode provided sufficiently large track slopes ($\Delta m_i \sim p_1^2$). Otherwise 2 rotations have to be fixed.

Rotations around Z-axis

For a small rotation γ (around the Z-axis) the measurement m_i changes according to the other coordinate n_i in the detector plane:

$$\Delta m_i = \gamma \cdot n_i = \gamma \cdot \left(p_2 + p_3 \cdot z_i \right), \quad n_i = p_2 + p_3 \cdot z_i$$

For constant γ the change Δm_i is linear in Z and the track Chi2 will not change resulting in a weak mode requiring the fixing of the rotation of one measurement plane. For $\gamma = \gamma_0 + \gamma_1 \cdot Z$ there will be a term $\gamma_1 \cdot p_3 \cdot z_i^2$ quadratic in Z in Δm_i. This will change the Chi2 and γ_1 is not a weak mode provided sufficiently large track slopes ($\Delta m_i \sim p_3$). Otherwise 2 rotations have to be fixed.

Summary

Offsets (linear in Z) and constant rotations are weak modes requiring the fixing of some parameters. Rotations proportional to Z may or may not be weak modes depending on the track slopes. For rotations in the plane of measurement direction and Z-axis the sensitivity is quadratic and for rotations around the Z-axis linear in the track slopes. Depending on the distribution of the track slopes the fixing of more parameters may be necessary.