

Exercise 1: Rotational diagram for methanol (CH₃OH)

The molecule methanol has been detected in the young low-mass (Class 0) protostar IRAS 16293–2422. In the datafile you can find parameters associated to each detected transition, such as

1. frequencies (ν in GHz)
2. energies of the upper level (E_u in K)
3. Einstein-A coefficients (A_{ul} in s⁻¹)
4. degeneracies (g_u)
5. integrated intensities ($W = \int T_{mb} dv$ in K km s⁻¹)
6. 1 σ error associated to W

The column density of the upper level, N_u , is

$$\frac{N_u}{g_u} = C \frac{(\nu[\text{GHz}])^2 W[\text{K km s}^{-1}]}{A_{ul}[\text{s}^{-1}] g_u},$$

where $C=1942.75$.

Q1: Write a small program (per example in Python), to plot a diagram of $\ln(N_u/g_u)$ as a function of E_u .

Q2: Fit a linear function to the data and obtain the values for a and b such that $\ln(N_u/g_u) = a + b E_u$.

Q3: If we consider LTE, we can write

$$\frac{N_u}{g_u} = \frac{N}{Q(T_{\text{rot}})} e^{-E_u/T_{\text{rot}}},$$

where $Q(T_{\text{rot}})$ is the partition function. Derive the relation between the parameter b and T_{rot} and calculate T_{rot} using your results from **Q2**.

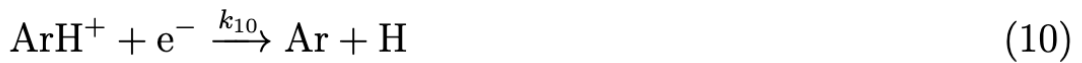
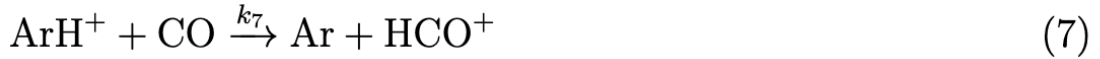
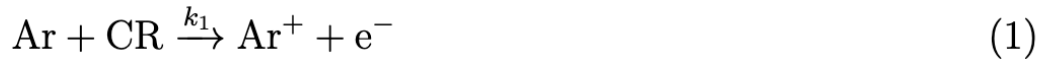
Q4: Derive the relation between the parameter a (from Q2) and N . Use the information given in the table below to find the partition function at the rotational temperature found in Q3, $Q(T_{\text{rot}})$. Finally derive the total column density N .

T(K)	300.0	225.0	150.0	75.0	37.5	18.75	9.375
Q(T)	6413.57 3	4165.81 5	2267.77 6	801.678	283.465	100.207	35.432

Exercise 2: Making ArH⁺ in the ISM

ArH⁺ was the first molecule containing a noble gas element detected in the ISM (Barlow et al. 2013, Science, 342, 134) and it is considered a good tracer of neutral gas.

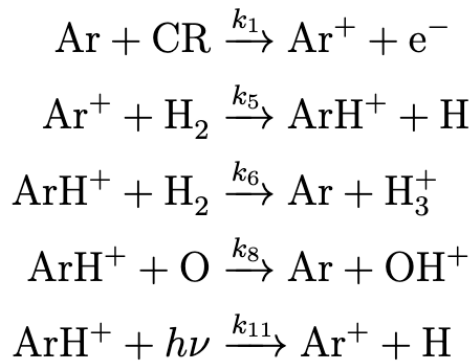
The most important reactions involved in the formation of ArH⁺ are



Q1: Classify each type of reactions above.

Q2: Write the ordinary differential equation (ODE) that describes the evolution of [ArH⁺] as function of time. (**TIP:** consider only the reactions that create and destroy ArH⁺)

Q3: Among the reactions related to ArH⁺, the most important according to Schilke et al (A&A, 566, A29) are the following



Assuming conditions of steady state ($dn/dt=0$), derive the following expression for the abundance ratio of ArH^+ and Ar in the diffuse ISM,

$$\frac{n(\text{ArH}^+)}{n(\text{Ar})} = \frac{\zeta(\text{Ar})}{k_6 n(\text{H}_2) + k_8 n(\text{O})}$$

where $\zeta(\text{Ar})=k_1$ is the cosmic-ray ionization rate of Ar . (**TIP:** You need the expressions describing the steady-state abundances of both ArH^+ and Ar^+)

Q4: Consider now a diffuse clouds with the following properties

$$\begin{aligned}
k_1 &= 11.4 \zeta_p(\text{H}) \text{ s}^{-1} \\
k_6 &= 8.0 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1} \\
k_8 &= 8.0 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1} \\
\zeta_p(\text{H}) &= 2.0 \times 10^{-16} \text{ s}^{-1} \\
n(\text{O}) &= 2.9 \times 10^{-4} n_{\text{H}} \\
n(\text{Ar}) &= 3.2 \times 10^{-6} n_{\text{H}}
\end{aligned}$$

- Write an expression that tells how the abundance of ArH^+ changes as a function of the logarithmic molecular abundance of H_2 that is $[\text{H}_2]=\log_{10} (2n(\text{H}_2)/n_{\text{H}})$.
- Evaluate how $[\text{ArH}^+]$ changes for the following values of $\log_{10} (2n(\text{H}_2)/n_{\text{H}}) \Rightarrow -1.0, -2.5, -2.5, -4.0$.
- Make a plot of how $[\text{ArH}^+]$ changes as function of $\log_{10} (2n(\text{H}_2)/n_{\text{H}})$. How can we conclude that ArH^+ is a good tracer of atomic gas? (**TIP:** density is key)