Afina Putri D.

1. $\det(D) = \begin{vmatrix} 2 & 5 & 2 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & 6 & 3 & 2 \\ 4 & 12 & 0 & 8 \end{vmatrix}$

$$= 2\begin{vmatrix} 3 & 3 & 4 \\ 6 & 3 & 2 \\ 12 & 0 & 8 \end{vmatrix} - 2\begin{vmatrix} 5 & 2 & 3 \\ 6 & 3 & 2 \\ 12 & 0 & 8 \end{vmatrix} + 3\begin{vmatrix} 5 & 2 & 3 \\ 3 & 3 & 4 \\ 12 & 0 & 8 \end{vmatrix} - 4\begin{vmatrix} 5 & 2 & 3 \\ 3 & 3 & 4 \\ 6 & 3 & 2 \end{vmatrix}$$

$$\Rightarrow 2\begin{vmatrix} 3 & 3 & 4 \\ 6 & 3 & 2 \\ 12 & 0 & 8 \end{vmatrix} = 2\left(3\begin{vmatrix} 3 & 2 \\ 0 & 8 \end{vmatrix} - 3\begin{vmatrix} 6 & 2 \\ 12 & 8 \end{vmatrix} + 4\begin{vmatrix} 6 & 3 \\ 12 & 0 \end{vmatrix}\right)$$

$$= 2(72 - 72 - 144) = -288$$

$$\Rightarrow -2\begin{vmatrix} 5 & 2 & 3 \\ 6 & 3 & 2 \\ 12 & 0 & 8 \end{vmatrix} = -2\left(5\begin{vmatrix} 3 & 2 \\ 0 & 8 \end{vmatrix} - 2\begin{vmatrix} 6 & 2 \\ 12 & 8 \end{vmatrix} + 3\begin{vmatrix} 6 & 3 \\ 12 & 0 \end{vmatrix}\right)$$

$$= -2(120 - 48 - 108) = 72$$

$$\Rightarrow 3\begin{vmatrix} 5 & 2 & 3 \\ 3 & 3 & 4 \\ 12 & 0 & 8 \end{vmatrix} = 3\left(5\begin{vmatrix} 3 & 4 \\ 0 & 8 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 12 & 8 \end{vmatrix} + 3\begin{vmatrix} 3 & 3 \\ 12 & 0 \end{vmatrix}\right)$$

$$= 3(120 + 48 - 108) = 180$$

$$\Rightarrow -4\begin{vmatrix} 5 & 2 & 3 \\ 3 & 3 & 4 \\ 6 & 3 & 2 \end{vmatrix} = -4\left(5\begin{vmatrix} 3 & 4 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 6 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & 3 \\ 6 & 3 \end{vmatrix}\right)$$

$$= -4(-30 + 36 - 27) = 84$$

$$\det(D) = -288 + 72 + 180 + 84 = \underline{\underline{48}}$$

2.

$$\det(B) = \begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -7 & 0 & -4 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 3 & 1 & 5 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 3 & 1 & 5 & 3 \\ -7 & 0 & -4 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} 3 & 1 & 5 & 3 \\ -7 & 0 & -4 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 3 & 1 & 5 & 3 \\ -7 & 0 & -4 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$$

* untuk matrix ke 3, 4, dan 5 tidak pernah dihitung, karena koefisiennya 0, sehingga apabila dikali hasilnya akan tetap 0

$$\Rightarrow 1 \begin{vmatrix} -7 & 0 & -4 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$1\left(-7 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -4 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & -4 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -4 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}\right)$$

* untuk matrix ke 2, 3, dan 4 tidak pernah dihitung, karena koefisienya 0, sehingga apabila dikali hasilnya akan tetap 0.

$$-7 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -7\left(1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 0 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}\right)$$

$$= -7(0 - 0 + 2) = -14$$

$$= 1(-14 - 0 + 0 - 0) = -14$$

$$\Rightarrow 2\begin{vmatrix} 3 & 1 & 5 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$2\left(3\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 5 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 5 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 5 & 3 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}\right)$$

* untuk matrix ke 2, 3, dan 4 tidak pernah dihitung, karena koefisiennya 0, sehingga apabila dikali hasilnya akan tetap 0

$$3\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 3\left(1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}\right)$$

$$= 3(0 - 0 + 2) = 6$$

$$= 2(6 - 0 + 0 - 0) = 12$$

$$\det(B) = -14 + 12 + 0 - 0 + 0 = \underline{\underline{-2}}$$

3) $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

* ekspansi baris ke-3

$= a^2(c-b) - b^2(c-a) + c^2(b-a)$

$= a^2c - a^2b - b^2c + ab^2 + bc^2 - ac^2$

$= (a^2c - ac^2) + (ab^2 - a^2b) + (bc^2 - b^2c)$

$= ac(a-c) + ab(b-a) + bc(c-b)$

$= \underline{(b-a)(c-a)(c-b)}$

jadi pembuktian tsb benar