CITA200H Assignment 2

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1 Region of Convergence for $z_{i+1} = z_i^2 + c$

1.1 Methods

This problem requires us to map out an image of where $z_{i+1} = z_i^2 + c$, where c is the complex plane in the region $-2 < x < 2$, $-2 < y < 2$, and $z_0 = 0$.

Using `numpy`, we can construct an array in $\mathbb{C}^2$. The iterations require a for loop; but only one, thanks to `numpy`'s vector algebra. Choosing an arbitrary ϵ value to define divergence; We then generate a subspace of convergence and plot.

Unfortunately, there does not exist an elegant method to record the iteration where a point is considered divergent. We are forced to use a nested for loop after every iteration, checking every point in our array for divergence. We then record this iteration number on a new array; this new array is then plotted.

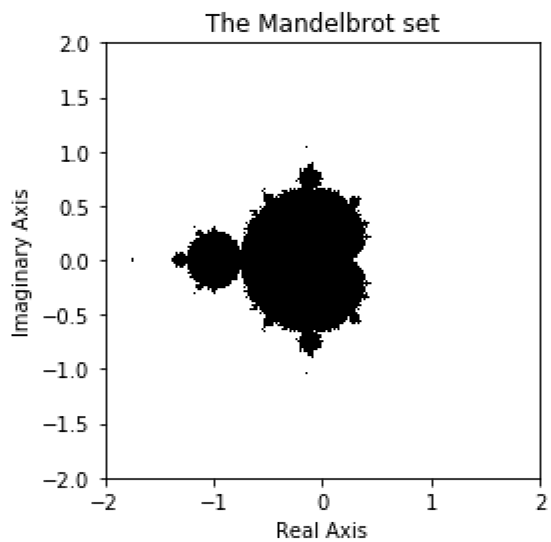

Figure 1: Convergence of the Mandelbrot set is in black

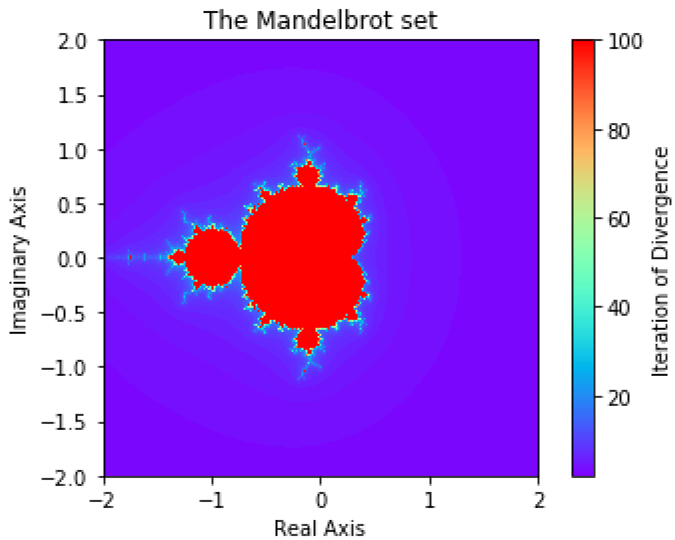

Figure 2: Divergence of the Mandelbrot set, by iteration

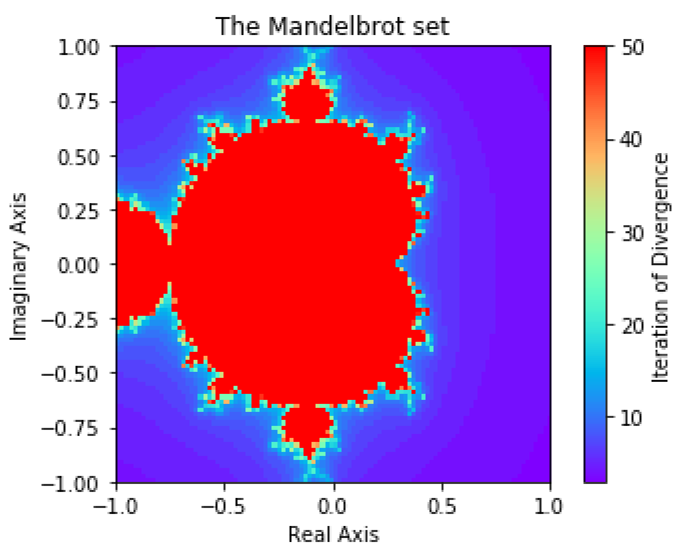

Figure 3: Zoom of the Mandelbrot set

1.2 Analysis

We recognize $z_{i+1} = z_i^2 + c$ as the Mandelbrot set. An obvious feature of the plots, is that they are symmetric across the x axis which is indicative of the complex plane. A less trivial feature, is that the set appear to be "repeating"; That is to say, the set is self similar, in a fractal pattern, when zoomed in.

2 The SIR Model

2.1 Methods

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \tag{1}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I, \tag{2}$$
$$\frac{dR}{dt} = \gamma I \tag{3}$$

This problem requires us to numerically solve the system of first order ODEs. A simple method, is to use `odeint` from `scipy.integrate`. `odeint` has us input the system, values of β and γ, and the domain for which to solve. `odeint` will then return us a vector of $S(t), I(t), R(t)$.

It is known $R_0 = \frac{\beta}{\gamma}$, the average number of people one infected person infects. Reasonable values for β and γ, can then be deduced by having a reasonable R_0 value.

2.2 Bonus

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I - \mu I, \tag{4}$$
$$\frac{dD}{dt} = \mu I \tag{5}$$

We can account for deaths, denoted as $D(t)$, by adding a $-\mu I$ term in equation (2), and adding a $\frac{dD}{dt} = \mu I$ equation to our system. The coefficient μ represents the fraction of the infected population that die. The $-\mu I$ in (4), is then to account for the decrease in the infected population due to deaths. We then solve as before, using the `odeint` function.

