Helix Derivatives

P. Hansson Adrian[1], M. Graham[1], S. Uemura[1]

August 31, 2012

1 Track based alignment

Each hit measurement, y_i, is assumed to be described by a (non-)linear track model $f(x_i, \mathbf{q})$ which depends on a small number of parameter $\mathbf{q}$. Typically these are the five track parameters (see above) for tracks in a uniform magnetic field,

$$y_i = f(x_i, \mathbf{q}) + \epsilon_i. \tag{1}$$

The coordinate x_i is the coordiante of the hit y_i and ϵ is the uncertainty on y_i. The local-fit function f is linearized, if needed, by expressing it as a linear function of the local parameter corrections $\Delta \mathbf{q}$ at some reference value $\mathbf{q}_k$,

$$f(x_i, \mathbf{q}_k + \Delta \mathbf{q}) = f(x_i, \mathbf{q}_k) + \frac{\partial f}{\partial q_1} \Delta q_1 + \frac{\partial f}{\partial q_2} \Delta q_2 + \dots \tag{2}$$

The local fit relies on minimizing the measured residual z_i for each hit,

$$z_i = y_i - f(x_i, \mathbf{q}_k). \tag{3}$$

By solving for $\Delta \mathbf{q}$ for each iteration k and updating with $q_{k+1} = q_k + \Delta \mathbf{q}$ convergence and optimal $\Delta \mathbf{q}$ can be obtained. These local parameter corrections $\Delta \mathbf{q}$ is used in the global fit. The derivatives of f w.r.t. the local parameters $\mathbf{q}$ is calculated in Sec. 3.

Describe the addition of global parameters here.

2 Helical Track Equations

Equations for trajectories in the XY plane

Point on helix (x, y) satisfies,

$$\mathrm{R}^2 = (x - x_c)^2 + (y - y_c)^2 \tag{4}$$

and the coordinate of the centre of the circle can be written,

$$
\begin{aligned}
x_c &= x + \mathrm{R}\sin\phi \tag{5}\\
y_c &= y - \mathrm{R}\cos\phi. \tag{6}
\end{aligned}
$$

Equation for trajectory in XZ plane

$$z = z_0 + \text{s} \times \text{slope} \tag{7}$$

3 Local Parameters

Many of these are deliberately not fully simplified but adhere to the way they are implemented in the software.

Local derivative of d_0

Using 21,

$$\frac{\partial x}{\partial d_0} = -\sin(\phi_0) + \text{sign}(\text{R}) \frac{\partial y_c}{\partial d_0} \tag{8}$$

$$= -\sin(\phi_0) + \text{sign}(\text{R}) \left(\frac{\text{sign}(\text{R})^2 \sin(\phi_0)(x - x_c)}{\sqrt{\text{R}^2 - \text{sign}(\text{R})^2 \left(\text{R}^2 - (x - x_c)^2 \right)}} \right) \tag{9}$$

Local derivative of z_0

Local derivative of slope

Local derivative of ϕ_0

Local derivative of R

4 Global Transformations

4.1 Translation

Using Eq. 4, 5, 6,we can write,

$$x = x_c + \text{sign}(\text{R}) \times \text{R} \times \sin \phi \tag{10}$$
$$y = y_c + \text{sign}(\text{R}) \times \text{R} \times \cos \phi \tag{11}$$
$$z = z_0 - \text{R} \times \text{slope} \times \Delta \phi. \tag{12}$$

Note that,

$$\phi = \text{atan2}\left(\sin \phi_0 - \frac{x - x_0}{\text{R}}, \cos \phi_0 + \frac{y - y_0}{\text{R}} \right)$$

$$\frac{\partial \phi}{\partial x} = \frac{\cos \phi_0 + \frac{y - y_0}{\text{R}}}{\left(\cos \phi_0 + \frac{y - y_0}{\text{R}} \right)^2 + \left(\sin \phi_0 - \frac{x - x_0}{\text{R}} \right)^2}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\sin \phi_0 - \frac{x - x_0}{\text{R}}}{\left(\cos \phi_0 + \frac{y - y_0}{\text{R}} \right)^2 + \left(\sin \phi_0 - \frac{x - x_0}{\text{R}} \right)^2}$$

$$\frac{\partial \phi}{\partial z} = -\frac{1}{\text{R} \times \text{slope}}. \tag{13}$$

4.1.1 Translation in x

Using Eq. 4, 5, 6,

$$
\begin{aligned}
\frac{\partial x}{\partial x} &= 1 \\
\frac{\partial y}{\partial x} &= -\operatorname{sign}(R) \times R \sin \phi \frac{\partial \phi}{\partial x} \\
\frac{\partial z}{\partial x} &= -R \times \operatorname{slope} \frac{\partial \phi}{\partial x},
\end{aligned}
\tag{14}
$$

with $\frac{\partial \phi}{\partial x}$ given byEq. 13.

4.1.2 Translation in y

Using Eq. 4, 5, 6,

$$
\begin{aligned}
\frac{\partial x}{\partial y} &= \operatorname{sign}(R) \times R \cos \phi \frac{\partial \phi}{\partial y} \\
\frac{\partial y}{\partial y} &= 1 \\
\frac{\partial z}{\partial y} &= -R \times \operatorname{slope} \frac{\partial \phi}{\partial y},
\end{aligned}
\tag{15}
$$

with $\frac{\partial \phi}{\partial y}$ given byEq. 13.

4.1.3 Translation in z

Using Eq. 4, 5, 6,

$$
\begin{aligned}
\frac{\partial x}{\partial z} &= \operatorname{sign}(R) \times R \cos \phi \frac{\partial \phi}{\partial z} \\
\frac{\partial y}{\partial z} &= -\operatorname{sign}(R) \times R \times \sin \phi \frac{\partial \phi}{\partial z} \\
\frac{\partial z}{\partial z} &= 1,
\end{aligned}
\tag{16}
$$

with $\frac{\partial \phi}{\partial z}$ given byEq. 13.

4.2 Rotation

4.2.1 Rotation around x - α

4.2.2 Rotation around y - β

4.2.3 Rotation around z - γ

Appendix

Practical Formulas

$$y = y_c + \text{sign}\left((\ )\,\text{R}\right) \times \sqrt{\left(\text{R}^2 - (x - x_c)^2\right)} \tag{17}$$

$$x = x_c + \text{sign}\left((\ )\,\text{R}\right) \times \sqrt{\left(\text{R}^2 - (y - y_c)^2\right)} \tag{18}$$

$$\Delta\phi = \text{atan2}(x - x_c, y - y_c) - \text{atan2}(x_0 - x_c, y_0 - y_c) \tag{19}$$

Arc length, s, can be written as:

$$\text{s} = -\Delta\phi \times \text{R}. \tag{20}$$

$$\frac{\partial y_c}{\partial d_0} = \frac{\text{sign}\left(\text{R}\right)^2 \sin\left(\phi_0\right)\left(x - x_c\right)}{\sqrt{\text{R}^2 - \text{sign}\left(\text{R}\right)^2 \left(\text{R}^2 - (x - x_c)^2\right)}} \tag{21}$$

4