

$$S = \int_{x_1}^{x_2} F(\bar{y}, \bar{y}', x) dx$$

$$\bar{y} = y(x) + \alpha \eta(x)$$

$$\bar{y}' = y'(x) + \alpha \eta'(x)$$

$$y(x) = \text{stationary}, \therefore \alpha = 0$$

$$\frac{dS}{d\alpha} = 0, \text{ for correct } y(x)$$

as $S = S(\alpha)$

$$\frac{dS}{d\alpha} = \int_{x_1}^{x_2} \frac{d}{d\alpha} F(\bar{y}, \bar{y}', x) dx$$

$$\frac{dF}{d\alpha} = \frac{\partial F}{\partial \bar{y}} \frac{d\bar{y}}{d\alpha} + \frac{\partial F}{\partial \bar{y}'} \frac{d\bar{y}'}{d\alpha}$$

$$= \eta \frac{\partial F}{\partial \bar{y}} + \eta' \frac{\partial F}{\partial \bar{y}'} \quad (A)$$

But, Boole has

$$= \eta \frac{df}{dy} + \eta' \frac{df}{dy'} \quad (B)$$

How do I go from (A) \rightarrow (B)?

That is, how do I know?

$$\frac{\partial F}{\partial \bar{y}} = \frac{\partial F}{\partial y}, \quad \frac{\partial F}{\partial \bar{y}'} = \frac{\partial F}{\partial y'}$$