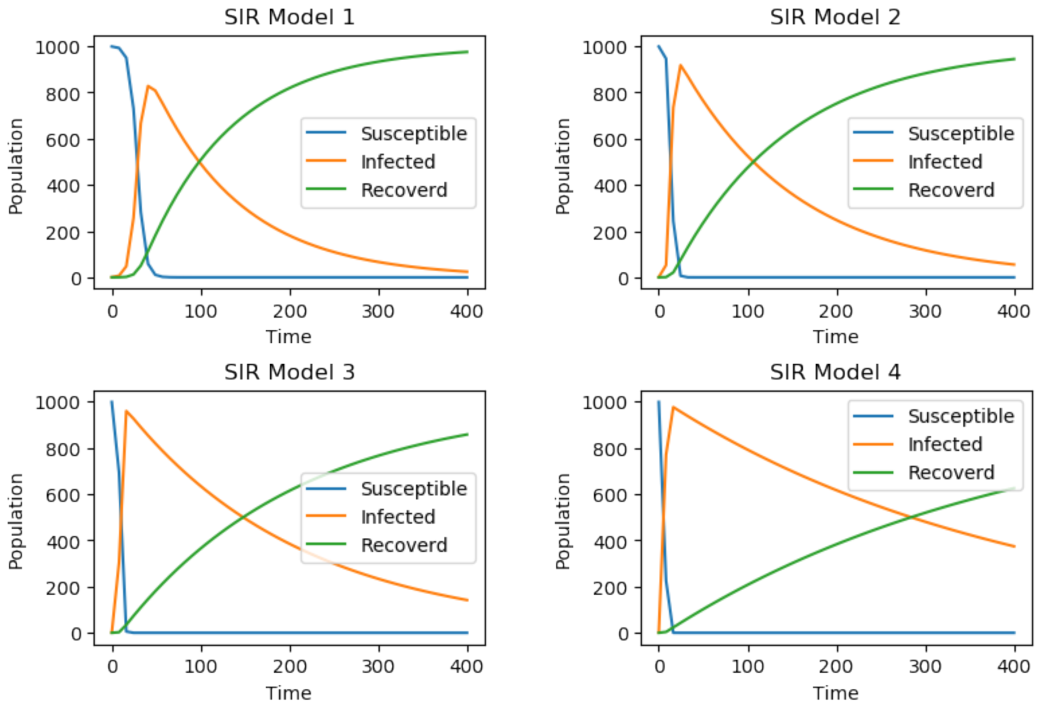

Figure 4: These SIR models use $\beta \approx 0.01$, and $\gamma \approx 0.001$. Time & Population units are arbitrary

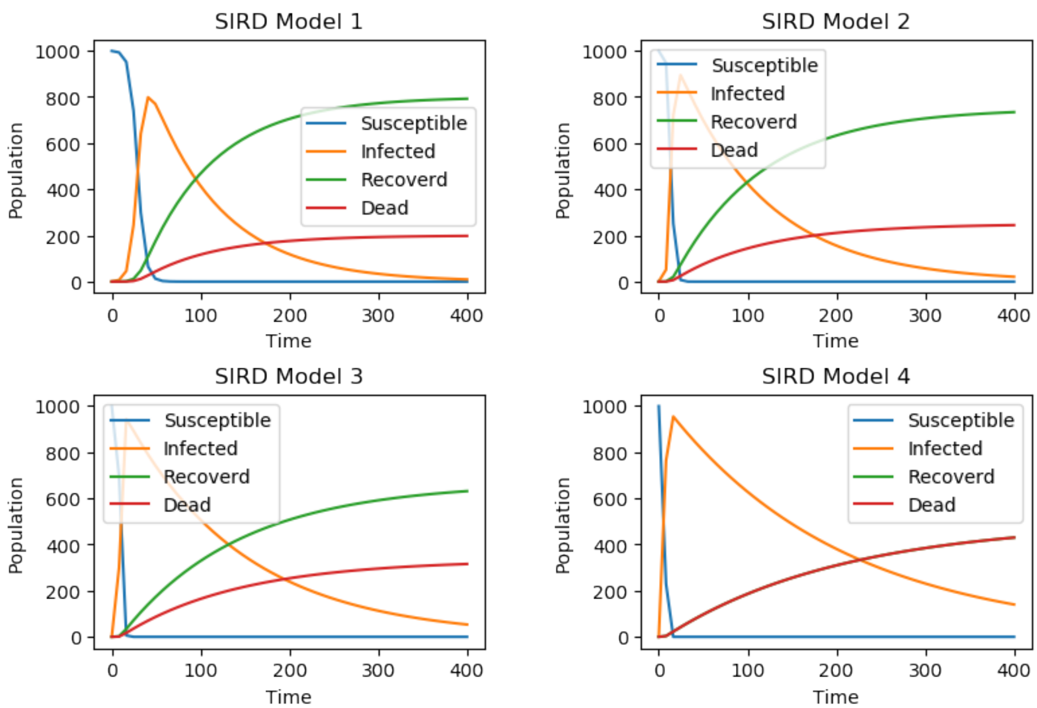

Figure 5: These SIRD models use $\beta \approx 0.01$, $\gamma \approx 0.001$, and $\mu \approx 0.001$. Time & Population units are arbitrary

2.3 Analysis

Looking at the solutions for the SIR and SIRD models; we note that infected rises very sharply, then decays like an exponential function. Recovered and death both behave similarly, increasing to a maximum value as time approaches infinity.